Cheatsheet - Vectors

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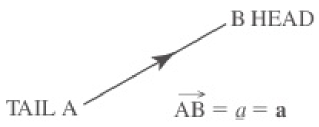
1. Intro

Quantities that can be described as single numbers (**magnitudes**) are known as **scalars**. Vectors are quantities that have a magnitude but also a **direction** and are represented as arrows, where the length represents the magnitude and the direction is given as points on a $x - y$ plane:

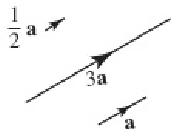


Here, the vector A to B is written as $\overrightarrow{\mathrm{AB}}$ (sometimes $\underset{\sim}{a}$ or $\mathbf{a}$), where A is the *tail* and B is the *head*. The **magnitude** of $\overrightarrow{\mathrm{AB}}$ is written as $\overrightarrow{|AB|}$ or $\underline{\mathbf{a}}$.

Two vectors are **equal** if they have the same magnitude **and** the same direction.

A **unit vector** has magnitude of 1, written as $\widehat{\mathbf{a}}$

2. Multiplying Vector by Scalar

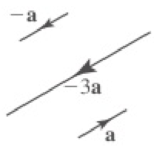


$$\mathbf{a} = 4$$

$$3\mathbf{a} = 12$$

$$\frac{1}{2}\mathbf{a} = 2$$

Multiplying by a negative number **reverses** the direction, but the magnitude stays **positive**.



$$\mathbf{a} = 4$$

$$-3\mathbf{a} = 12$$

The unit vector of $\mathbf{a}$ is given by:

$$\widehat{\mathbf{a}} = \frac{1}{|\mathbf{a}|}\mathbf{a}$$

For example:

$$\mathbf{a} = 4$$

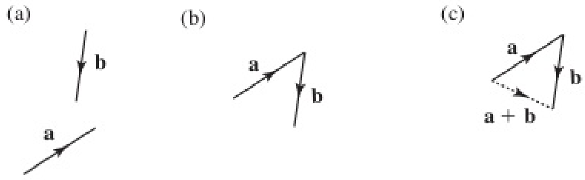
$$\widehat{\mathbf{a}} = \frac{1}{4}\mathbf{a}$$

$$\mathbf{b} = \frac{1}{2}$$

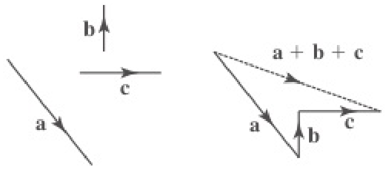
$$\widehat{\mathbf{b}} = \frac{1}{\frac{1}{2}}\mathbf{b} = 2\mathbf{b}$$

3. Adding and Subtracting Vectors

If $\mathbf{a}$ and $\mathbf{b}$ are two vectors then we can form the vector $\mathbf{a} + \mathbf{b}$. We take $\mathbf{b}$ by its tail and move it to the head of $\mathbf{a}$ while still maintaining magnitude and direction.



Or with more vectors:

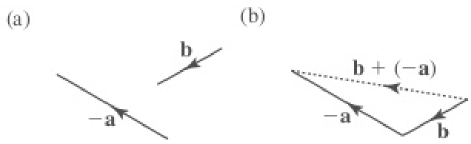


When subtracting vectors, we consider that:

$$\mathbf{a} - \mathbf{b} = \mathbf{a} + (-\mathbf{b})$$

Therefore, we reverse the **direction** of $\mathbf{b}$ and move the tail of $-\mathbf{b}$ to the head of $\mathbf{a}$.

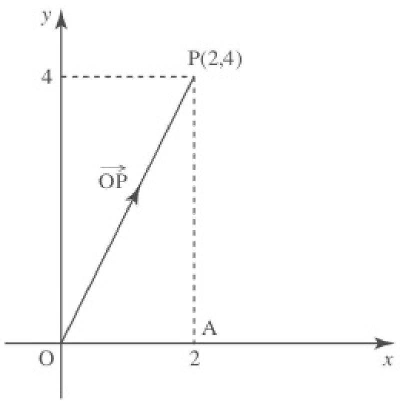
Additionally:



TODO: Add clear rules for head/tail usage.

4. Cartesian Components

We have a $x - y$ plane, where unit vector of the *x* axis is $\mathbf{i}$ and the unit vector of the *y* axis is $\mathbf{j}$.



We can see that:

$$\overrightarrow{\mathrm{OP}} = \overrightarrow{\mathrm{OA}} + \overrightarrow{\mathrm{AP}}$$

$$\overrightarrow{\mathrm{OA}} = 2\mathbf{i}$$

$$\overrightarrow{\mathrm{AP}} = 4\mathbf{j}$$

$$\overrightarrow{\mathrm{OP}} = 2\mathbf{i} + 4\mathbf{j}$$

The quantities 2 and 4 are **Cartesian components** of the vector $\overrightarrow{|OP|}$.

By using Pythagoras' theorem we can calculate the magnitude of $\overrightarrow{\mathrm{OP}}$:

$$\overrightarrow{|OP|} = \sqrt{x^2 + y^2}$$

5. Scalar Product

We have already seen how to multiply a vector by a scalar (single number). To multiply a vector by a vector:

$$\mathbf{a} \times \mathbf{b} = |\mathbf{a}| \times |\mathbf{b}| \times \cos\theta$$

where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$. This scalar product is also know as the **dot product**.

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