

# 新概念力学习题答案

## 第一章

1-1 位移  $\Delta x = x(t) - x(0) = 3 \sin \frac{p}{6} t$ ,

速度  $v = \frac{dx}{dt} = \frac{p}{2} \cos \frac{p}{6} t$ ,

加速度  $a = \frac{dv}{dt} = -\frac{p^2}{12} \sin \frac{p}{6} t$ 。

1-2 (1)  $\mathbf{Q} x = R \cos wt, y = R \sin wt; \mathbf{Q} x^2 + y^2 = R^2$ , 质点轨迹是圆心在圆点的圆。

(2)  $\mathbf{v} = \frac{d\mathbf{r}}{dt} = wR(-\sin wti + \cos wtj)$   
 $\mathbf{a} = \frac{d\mathbf{v}}{dt} = -wR(\cos wti + \sin wtj) = -w^2\mathbf{r}$  方向恒指向圆心

1-3 (1)  $x = 4t^2, y = 2t + 3, x = (y - 3)^2$  故  $x \geq 0, y \geq 3$ , 质点轨迹为抛物线的一段。

(2)  $\Delta \mathbf{r} = \mathbf{r}(1) - \mathbf{r}(0) = 4\mathbf{i} + 2\mathbf{j}$ ; 大小为  $|\Delta \mathbf{r}| = \sqrt{4^2 + 2^2} = 2\sqrt{5}m$ , 与  $x$  轴夹角  $q = \tan^{-1} \frac{2}{4} = 26.6^\circ$

(3)  $\mathbf{v} = \frac{d\mathbf{r}}{dt} = 8t\mathbf{i} + 2\mathbf{j}, \mathbf{a} = \frac{d\mathbf{v}}{dt} = 8\mathbf{i}$ 。

1-4  $\Delta t_n = t_n - t_{n-1} = (\sqrt{n} - \sqrt{n-1})\Delta t_1 = 4 \times (\sqrt{7} - \sqrt{6}) = 0.785s$

1-5  $v_0 = \frac{h}{t} = \sqrt{gh}$

1-6  $y = \frac{v_0^2}{2g} - \frac{1}{8}gt_0^2$

1-7 由 7, 由  $\Delta s = v_0\Delta t_1 + \frac{1}{2}a\Delta t_1^2$ , 及  $2\Delta s = v_0(\Delta t_1 + \Delta t_2) + \frac{1}{2}a(\Delta t_1 + \Delta t_2)^2$  即可证。

1-8  $v_2 = \frac{h_1}{h_1 - h_2} v_1, a_2 = \frac{dx_2}{dt} = 0$ 。

1-9 由  $y_m = \frac{v_0^2 \sin^2 b}{2g}, x_m = \frac{v_0^2 2 \sin b \cos b}{g}$ ; 及  $tga = \frac{y_m}{x_m/2}$  即可证。

1-10  $\overline{AB} = \frac{9.8^2}{2 \times 9.8} \cdot \frac{\sqrt{3}}{2} \cdot \frac{2}{3} = 2.83m$

$$1-11 \quad (1) \quad s = 100 \times \sqrt{\frac{2 \times 9.8}{9.8}} = \underline{200\sqrt{5}} = \underline{447.2m}$$

$$tga = \frac{s}{h}, a = tg^{-1}(100 \times \sqrt{\frac{2}{98 \times 9.8}}) = 77.64^\circ = 77^\circ 38' 24''$$

$$(2) \quad a_t = g \cos q = g \cdot \frac{v_y}{v} = 0.96m/s; \quad a_n = g \sin q = g \cdot \frac{v_x}{v} = \underline{9.75m/s^2}$$

$$1-12 \quad r = \frac{v^3}{gv_x} = \frac{1}{v_0 g \cos q} (v_0^2 - 2gy)^{\frac{3}{2}}$$

$$1-13 \quad AB = \frac{2v_0^2}{g} = 4h = 4 \times 0.20 = 0.80m$$

$$1-14 \quad t = \sqrt{\frac{R}{a_t}} = \sqrt{\frac{300}{3.00}} = 10s$$

$$1-15 \quad v_{\text{物}} = v_0 - gt = 49 - 9.8t, \quad v_{\text{测物}} = v_{\text{物}} - v = 29.4 - 9.8t$$

## 第二章

$$2-1 \quad P_B = \sqrt{P_e^2 + P_v^2} = 10.65 \times 10^{-16} g \cdot cm/s. \quad q = 30^\circ.$$

$$2-2 \quad (1) \text{木块的速率 } v = \frac{m}{M+m} v_0 \text{ 和动量 } p_{\text{木}} = \frac{Mm}{M+m} v_0; \quad \text{子弹的动量 } p_{\text{子}} = \frac{m^2}{M+m} v_0.$$

$$(2) \text{子弹施予木块的动量 } I_{\text{木}} = \frac{Mm}{M+m} v_0.$$

$$2-3 \quad I = \sqrt{m(T_0 - mg)l} = 0.86kg \cdot m/s$$

$$2-4 \quad v_1 = \frac{ft_1}{m_1 + m_2}, \quad v_2 = \frac{ft_1}{m_1 + m_2} + \frac{ft_2}{m_2}.$$

$$2-5 \quad S_{\text{船}} = 1.4m. \text{ (对岸)}, \quad S_{\text{人}} = -S_{\text{船}} + S_{\text{人对船}} = 2.6m. \text{ (对岸)}.$$

$$2-6 \quad m_{\text{乙}} = \frac{v_0 + v_{\text{乙}}}{v_0 - v_{\text{乙}}} m_{\text{货}} = 300kg.$$

$$2-7 \quad v_{\text{前}} = v + \frac{m}{M+m} u, \quad v_{\text{中}} = v, \quad v_{\text{后}} = v - \frac{m}{M+m} u$$

$$2-8 \quad (1) \quad \vec{v}_{\text{车}} = -\frac{Nm}{M+Nm} \vec{u}$$

$$(2) \quad \vec{v}_{\text{车N}} = -m \left[ \frac{1}{M+Nm} + \frac{1}{M+(N-1)m} + \mathbf{L} + \frac{1}{M+2m} + \frac{1}{M+m} \right] \vec{u}$$

$$(3) \text{ 比较(1)和(2), 显然有 } |\vec{v}_{\text{车N}}| > |\vec{v}_{\text{车}}|.$$

$$2-9 \quad v_2 = \sqrt{v_1^2 + 4v_0^2 \cos^2 q_0}, \quad a = tg^{-1} \frac{v_1}{2v_0 \cos q_0} = \sin^{-1} \frac{v_1}{v_2} = \cos^{-1} \frac{2v_0 \cos q_0}{v_2}.$$

$$2-10 \quad \overline{F}_t = \frac{240mv}{t} = \frac{240 \times 10 \times 10^{-3} \times 900}{60} = 36N$$

$$2-11 \quad (1) a_0 = \frac{v_0}{M_0} m - g; \quad (2) m = \frac{M_0}{v_0} (a_0 + g) = 735kg/s.$$

$$2-12 \quad (1) v_1 = c \ln \frac{m_0}{m} = 2500 \ln 3; \quad v_2 = 5000 \ln 3; \quad v_3 = 7500 \ln 3 = 8239.6m/s.$$

$$(2) v = 2500 \ln \frac{60}{60-48} = 2500 \ln 5 = 4023.6m/s.$$

$$2-13 \quad F = (v+u) \frac{dm}{dt} \text{ 为向前的推力, 此式的 } v、u \text{ 为绝对值.}$$

$$2-14 \quad (1) \text{ 水平总推力为 } F = v \frac{dm}{dt} \text{ (向前)}$$

(2) 以上问题的答案不改变

$$2-15 \quad \text{质点受力 } \vec{f} = m\vec{a} = -m\omega^2 \vec{r}, \text{ 恒指向原点.}$$

