

# Cheatsheet - Gradients of Curves & Differentiation

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## 1. Gradient Function

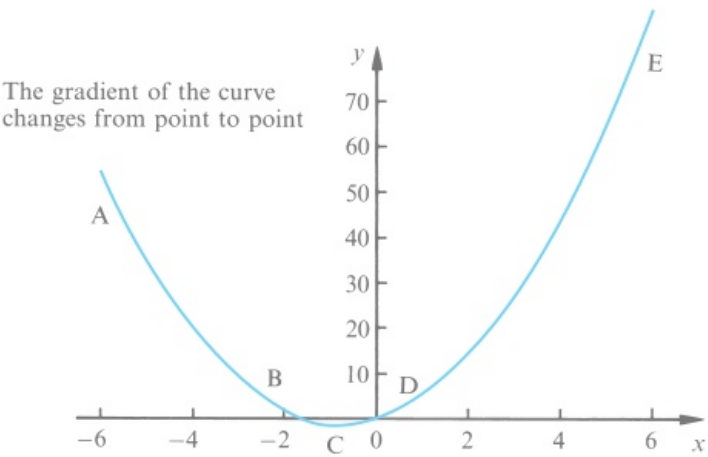
The **gradient** (or "slope") of a graph tells us something about the **rate of change** and "steepness" of a function. Given a function  $y = f(x)$  we denote its gradient function by "dee y by dee x" or simply by "y dash".

$$\frac{dy}{dx} = y'$$

IMPORTANT

This is **not** defined as "dy divided by dx", respectively  $dy$  and  $dx$  don't have any meaning here. Rather, we take  $\frac{dy}{dx}$  as a symbol of its own, such as  $y'$ .

34.2 ■ Gradient function of  $y = x^n$



The gradient function is also called **first derivative**. The process of obtaining this is also known as **differentiation**. Saying to differentiate  $y = x^5$  means to find its gradient function  $y'$ . *Differential calculus* studies this more in depth.

## 2. Gradient function of $y = x^n$

For any function of the form  $y = x^n$  the gradient function is found from the following formula:

$$y = x^n \text{ then } y' = nx^{n-1}$$

For example:

$$\text{if } y = x^3 \text{ then } y' = 3x^{3-1} = 3x^2$$

Respectively:

$$y' = f'(x^3) = 3x^2$$

When we substitute  $x$  and the result is negative, the curve is falling. If the result is positive, the curve is rising. We write  $y'(x = 2)$  or simply  $y'(2)$  to denote the value of the gradient function when  $x = 2$ .

The gradient function of some common functions:

| For: $y = f(x)$ | For: $y' = f'(x)$ | Notes             |
|-----------------|-------------------|-------------------|
| constant        | 0                 |                   |
| $x$             | 1                 |                   |
| $x^2$           | $2x$              |                   |
| $x^n$           | $nx^{n-1}$        |                   |
| $e^x$           | $e^x$             |                   |
| $e^{kx}$        | $ke^{kx}$         | $k$ is a constant |
| $\sin x$        | $\cos x$          |                   |

|           |              |                   |
|-----------|--------------|-------------------|
| $\cos x$  | $-\sin x$    |                   |
| $\sin kx$ | $k \cos kx$  | $k$ is a constant |
| $\cos kx$ | $-k \sin kx$ | $k$ is a constant |
| $\ln kx$  | $1/x$        | $k$ is a constant |

### 3. Rules for Finding Gradient Functions

Rules for finding gradients of non-single terms.

#### 3.1. Rule 1

To find the gradient function of a sum of two functions we can simply find the two gradient functions separately and those together.

$$y = f(x) + g(x) \quad \text{then} \quad y' = f'(x) + g'(x)$$

For example:

$$\begin{aligned} y &= x^2 + x^4 \\ f'(x^2) &= 2x \\ f'(x^4) &= 4x^3 \\ \text{hence } y' &= f'(x^2 + x^4) = 2x + 4x^3 \end{aligned}$$

#### 3.2. Rule 2

Extension of the first rule.

$$y = f(x) - g(x) \quad \text{then} \quad y' = f'(x) - g'(x)$$

For example:

$$\begin{aligned} y &= x^5 - x^7 \\ f'(x^5) &= 5x^4 \\ f'(x^7) &= 7x^6 \\ \text{hence } y' &= f'(x^5 - x^7) = 5x^4 - 7x^6 \end{aligned}$$

#### 3.3. Rule 3

$$y = kf(x) \quad \text{then} \quad y' = kf'(x)$$

where  $k$  is a number.

For example:

$$\begin{aligned} y &= 3x^2 = 3(x^2) \\ x^2 &= 2x \\ \text{hence } y' &= f'(3x^2) = 3(2x) = 6x \end{aligned}$$

### 4. Higher Derivatives

To find the derivative of the derivative itself, known as the **second derivative** and denoted as  $y''$ , we define:

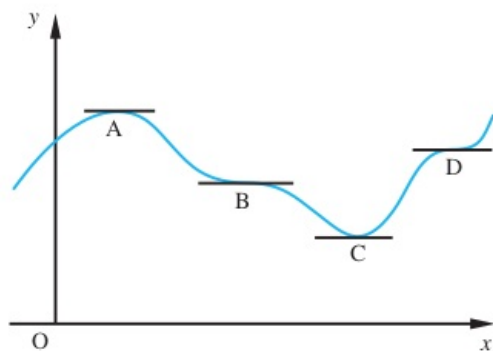
$$y'' = \frac{d^2y}{dx^2}$$

$y''$  is found by differentiating  $y'$ .

For example:

$$\text{if } y' = 4x^3 \quad \text{then} \quad y'' = 4(3x^2) = 12x^2$$

### 5. Maximum and Minimum Points



Points where the gradient is zero are known as **stationary points**, such as points  $A$ ,  $B$ ,  $C$  and  $D$  (seen in the graph above). A point like  $A$  is the **maximum turning point** (or just **maximum**). A point like  $C$  is the **minimum turning point** (or just **minimum**). Points like  $B$  and  $D$  are known as **points of inflexion**, where the curve falls and rises (unlike  $A$  where the curve only falls and  $C$  where the curve only rises).

#### IMPORTANT

Stationary points are located by setting the gradient function equal to zero, that is  $y' = 0$ .

For example, to find the stationary points of:

$$y = 3x^2 - 6x + 8$$

we determine the gradient function  $y'$  by differentiating  $y$ :

$$y' = nx^{n-1}$$

$$y' = 3x^{3-1} = 3x^2$$

$$y' = 9x$$