

# Cheatsheet - Recursion

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## 1. Intro

When a function calls itself, we refer to this process as "recursion". A recursive formula can be useful to solve certain kinds of problems.

### 1.1. Example Exercise

*Example 1. Disclaimer: The following exercise was solved by ChatGPT and includes direct quotes from its output.*

“There are 10 steps and you can take either one step at the time or two steps at the time. How many ways are there to walk all 10 steps?

Let  $S(n)$  be the number of ways to walk  $n$  steps by taking either one or two steps at a time. Then:

$$S(n) = S(n - 1) + S(n - 2)$$

The intuition behind this formula is that to walk  $n$  steps, we can either take a single step and then walk the remaining  $n - 1$  steps, or take two steps and then walk the remaining  $n - 2$  steps. Therefore, the total number of ways to walk  $n$  steps is the sum of the number of ways to walk  $n - 1$  steps and the number of ways to walk  $n - 2$  steps.

We can use this recursive formula to calculate  $S(10)$ :

$$S(1) = 1$$

$$S(2) = 2$$

$$S(3) = S(2) + S(1) = 3$$

$$S(4) = S(3) + S(2) = 5$$

$$S(5) = S(4) + S(3) = 8$$

...

$$S(10) = 89$$

#### NOTE

We define  $S(1) = 1$  (only one way to walk one step) and  $S(2) = 2$  (two ways to walk two steps, either one step at the time or two steps at once). Therefore, we don't use the recursive formula directly on  $S(1)$  and  $S(2)$ .

Therefore, there are 89 ways to walk all 10 steps by taking either one or two steps at a time.

## 2. Euclid's Algorithm

Consider the following: there are two non-zero integers  $a$  and  $b$ , now let's find the greatest integer that divides  $a$  and  $b$  without leaving a remainder.

With Euclid's algorithm, let's first assume that  $a \geq b$ , then let's divide  $a$  by  $b$  and the remainder is  $r$ .

**Importantly:**

$$\gcd(a, b) = \gcd(b, r) = \gcd(a, r)$$

By using division once, we have  $b \leq a$ ,  $r < b$  since the arguments of  $\gcd$  have decreased. We continue with this until  $r = 0$ , then  $b$  is the final  $\gcd$ .

More formally:

$$\gcd(a, b) = \begin{cases} \gcd(b, |a \bmod b|) & \text{if } b > 0 \\ a & \text{if } b = 0 \end{cases}$$

For example, if we want to solve  $\gcd(27, 36)$ :

$$\gcd(27, 36) = \gcd(36, 27)$$

$$\gcd(36, 27) = \gcd(27, 9)$$

$$\gcd(27, 9) = \gcd(9, 0)$$

$$\gcd(9, 0) = 9$$

Or, if the numbers were reversed:

$$\gcd(36, 27) = \gcd(27, 9)$$

$$\gcd(27, 9) = \gcd(9, 0)$$

$$\gcd(9, 0) = 9$$

Hence,  $\gcd(27, 36) = \gcd(36, 27) = 9$ .

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