$$2-16 \quad F > m(m_A + m_B)g$$

2-17

$$\begin{cases} a_{1x} = -\frac{m_2 g \sin q \cos q}{m_2 + m_1 \sin^2 q} = -\frac{m_2 g}{(m_1 + m_2) \operatorname{tg} q + m_2 \operatorname{ctg} q} \\ a_{1y} = -\frac{(m_1 + m_2)g}{m_2 + m_1 \sin^2 q} \sin^2 q = -\frac{(m_1 + m_2)g \operatorname{tg} q}{(m_1 + m_2) \operatorname{tg} q + m_2 \operatorname{ctg} q} \\ a_{2x} = \frac{m_1 g \sin q \cos q}{m_2 + m_1 \sin^2 q} = \frac{m_1 g}{(m_1 + m_2) \operatorname{tg} q + m_2 \operatorname{ctg} q} \\ a_{2y} = 0 \end{cases}$$

$$2-18 \quad F > (m_1 + m_2)(m_1 + m_2)g$$

$$2-19 \quad (1) t = \left[ \frac{2d}{g \cos q (\sin q - m \cos q)} \right]^{\frac{1}{2}}$$

$$(2) m = \frac{\cos 60^\circ \sin 60^\circ - \cos 45^\circ \sin 45^\circ}{\cos^2 60^\circ - \cos^2 45^\circ} = 2 - \sqrt{3} = 0.268$$

$$2-20 \quad a_1' = \frac{m_1(m_2 + m_3) - 4m_2 m_3}{m_1(m_2 + m_3) + 4m_2 m_3} g = \frac{1}{17} g = 0.58m/s^2$$

$$2-21 \quad \operatorname{tg} q > \frac{3m_1 + m_2}{m_1 - m_2} m.$$

$$2-22 \quad F = \frac{m_3}{m_2} (m_1 + m_2 + m_3)g$$

$$2-23 \quad (1) \begin{cases} T = m(g \sin q + a \cos q) \\ N = m(g \cos q - a \sin q) \end{cases}$$

$$(2) a = g \operatorname{ctg} q$$

$$2-24 \quad v_{\min} = \sqrt{\frac{\sin q - m \cos q}{\cos q + m \sin q}} kg = \sqrt{\frac{\operatorname{tg} q - m}{1 + m \operatorname{tg} q}} kg, \quad v_{\max} = \sqrt{\frac{\sin q + m \cos q}{\cos q - m \sin q}} kg = \sqrt{\frac{\operatorname{tg} q + m}{1 - m \operatorname{tg} q}} kg$$

$$2-25 \quad f = \frac{Mm}{M+m}(2g - a')$$

$$2-26 \quad \text{从机内看: } a = 3/4g$$

$$\text{从地面上的人看: } \begin{cases} a_{Ax} = 3/4g \\ a_{By} = 1/2g \end{cases}; \begin{cases} a_{Bx} = 0 \\ a_{By} = -1/4g \end{cases}.$$

$$2-27 \quad (1) \quad v = \sqrt{gl}$$

$$(2) \quad \begin{cases} N_t = mg = 4.9N \\ N_n = m \frac{v^2}{l} = 0.16N \end{cases}$$

$$2-28 \quad \text{由 } dr \text{ 这一段, 所需向心力 } dT = dm\omega^2 r = \frac{m}{l} \omega^2 r dr \text{ 易证.}$$

$$2-29 \quad w = \sqrt{\frac{kg}{kl_0 \cos q + mg}}, \quad \Delta l = \frac{mg}{k \cos q + mg}$$

$$2-30 \quad (1) \quad w = \sqrt{2ag}; (2) \text{ 相对弯管静止的角速度为 } w = \sqrt{\frac{g}{R-y}}, \text{ 即没有唯一的角速度.}$$

$$2-31 \quad f = mM(2a - a')$$

$$2-34 \quad \mathbf{f}_c = 2m\mathbf{v} \times \mathbf{w}; \quad f_c = 2mw \sin 30^\circ = 91N, \text{ 压向东边.}$$

### 第三章

$$3-1 \quad \text{最后一节车厢与列车后端相距 } \Delta s = s' + s = Ms/(M-m)$$

$$3-2 \quad h = R/3$$

$$3-3 \quad h \geq 5R/2$$

$$3-4 \quad h = (v_0^2 + v_1^2)/4g$$

$$3-5 \quad v_B = \left[ \frac{2(m_B - m_A)gh}{m_A + m_B} \right]^{\frac{1}{2}}$$

$$3-6 \quad q_{\min} = \cos^{-1} \left( \frac{1}{3} + \frac{1}{3} \sqrt{1 - \frac{3M}{2M}} \right)$$

$$3-7 \quad v_m = \frac{m_2}{m_1} g \sqrt{\frac{m_1}{k}} = \frac{m_2}{m_1} \frac{g}{w_1}, \quad w_1 = \sqrt{\frac{k}{m_1}}$$

$$3-8 \quad (1) \quad v_B = \sqrt{\frac{k}{m_A + m_B}} x_0; \quad (2) \quad x_{A\max} = x_0 + x_0' = \left( 1 + \sqrt{\frac{m_A}{m_A + m_B}} \right) x_0$$

$$3-9 \quad (1) \quad a_{c\max} = F_{\text{外}}/m = kx_0/(m_A + m_B); \quad (2) \quad v_{c\max} = \frac{\sum u_i v_i}{m} = \frac{m_B}{m_A + m_B} \sqrt{\frac{k}{m_B}} x_0.$$

$$3-10 \quad (1) \quad F = (m_2 + m_2)g$$

$$(2) \text{ 当 } F \text{ 刚撤除时, } a_{c\max} = g \text{ (方向向上); 当 } l_0 - x = l_1 \text{ 时, } F_{\text{外}} = 0, a_c = 0; \text{ 当 } l_0 - x = -l_1, \\ F_{\text{外}} = -(m_1 + m_2)g, a_c = -g, \text{ 是 } m_2 \text{ 刚要离地时的质心力加速度, 方向向下.}$$

$$3-11 \quad \text{证明的关键是作用力和反作用力在任何参考系中都相等, 即 } N=N'.$$

$$3-12 \quad \text{两物体的速度为 } V = \frac{m}{M+m} \sqrt{2gh}; \text{ 上升的最大高度为 } H = \frac{m^2}{M^2 - m^2} h.$$

3-13 (a)  $A = \frac{1}{2} \frac{k_1 k_2}{k_1 + k_2} l^2$ ; (b) 如非常缓慢地拉长, 则 A 被分配到两弹簧上, 此时 A 如上最小;

若非常急速地拉,  $k_1$  及  $m$  都来不及变化和运动, 故  $A_{\max} = \frac{1}{2} k_2 l^2$ 。一般地有:

$$\frac{1}{2} \frac{k_1 k_2}{k_1 + k_2} l^2 \leq A \leq \frac{1}{2} k_2 l^2.$$

3-14  $V = \frac{m}{M+m} v_0 = 16 \text{ m/s}$ . (1)  $A_{\text{木块} \rightarrow \text{子弹}} = \frac{1}{2} m V^2 - \frac{1}{2} m V_0^2 = -6397.44 \text{ J}$ ; (2)

$A_{\text{子弹} \rightarrow \text{木块}} = \frac{1}{2} M V^2 = 125.44 \text{ J}$ ; (3) 耗散掉的机能:  $\Delta E = -A_{\text{木块} \rightarrow \text{子弹}} - A_{\text{子弹} \rightarrow \text{木块}} = 6272 \text{ J}$ .

3-15  $x_{\max} = \sqrt{\frac{m_2}{(m+m_1)(m+m_1+m_2)}} \cdot m v_0$

3-16 A 球第一次碰撞后返回的高度是  $h_A = \frac{1}{4} (1-e)^2 h_1$ .

3-17  $m_B > 3m_A$ .

