Cheatsheet - Graphs of Functions and Kinematics

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1. Intro

We introduce horizontal and vertical axes. These axes intersect at a point O called the origin. The horizontal axis is used to represent the **independent** variable, commonly x, and the vertical axis is used to represent the **dependent** variable, commonly y. The region shown is then referred to as the $x-y$ plane.

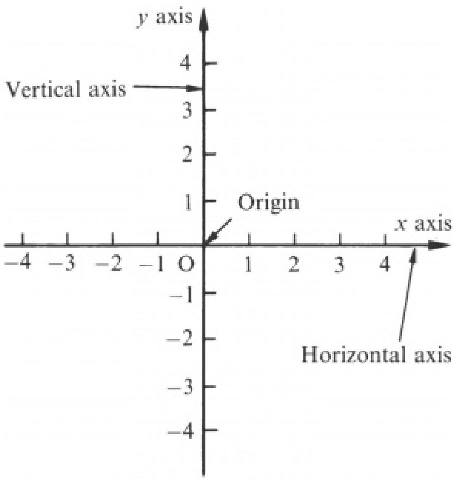


Figure 17.1
The $x-y$ plane

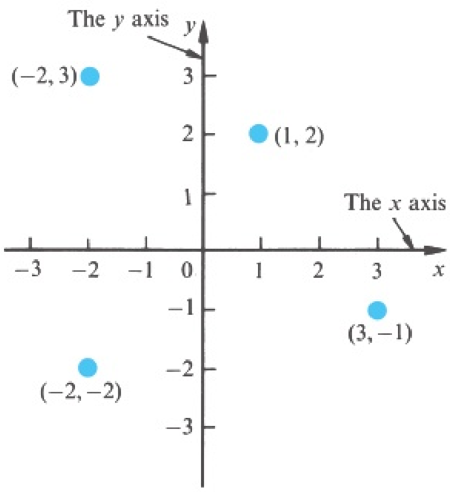


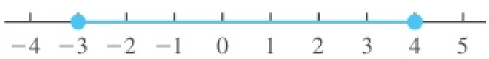
Figure 17.2
Several points in the $x-y$ plane

2. Intervals

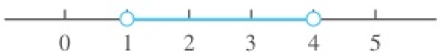
We only need part of the x/y axis when plotting graphs. Each coordinate on the axis is called an **interval**. There are three types of intervals:

- *Closed interval*: includes its end-points, e.g. $\{x : x \in \mathbb{R}, 1 \leq x \leq 3\}$, denoted as $[1, 3]$ (square brackets).
- *Open interval*: does not include its end-points, e.g. $\{x : x \in \mathbb{R}, 1 < x < 3\}$, denoted as $(1, 3)$ (round brackets).
- *Semi-open and semi-closed interval*: may be open at one end and closed at the other, e.g. $\{x : x \in \mathbb{R}, 1 < x \leq 3\}$, denoted as $(1, 3]$ (round and square bracket).

Closed intervals use the filled marking, $[-3, 4]$:



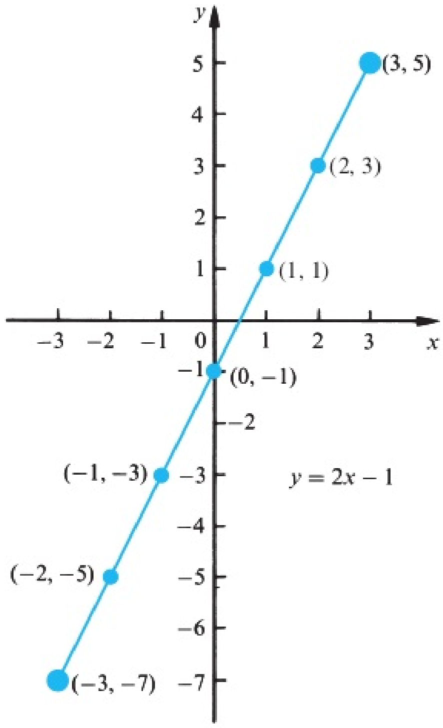
Open intervals used the hollow marking, $(1, 4)$:



3. Plotting Graphs of a Function

Plotting a graph of $y = 2x - 1$ for $-3 \leq x \leq 3$:

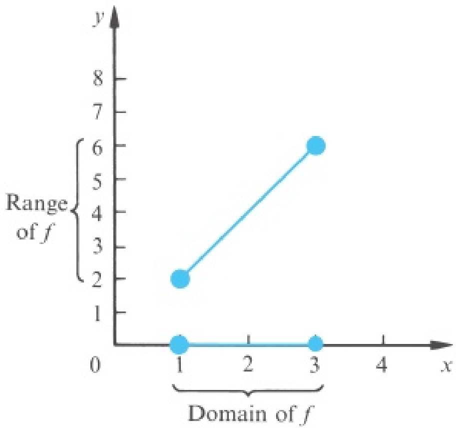
x	-3	-2	-1	0	1	2	3
y	-7	-5	-3	-1	1	3	5



4. Domain and Range of Function

The set of values that we allow the independent variable to take is called the **domain** of the function. If the domain is not specified, we take the largest set possible. The set of values taken by the output is called the **range** of function. The domain and range can extend indefinitely (infinite) in one or both directions.

For example, the function f is given by $y = f(x) = 2x$, for $1 \le x \le 3$.



5. Kinematics

Describes the motion of objects without reference to forces, hence **acceleration will always be constant**.

Basic definitions:

$$d = \text{displacement}$$

$$v = \text{velocity}$$

$$a = \text{acceleration}$$

$$t = \text{time}$$

Velocity and **displacement** can have subscripts that indicate initial conditions:

$$v_o = v_i = \text{initial velocity}$$

$$v_t = v_f = \text{final velocity}$$

Fundamental equations:

- $v_t = v_i + at$
  - The velocity of any object at time t is equal to the *initial* velocity plus the acceleration times t

- $x_t = x_o \times v_i t + \frac{1}{2}at^2$
  - The position of the object at time t is qual to the *initial* position x_o plus its *initial* velocity v_i multiplied by time t plus $\frac{1}{2}at^2$.
- $v_f^2 = v_i^2 + 2ad$
  - Velocity squared at time t is equal to the *initial* velocity squared plus $2ad$.

Additionally:

$$d = v_i t + \frac{1}{2}at^2$$

$$v_f^2 = v_i^2 + 2ad$$

$$v_f = v_i + at$$

$$d = \left(\frac{v_i + v_f}{2}\right)t$$