# More on Regression Discontinuity Designs

Advanced Quantitative Methods (PLSC 504)

Fall 2017

# Today

- Two different frameworks for understanding RD
  1. The continuity based framework (most common)
     - Relies on continuity assumptions for identification
     - Relies on extrapolation for estimation and inference
  2. The local randomization framework
     - RD is a randomized experiment *near* the cutoff
     - Imposes *explicit* (stronger) "as good as" random assumptions
     - But if forcing variable is discrete, may be only option
- More details on local randomization approach in:
  1. "Understanding Regression Discontinuity Designs As Observational Studies", Sekhon and Titiunik (2016)
  2. "On Interpreting the Regression Discontinuity Design as a Local Experiment", Sekhon and Titiunik (2017)
- **Great reference**: "A Practical Introduction to Regression Discontinuity Designs: Part I" by Cattaneo, Idrobo and Titiunik.

# Review: the (sharp) RD setup

- Some deterministic rule governs treatment $D_i$

$$D_i = \begin{cases} 1 & : V_i \geq c \\ 0 & : V_i < c \end{cases}$$

- Where $V_i$ is some *continuous* "forcing variable" or "running variable" and $c$ is defined here as the "cutpoint."
- Unit $i$ receives the treatment iff $V_i \geq c$.
- The AM is deterministic because once we know $V_i$, we know $D_i$.
- Treatment is "discontinuous" because treatment status is unchanged until $V_i = c$.

# Review: the (sharp) RD setup

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# Review: identification

1. SUTVA: $Y_i = Y_i(1) \cdot D_i + Y_i(0) \cdot (1 - D_i)$.
2. Continuity of the CEFs of potential outcomes in $V_i$ near the cutpoint:

$$\lim_{\epsilon \to 0} \mathbb{E}[Y_i(0)|V_i = c + \epsilon] = \lim_{\epsilon \to 0} \mathbb{E}[Y_i(0)|V_i = c - \epsilon] = \mathbb{E}[Y_i(0)|V_i = c],$$
$$\lim_{\epsilon \to 0} \mathbb{E}[Y_i(1)|V_i = c + \epsilon] = \lim_{\epsilon \to 0} \mathbb{E}[Y_i(1)|V_i = c - \epsilon] = \mathbb{E}[Y_i(1)|V_i = c],$$

3. Continuity of PDF of $V_i$ and positivity near the cutpoint.

$$\lim_{\epsilon \to 0} f(V_i = c + \epsilon) = \lim_{\epsilon \to 0} f(V_i = c - \epsilon) > 0$$

- As mentioned in readings and lecture, Assumption 2 – the "continuity based assumption" (CBA) – is crucial for identification.

# Review: identification

- The estimand identified by the (sharp) RD is the ATE for units *at the cutpoint*:

$$\lim_{\epsilon \to 0^+} \mathbb{E}[Y_i | V_i = c + \epsilon] - \lim_{\epsilon \to 0^-} \mathbb{E}[Y_i | V_i = c + \epsilon]$$

$$\lim_{\epsilon \to 0^+} \mathbb{E}[Y_i | D_i = 1, V_i = c + \epsilon] - \lim_{\epsilon \to 0^-} \mathbb{E}[Y_i | D_i = 0, V_i = c + \epsilon]$$

$$\lim_{\epsilon \to 0^+} \mathbb{E}[Y_i(1) | V_i = c + \epsilon] - \lim_{\epsilon \to 0^-} \mathbb{E}[Y_i(0) | V_i = c + \epsilon]$$