3-18 
$$\begin{cases} v_m = \frac{m-M}{m+M} v_0 = \frac{r-1}{r+1} v_0 \\ v_M = \frac{2m}{m+M} v_0 = \frac{2r}{r+1} v_0 \end{cases}, \text{ 式中 } r = m/M.$$

3-19 (1)  $v = u_1/2$ ,  $m_2 = 3m_1$ ; (2)  $v_c = \frac{1}{4} u_1$ ; (3)  $E_k^c = \frac{3}{4} \cdot \frac{1}{2} m_1 u_1^2$ ; (4)  $E_{k1} = \frac{1}{4} \cdot \frac{1}{2} m_1 u_1^2$ .

3-20  $m_n = \frac{14v_N - v_H}{v_H - v_N} m_H = 1.159 m_H$ ;  $v_0 = 3.07 \times 10^7 \text{ m/s}$ .

经误差分析后得:  $m_n = (1.159 \pm 0.252) m_H$ ;  $v_0 = (3.07 \pm 0.31) \times 10^7 \text{ m/s}$ .

3-21  $v_1 = \frac{28}{27} v_0$ ,  $v_2 = \frac{13}{27} v_0$ .

3-22  $v = \frac{1}{4} v_0 \sqrt{5-2\sqrt{2}} = 0.368 v_0$ ,  $q = \tan^{-1} \frac{\sqrt{2}}{4-\sqrt{2}} = 28.68^\circ$

末态动能  $E_k = 0.52 \times \frac{1}{2} m_0 v_0^2 < \frac{1}{2} m_0 v_0^2$  不守恒!

3-23 (1) 由此得  $v_{\text{汽}} > 120 \text{ km/h}$ , 目击者的判断不可信。

(2) 总初始动能的 5/8 由于碰撞而转换成了其他形式的能量。

3-24  $m_Z = 300 \text{ kg}$ ; 总动能减少了。

3-25 由 (3.63) 式可以证明:  $v \approx (1 - e^{-na}) v_0$ ; 且在  $an \ll 1$ , 和  $an \rightarrow \infty$ , 时有效。

3-26 十人一个一个地往后跳跳时, 车子获得最大的动能。

3-27  $y_c = 0$ ,  $x_c = \frac{R \sin q}{q}$

3-28  $x_c = 0, z_c = 0$ ,  $y_c = \frac{3}{8} R$ .

3-29 (a) 当  $mg < kR - kl/2$  时,  $E_p$  有两个稳定平衡点 ( $q_{\pm} = \cos^{-1} \frac{kl}{2(kR - mg)}$ ), 有一个不

稳定平衡点 ( $q_1 = 0$ ); (b) 当  $mg \geq kR - kl/2$  时,  $E_p$  只有一个稳定平衡点 ( $q_1 = 0$ )。

3-30 从略。

## 第四章

4-1 (1)  $J_A = mvd_1, J_B = mvd_1$ , 方向向纸里;  $\dot{J}_C = m\dot{d}_3 \times \dot{\mathbf{r}} = 0$ .

(2)  $M_A = mgd_1, M_B = mgd_1$ , 方向向纸里;  $\dot{M}_C = \dot{d}_3 \times m\dot{\mathbf{g}} = 0$ .

4-2  $\dot{\mathbf{J}} = m\dot{\mathbf{r}} \times \dot{\mathbf{v}} = m(xv_y - yv_x)\dot{\mathbf{k}}, \quad \dot{\mathbf{M}} = \dot{\mathbf{r}} \times \dot{\mathbf{f}} = yf\dot{\mathbf{k}}.$

4-3  $w = \frac{1}{mr^2} \frac{h}{2p} = 4.13 \times 10^{16} \text{ md/s}$

4-4  $v_2 = \frac{r_1}{r_2} v_1, \tan q_2 = \frac{v_2^3}{gr_1 v_1} \propto v_2^3$ ; 即  $v_2$  增大, 故  $q_2$  亦增大,  $q_2 > q_1$ .

4-5  $w' = \frac{8}{5} w; \quad \Delta E_k = \frac{39}{25} E_{k0}$ , 增加的能量来自汽车的动力。

4-6  $w = \frac{v}{2R}$  (这是转台反方向旋转地角速度)。

4-7  $m_2$  对质心的角动量更大,  $\frac{J_{c2}}{J_{c1}} = \frac{m_1}{m_2}$ .

4-8  $v_c = w l_2 = \frac{m_1 l}{m_1 + m_2} w$  沿切线方向做匀速直线运动;  $T = T_1 = m l w^2$ .

4-9 (1)  $J_{\text{前}} = J_{\text{后}}$  (角动量守恒); (2)  $v' = \frac{r_{10}}{r_5} v_0 = \frac{5}{2.5} \times 6.5 = 13 \text{ m/s}$ ;

(3)  $T' = m \frac{v'^2}{r_5} = 4056 \text{ N}$ ; (4)  $A = 3802.5 \text{ J} = \Delta E_k$ .

4-10 (1)  $\frac{E_k}{E_{k0}} = \frac{b^2}{l^2} < 1$ , 其他能量转变为绳子的弹性势能, 以后转化为分子内能.

(2) 绳子断后, 质点将按速度  $v = v_0 \frac{b}{l}$  沿切线方向飞出, 做匀速直线运动;  
质点对 O 点的角动量  $J = mv_0 b = \text{恒量}$ .

4-11 (1)  $U_{(r)} = \frac{k}{r}$ , 选  $r = \infty$  处为  $U$  的零点; (2)  $R = \frac{mv_0^2 b^2}{\sqrt{k^2 + m^2 v_0^4 b^2} - k} = \frac{\sqrt{k^2 + m^2 v_0^4 b^2} + k}{mv_0^2}$ .

4-12 把  $k$  换成  $-k$ .

4-13 地月之间距离增大了 0.28 倍

4-14  $V_C = \frac{4}{7} V_0, \quad V_1 = -\frac{1}{7} V_0; \quad w = \frac{4\sqrt{2}}{7} \frac{V_0}{l}$ .

4-15  $v' = \frac{3m-M}{3m+M} v; w = \frac{6v}{L} \bullet \frac{m}{3m+M}$ . 讨论: (1)  $M \leq 3m, v \geq 0$ , 小球向前运动; (2)  $M = 3m, v = 0$ , 小球不动; (3)  $M \geq 3m, v' \leq 0$ , 小球向后运动。三种情况下, 薄板匀绕轴向前转动, 此题中系统的动量不守恒, 因为轴对薄板有做用力

4-16  $I = I_B + I_C + I_{BC} = \frac{1}{3} ml^2 \times 2 + \frac{5}{6} ml^2 = \frac{3}{2} ml^2$

4-17 (1)  $I = 6ml^2 = 3.6 \times 10^{-5} \text{ kgm}^2$ ; (2)  $I = 4m(\frac{\sqrt{3}}{2} l)^2 = 1.8 \times 10^{-5} \text{ kgm}^2$ ; (3)  $I = 12ml^2 = 7.2 \times 10^{-5} \text{ kgm}^2$ .

