

Cheatsheet - Gradients of Curves & Differentiation

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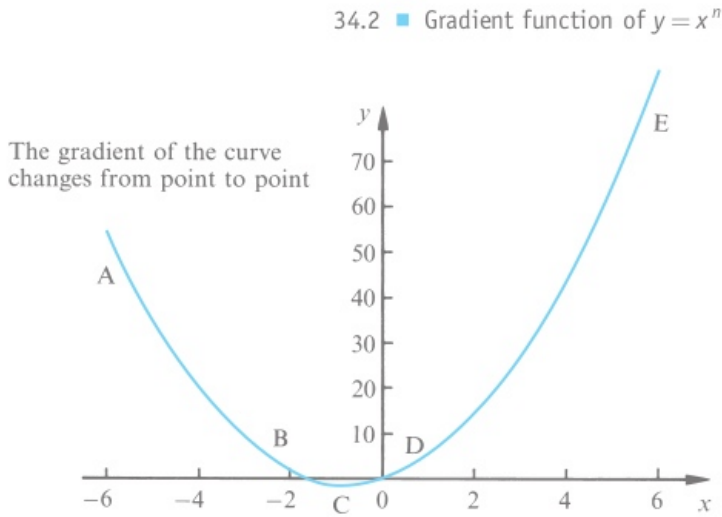
1. Gradient Function

The **gradient** (or "slope") of a graph tells us something about the **rate of change** and "steepness" of a function. Given a function $y = f(x)$ we denote its gradient function by "dee y by dee x" or simply by "y dash".

$$\frac{dy}{dx} = y'$$

IMPORTANT

This is **not** defined as "dy divided by dx", respectively dy and dx don't have any meaning here. Rather, we take $\frac{dy}{dx}$ as a symbol of its own, such as y' .



The gradient function is also called **first derivative**. The process of obtaining this is also known as **differentiation**. Saying to differentiate $y = x^5$ means to find its gradient function y' . *Differential calculus* studies this more in depth.

2. Gradient function of $y = x^n$

For any function of the form $y = x^n$ the gradient function is found from the following formula:

$$y = x^n \text{ then } y' = nx^{n-1}$$

For example:

$$\text{if } y = x^3 \text{ then } y' = 3x^{3-1} = 3x^2$$

Respectively:

$$y' = f'(x^3) = 3x^2$$

When we substitute x and the result is negative, the curve is falling. If the result is positive, the curve is rising. We write $y'(x = 2)$ or simply $y'(2)$ to denote the value of the gradient function when $x = 2$.

The gradient function of some common functions:

| For: $y = f(x)$ | For: $y' = f'(x)$ | Notes |
|-----------------|-------------------|-------------------|
| constant | 0 | |
| x | 1 | |
| x^2 | $2x$ | |
| x^n | nx^{n-1} | |
| e^x | e^x | |
| e^{kx} | ke^{kx} | k is a constant |
| $\sin x$ | $\cos x$ | |

| | | |
|-----------|--------------|-------------------|
| $\cos x$ | $-\sin x$ | |
| $\sin kx$ | $k \cos kx$ | k is a constant |
| $\cos kx$ | $-k \sin kx$ | k is a constant |
| $\ln kx$ | $1/x$ | k is a constant |

3. Rules for Finding Gradient Functions

Rules for finding gradients of non-single terms.

3.1. Rule 1

To find the gradient function of a sum of two functions we can simply find the two gradient functions separately and those together.

$$y = f(x) + g(x) \quad \text{then} \quad y' = f'(x) + g'(x)$$

For example:

$$\begin{aligned} y &= x^2 + x^4 \\ f'(x^2) &= 2x \\ f'(x^4) &= 4x^3 \\ \text{hence } y' &= f'(x^2 + x^4) = 2x + 4x^3 \end{aligned}$$

3.2. Rule 2

Extension of the first rule.

$$y = f(x) - g(x) \quad \text{then} \quad y' = f'(x) - g'(x)$$

For example:

$$\begin{aligned} y &= x^5 - x^7 \\ f'(x^5) &= 5x^4 \\ f'(x^7) &= 7x^6 \\ \text{hence } y' &= f'(x^5 - x^7) = 5x^4 - 7x^6 \end{aligned}$$

3.3. Rule 3

$$y = kf(x) \quad \text{then} \quad y' = kf'(x)$$

where k is a number.

For example:

$$\begin{aligned} y &= 3x^2 = 3(x^2) \\ x^2 &= 2x \\ \text{hence } y' &= f'(3x^2) = 3(2x) = 6x \end{aligned}$$

4. Higher Derivatives

To find the derivative of the derivative itself, known as the **second derivative** and denoted as y'' , we define:

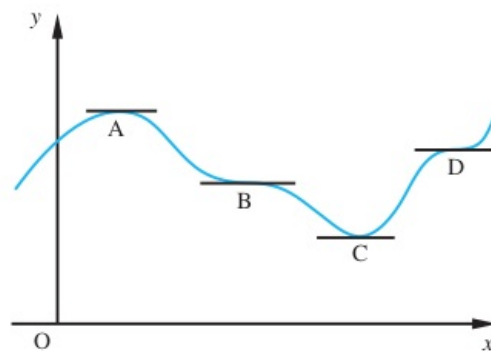
$$y'' = \frac{d^2y}{dx^2}$$

y'' is found by differentiating y' .

For example:

$$\text{if } y' = 4x^3 \quad \text{then} \quad y'' = 4(3x^2) = 12x^2$$

5. Maximum and Minimum Points



Points where the gradient is zero are known as **stationary points**, such as points A , B , C and D (seen in the graph above). A point like A is the **maximum turning point** (or just **maximum**). A point like C is the **minimum turning point** (or just **minimum**). Points like B and D are known as **points of inflexion**, where the curve falls and rises (unlike A where the curve only falls and C where the curve only rises).

IMPORTANT

Stationary points are located by setting the gradient function equal to zero, that is $y' = 0$.

For example, to find the stationary points of:

$$y = 3x^2 - 6x + 8$$

we determine the gradient function y' by differentiating y :

$$y' = nx^{n-1}$$

$$y' = 3x^{3-1} = 3x^2$$

$$y' = 9x$$