

Cheatsheet - Universal Set, Complement and Laws

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1. Intro

A universal set is a set that contains everything. We note the universal set with the letter U .

$$A \subseteq U$$

2. Complement of a Set

The complement of set A contains all elements of U but not A .

$$\bar{A} = U - A$$

and therefore:

$$\bar{\bar{A}} \cup A = U$$

For example:

$$U = \{1, 2, 3, 4\}$$

$$A = \{1, 2\}$$

$$\bar{A} = \{3, 4\}$$

$$A \cup \bar{A} = \{1, 2, 3, 4\} = U$$

3. De Morgan's Law

The complement of the **union** of two sets A and B is equal to the **intersection** of their complements.

$$\overline{A \cup B} = \bar{A} \cap \bar{B}$$

The complement of the **intersection** of two sets A and B is equal to the **union** of their complements.

$$\overline{A \cap B} = \bar{A} \cup \bar{B}$$

For example:

$$U = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

$$A = \{1, 2, 3, 4\} \text{ and } B = \{4, 5, 6\}$$

$$A \cup B = \{1, 2, 3, 4, 5, 6\} \text{ and } A \cap B = \{4\}$$

$$\overline{A \cup B} = \{7, 8\} \text{ and } \overline{A \cap B} = \{1, 2, 3, 5, 6, 7, 8\}$$

$$\bar{A} = \{5, 6, 7, 8\} \text{ and } \bar{B} = \{1, 2, 3, 7, 8\}$$

We conclude:

$$\overline{A \cup B} = \{7, 8\} = \bar{A} \cap \bar{B}$$

$$\overline{A \cap B} = \{1, 2, 3, 5, 6, 7, 8\} = \bar{A} \cup \bar{B}$$

4. Laws of Sets

4.1. Commutativity

Unions, intersections and symmetric differences are commutative.

$$A \cup B = B \cup A$$

$$A \cap B = B \cap A$$

$$A \oplus B = B \oplus A$$

Set difference is **not** commutative.

$$A - B \neq B - A$$

For example:

$$A = \{1, 2, 3\}$$

$$B = \{3, 4, 5\}$$

$$A - B = \{1, 2\}$$

$$B - A = \{4, 5\}$$

4.2. Associativity

Unions, intersections and symmetric differences are associative.

$$A \cup (B \cup C) = (A \cup B) \cup C$$

$$A \cap (B \cap C) = (A \cap B) \cap C$$

$$A \oplus (B \oplus C) = (A \oplus B) \oplus C$$

Set difference is **not** associative:

$$A - (B - C) \neq (A - B) - C$$

For example:

$$A = \{1, 2, 3\}$$

$$B = \{3, 4, 5\}$$

$$C = \{3\}$$

$$A - (B - C) = \{1, 2, 3\}$$

$$(A - B) - C = \{1, 2\}$$

4.3. Distributive

Unions and intersections are distributive.

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

5. Set Identities

5.1. Unions

$$A \cup B = B \cup A$$

$$(A \cup B) \cup C = A \cup (B \cup C)$$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$

$$A \cup \emptyset = A$$

$$A \cup U = U$$

$$A \cup \overline{A} = U$$

$$\overline{\overline{U}} = \emptyset$$

$$\overline{\overline{A}} = A$$

$$A \cup (A \cap B) = A$$

$$A - B = A \cap \overline{B}$$

5.2. Intersections

$$A \cap B = B \cap A$$

$$(A \cap B) \cap C = A \cap (B \cap C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$\overline{\overline{A \cap B}} = \overline{A} \cup \overline{B}$$

$$A \cap \emptyset = \emptyset$$

$$A \cap U = A$$

$$A \cap \overline{A} = \emptyset$$

$$\overline{\emptyset} = U$$

$$A \cap (A \cup B) = A$$

6. Partition Set

The two sets A and B are **disjoint** if and only if $A \cap B = \emptyset$.

For example:

$$\begin{aligned} A &= \{1, 2, 3\} \\ B &= \{4, 5, 6\} \\ A \cap B &= \emptyset \quad (\text{disjoint}) \end{aligned}$$

The set A consists of subsets A_1 , A_2 and A_3 such that those subsets are disjoint and the union of all those subsets is equal to A :

$$\begin{aligned} A &= \{1, 2, 3, 4, 5, 6, 7, 8\} \\ A_1 &= \{1, 2, 3\} \\ A_2 &= \{4, 5, 6\} \\ A_3 &= \{7, 8\} \\ A_1 \cap A_2 \cap A_3 &= \emptyset \\ A &= A_1 \cup A_2 \cup A_3 \end{aligned}$$

Hence, A_1 , A_2 and A_3 are **partitions** of set A .