4-18 (1)  $I = \frac{1}{3} ml^2 + \frac{1}{2} MR^2 + M(l+R)^2$

$$(2) \quad r_c = \frac{l}{2} + \frac{M}{M+m} \left( \frac{l}{2} + R \right) = l + R - \frac{m}{M+m} \left( \frac{l}{2} + R \right)$$

$$I_c = \frac{1}{12} m l^2 + \frac{1}{2} M R^2 + \frac{mM}{M+m} \left( R + \frac{l}{2} \right)^2$$

$$4-19 \quad I_{\text{余}} = \frac{1}{2} M R^2 \left( 1 - \frac{r^2}{R^2} - 2 \frac{r^4}{R^4} \right)$$

$$4-20 \quad I = \frac{M_{\text{摩}} t_1}{w_0} = \frac{100 \times 240}{40p} = 191 \text{ kgm}^2$$

$$4-21 \quad m = \frac{m R w_0}{2 N t} = 0.098$$

$$4-22 \quad \text{制动力矩: } N' = \frac{m R w_0}{m t} = 2.09 N; \quad \text{制动力: } F = \frac{N'}{1.25} = 0.836.$$

$$4-23 \quad (1) \quad t = \frac{R_A}{R_B} = \frac{w_A}{b_B} = \frac{30}{75} \times \frac{20p}{0.8p} = 10s$$

$$(2) \quad b_A = \frac{w_A - w_{A0}}{t'} = -\frac{1}{6} p \text{ rad/s}^2; \quad b_B = \frac{w_B - w_{B0}}{t'} = -\frac{1}{15} p \text{ rad/s}^2.$$

$$4-24 \quad (1) \quad t = \frac{I_c w}{M} = 0.105s$$

$$(2) \quad F_r = T_1 - T_2 = T_1 (1 - e^{-\mu p}) = \frac{M}{2mR} = 83.3 N;$$

$$T_1 = \frac{83.3}{1 - e^{-0.3p}} = 136.6 N, \quad T_2 = T_1 e^{-0.3p} = 53.3 N.$$

4-25

$$a_1 = \frac{(m_1 R - m_2 r) R}{I_c + m_1 R^2 + m_2 r^2} g, \quad a_2 = \frac{r}{R} a_1 = \frac{(m_1 R - m_2 r) r}{I_c + m_1 R^2 + m_2 r^2} g;$$

$$T_1 = \frac{I_c + m_2 r(r+R)}{I_c + m_1 R^2 + m_2 r^2} m_1 g, \quad T_2 = \frac{I_c + m_1 r(r+R)}{I_c + m_1 R^2 + m_2 r^2} m_2 g.$$

$$4-26 \quad \text{小幅摆动的周期: } T = 2p \sqrt{\frac{l_1^2 + l_2^2}{g(l_2 - l_1)}}; \quad \text{等值摆长: } l_0 = \frac{l_1^2 + l_2^2}{l_2 - l_1} > l_1 + l_2.$$

$$4-27 \quad I = \frac{T_1^2}{T_2^2 - T_1^2} m l \left( l - \frac{T_2^2}{4p^2} g \right) = 1.21 \times 10^3 g \cdot \text{cm}^2$$

$$4-28 \quad (1) \quad \frac{T}{T_0} = \sqrt{\frac{l^2 + 3h^2}{l^2 + 2lh}}; \quad h = 0.5m \text{ 时}, \frac{T}{T_0} = \sqrt{\frac{7}{8}}, \quad h = 1m \text{ 时}, \frac{T}{T_0} = \sqrt{\frac{4}{3}}.$$

$$(2) \quad \text{当 } h = \frac{2}{3} l \text{ 时}, \quad \frac{T}{T_0} = 1.$$

$$4-29 \quad h = \frac{1}{10} (27R - 17r)$$

$$4-30 \quad a_c = \frac{m r^2}{I_c + m r^2} g; \quad T = \frac{I_c}{2(I_c + m r^2)} m g.$$

$$4-31 \quad (1) \quad a \leq 2mg, \quad \text{或 } m \geq \frac{a}{2g}.$$

$$(2) \quad a_c = \frac{1}{2} a, \quad b = \frac{a_c}{R} = \frac{a}{2R}.$$

4-32 前后轮对地面的压力:  $N_1 = \frac{L-l+mh}{L}mg, N_2 = \frac{l-mh}{L}mg$

4-33  $v_c = v_0 - mgt, \quad w = w_0 - \frac{3}{2} \frac{mg}{R} t.$

讨论: (1) 当  $t = t_1 = \frac{2v_0}{3mg}$  时,  $w = 0, v_c = v_0 - \frac{2}{3}v_0, v_p = R\omega + v_c = \frac{1}{3}v_0$ , 球不转, 只是滑动;

当  $t < t_1$  时,  $w > 0$  球还是倒转; 当  $t > t_1$  时,  $w < 0$ , 在摩擦力矩作用下, 足球按顺时针转动。 4-34

(2) 当  $t = t_2 = \frac{4v_0}{5mg}$  时,  $v_p = v_c + R\omega = R(w_0 - \frac{3}{2} \frac{mg}{R} t_2) + (v_0 - mgt_2) = 0$ , 亦即球只滚不

滑, 此时,  $v_c = v_0 - mgt_2 = \frac{1}{5}v_0, w = -\frac{v_0}{R} = -\frac{1}{5}w_0 < 0$ , 若不计滚动摩擦, 此后  $v_c, w$  保持不变。

球的反弹速度为  $v_f = ev_0$ , 还受墙的摩擦冲力有一向上的速度  $v_1$ , 形成以墙角为原点的抛体运动; 落地后受摩擦及非弹性碰撞的影响, 两速度分量减小, 形成一个较小轨迹的抛体运动; 如此等等……。球向上的可能最大高度由机械能量守恒确定, 为  $h_m = v_0^2/3g$ 。

4-35  $v_{\text{底}} = \sqrt{\frac{10}{7}g(R-r)}, w_{\text{低}} = \frac{1}{r}\sqrt{\frac{10}{7}g(R-r)}, N = \frac{17}{7}mg$ 。

4-36 在  $m \geq \frac{2a}{7g}$  条件下, 用力将平板抽出时, 球一边向前运动; 若  $m < \frac{2a}{7g}$ , 则球只会滑动, 没有滚动 (其中  $a$  为平板的角速度)。

4-37 (1)  $h \geq \frac{R}{5}(2 - \frac{7mng}{F})$ ;  $QFmng, \therefore h \geq \frac{2}{5}R$ 。

(2) 台球只有滑动, 没有滚动, 此时  $a_c = \frac{F - mng}{m}$ 。之后, 在摩擦力作用下, 球渐渐滚动起来。

4-38 12秒钟后, 雪球会碰到滑雪者, 人不能逃脱。

4-39 当  $m > \frac{b}{a}$  时,  $q_1 < q_2$ , 则  $q$  在到达  $q_1 = \arctg \frac{b}{a}$  时发生翻倒;

当  $m < \frac{b}{a}$  时,  $q_1 > q_2$ , 则  $q$  在到达  $q_2 = \arctg \frac{b}{a}$  时发生滑动;

当  $m = \frac{b}{a}$  时,  $q_1 = q_2$ , 则  $q$  在到达  $q_1 = q_2 = \arctgm = \arctg \frac{b}{a}$  时滑动和翻倒同时发生。

4-40 细杆平衡条件:  $\frac{l}{2}\cos^3 q + R\sin q = d (0 < q < \frac{p}{2})$

4-41  $tga \leq 2m$  时,  $m_1$  与  $m_2$  平衡;  $\sin a > \frac{2m_2}{m_1}$  时, 圆柱下滚 ( $f \leq mN$ , 只滚不滑)。

4-42  $N_A = \frac{l_1}{l_2}mg = 12544N; N_B = \frac{\sqrt{l_1^2 + l_2^2}}{l_2}mg = 15680N > N_A$ 。

4-43 铰链落地时,  $v_{\text{铰链}} = \sqrt{3gl/2}$ 。

4-44 进动角速度为  $\Omega = \frac{M}{I_c \omega} = \frac{mgl}{I_c \omega}$ ;

绳子与铅垂线所成的夹角  $q$  由下述超越方程给出:  $L\sin q + l = \frac{I_c^2 \omega^2}{m^2 gl^2} \tan q$ 。

## 第五章 连续体力学

5-1 张力  $T = Dp/2d$ 。



5-2 (1)  $QV = lab = l_0(1+e) \cdot a_0(1-se) \cdot b_0(1-se) \approx l_0a_0b_0(1+e-2se)$ ; 而

$$l_0a_0b_0 = V_0, \therefore (V - V_0)/V_0 = e(1-2s).$$

(2) 压缩时  $(V - V_0)/V_0 = -e(1-2s)$ .

$$(3) (V - V_0)/V_0 = -\frac{t}{Y}(1-2s) = -\frac{1.37}{19.6 \times 10^{10}}(1-0.3) = -4.9 \times 10^{-12}$$

