Cheatsheet - Number Bases

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1. Decomposing Number Bases

1.1. The Decimal System

The decimal system is the "base 10" system, which only has the numbers $(0, 1, ..., 9)$.

$$\begin{aligned} 253 &= 200 + 50 + 3 \\ &= 2(100) + 5(10) + 3(1) \\ &= 2(10^2) + 5(10^1) + 3(10^0) \end{aligned}$$

1.2. The Binary System

The binary system is the base 2 system, which only has the numbers $(0, 1)$.

$$\begin{aligned} 10001 &= 1(2^4) + 0(2^3) + 0(2^2) + 0(2^1) + 1(2^0) \\ &= 1(16) + 0(8) + 0(4) + 0(2) + 1(1) \\ &= 16 + 1 \\ &= 17_{10} \end{aligned}$$

2. Conversion To Base

Example: convert 58_{10} to base 2.

$$\begin{aligned} 58 \div 2 &= 29, \text{remainder} \quad 0 \\ 29 \div 2 &= 14, \text{remainder} \quad 1 \\ 14 \div 2 &= 7, \text{remainder} \quad 0 \\ 7 \div 2 &= 3, \text{remainder} \quad 1 \\ 3 \div 2 &= 1, \text{remainder} \quad 1 \\ 1 \div 2 &= 0, \text{remainder} \quad 1 \end{aligned}$$

Hence $58_{10} = 111010_2$ (read remainder from bottom up).

Example: convert 558_{10} to base 5.

$$\begin{aligned} 558 \div 5 &= 111, \text{remainder} \quad 3 \\ 111 \div 5 &= 22, \text{remainder} \quad 1 \\ 22 \div 5 &= 4, \text{remainder} \quad 2 \\ 4 \div 5 &= 0, \text{remainder} \quad 4 \end{aligned}$$

Hence $558_{10} = 4213_5$.

3. Non-integer Numbers

3.1. Decimal to Binary

$$\begin{aligned} 17.375_{10} &= 1 \times 10^1 + 7 \times 10^0 + 3 \times 10^{-1} + 7 \times 10^{-2} + 5 \times 10^{-3} \\ &= 10 + 7 + \frac{3}{10} + \frac{7}{100} + \frac{5}{1000} \end{aligned}$$

First, convert the integer:

$$17_{10} = 10001_2$$

Now the fractional part:

$$\begin{aligned} 0.375 \times 2 &= 0.75 = 0 + 0.75 \\ 0.75 \times 2 &= 1.5 = 1 + 0.5 \\ 0.5 \times 2 &= 1.0 = 1 + 0 \end{aligned}$$

Stop at 0 (e.g. $1 + 0$). Now combine the final integers, top to bottom: $0.375_{10} = 0.011_2$.

Hence:

$$17.375_{10} = 10001.011_2$$

3.2. Binary to Decimal

Example: 1101.101_2.

$$\begin{aligned} 1101 &= 1(2^3) + 1(2^2) + 0(2^1) + 1(2^0) + 1(2^{-1}) + 0(2^{-2}) + 1(2^{-3}) \\ &= 8 + 4 + 1 + \frac{1}{2} + \frac{1}{8} \\ &= 8 + 4 + 1 + 0.5 + 0.125 \\ &= 13.625 \end{aligned}$$

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