Cheatsheet - Matrices

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1. Intro

A **matrix** is a set of numbers ("elements") arranged in the form of a rectangle and enclosed in curved brackets. The matrix A is of size **"two by three"**:

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 4 & 6 \end{pmatrix}$$

A **square** matrix has the same number of rows as colummns:

$$\begin{pmatrix} 2 & -7 \\ -1 & 6 \end{pmatrix}$$

A **diagnoal** matrix is a square matrix where all elements are *0* except those on the diagonal from the top left to the bottom right.

$$\begin{pmatrix} 7 & 0 \\ 0 & 9 \end{pmatrix}, \begin{pmatrix} 9 & 0 \\ 0 & 0 \end{pmatrix} \text{ and } \begin{pmatrix} -1 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & -17 \end{pmatrix}$$

An **identity** matrix is a diagonal matrix where all the diagonal elements are equal to *1*.

2. Addition and Subtraction

Two matrices that have the same size can be added/subtracted by simply adding/subtracting the corresponding elements.

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} 5 & 2 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1+5 & 2+2 \\ 3+1 & 4+0 \end{pmatrix} = \begin{pmatrix} 6 & 4 \\ 4 & 4 \end{pmatrix}$$

Respectively:

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} 5 & 2 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1-5 & 2-2 \\ 3-1 & 4-0 \end{pmatrix} = \begin{pmatrix} -4 & 0 \\ 2 & 4 \end{pmatrix}$$

3. Multiplication and Division by Number

A matrix is multiplied by a number by multiplying/dividing each element by that number.

$$4\begin{pmatrix} 1 & 2 \\ 3 & -9 \end{pmatrix} = \begin{pmatrix} 4 \times 1 & 4 \times 2 \\ 4 \times 3 & 4 \times -9 \end{pmatrix} = \begin{pmatrix} 4 & 8 \\ 12 & -36 \end{pmatrix}$$

Respectively:

$$\frac{1}{4}\begin{pmatrix} 16 \\ 8 \end{pmatrix} = \begin{pmatrix} \frac{1}{4} \times 16 \\ \frac{1}{4} \times 8 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

4. Multiplying two Matrices together

Two matrices can only be multiplied together if the number of colums in the first matrix is equal to the number of rows in the second matrix. The product of two such matrices is a matrix that has the same number of rows as the first matrix and the same number of columns as the second. If matrix A has size $p \times q$ and B has size $q \times s$, then AB has size $p \times s$.

$$\begin{pmatrix} 1 & 4 \\ 6 & 3 \end{pmatrix} \times \begin{pmatrix} 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 1 \times 2 + 4 \times 5 \\ 6 \times 2 + 3 \times 5 \end{pmatrix} = \begin{pmatrix} 22 \\ 27 \end{pmatrix}$$

And

$$\begin{pmatrix} 1 & 4 & 9 \\ 2 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 9 \\ 8 & 7 \\ -7 & 3 \end{pmatrix} = \begin{pmatrix} 1 \times 1 + 4 \times 8 + 9 \times -7 & 1 \times 9 + 4 \times 7 + 9 \times 3 \\ 2 \times 1 + 0 \times 8 + 1 \times -7 & 2 \times 9 + 0 \times 7 + 1 \times 3 \end{pmatrix} = \begin{pmatrix} -30 & 64 \\ -5 & 21 \end{pmatrix}$$

5. The Inverse of a 2×2 Matrix

The matrix A^{-1} is the **inverse** of matrix A (note that the -1 superscript should not be read as a power).

$$\text{If } A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \text{ then } A^{-1} = \frac{1}{ad - bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

Note that the elements on the leading diagonal are interchanged.

Additinally:

$$A \times A^{-1} = A^{-1} \times A = I$$

Where I is the identity matrix.

5.1. Determinant

If quantity $ad - bc$ in the formula for the inverse is known as the **determinant** of the matrix, indicated by $|A|$.

$$\text{If} \quad A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad \text{then its determinant is} \quad |A| = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$$

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