Cheatsheet - Automata Theory

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1. Basics of (finite) Automata

An **alphabet**, Σ, is a non-empty set of symbols.

$$\Sigma = \{0, 1\} \quad \text{binary alphabet}$$
$$\Sigma = \{a, b, \ldots, z\} \quad \text{collection of lowercase letters}$$

A **string** or word is a finite sequence of letters drawn from an alphabet. **Empty strings**, ε, are strings with zero occurrences of letters. Empty strings can be from any alphabet.

The **length** of a string x is denoted as $|x|$:

$$x = \text{'hello'}$$
$$|x| = 5$$

Other string related notations:

- The set of **all strings** composed from the letters in Σ is denoted by Σ^*.
- The set of **all non-empty strings** composed from letters in Σ is denoted by Σ^+
- The set of **all strings of length** k composed from letters in Σ is denoted by Σ^k.

$$\Sigma = \{0, 1\}$$
$$\Sigma^* = \{\varepsilon, 0, 1, 00, 01, 10, 11, \ldots\}$$
$$\Sigma^+ = \{0, 1, 00, 01, 10, 11, \ldots\}$$
$$\Sigma^2 = \{00, 01, 10, 11\}$$

Note that the size of Σ^k is denoted as $|\Sigma|^k$.

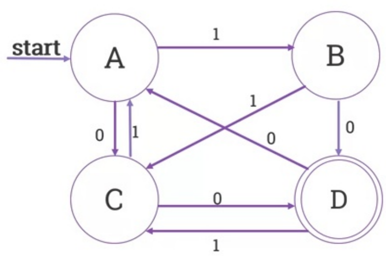
2. Automaton

A **finite automaton** is a simple mathematical machine; it is a representation of how computations are performed with *limited memory* space. It is a model of computation, which consists of a set of states that are connected by transitions. It has an input and it has an output.

An automaton M is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$ where:

- Q is a finite set called the **states**.
- Σ is a finite set called **the alphabet**.
- $\delta : Q \times E \to Q$ is the **transition function**.
- $q_0 \in Q$ is the **start state**.
- $F \subseteq Q$ is the set of **accepted states**.

Example



- $Q = \{A, B, C, D\}$
- $\Sigma = \{0, 1\}$
- $q_0 = A$
- $F = \{D\}$
- $\delta = ?$

δ	0	1
A	C	B
B	D	C
C	D	A
*D	A	C

Figure 1. Note: Bottom right is the transition table. D is the only accepted state.

For example, based on the automaton in the picture above:

- Input 0010 results in state D; the input is **accepted** ($D \in F$).
- Input 0011 results in state A; the input is **rejected** ($A \notin F$).

2.1. Language of Automaton

The set of all strings accepted by an automaton is called the **language** of that automaton. If M is an automaton on alphabet Σ, then $\mathscr{L}(M)$ is the language of M:

$$\mathscr{L}(M) = \{x \in \Sigma^* \mid M \quad \text{accepts} \quad x\}$$

3. Determinism

The difference between Deterministic Finite Automata (DFA) and Nondeterministic Finite Automata (NFA) is that DFA has exactly one transition and there is a unique starting state, which is not the case in NFA (hence, **nondeterministic**). Generally, in NFA there are many choices at one particular point and an input is accepted if at least one sequence of choices leads to an accepting state.

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