5-3 (1)  $t_{II} = F/S = 7.8 \times 10^7 \text{ N/m}^2$ ;

$$(2) e_{\text{剪}} = t_{II}/G = 9.7 \times 10^{-4};$$

$$(3) \Delta d = e_{\text{剪}}d = 4.9 \times 10^{-4} \text{ cm}.$$

$$5-4 \frac{R_H}{R_b} = \frac{Ybh^3/12M_{\text{外}}}{Yhb^3/12M_{\text{外}}} = \left(\frac{h}{b}\right)^2 = \left(\frac{3}{2}\right)^2 = 2.25$$

$$5-5 M = \frac{pGj}{2l}(R_2^4 - R_1^4) = Dj, \text{ 其中 } D = \frac{pG}{2l}(R_2^4 - R_1^4).$$

$$5-6 QD_{Al} = p \times 2.65 \times 10^{10} \times (0.020^4 - 0.019^4)/(2 \times 10) = 1.25 \times 10^2,$$

$$D_{Fe} = p \times 8.0 \times 10^{10} \times (0.020^4 - 0.019^4)/(2 \times 10) = 3.77 \times 10^2;$$

$$\therefore q_{Al} = M/D_{Al} = 50/125 = 0.4 = 0.4 \times 180^\circ/p = 23^\circ,$$

$$q_{Fe} = M/D_{Fe} = 50/377 = 0.13 = 0.13 \times 180^\circ/p = 7.4^\circ.$$

5-7 论证从略.

$$5-8 Qv_1 = Q/S_1 = 6.7 \text{ m/s}, \quad v_2 = \sqrt{v_1^2 - 2gh} = 2.3 \text{ m/s}; \quad \therefore S_2 = v_1S_1/v_2 = 4.35 \text{ m/s}.$$

$$5-9 p = p_0 - rg(h_1 + h_2) = 0.856 \times 10^5 \text{ Pa}; \quad Q = vS = \sqrt{2gh_2} S = 1.71 \times 10^{-4} \text{ m}^3.$$

$$5-10 v = \sqrt{2g[(r_1/r_2)h_2 + h_2]} = 9.5 \text{ m/s}.$$

$$5-11 \text{ 一半需时 } t_1 = (\sqrt{2}-1)\frac{A}{S}\sqrt{\frac{H}{g}}; \text{ 全部需时 } t_2 = \sqrt{2}\frac{A}{S}\sqrt{\frac{H}{g}}.$$

$$5-12 t = (\sqrt{2}-1)\frac{S_1}{S_2}\sqrt{\frac{H}{g}} = 28.1 \text{ s}.$$

$$5-13 \text{ 由射程公式 } l = vt = \sqrt{2gh} \cdot \sqrt{2(H-h)/g} = 2\sqrt{h(H-h)} \text{ 求极大值可证.}$$

5-14 推导从略.

5-15 在距水面下  $h = h_1 + h_2 = 75 \text{ cm}$  处相交.

5-16 (1) 压力计的水面与出水口等高; (2) 压力计的水面升高.

$$5-17 \quad v = \sqrt{2(g+a)h} = \sqrt{2(9.8+120) \times 0.30} = 8.8 \text{ m/s}.$$

5-18 加速度  $a$  和液面倾斜角  $\theta$  分别为:  $a = 2(H-h)/l$ ;  $q = \arctan 2(H-h)/l$ .

$$5-19 \quad v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 5.1} = 10 \text{ m/s}.$$

5-20 水流速度  $v_0 = \sqrt{2gh}$ , 水轮机的功率  $P = rS(v_0 - v)^2 v$ ; 其  $v$  中为轮机叶片的速度.

$$\text{令 } dP/dv = 0, \text{ 可得 } P_{\min}=0, (\text{对应 } v=0); P_{\max} = rS \cdot \left(\frac{4}{3}\sqrt{2gh}\right)^2 \cdot \frac{1}{3}\sqrt{2gh} = 3.4 \times 10^4 \text{ W}$$

(对应  $v = v_0/3$ ). 水轮机最大功率时的转速为  $n_m = (v_0/3)/2\pi R = 0.21 \text{ s}^{-1} = 12.4 \text{ m}^{-1}$ .

5-21 大小:  $F = r v S |\Delta \mathbf{v}| = r v S \cdot 2v \sin \alpha / 2 = 55 \text{ N}$ ; 方向 (与原水流方向夹角):  $q = 127.5^\circ$ .

$$5-22 \quad (1) \quad h = \frac{prgR^4}{8Q}; \quad (2) \quad v_0 = \frac{P_a - P_b}{4hl} R^2 = \frac{2Q}{pR^2}.$$

5-23  $v = \frac{p_x}{2h}(d^2 - z^2)$ ; 其中  $p_x = dP/dx$  为与坐标无关的压强沿流速方向的梯度,  $2d$  为两平面间的

距离,  $z$  为坐标原点在两板中间、坐标轴与平板正交的坐标.

$$5-24 \quad h = gd^2(r_1 - r_2)/18v = 0.82 \text{ Pa} \cdot \text{S}.$$

$$5-25 \quad v_m = \frac{2}{9h} r^2 rg = 1.43 \times 10^{-2} \text{ m/s}.$$

$$5-26 \quad v_{m1} = \frac{2r_1^2(r - r')}{9h} g = 1.2 \times 10^{-4} \text{ m/s}; \quad v_{m1} = \frac{2r_1^2(r - r')}{9h} g = 3.0 \times 10^{-1} \text{ m/s}.$$

$$5-27 \quad \text{水滴所受的重力为 } mg = r\left(\frac{1}{6}pd^3\right)g = 4.1 \times 10^{-11} \text{ N};$$

而它受气流向上的曳引力为  $f = 6ph(d/2)v = 6.8 \times 10^{-11} \text{ N}$ .

由于  $f > mg$ , 故水滴不能回落地面.

$$5-28 \quad \text{由雷诺数 } R = \frac{rvl}{h} = \frac{(Q/\frac{pd^2}{4 \times 5})(\frac{d}{\sqrt{5}})}{h/r} = \frac{4\sqrt{5}Q}{pd(h/r)} = 6.3 \times 10^3 > 2600, \text{ 可断定为湍流}.$$

## 第六章 振动和波

$$6-1 \quad (1) \quad j_0 = 2p - \frac{p}{3} = \frac{5p}{3} \quad \text{或} \quad -\frac{p}{3}.$$

$$(2) \quad x_{(0.5)} = 6\sqrt{3}cm; \quad v_{(0.5)} = -6p \text{ cm/s}; \quad a_{(0.5)} = -6\sqrt{3}p^2 \text{ cm/s}^2.$$

$$(3) \text{ 相当于 } t=1s: \quad v_{(1)} = -6\sqrt{3}p \text{ cm/s}; \quad a_{(1)} = 6p^2 \text{ cm/s}^2.$$

6-2 当  $a=0$  时,  $x = \cos pt$ ; 当  $a = \frac{p}{3}$  时,  $x = \cos(pt + \frac{p}{3}) = \cos pt_1$ ,  $t_1 = t + \frac{1}{3}$ ;

当  $a = \frac{p}{2}$  时,  $x = \cos pt_2$ ,  $t_2 = t + \frac{1}{2}$ ; 当  $a = -\frac{p}{3}$  时,  $x = \cos pt_3$ ,  $t_3 = t - \frac{1}{3}$ .

由上各式可见, 各轨迹均为余弦曲线, 只不过原点在  $t$  轴上作相应的平移。

6-3  $S_{1(t)} = A \cos(\frac{2p}{T}t + a_1)$ ;  $S_{2(t)} = A \cos(\frac{2p}{T}t_1 + a)$ ,  $t_1 = t' - \frac{T}{3}$ ;

$$S_{3(t)} = A \cos(\frac{2p}{T}t_2 + a), \quad t_2 = t + \frac{T}{3}.$$

由上各式可见, 各轨迹均为余弦曲线, 只不过原点在  $t$  轴上作相应的平移。

6-4 (1)  $T = \frac{2p}{w} = \frac{2p}{5} = 1.26s$ ; (2)  $f = m\omega(0) = 37.5 \text{ dyn}$ ; (3)  $E = \frac{1}{2}mw^2A^2 = 112.5 \text{ erg}$ .

