Cheatsheet - Universal Set, Complement and Laws

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1. Intro

A universal set is a set that contains everything. We note the universal set with the letter U.

$$A \subseteq U$$

2. Complement of a Set

The complement of set A contains all elements of U but not A.

$$\overline{A} = U - A$$

and therefore:

$$\overline{A} \cup A = U$$

For example:

$$\begin{aligned} U &= \{1, 2, 3, 4\} \\ A &= \{1, 2\} \\ \overline{A} &= \{3, 4\} \\ A \cup \overline{A} &= \{1, 2, 3, 4\} = U \end{aligned}$$

3. De Morgan's Law

The complement of the **union** of two sets A and B is equal to the **intersection** of their complements.

$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$

The complement of the **intersection** of two sets A and B is equal to the **union** of their complements.

$$\overline{A \cap B} = \overline{A} \cup \overline{B}$$

For example:

$$\begin{aligned} U &= \{1, 2, 3, 4, 5, 6, 7, 8\} \\ A &= \{1, 2, 3, 4\} \quad \text{and} \quad B = \{4, 5, 6\} \\ A \cup B &= \{1, 2, 3, 4, 5, 6\} \quad \text{and} \quad A \cap B = \{4\} \\ \overline{A \cup B} &= \{7, 8\} \quad \text{and} \quad \overline{A \cap B} = \{1, 2, 3, 5, 6, 7, 8\} \\ \overline{A} &= \{5, 6, 7, 8\} \quad \text{and} \quad \overline{B} = \{1, 2, 3, 7, 8\} \end{aligned}$$

We conclude:

$$\begin{aligned} \overline{A \cup B} &= \{7, 8\} = \overline{A} \cap \overline{B} \\ \overline{A \cap B} &= \{1, 2, 3, 5, 6, 7, 8\} = \overline{A} \cup \overline{B} \end{aligned}$$

4. Laws of Sets

4.1. Commutativity

Unions, intersections and symmetric differences are commutative.

$$\begin{aligned} A \cup B &= B \cup A \\ A \cap B &= B \cap A \\ A \oplus B &= B \oplus A \end{aligned}$$

Set difference is **not** commutative.

$$A - B \neq B - A$$

For example:

$$\begin{aligned} A &= \{1, 2, 3\} \\ B &= \{3, 4, 5\} \end{aligned}$$

$$A - B = \{1, 2\}$$
$$B - A = \{4, 5\}$$

4.2. Associativity

Unions, intersections and symmetric differences are associative.

$$A \cup (B \cup C) = (A \cup B) \cup C$$
$$A \cap (B \cap C) = (A \cap B) \cap C$$
$$A \oplus (B \oplus C) = (A \oplus B) \oplus C$$

Set difference is **not** associative:

$$A - (B - C) \neq (A - B - C)$$

For example:

$$A = \{1, 2, 3\}$$
$$B = \{3, 4, 5\}$$
$$C = \{3\}$$
$$A - (B - C) = \{1, 2, 3\}$$
$$(A - B) - C = \{1, 2\}$$

4.3. Distributive

Unions and intersections are distributive.

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

5. Set Identities

5.1. Unions

$$A \cup B = B \cup A$$
$$(A \cup B) \cup C = A \cup (B \cup C)$$
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$
$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$
$$A \cup \phi = A$$
$$A \cup U = U$$
$$A \cup \overline{A} = U$$
$$\overline{U} = \phi$$
$$\overline{\overline{A}} = A$$
$$A \cup (A \cap B) = A$$
$$A - B = A \cap \overline{B}$$

5.2. Intersections

$$A \cap B = B \cap A$$
$$(A \cap B) \cap C = A \cap (B \cap C)$$
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$
$$\overline{A \cap B} = \overline{A} \cup \overline{B}$$
$$A \cap \phi = \phi$$
$$A \cap U = A$$
$$A \cap \overline{A} = \phi$$
$$\bar{\phi} = U$$
$$A \cap (A \cup B) = A$$

6. Partition Set

The two sets A and B are **disjoint** if and only if $A \cap B = \phi$.

For example:

$$A = \{1, 2, 3\}$$
$$B = \{4, 5, 6\}$$
$$A \cap B = \phi \quad \text{(disjoint)}$$

The set A consists of subsets A_1, A_2 and A_3 such that those subsets are disjoint and the union of all those subsets is equal to A:

$$A = \{1, 2, 3, 4, 5, 6, 7, 8\}$$
$$A_1 = \{1, 2, 3\}$$
$$A_2 = \{4, 5, 6\}$$
$$A_3 = \{7, 8\}$$
$$A_1 \cap A_2 \cap A_3 = \phi$$
$$A = A_1 \cup A_2 \cup A_3$$

Hence, A_1, A_2 and A_3 are **partitions** of set A.

Last updated 2022-10-29 15:28:36 UTC