Cheatsheet - Gradients of Curves & Differentiation

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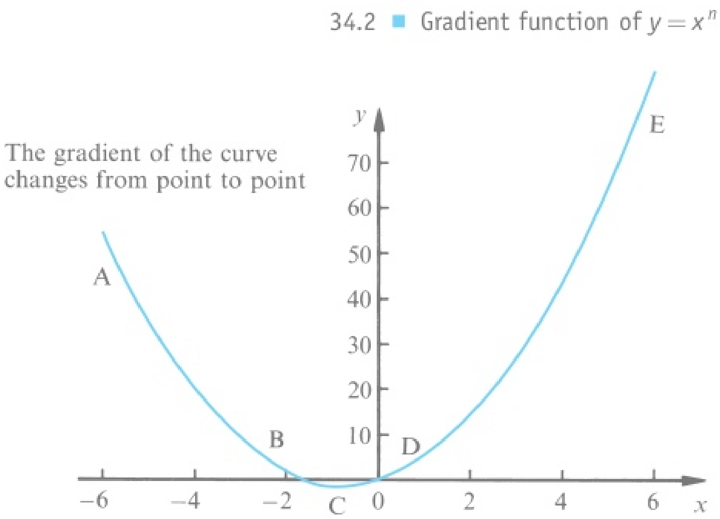
1. Gradient Function

The **gradient** (or "slope") of a graph tells us something about the **rate of change** and "steepness" of a function. Given a function $y = f(x)$ we denote its gradient function by "dee *y* by dee *x*" or simply by "*y* dash".

$$\frac{dy}{dx} = y'$$

IMPORTANT This is **not** defined as "*dy* divided by *dx*", respectively *dy* and *dx* don't have any meaning here. Rather, we take $\frac{dy}{dx}$ as a symbol of its own, such as y'.



The gradient function is also called **first derivative**. The process of obtaining this is also known as **differentiation**. Saying to differentiate $y = x^5$ means to find its gradient function *y*. *Differential calculus* studies this more in depth.

2. Gradient function of $y = x^n$

For any function of the form $y = x^n$ the gradient function is found from the following formula:

$$y = x^n \quad \text{then} \quad y' = nx^{n-1}$$

For example:

$$\text{if} \quad y = x^3 \quad \text{then} \quad y' = 3x^{3-1} = 3x^2$$

Respectively:

$$y' = f'(x^3) = 3x^2$$

When we substitute *x* and the result is negative, the curve is falling. If the result is positive, the curve is rising. We write $y'(x = 2)$ or simply $y'(2)$ to denote the value of the gradient function when $x = 2$.

The gradient function of some common functions:

For: $y = f(x)$	**For:** $y' = f'(x)$	**Notes**
constant	0	
x	1	
x^2	2x	
x^n	nx^{n-1}	
e^x	e^x	
e^{kx}	ke^{kx}	*k* is a constant
$\sin x$	$\cos x$	

$\cos x$	$-\sin x$	
$\sin kx$	$k\cos kx$	k is a constant
$\cos kx$	$-k\sin kx$	k is a constant
$\ln kx$	$1/x$	k is a constant

3. Rules for Finding Gradient Functions

Rules for finding gradients of non-single terms.

3.1. Rule 1

To find the gradient function of a sum of two functions we can simply find the two gradient functions separately and those together.

$$y = f(x) + g(x) \text{ then } y' = f'(x) + g'(x)$$

For example:

$$y = x^2 + x^4$$
$$f'\left(x^2\right) = 2x$$
$$f'\left(x^4\right) = 4x^3$$
$$\text{hence } y' = f'\left(x^2 + x^4\right) = 2x + 4x^3$$

3.2. Rule 2

Extension of the first rule.

$$y = f(x) - g(x) \text{ then } y' = f'(x) - g'(x)$$

For example:

$$y = x^5 - x^7$$
$$f'\left(x^5\right) = 5x^4$$
$$f'\left(x^7\right) = 7x^6$$
$$\text{hence } y' = f'\left(x^5 - x^7\right) = 5x^4 - 7x^6$$

3.3. Rule 3

$$y = kf(x) \text{ then } y' = kf'(x)$$

where k is a number.

For example:

$$y = 3x^2 = 3\left(x^2\right)$$
$$x^2 = 2x$$
$$\text{hence } y' = f'\left(3x^2\right) = 3(2x) = 6x$$

4. Higher Derivatives

To find the derivative of the derivative itself, known as the **second derivative** and denoted as y'', we define:

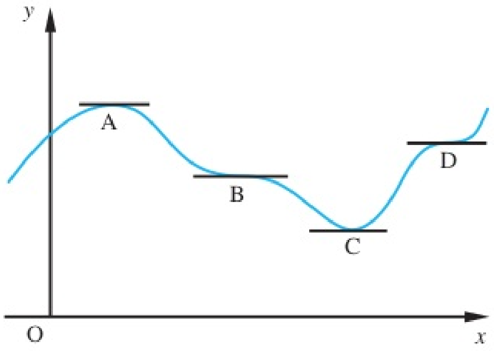
$$y'' = \frac{d^2y}{dx^2}$$

y'' is found by differentiating y'.

For example:

$$\text{if } y' = 4x^3 \text{ then } y'' = 4\left(3x^2\right) = 12x^2$$

5. Maximum and Minimum Points



Points where the gradient is zero are known as **stationary points**, such as points *A*, *B*, *C* and *D* (seen in the graph above). A point like *A* is the **maximum turning point** (or just **maximum**). A point like *C* is the **minimum turning point** (or just **minimum**). Points like *B* and *D* are known as **points of inflexion**, where the curve falls and rises (unlike *A* where the curve only falls and *C* where the curve only rises).

IMPORTANT — Stationary points are located by setting the gradient function equal to zero, that is $y' = 0$.

For example, to find the stationary points of:

$$y = 3x^2 - 6x + 8$$

we deterime the gradient function y' by differentiating *y*:

$$y' = nx^{n-1}$$
$$y' = 3x^{3-1} = 3x^n$$
$$y' = 9x$$

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