6-5 液体的振荡是简谐振动, 周期为  $T = 2p\sqrt{\frac{L_{\text{总}}}{2g}} = p\sqrt{\frac{2L_{\text{总}}}{g}}$ .

6-6  $w = \sqrt{\frac{k_1 + k_2}{m}}$

6-7 (1)  $w = \sqrt{\frac{g}{5}} = 14 \text{ rad/s}$ ,  $v = \frac{w}{2p} = 2.23 \text{ Hz}$ ; (2)  $v_{(x=3cm)} = -w5 \sin(14t + p) = 56 \text{ cm/s}$ ;

(3)  $m = \frac{1}{3}\Delta m = 100g$ ; (4)  $x_o' = \frac{4mg}{k} = 4x_0 = 20cm$ .

6-8 (1)  $j_0 = \frac{3p}{2}$ ,  $q_0 = \frac{v_0}{lw} = \frac{10^{-2}}{\sqrt{1 \times 9.8}} = 3.19 \times 10^{-3} \text{ rad}$ ; (2) 若  $F\Delta t$  向左, 则初相位为  $j_0 = \frac{p}{2}$ .

6-9 振幅  $A = \frac{mg}{k} \sqrt{1 + \frac{2kh}{(M+m)g}}$ , 初相位  $j_0 = \arctg \sqrt{\frac{2hk}{(M+m)g}}$ .

6-9  $\because T - mg \cos q = ml\dot{q}^2$ ,  $\therefore T = mg \cos[q_0 \cos(\omega t + j_0)] + mlq_0^2 \frac{g}{l} \sin^2(\omega t + j_0)$ ;

在  $\omega t + j_0 = 2np \pm \frac{p}{2}$  时,  $T = T_{\text{max}} = mg(1 + q_0^2)$ .

6-11  $T = \frac{2p}{w} = 2p\sqrt{\frac{l}{g \cos a}} = \frac{T_0}{\sqrt{\cos a}} > T_0$ ,  $T_0 = 2p\sqrt{\frac{l}{g}}$ , 周期变大

6-12 是简谐运动, 周期为  $T = \frac{2p}{w} = 2p\sqrt{\frac{l}{g}}$ .

6-13 系统作简谐振动, 周期是  $T = \frac{2p}{w} = 2p\sqrt{\frac{M + 3m/4}{2k}}$ .

6-14 (1) 木板的运动方程为  $M\ddot{x} = m(\frac{l-x}{2l} - \frac{l+x}{2l})Mg = -\frac{m}{l}Mgx$ , 故木板作简谐运动, 其固

有角频率为  $w = \sqrt{\frac{mg}{l}}$ ;

(2) 木板不作简谐振动, 而是向右(或向左)滑出。

6-15 弹簧的运动比较复杂, 较严格的分析可参见:

(1) 罗蔚茵, 《力学简明教程》, 广州, 中山大学出版社, 1985, 340~346。

(2) 钱伯初, 美国研究生考题分析(三)——近似处理, 大学物理, 1983年第3期, 第28页, 例1。

6-16  $w_1 = 0, w_2 = w_3 = \sqrt{\frac{3k}{m}}$ 。讨论:

(1)  $w_1 = 0 \Rightarrow a_1 = a_2 = a_3$  体系各质点给圆心作纯转动;

(2)  $w_2 = w_3 = \sqrt{\frac{3k}{m}}$ , 可能形况为: ①  $a_1 = 0, a_2 = -a_3 = \pm \frac{1}{\sqrt{2}}a$ , ②  $a_1 = \pm \sqrt{\frac{2}{3}}a$ ,

$a_2 = a_3 = -\frac{1}{2}a_1 = \pm \frac{1}{2}\sqrt{\frac{2}{3}}a$ ; 其余情况比较复杂, 此处从略。

6-17  $t = \frac{1}{b} \ln \frac{A}{A_{(t)}} t = \frac{1}{\ln 3/10} \ln \frac{3}{0.3} = 21s$

6-18  $Q = \frac{1}{2\Lambda} = \frac{w_0}{2b} = \frac{880p}{2 \times \ln 5/8} = 6.87 \times 10^3$

6-19  $k = mw^2 = 5p^2 = 49.3 \frac{N}{m}$  (劲度系数),  $b = \frac{w_0 l}{\sqrt{l^2 + 4p^2}} = 0.01/s$  (阻尼系数)。

6-20 阻力系数  $g = \frac{F}{A_{\max} w_0} = \frac{1}{200p} \frac{kg}{s}$ ; 阻力的幅度  $F_0 = g v_{\max} = g \frac{F}{2mb} = F = 10^{-3} N$ 。

6-21  $A_{\text{合}} = \sqrt{A^2 + (\sqrt{3}A)^2} = 2A$ ,  $j_{\text{合}} = \frac{7}{12}p = 105^\circ$ 。

6-22 (1) 轨迹为  $\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$  顺时针方向旋转的正椭圆;

(2) 轨迹同上, 但为逆时针方向旋转的正椭圆。

6-23  $T_{\text{拍}} = \frac{t}{n} = 2s$ ,  $\Delta u = 0.5H_z$ ,  $u = u_0 \pm \Delta u = 256 \pm 0.5H_z$ 。

6-24 从左自右依次为:  $w_{\text{纵}} = 2w$ ,  $\frac{3}{2}w$ ,  $\frac{3}{2}w$ ,  $\frac{4}{3}w$ ,  $3w$ ,  $3w$ 。

6-25 (1) 1)  $u(x, 0) = A \sin \frac{2px}{l}$ , 2)  $u(x, \frac{T}{4}) = -A \cos \frac{2px}{l}$  3)  $u(x, \frac{T}{2}) = -A \sin \frac{2px}{l}$ ,

4)  $u(x, \frac{3T}{4}) = A \cos \frac{2px}{l}$ ;

(2) 1)  $u(0, t) = -A \sin \frac{2pt}{T}$ , 2)  $u(\frac{l}{4}, t) = A \cos \frac{2pt}{T}$ , 3)  $u(\frac{l}{2}, t) = \sin \frac{2pt}{l}$ ,

4)  $u(\frac{3l}{4}, t) = -A \cos \frac{2pt}{T}$

6-26  $u(x, t) = 0.001 \cos(3300pt + 10px + \frac{p}{2}) = 0.001 \sin(3300pt + 10px + p)$

6-27  $A = 2.0cm$ ,  $l = 30cm$ ,  $v = 100Hz$ ,  $c = vl = 3000cm/s$ ;

当  $x = 10cm$  时, 初相位为  $y_0' = -\frac{2p}{3}$  或  $\frac{4p}{3}$

6-28  $u(x, t) = A \cos[w(t - \frac{x}{c}) + y_0] = A \cos[2pv(t - \frac{x}{c}) + y_0]$ 。

6-29  $I_1 = \frac{c}{v_1} = 16.5m$ ,  $I_2 = \frac{c}{v_2} = 1.65cm$ 。

6-30 可见光的频率范围为:  $7.5 \times 10^{14} \sim 3.95 \times 10^{14} Hz$

6-31 从略。

$$6-32 \quad Q \quad w \frac{d}{c} = \frac{2pd}{l} = \frac{5}{2}p, \quad \therefore u(0, t) = A \cos(\omega t + j_0).$$

(1) 无半波相位突: (a) O 点左边:  $u = -2A \sin(\omega t + j_0) \sin w \frac{x}{t}$ , 是驻波;

(b) O 点右边:  $u = -2A \sin[w(t - \frac{x}{c}) + j_0] \cos \frac{p}{2} = 0$ , 不动。

(2) 半波相位突变: (a) O 点左边:  $u = 2A \cos(\omega t + j_0) \cos w \frac{x}{c}$ , 是驻波;

