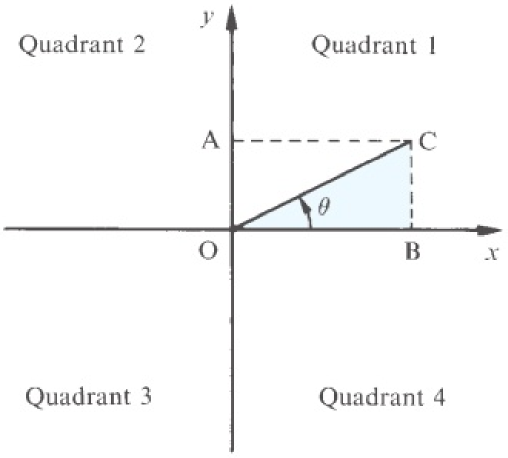
Cheatsheet - Trigonometrical Functions & Identities

Fabio Lama – fabio.lama@pm.me

1. Quadrants & Projections

The *x* and *y* axes divide the plane into four quadrants. The angle θ is of the following degree based where the arm is located:

- First quadrant, then $0^\circ \leq \theta \leq 90^\circ$
- Second quadrant, then $90^\circ \leq \theta \leq 180^\circ$
- Third quadrant, then $180^\circ \leq \theta \leq 270^\circ$
- Fourth quadrant, then $270^\circ \leq \theta \leq 360^\circ$



Consider the projection of the arm to be *OC* and label the *x* projection *OB* and the *y* projection *OA*. The trigonometrical ratios are defined as:

$$\sin\theta = \frac{\mathrm{OA}}{\mathrm{OC}} = \frac{\mathrm{BC}}{\mathrm{OC}}$$

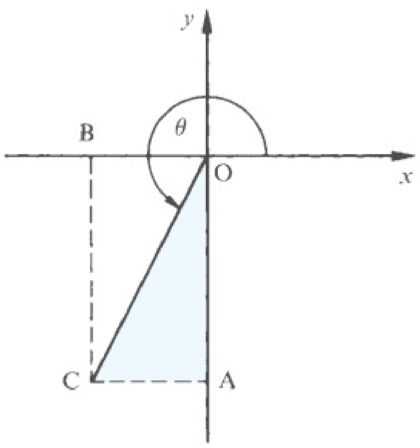
$$\cos\theta = \frac{\mathrm{OB}}{\mathrm{OC}} = \frac{\mathrm{AC}}{\mathrm{OC}}$$

$$\tan\theta = \frac{\mathrm{OA}}{\mathrm{OB}} = \frac{\mathrm{BC}}{\mathrm{AC}}$$

(Note that in the image above, $\mathrm{OA}{=}\mathrm{BC}$ *and* $\mathrm{OB}{=}\mathrm{AC}$ *. The rules for those trigonometrical functions are not any different than what's already covered in 'Cheatsheet - Trigonometry')*

Given that *x* and *y* can be negative, this also means that $\sin\theta$, $\cos\theta$ and/or $\tan\theta$ can either be positive or negative, depending on in which quadrant the arm is located and which projection is used for the calculation.

For example, in this graph:



$\sin\theta$ and $\cos\theta$ are negative and $\tan\theta$ is positive, because *x* (*OB*) and *y* (OA) are negative, but *OC* is positive. Respectively:

$$\sin\theta = \frac{\mathrm{AC}}{\mathrm{OC}} = \frac{\mathrm{OB}}{\mathrm{OC}}$$

$$\cos\theta = \frac{\mathrm{OA}}{\mathrm{OC}} = \frac{\mathrm{BC}}{\mathrm{OC}}$$

$$\tan\theta = \frac{AC}{OA} = \frac{AC}{BC} = \frac{OB}{OA} = \frac{OB}{BC}$$

Given that $OB = AC$ and $BC = OA$.

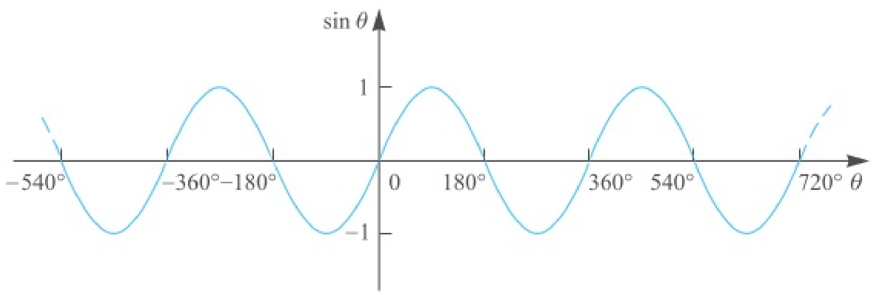
2. Adding/Subtracting 360°

Adding or subtracting 360° to/from the arm does not alter the ratios, since it's a full rotation. Hence:

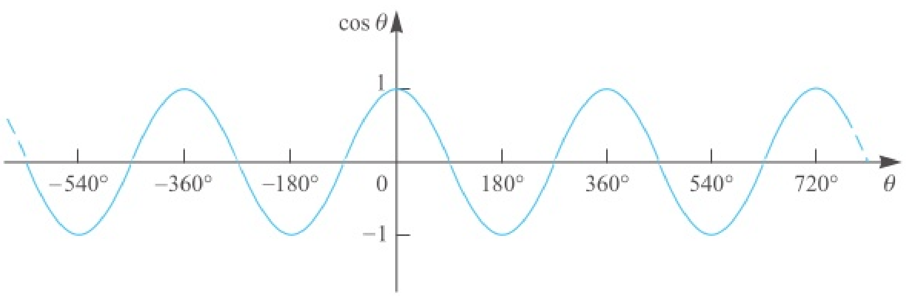
$$\sin\theta = \sin(\theta + 360^\circ) = \sin(\theta - 360^\circ)$$
$$\cos\theta = \cos(\theta + 360^\circ) = \cos(\theta - 360^\circ)$$
$$\tan\theta = \tan(\theta + 360^\circ) = \tan(\theta - 360^\circ)$$

3. The Sine, Cosine and Tans Functions

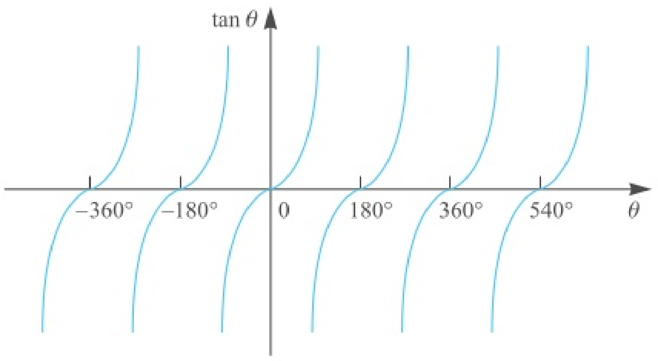
The sine function is $y = \sin\theta$, which repeats every 360°:



This cosine function is $y = \cos\theta$, which repeats every 360°:



The tangent function is $y = \tan\theta$, which repeats every 180°:



4. Identities

Identities imply equations that are true for every value of the involved variables. Important ones are:

$$\frac{\sin\theta}{\cos\theta} = \tan\theta$$

and:

$$(\sin A)^2 + (\cos A)^2$$
$$= \sin^2 A + \cos^2 A = 1$$

Additionally:

$$\sin\theta = -\sin(\theta - 180^\circ)$$

$$\sin\theta = \sin(180^\circ - \theta)$$
$$\sin\theta = -\sin(360^\circ - \theta)$$

$$\cos\theta = -\cos(\theta - 180^\circ)$$
$$\cos\theta = -\cos(180^\circ - \theta)$$
$$\cos\theta = \cos(360^\circ - \theta)$$

$$\tan\theta = -\tan(180^\circ - \theta)$$
$$\tan\theta = \tan(\theta - 180^\circ)$$
$$\tan\theta = -\tan(360^\circ - \theta)$$

IMPORTANT This means that trigonometric functions can have more than one solution.

4.1. Common Identities

$$\frac{\sin A}{\cos A} = \tan A$$
$$\sin(A+B) = \sin A\cos B + \sin B\cos A$$
$$\sin(A-B) = \sin A\cos B - \sin B\cos A$$
$$\sin 2A = 2\sin A\cos A$$
$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$
$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$
$$\cos 2A = (\cos A)^2 - (\sin A)^2 = \cos^2 A - \sin^2 A$$
$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$
$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$
$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$
$$\sin A - \sin B = 2\sin\left(\frac{A-B}{2}\right)\cos\left(\frac{A+B}{2}\right)$$
$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$
$$\cos A - \cos B = -2\sin\left(\frac{A-B}{2}\right)\sin\left(\frac{A+B}{2}\right)$$
$$\sin A = -\sin(-A)$$
$$\cos A = \cos(-A)$$
$$\tan A = -\tan(-A)$$

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