Cheatsheet - Set Theory

Fabio Lama – fabio.lama@pm.me

1. Definition

A set refers to an **unordered** collection of any kind of (abstract) **unique** objects, so called "elements", such as numbers, figures, things, etc.

1.1. Basic Notation

We have the set E containing four elements:

$$E = \{1, 2, 3, 4\}$$

or set V:

$$V = \{a, b, c, d, e, f\}$$

We can see that the element 2 *is in* E, respectively:

$$2 \in E$$

but *not in* V

$$2 \notin V$$

2. Cardinality

Given the set S, the **cardinality** of S is the number of elements contained in S. We write the cardinality of S as $|S|$.

$$S = \{a, b, c\}$$
$$|S| = 3$$

and the following **empty** set:

$$S = \phi = \{\}$$
$$|S| = 0$$

3. Subset and Superset

We have set E and $\{1, 2\}$ is a **subset** ($\subseteq$) of E. We can also say that E is a **superset** ($\supseteq$) of $\{1, 2\}$. The set $\{a, b\}$ is **not** a subset ($\nsubseteq$) of E, however.

$$\begin{gathered} E = \{1,2,3,4\} \\ E \subseteq E \\ E \supseteq E \\ \{1,2\} \subseteq E \\ E \supseteq \{1,2\} \\ \{a,b\} \nsubseteq E \\ E \nsupseteq \{a,b\} \\ E \nsupseteq \{1,2,3,4,5\} \end{gathered}$$

Importantly, we distinguish between a subset ($\subseteq$) and a **proper subset** ($\subset$):

$$\begin{gathered} \{1,2,3\} \subset E \\ \{1,2,3,4\} \not\subset E \\ \{1,2,3,4\} \subseteq E \end{gathered}$$

Do note that any set also contains an empty subset:

$$(\{\} = \phi) \subseteq E$$

4. Special Sets

Commonly defined and used sets in mathematics:

$$\mathbb{N} = \{1, 2, 3, 4, ...\} \quad \text{natural numbers}$$

$$\mathbb{W} = \{0, 1, 2, 3, ...\} \quad \text{whole numbers}$$

$$\mathbb{Z} = \{..., -1, 0, 1, 2, ...\} \quad \text{integers (no fractions)}$$

$$\mathbb{Q} = \left\{..., -1, -\left(\frac{1}{2}\right), 0, \frac{2}{3}, 1, ...\right\} \quad \text{rational numbers}$$

$$\mathbb{R} = \quad \text{real numbers (non complex-numbers)}$$

Note that infinite numbers are *irrational numbers* and are not part of $\mathbb{Q}$, such as $\pi \notin \mathbb{Q}$.

Additionally:

$$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$$

5. Set Builder

We define:

$$E = \{1, 2, 3, 4\}$$

and use the set builder to create a new set, for example:

$$V = \{2n \mid n \in E\}$$
$$V = \{2, 4, 6, 8\}$$

Read as "each element set V is two times n for each n in set E".

We now define the set J:

$$J = \{2n \mid n \in E \quad \text{and} \quad n < 3\}$$
$$J = \{2, 4\}$$

6. Powerset

Given a set S, the powerset of that set is denoted as $P(S)$ and contains all subsets of S:

$$S = \{a, b\}$$
$$P(S) = \{\phi, \{a\}, \{b\}, \{a, b\}\}$$

since

$$\phi \subseteq S$$
$$\{a\} \subseteq S$$
$$\{b\} \subseteq S$$
$$\{a, b\} \subseteq S$$

6.1. Cardinality of a powerset

We define:

$$|P(S)| = 2^{|S|}$$

For example:

$$S = \{1, 2, 3\}$$
$$|S| = 3$$
$$P(S) = \{\phi, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$$
$$|P(S)| = 2^3 = 8$$

7. Set Operations

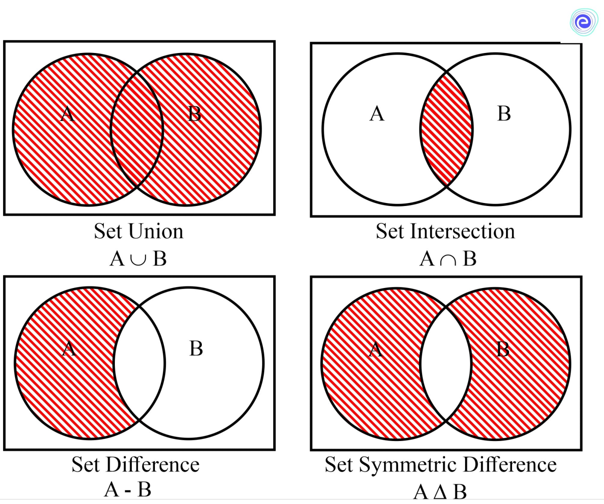

Figure 1. Venn Diagram, source: https://www.embibe.com/exams/set-theoretic-approach/

7.1. Union

Given two sets A and B, the union of A and B, $A \cup B$, contains all the element in **either** A and B.

$$A \cup B = \{x \mid x \in A \quad \text{or} \quad x \in B\}$$

For example:

$$A = \{1, 2, 3\}$$
$$B = \{4, 5, 6\}$$
$$A \cup B = \{1, 2, 3, 4, 5, 6\}$$

and

$$A = \{1, 2, 3\}$$
$$C = \{2, 3, 4\}$$
$$A \cup C = \{1, 2, 3, 4\}$$

7.2. Intersection

Given two sets A and B, the intersection of A and B, $A \cap B$, contains all the elements in both A **and** B.

$$A \cap B = \{x \mid x \in A \quad \text{and} \quad x \in B\}$$

For example:

$$A = \{1, 2, 3\}$$
$$B = \{2, 3, 4\}$$
$$A \cap B = \{2, 3\}$$

7.3. Set Difference

Given two sets A and B, the set difference, $A - B$, contains the elements that are in A but not in B.

$$A - B = \{x \mid x \in A \quad \text{and} \quad x \notin B\}$$

For example:

$$A = \{1, 2, 3\}$$
$$B = \{2, 3, 4\}$$
$$A - B = \{1\}$$

NOTE $A - B = A \backslash B$

7.4. Symmetric Difference

Given two sets A and B, the symmetric difference, $A \oplus B$, contains the elements that are in A or in B but **not in both**.

$$A \oplus B = \{x \mid (x \in A \quad \text{or} \quad x \in B) \quad \text{and} \quad x \notin A \cap B\}$$

For example

$$A = \{1, 2, 3\}$$
$$B = \{2, 3, 4\}$$
$$A \oplus B = \{1, 4\}$$

NOTE $A \oplus B = A \Delta B$

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