Cheatsheet - First-Order Logic

Fabio Lama – fabio.lama@pm.me

1. Predicates

Predicates describe properties of objects.

For example:

$$\text{odd}(3)$$

$\text{odd}(3)$ means 3 is an odd number. odd is a predicate, 3 is an object. Predicates take arguments and become **propositions**.

Connectives can be applied:

$$\text{odd}(3) \land \text{prime}(3)$$

This means that 3 is odd but also prime.

2. Quantifiers

We use the symbol $\exists$ to indicate the existence of something (**existential quantifier**).

$$\exists x \quad \text{odd}(x)$$

This means that there exists some x that is odd.

We denote the **universal quantifier** as $\forall$.

$$\forall x(\text{odd}(x) \lor \text{even}(x))$$

This meant that **for all** x the number is either even or odd.

Other examples, "All Ps are Qs":

$$\forall x(P(x) \rightarrow Q(x))$$

And "No Ps are Qs":

$$\forall x(P(x) \rightarrow \neg Q(x))$$

3. Quantifiers to Connectives

$\exists x, P(x)$ where $x \in \{x_1, x_2, ..., x_n\}$ means that there exists some x for which $P(x)$ is true.

Denoted alternatively:

$$\exists x, P(x) \equiv P(x_1) \lor P(x_2) \lor ... \lor P(x_3)$$

We can also conclude:

$$\neg \exists x, P(x) \equiv \neg(P(x_1) \lor P(x_2) \lor ... \lor P(x_3))$$
$$\neg \exists x, P(x) \equiv \neg P(x_1) \land \neg P(x_2) \land ... \land \neg P(x_3)$$
$$\neg \exists x, P(x) \equiv \forall x, \neg P(x)$$

Last updated 2022-10-29 15:21:30 UTC