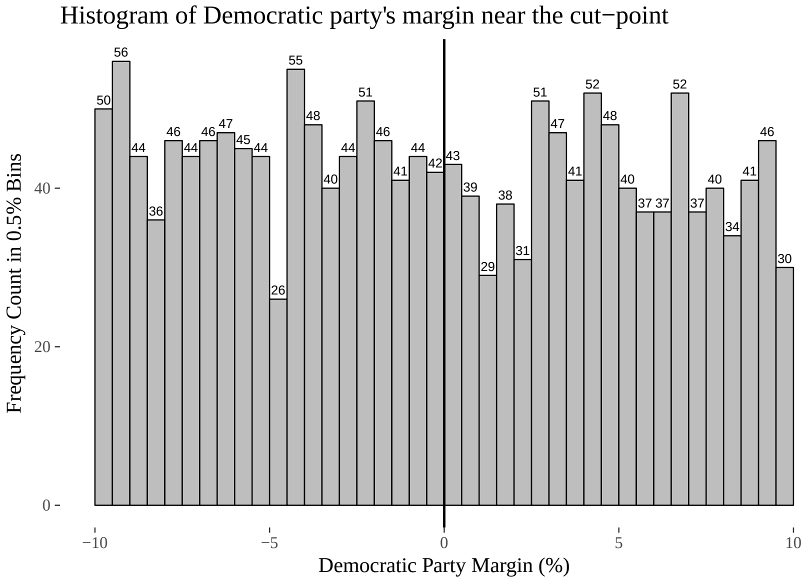
$$\mathbb{E}[Y_i(1) | V_i = c] - \mathbb{E}[Y_i(0) | V_i = c]$$

$$\mathbb{E}[\tau_i | V_i = c]$$

# Critical thinking about assumptions

- The CBA (continuity of potential outcome functions): do we see systematic differences in $X_i$ on either side of the cutpoint? Regress $X_i$ on $D_i$, $V_i$ and $D_i \cdot V_i$.
  - This is frequently violated by sorting, e.g. units *can decide* the precise value of $V_i$ based on their knowledge about $c$.
  - Q: if $c$ is a known value of SAT score, is CBA violated?
- Assumption 3 (continuity of PDF of $V_i$): do we see an unusual number of people just above/below the cutpoint? E.g. 500 people above cutpoint and 20 below.
  - The McCrary test is one option. See the `rdd` package for implementation.
  - The local polynomial approach via the `rddensity` function in the `rdrobust` package is better.

# Critical thinking about assumptions



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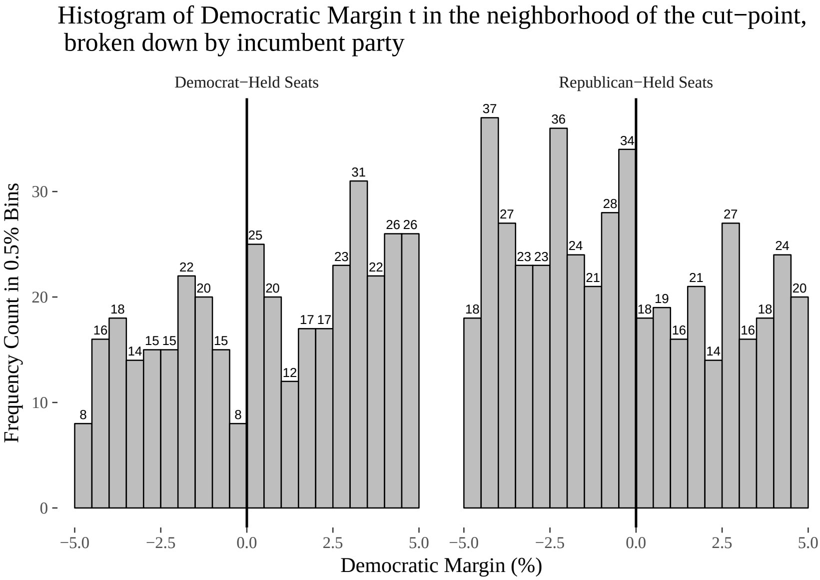
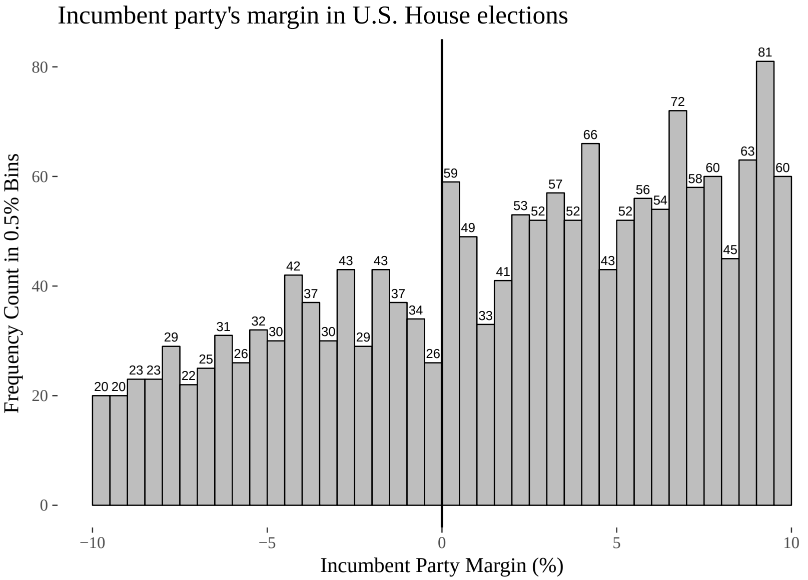