(b) O 点右边:  $u = 2A \cos[w(t - \frac{x}{c}) + j_0]$ , 是一振幅加倍的行波。

6-33 反射波(向左)在固定端有  $180^\circ$  的相位跃变。

$$6-34 \quad u_{\text{反}}(x, t) = A \cos[2p(\frac{t}{T} - \frac{x}{l}) + \frac{p}{4}].$$

$$6-35 \quad u = 2A \cos 2p \frac{t}{T} \cos 2p \frac{x}{l}; \text{波腹: } x = n \frac{l}{2}, \text{波节: } x = n \frac{l}{2} - \frac{l}{4}, n = 1, 2, 3, \dots$$

$$6-36 \quad \text{AB 上不动点即为波节 } x = n \frac{l}{2}, n = 0, 1, 2, 3, \dots, 20 \text{ 共 } 21 \text{ 个点 } (\lambda/2 = 1 \text{ m}).$$

$$6-37 \quad \text{群速: } v_g = \frac{dw}{dk} = \frac{g + 3rk^2/r}{2w}. \text{ 当 } v_g = c = \frac{w}{k} \text{ 时, } k = \sqrt{\frac{rg}{r}}; \text{ 此时}$$

$$\frac{dc}{dk} = \frac{1}{2} \frac{1}{\sqrt{g/k + rk/r}} (-\frac{g}{k^2} + \frac{r}{r}) = 0, \text{ 相速 } c \text{ 有极小值, 也是最小值 } (c_{\min} = \sqrt{2}(\frac{rg}{r})^{\frac{1}{4}}).$$

$$6-38 \quad (1) l = 24 \text{ cm}, c = 240 \text{ cm/s}; (2) \text{ B 点比 A 点的相位落后: } \Delta j = \frac{p}{5}.$$

$$6-39 \quad \text{A 听到的拍频为: } \Delta \nu_A = 30.3 \text{ Hz}; \text{ B 听到的拍频为 } \Delta \nu_B = 29.4 \text{ Hz}.$$

$$6-40 \quad \nu = \frac{c^2 - v_s^2}{2cv_s} \Delta \nu_{\text{拍}} = 204 \text{ Hz}$$

$$6-41 \quad \text{潜水艇的速率 } v = \frac{\Delta \nu_{\text{拍}}}{2\nu + \Delta \nu_{\text{拍}}} c = 6 \text{ m/s}.$$

$$6-42 \quad \text{马赫锥半顶角 } \alpha = \arcsin \frac{c}{v_s} = \arcsin \frac{c}{1.5c} = 41.8^\circ.$$

## 第七章 万有引力

$$7-1 \quad M = \frac{4p^2}{GT^2} r^3 = 6.06 \times 10^{24} \text{ kg}.$$

$$7-2 \quad K = \frac{r^3}{T^2} = \frac{G}{4p^2}(M+m).$$

$$7-3 \quad T' = T\sqrt{\frac{M+m}{M}} = 27.3\sqrt{\frac{81m+m}{81m}} = 27.5 \text{ d}.$$

$$7-4 \quad a_1 = 2.5 \text{ AU}: T_1 = (a_1/a)^{3/2}T = 3.95 \text{ y}; \quad a_2 = 3.0 \text{ AU}: T_2 = (a_2/a)^{3/2}T = 5.18 \text{ y}.$$

$$7-5 \quad \bar{r} \approx \frac{24p}{GT^2q^3} = 1.29 \times 10^3 \text{ kg/m}^3.$$

$$7-6 \quad \text{由 } T^2 = \frac{4p^2}{GM}r^3, \quad r \approx R, \quad \text{及} \quad M = \bar{r} \cdot \frac{4p}{3}R^3, \quad \text{得} \quad T = \sqrt{\frac{3p}{G\bar{r}}} \propto \frac{1}{\bar{r}}.$$

$$7-7 \quad (1) \quad \frac{r_M}{r_E} = \frac{M_M}{M_E} \cdot \frac{d_E^3}{d_M^3} = 0.74; \quad (2) \quad g_M = \frac{M_M d_E^2}{M_E d_M^2} g_E = 0.207 g_E = 2.03 \text{ m/s}^2.$$

$$7-8 \quad (1) \quad h = r - R = \sqrt[3]{\frac{GMT^2}{4p^2}} - R = 1.69 \times 10^6 \text{ m}; \quad (2) \quad t = \frac{2 \arccos(R/r)}{\Delta \omega} = 2.63 \times 10^2 \text{ s}.$$

$$7-9 \quad \text{同步卫星圆形轨道的半径 } r = \sqrt[3]{\frac{GM}{\omega^2}} = 4.23 \times 10^7 \text{ m}, \quad \text{容许的半径误差为}$$

$$|\Delta r| = \frac{2r}{3} \cdot \frac{\Delta \omega}{\omega} = \frac{2 \times 4.23 \times 10^7}{3} \cdot \frac{10^{-10}}{10 \times 365 \times 24 \times 60 \times 60} = 214 \text{ m}.$$

$$7-10 \quad h = r - R_{\text{木}} = \sqrt[3]{\frac{GM_{\text{木}}T}{4p^2}} - R_{\text{木}} = 8.77 \times 10^7 \text{ m}.$$

$$7-11 \quad (1) \quad T = 2pD\sqrt{\frac{r_{2C}}{GM}} = 2pD\sqrt{\frac{10D}{11GM}}; \quad (2) \quad \frac{mv_2^2/2}{Mv_1^2/2 + mv_2^2/2} = \frac{v_2^2}{10v_1^2 + v_2^2} = \frac{10}{11}.$$

$$7-12 \quad \text{轨道椭圆长轴 } a = \left(\frac{GM_s T^2}{4p^2}\right)^{1/3} = 2.69 \times 10^{14} \text{ m}, \quad \text{远日点 } r_+ \approx 2a = 5.38 \times 10^{14} \text{ m}.$$

$$7-13 \quad G = \frac{Dr^2 q}{Mml} = 6.61 \times 10^{-11} \text{ m}^3/\text{kg} \cdot \text{s}^2.$$

$$7-14 \quad (1) \quad x = GM^2/k[l_0 + 2\left(\frac{3M}{4pr}\right)^{1/3}]^2 = 5.90 \times 10^{-6} \text{ m};$$

$$(2) \quad \omega = \sqrt{GM/(R+l_0/2)(2R+l_0)} = 6.23 \times 10^{-4} \text{ rad/s}.$$

$$7-15 \quad r_M = \left(\frac{5}{2} \times 0.33R_E - r_c\right)M_E / \frac{4}{3}pR_E^3(R_E^2 - r_c^2) = 4.17 \times 10^3 \text{ kg/m}^3,$$

$$r_c = [M_E - \frac{4}{3}p(R_E^3 - r_c^3)r_M] / \frac{4}{3}pr_c^3 = 12.7 \times 10^3 \text{ kg/m}^3.$$

7-16 地幔和地核交界处, 重力加速度最大:  $g_c = \frac{4}{3}pGr_c r_c = 12.3 \text{ m/s}^2$ .

7-17 (a) 逃逸速度  $v_2 = \sqrt{2GM/R}$ , 按能量守恒, 可求得离球心的最大距离为  $r = 16R/7$ ;

(b)  $\frac{3}{4}v_2 = \frac{3}{4}\sqrt{\frac{2GM}{R}} > \sqrt{\frac{GM}{R}} = v_1$ , 即此速度大于第一宇宙速度, 粒子此情况下已成为卫

星, 但可求得离球心的最大距离为  $r = 9R/7$ .

7-18 由角动量守恒和能量守恒定律可证.

