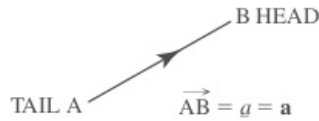


# Cheatsheet - Vectors

Fabio Lama – fabio.lama@pm.me

## 1. Intro

Quantities that can be described as single numbers (**magnitudes**) are known as **scalars**. Vectors are quantities that have a magnitude but also a **direction** and are represented as arrows, where the length represents the magnitude and the direction is given as points on a  $x - y$  plane:

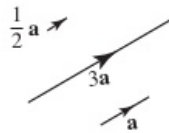


Here, the vector  $A$  to  $B$  is written as  $\vec{AB}$  (sometimes  $\underline{a}$  or  $\mathbf{a}$ ), where  $A$  is the *tail* and  $B$  is the *head*. The **magnitude** of  $\vec{AB}$  is written as  $|\vec{AB}|$  or  $\underline{a}$ .

Two vectors are **equal** if they have the same magnitude **and** the same direction.

A **unit vector** has magnitude of 1, written as  $\hat{\mathbf{a}}$

## 2. Multiplying Vector by Scalar

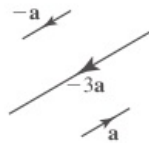


$$\mathbf{a} = 4$$

$$3\mathbf{a} = 12$$

$$\frac{1}{2}\mathbf{a} = 2$$

Multiplying by a negative number **reverses** the direction, but the magnitude stays **positive**.



$$\mathbf{a} = 4$$

$$-3\mathbf{a} = 12$$

The unit vector of  $\mathbf{a}$  is given by:

$$\hat{\mathbf{a}} = \frac{1}{|\mathbf{a}|}\mathbf{a}$$

For example:

$$\mathbf{a} = 4$$

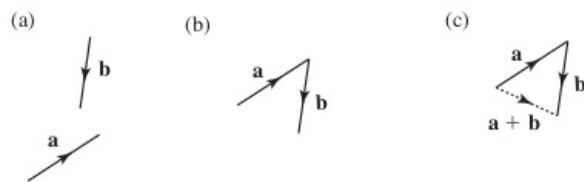
$$\hat{\mathbf{a}} = \frac{1}{4}\mathbf{a}$$

$$\mathbf{b} = \frac{1}{2}$$

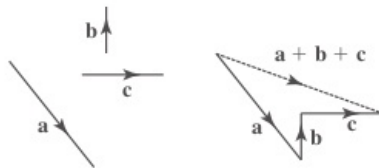
$$\hat{\mathbf{b}} = \frac{1}{\frac{1}{2}}\mathbf{b} = 2\mathbf{b}$$

## 3. Adding and Subtracting Vectors

If  $\mathbf{a}$  and  $\mathbf{b}$  are two vectors then we can form the vector  $\mathbf{a} + \mathbf{b}$ . We take  $\mathbf{b}$  by its tail and move it to the head of  $\mathbf{a}$  while still maintaining magnitude and direction.



Or with more vectors:

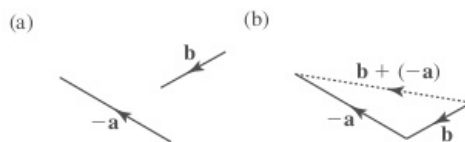


When subtracting vectors, we consider that:

$$\mathbf{a} - \mathbf{b} = \mathbf{a} + (-\mathbf{b})$$

Therefore, we reverse the **direction** of **b** and move the tail of  $-\mathbf{b}$  to the head of **a**.

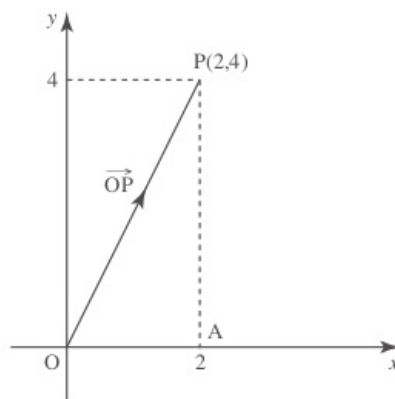
Additionally:



TODO: Add clear rules for head/tail usage.

## 4. Cartesian Components

We have a  $x - y$  plane, where unit vector of the  $x$  axis is  $\mathbf{i}$  and the unit vector of the  $y$  axis is  $\mathbf{j}$ .



We can see that:

$$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$$

$$\overrightarrow{OA} = 2\mathbf{i}$$

$$\overrightarrow{AP} = 4\mathbf{j}$$

$$\overrightarrow{OP} = 2\mathbf{i} + 4\mathbf{j}$$

The quantities 2 and 4 are **Cartesian components** of the vector  $\overrightarrow{OP}$ .

By using Pythagoras' theorem we can calculate the magnitude of  $\overrightarrow{OP}$ :

$$|\overrightarrow{OP}| = \sqrt{x^2 + y^2}$$

## 5. Scalar Product

We have already seen how to multiply a vector by a scalar (single number). To multiply a vector by a vector:

$$\mathbf{a} \times \mathbf{b} = |\mathbf{a}| \times |\mathbf{b}| \times \cos \theta$$

where  $\theta$  is the angle between  $\mathbf{a}$  and  $\mathbf{b}$ . This scalar product is also know as the **dot product**.

Last updated 2022-08-11 20:20:26 UTC