# Critical thinking about assumptions

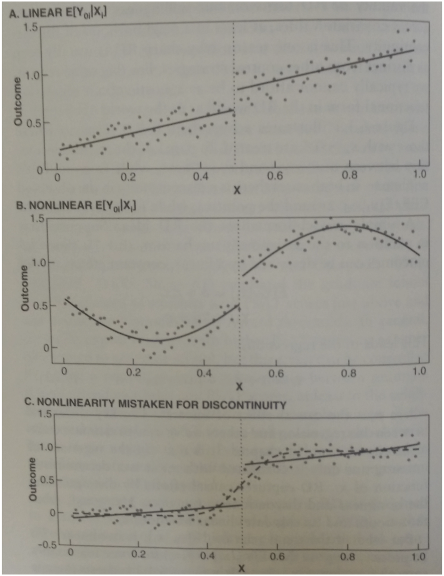


# Estimation

- In general, there are no observations where $V_i = c$. Why?
  - For a r.v. $X$ w/ a truly continuous PDF, $\Pr(X = x) = 0$
- How many observations are "near" the cutpoint?
  - Depends on how we define "near"
- Key issue: RD depends crucially on extrapolation for estimation, atleast in some neighborhood near the cutpoint
  - Approximating $\mathbb{E}[Y_i(1)|V_i = c]$, $\mathbb{E}[Y_i(0)|V_i = c]$ in practice relies on the observations "near"" $c$

# Estimation

From Mostly Harmless (p. 254):



# Estimation

- **Weierstrass approximation theorem**: every continuous function on a closed interval, e.g. $[c-h, c+h]$ can be uniformly approximated as closely as desired by a polynomial function
  - Problem: RD treatment effects are defined at boundry points and global approximations do a bad job here (**Runge's phenomenon**)
  - Solution (from analysis): convolve the function we want to approximate via a polynomial kernel
  - Implementation: use polynomial approximation with kernel for weighting observations in $[c-h, c+h]$

# Estimation

- A local-polynomial recipe:
  1. Choose a polynomial order $p$ and a kernel function $K(\cdot)$
  2. Choose a bandwidth $h$
  3. For $V_i \geq c$, regress $Y_i$ on $(V_i - c)$, $(V_i - c)^2, \ldots, (V_i - c)^p$ via WLS with weight $K\left(\frac{v_i - c}{h}\right)$.
  4. For $V_i < c$, regress $Y_i$ on $(V_i - c)$, $(V_i - c)^2, \ldots, (V_i - c)^p$ via WLS with weight $K\left(\frac{v_i - c}{h}\right)$
  5. Sharp RD point estimate in the intercept in 3) minus the intercept in 4)
- Researcher degrees of freedom: $K(\cdot)$, $p$, and $h$
  - People often recommend weighting observations closer to the cutoff more heavily; hence a triangular kernel
  - A trade-off exists between $p$ and $h$
  - Imbens and Kalyanaraman (2012) propose a method for choosing bandwidth that is (asymptotically) MSE-optimal
- None of this has **anything** to do with identification

# Summary of continuity based framework

- Key identification assumption: continuity of CEFs of potential outcomes in $V_i$ near $c$ (the "CBA")
- Under random assignment we shouldn't see differences in pretreatment characteristics on either side of the cutpoint
  - We can't test continuity directly (potential outcomes are unobservable)
  - We can test a stronger version: continuity of the CEFs of potential outcomes **and** a background covariate
  - Failure to find evidence of differences **does not** imply continuity
- We also shouldn't see an unusual number of units on either side of the cutpoint
  - We have a direct test of Assumption 3 (continuity of PDF of $V_i$ at $c$)
  - The null is that there is no discontinuity in $f(V_i)$, e.g. $\lim_{\epsilon \to 0^+} f(V_i = c + \epsilon) = \lim_{\epsilon \to 0^+} f(V_i = c + \epsilon)$
  - McCrary test (`rdd`) or a local polynomial approach (`rdrobust`)

