

Cheatsheet - Logic Gates

Fabio Lama – fabio.lama@pm.me

1. Intro

(NOTE: Reading the *Postulates of Boolean Algebra* cheatsheet is recommended here)




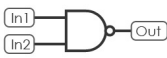




Logic gates are basic elements of circuits implementing Boolean operations. The most basic circuits are **OR** gates, **AND** gates and invertors (**NOT** gates). All boolean functions can be written in terms of these three logic operations.

- **AND** operation is represented as $f = x \cdot y$ or $f = xy$.
- **OR** operation is represented as $f = x + y$.
- **NOT** operation is represented as $f = \bar{x}$.

Other gates:

- **XOR** operation is *true* only when the value of the inputs differ.
- **NAND** operations is equivalent to "not AND".
- **NOR** operation is equivalent to "not OR".
- **XNOR** operation is equivalent to a "not XOR".

AND, OR, XOR and XNOR are **commutative** (e.g. $a + b = b + a$) and **associative** (e.g. $a + (b + c) = (a + b) + c$). NAND and NOR are commutative but not associative.

Logic Gates - Symbols and Truth Tables									
<div>BUF (Buffer)</div> <div></div>	In		Out		<div>NOT (Inverter)</div> <div></div>	In		Out	
	0		0			0		1	
	1		1			1		0	
<div>AND</div> <div></div>	In1	In2	Out		<div>NAND (NOT AND)</div> <div></div>	In1	In2	Out	
	0	0	0			0	0	1	
	0	1	0			0	1	1	
	1	0	0			1	0	1	
	1	1	1			1	1	0	
<div>OR</div> <div></div>	In1	In2	Out		<div>NOR (NOT OR)</div> <div></div>	In1	In2	Out	
	0	0	0			0	0	1	
	0	1	1			0	1	0	
	1	0	1			1	0	0	
	1	1	1			1	1	0	
<div>XOR (Exclusive Or)</div> <div></div>	In1	In2	Out		<div>XNOR (NOT XOR)</div> <div></div>	In1	In2	Out	
	0	0	0			0	0	1	
	0	1	1			1	0	0	
	1	0	1			1	0	0	
	1	1	0			0	1	1	

A circle behind a symbol indicates that the output signal is inverted.

Figure 1. Source: http://www.exclusivearchitecture.com/?page_id=2425

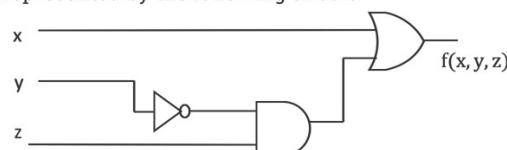
2. Circuits

We describe the combination of logic gates as a **circuit**.

- Let's consider the Boolean function f defined as:

$$f(x, y, z) = x + y'z$$

- f can be represented by the following circuit:



A circuit that's used for the **addition** of inputs is called an **adder**. A **half adder** takes two inputs and generates a **carry** and a **sum**. A **full adder** takes three inputs and generates a carry and a sum.

For example, an **half adder**:

$$\text{sum} = xy' + x'y = x \oplus y$$

$$\text{carry} = xy$$

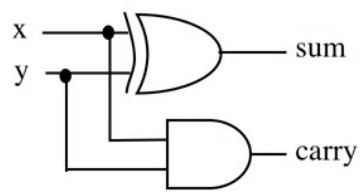


Figure 2. An half adder

And a **full adder**:

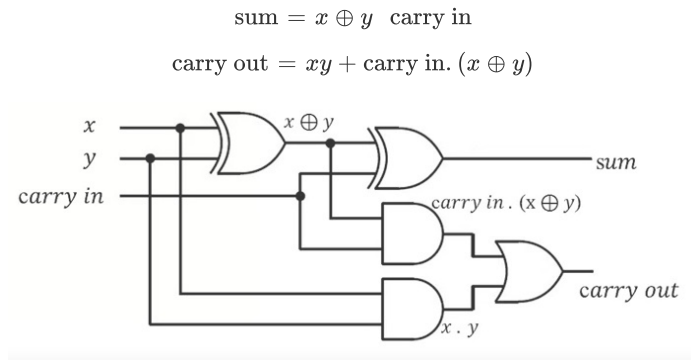


Figure 3. An half adder