Cheatsheet - Exponential & Logarithm Functions

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1. Exponential Expressions

An expression of the form a^x is called an **exponential expression**, where a is the **base** and x is the **exponent,power** or **index**.

NOTE | The most common exponential expression is e^x, where e is the constant *2.71828....* Also known as "Euler's number". This number is found in many natural phenomena.

2. Exponential Functions

An exponential function has the form of $y = e^x$. This function has important properties:

- y is never negative.
- When $x = 0$, then $y = 1$
- As x increases, then y increases (**exponential growth**).

x	-3	-2	-1	0	1	2	3
e^x	0.0498	0.1353	0.3679	1	2.7183	7.3891	20.086
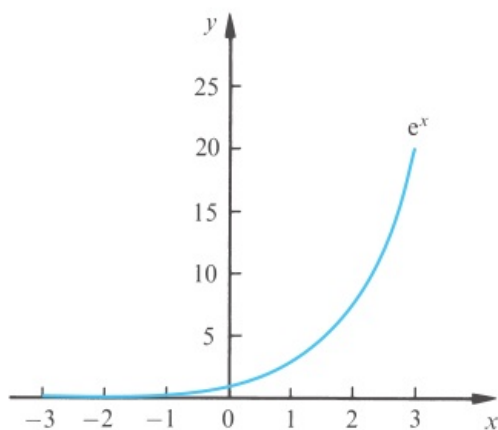
3. Logarithms

The logarithm is the inverse function to the exponentiation. The following two equations are equivalent.

$$y = a^x \Leftrightarrow \log_a(y) = x$$

For example:

$$5^3 = 125 \Leftrightarrow \log_5(125) = 3$$

Also, conventionally we define **natural logarithms** as:

$$\log = \log_{10}$$

$$\ln = \log_e$$

3.1. Calculating Logarithms to any Base

$$\log_a X = \frac{\log_{10} X}{\log_{10} a}$$

$$\log_a X = \frac{\ln X}{\ln a}$$

This also means that - for example:

$$\log_5(125) = \frac{\log(125)}{\log(5)}$$

Additionally:

$$\log_a a = 1$$

3.2. Laws

First Law:

$$\log A + \log B = \log AB$$

Second Law:

$$\log A - \log B = \log\left(\frac{A}{B}\right)$$

Also note that:

$$\log 1 = 0$$

Third Law

$$n \log A = \log A^n$$

such as $3\log 2 = \log 2^3 = \log 8$. This applies for all $n \in \mathbb{R}$.