# Summary of continuity based framework

- Assumption checks are important, but hard to implement. Why?
  - The estimand is defined by a measure zero event, e.g. a continuous variable taking on a specific value $c$
  - Unlike in experiment, covariate balance is only implied at the cut-point
- Estimation of RD hinges crucially on extrapolation, so we have to think carefully about functional form
  - As Fredrik explained in lecture, estimation can be a nightmare
  - A smoother functional form requires more data, but we are also pruning units that aren't "near" the cutpoint!
  - Bandwidth selection and so on has nothing to do with identification

# Local randomization framework

- Treatment is "as if" randomly assigned in some neighborhood around the cutoff
- Units with $V_i$ in some "small" window around $c$ are "as if" randomly assigned to treatment or control
- Formally, in some neighborhood $W \in [c - w, c + w]$ for $w > 0$
  - Assume $Y_i(1), Y_i(0) \perp\!\!\!\perp D_i \mid V_i \in W$
  - Then

$$\mathbb{E}[Y_i(1) - Y_i(0) | V_i \in W] = \mathbb{E}[Y_i | D_i = 1, V_i \in W] - \mathbb{E}[Y_i | D_i = 0, V_i \in W]$$

- Identification is based on this "Local Randomization Assumption" (LRA), rather than continuity
  - Target parameter is the ATE inside $W$ rather than the ATE at $c$
  - Estimation is not concerned w/ approximating $\mathbb{E}[Y_i(1)|V_i]$ and $\mathbb{E}[Y_i(0)|V_i]$

# Local randomization framework

- LRA is **not** part of what is required in the continuity based case
  - Indeed, it's typically not plausible
  - Units with higher scores on $V_i$ are generally systematically different from those with lower scores
- LRA implies a LATE, so invokes an exclusion restriction assumption for sharp RD
  - Namely, inside $W$, $D_i = D_i(1)V_i + D_i(0)(1 - V_i)$
  - Thus, $V_i$ is excluded from $Y_i = Y_i(1)D_i + Y_i(0)(1 - D_i)$
- Other problems:
  - How do we know what $W$ is supposed to be?
  - Methods for estimation and inference assume $W$ is fixed as $n \to \infty$

# Sekhon and Titiunik (2016)

- A conceptual critique of the local randomization approach.
- Misconception that treatment assignment rule in RD designs is "as good as" an RCT near the cutoff.
- RD is an **observational study**, but they are often not interpreted as such.
- But observational studies are often defined by not knowing AM, and in RD we know it!
- Key issues:
  - Distribution of $V_i$ is fundamentally unknown
  - Unlike in experiment, knowing a (deterministic) AM in the RD design is not enough. Why?
    - AM determines $D_i$, but not $V_i$!
  - $V_i$ is often an important determinant of potential outcomes
  - $V_i$ may have a **direct effect** on potential outcomes

# Sekhon and Titiunik (2017)

- A more formal critique of the local randomization approach
- Key points:
  - Continuity of the CEFs of potential outcomes in $V_i$ does not imply LRA
  - LRA does not imply $D_i$ and $Y_i(0), Y_i(1)$ are independent
  - LRA does not imply $V_i$ and $Y_i(0), Y_i(1)$ are independent
  - Assuming **local independence** between $Y_i(0), Y_i(1)$ and $D_i$ does not imply $V_i$ only affects $Y_i$ through $D_i$ (exclusion restriction)

# Sekhon and Titiunik (2017)

- Paper shows:
  - Even if we randomize $V_i$ inside $W$, the LRA is not guaranteed to hold!
  - If (randomly assigned) $V_i$ not independent of potential outcomes, treatment may fail to be independent of potential outcomes
  - Why? $D_i$ is a (deterministic) function of $V_i$. LRA does not imply exclusion restriction
- Conclusions:
  - If exclusion restriction fails, we can still capture some re-paramaterized causal effect like an ITT
  - This is the effect of obtaining $V_i \in [c, c + w]$ versus $V_i \in [c - w, c)$
  - Appealing because it seems to avoid estimation and inference issues of continuity based framework
  - However! It requires additional assumptions that are hard to swallow

# The End

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