7-19 (1)  $v_1^2 = v_2^2 = v_0^2 + (v_0/2)^2 = 5v_0^2/4$ ;  $E_1 = E_2 = \frac{1}{2} \cdot \frac{m}{2} v_1^2 - G \frac{Mm/2}{r} = -\frac{3}{16} G \frac{mM}{r}$ ;

$$L_1 = L_2 = (m/2)v_0 r = (m/2)\sqrt{GM}r.$$

(2) 两碎块均为以圆心为焦点的镜像对称椭圆; 其长半轴为  $a = -\frac{GM(m/2)}{2E_1} = \frac{4}{3}r$ ,

偏心率为  $e = \sqrt{1 - \frac{2|E_1|L_1^2}{G^2 M^2 (m/2)^3}} = \frac{1}{2}$ , 短半轴为  $b = \sqrt{a^2(1-e^2)} = \frac{2}{3}\sqrt{3}r$ .

7-20 圆轨道上行星的速度为  $v_0 = \sqrt{\frac{GM}{r}}$ ; 彗星在近日点的速度接近逃逸速度, 即

$$v_{\text{近}} \approx v_{\text{逃}} = \sqrt{\frac{2GM}{r}}; \text{ 故 } v_{\text{近}}: v_0 = \sqrt{2}.$$

7-21 按洛希公式,  $r_c = 2.45539R(r/r')^{1/3} = 2.45539R_{\text{木}}$ ; 即撕裂发生在木星上空高

$$h = r_c - R_{\text{木}} = (2.45539 - 1) \times 7.154 \times 10^7 = 1.041 \times 10^8 \text{ m} = 1.041 \times 10^5 \text{ km} \text{ 处}.$$

7-22 相对速度近似等于木星的逃逸速度, 即

$$V \approx v_{\text{木逃}} = \sqrt{\frac{2GM_{\text{木}}}{R_{\text{木}}}} \approx \sqrt{\frac{2G(320M_{\text{地}})}{(11R_{\text{地}})}} = v_{\text{地逃}} \sqrt{\frac{320}{11}} = 11.2 \times 5.4 \approx 60 \text{ km/s}.$$

## 第八章 相对论

8-1  $\Delta t = \frac{\Delta t}{\sqrt{1-v^2/c^2}} = 14 \times 10^{-6} \text{ s}$ ,  $\Delta l = v\Delta t = 4200 \text{ m}$ .

8-2  $\Delta t = g\Delta t = 2.5 \times 10^{-5} \text{ s}$ ,  $l = 7.5 \times 10^3 \text{ m}$ .

8-3  $v = \sqrt{1 - (\Delta t / \Delta t)^2} \cdot c = \frac{\sqrt{5}}{3} \cdot c$ ,  $l = v\Delta t = \sqrt{5}c$ .

$$8-4 \quad v = \sqrt{1 - (\Delta x / \Delta x')^2} \cdot c = \frac{\sqrt{8}}{3} \cdot c, \quad \Delta t' = -0.94 \times 10^{-8} \text{ s}.$$

$$8-5 \quad \text{质点的轨迹为一椭圆: } \frac{x'^2}{(1 - v^2/c^2)a^2} + \frac{y'^2}{a^2} = 1.$$

$$8-6 \quad q = \arctan(\tan q' / \sqrt{1 - v^2/c^2}).$$

$$8-7 \quad v_x = \frac{v' \cos q' + V}{1 + Vv' \cos q' / c^2}, \quad v_y = \frac{v' \sin q' \sqrt{1 - V^2/c^2}}{1 - Vv' \cos q' / c^2}; \quad q = \arctan \frac{v' \sin q' \sqrt{1 - V^2/c^2}}{V + v' \cos q'}.$$

$$8-8 \quad v_g = -c, \quad v_e = 0.924c.$$

$$8-9 \quad v' = (v - V) / [1 - vV/c^2] = 0.976c.$$

$$8-10 \quad (1) \text{ 对 K 系, 小包的速度为 } v_x = -c/2, \quad v_y = \sqrt{v^2 - v_x^2} = \sqrt{5}c/4;$$

$$\text{对 a 船, 小包的速度为 } v_x' = (v_x - V) / (1 - v_x V/c^2) = -4c/5,$$

$$v_y' = v_y \sqrt{1 - V^2/c^2} / (1 - v_x V/c^2) = \sqrt{15}c/10,$$

$$\text{故 a 船的瞄准角为 } a' = \arctan v_y' / v_x' = 154.2^\circ.$$

$$(2) \quad v' = \sqrt{v_x'^2 + v_y'^2} = \sqrt{(-4/5)^2 + (\sqrt{15}/10)^2} c = 0.888c.$$

$$(3) \quad v_x'' = (v_x + V) / (1 + v_x V/c^2) = 0, \quad v_y'' = v_y \sqrt{1 - V^2/c^2} / (1 + v_x V/c^2) = \sqrt{15}c/6;$$

$$\text{即 } a'' = 0, \quad v'' = v_y'' = (\sqrt{15}/6)c.$$

$$8-11 \quad A_1 = (m_{0.1c} - m_0)c^2 = 0.005m_0c^2 = 0.14 \times 10^{-15} \text{ J},$$

$$A_2 = (m_{0.9c} - m_{0.8c})c^2 = 0.085m_0c^2 = 6.97 \times 10^{-15} \text{ J} = 17A_1.$$

$$8-12 \quad \text{由 } mv = m_0 v / \sqrt{1 - v^2/c^2} = 2m_0 v, \text{ 解得 } v = \frac{\sqrt{3}}{2} c.$$

$$8-13 \quad \Delta m = \Delta E / c^2 \approx \frac{1}{2} m_0 v^2 / c^2 = 6.7 \times 10^{-10} m_0 = 6.7 \times 10^{-5} \text{ kg}.$$

$$8-14 \quad \Delta E = mC\Delta T = 2000 \text{ kC} = 8.4 \text{ J}, \quad \Delta m = \Delta E / c^2 = 9.3 \text{ kg}.$$

$$8-15 \quad E = 2m_0c^2 = 1.64 \times 10^{-13} \text{ J}.$$

$$8-16 \quad (1) \quad \Delta E = \Delta mc^2 = 1.8 \times 10^{14} \text{ J}; \quad (2) \quad \bar{P} = \Delta E / \Delta t = 1.8 \times 10^{20} \text{ J/s}.$$

$$8-17 \quad \Delta E = (4m_H - m_{H_e})c^2 = 4.26 \times 10^{-12} \text{ J}.$$



$$8-18 \quad v_c = \frac{pc^2}{E} = \frac{E_g \cdot c}{E_g + m_0 c^2}$$

$$8-19 \quad E_m = \frac{(m_p^2 + m_m^2)c^2}{2m_p}, \quad E_n = \frac{(m_p^2 - m_m^2)c^2}{2m_p}.$$

$$8-20 \quad p_2 = \frac{m_{10} v_1}{\sqrt{1 - v_1^2 / c^2}} = 1.13 \times 10^{-18} \text{ N}\cdot\text{s}, \quad E_2 = m_2 c^2 = m_0 c^2 - m_1 c^2 = 1.76 \times 10^{-9} \text{ J},$$

$$m_{20} = \sqrt{(E_2^2 - c^2 p_2^2) / c^4} = 1.95 \times 10^{-24} \text{ kg} = 11.7 \text{ 原子单位}.$$

$$8-21 \quad v = (\sqrt{3})c / 2.$$

$$8-22 \quad (1) E_{g\min} = \frac{(m_K + m_\Lambda)^2 - m_p^2}{2m_p} c = 917 \text{ Mev}.$$

(2)

$$8-23 \quad T = \frac{2pDI}{c\Delta I} = 1.526 \times 10^6 \text{ s} = 17.66 \text{ 天}.$$

$$8-24 \quad \mathbf{Q}n' = \sqrt{1 - b^2} / (1 - b \cos q) \approx n / (1 + v \cos q / c), \quad \therefore n_{\text{拍}} = n' - n \approx (-v \cos q / c)n.$$

$$8-25 \quad v = \frac{\Delta I}{I} c = \frac{0.10}{6.00 \times 10^3} \times 3.0 \times 10^8 = 5.0 \times 10^3 \text{ km/s}.$$