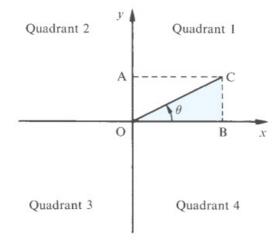
### 1. Quadrants & Projections

The x and y axes divide the plane into four quadrants. The angle  $\theta$  is of the following degree based where the arm is located:

- First quadrant, then  $0^{\circ} \leq \theta \leq 90^{\circ}$
- ullet Second quadrant, then  $90^\circ \le heta \le 180^\circ$
- ullet Third quadrant, then  $180^\circ \le heta \le 270^\circ$
- Fourth quadrant, then  $270^{\circ} \leq \theta \leq 360^{\circ}$



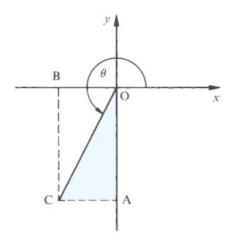
Consider the projection of the arm to be OC and label the x projection OB and the y projection OA. The trigonometrical ratios are defined as:

$$\sin \theta = \frac{\mathrm{OA}}{\mathrm{OC}} = \frac{\mathrm{BC}}{\mathrm{OC}}$$
$$\cos \theta = \frac{\mathrm{OB}}{\mathrm{OC}} = \frac{\mathrm{AC}}{\mathrm{OC}}$$
$$\tan \theta = \frac{\mathrm{OA}}{\mathrm{OB}} = \frac{\mathrm{BC}}{\mathrm{AC}}$$

(Note that in the image above, OA=BC and OB=AC. The rules for those trigonometrical functions are not any different than what's already covered in 'Cheatsheet - Trigonometry')

Given that x and y can be negative, this also means that  $\sin \theta$ ,  $\cos \theta$  and/or  $\tan \theta$  can either be positive or negative, depending on in which quadrant the arm is located and which projection is used for the calculation.

For example, in this graph:



 $\sin \theta$  and  $\cos \theta$  are negative and  $\tan \theta$  is positive, because x (OB) and y (OA) are negative, but OC is positive. Respectively:

$$\sin \theta = \frac{AC}{OC} = \frac{OB}{OC}$$
$$\cos \theta = \frac{OA}{OC} = \frac{BC}{OC}$$

$$\tan\theta = \frac{AC}{OA} = \frac{AC}{BC} = \frac{OB}{OA} = \frac{OB}{BC}$$

Given that  $OB{=}AC$  and  $BC{=}OA$  .

# 2. Adding/Subtracting 360°

Adding or subtracting 360° to/from the arm does not alter the ratios, since it's a full rotation. Hence:

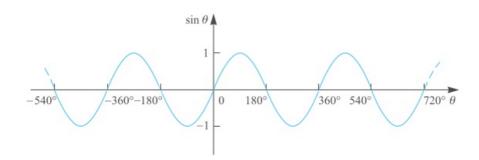
$$\sin \theta = \sin(\theta + 360^{\circ}) = \sin(\theta - 360^{\circ})$$

$$\cos \theta = \cos(\theta + 360^{\circ}) = \cos(\theta - 360^{\circ})$$

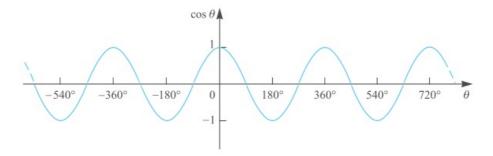
$$\tan \theta = \tan(\theta + 360^{\circ}) = \tan(\theta - 360^{\circ})$$

## 3. The Sine, Cosine and Tans Functions

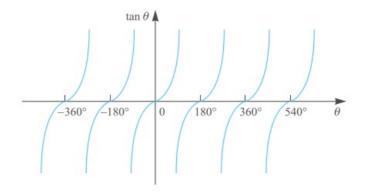
The sine function is  $y=\sin heta$  , which repeats every 360°:



This cosine function is  $y = \cos \theta$ , which repeats every 360°:



The tangent function is  $y = \tan \theta$ , which repeats every 180°:



#### 4. Identities

Identities imply equations that are true for every value of the involved variables. Important ones are:

$$\frac{\sin\theta}{\cos\theta}=\tan\theta$$

and:

$$(\sin A)^2 + (\cos A)^2$$
  
=  $\sin^2 A + \cos^2 A = 1$ 

Additionally:

$$\sin \theta = -\sin(\theta - 180^{\circ})$$

$$\sin \theta = \sin(180^{\circ} - \theta)$$

$$\sin \theta = -\sin(360^{\circ} - \theta)$$

$$\cos \theta = -\cos(\theta - 180^{\circ})$$

$$\cos \theta = -\cos(180^{\circ} - \theta)$$

$$\cos \theta = \cos(360^{\circ} - \theta)$$

$$\tan \theta = -\tan(180^{\circ} - \theta)$$

$$\tan \theta = \tan(\theta - 180^{\circ})$$

$$\tan \theta = -\tan(360^{\circ} - \theta)$$

IMPORTANT

This means that trigonometric functions can have more than one solution.

#### 4.1. Common Identities

$$\frac{\sin A}{\cos A} = \tan A$$

$$\sin(A+B) = \sin A \cos B + \sin B \cos A$$

$$\sin(A-B) = \sin A \cos B - \sin B \cos A$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\cos 2A = (\cos A)^2 - (\sin A)^2 = \cos^2 A - \sin^2 A$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\sin A - \sin B = 2\sin\left(\frac{A-B}{2}\right)\cos\left(\frac{A+B}{2}\right)$$

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\cos A - \cos B = -2\sin\left(\frac{A-B}{2}\right)\sin\left(\frac{A+B}{2}\right)$$

$$\sin A = -\sin(-A)$$

$$\cos A = \cos(-A)$$

$$\tan A = -\tan(-A)$$

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