Cheatsheet - Set Theory

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1. Definition

A set refers to an unordered collection of any kind of (abstract) unique objects, so called "elements", such as numbers, figures, things, etc.

1.1. Basic Notation

We have the set ${\cal E}$ containing four elements:

$$E = \{1, 2, 3, 4\}$$

or set V:

$$V = \{a, b, c, d, e, f\}$$

We can see that the element 2 is in E, respectively:

$$2 \in E$$

but not in V

$$2 \notin V$$

2. Cardinality

Given the set S, the cardinality of S is the number of elements contained in S. We write the cardinality of S as |S|.

$$S = \{a, b, c\}$$
$$|S| = 3$$

and the following empty set:

$$S=\phi=\{\}$$

$$|S| = 0$$

3. Subset and Superset

We have set E and $\{1,2\}$ is a **subset** (\subseteq) of E. We can also say that E is a **superset** (\supseteq) of $\{1,2\}$. The set $\{a,b\}$ is **not** a subset (\nsubseteq) of E, however.

$$E = \{1, 2, 3, 4\}$$
 $E \subseteq E$
 $E \supseteq E$
 $\{1, 2\} \subseteq E$
 $E \supseteq \{1, 2\}$
 $\{a, b\} \nsubseteq E$
 $E \not\supseteq \{a, b\}$
 $E \not\supseteq \{a, b\}$

Importantly, we distinguish between a subset (\subseteq) and a **proper subset** (\subseteq):

$$\{1,2,3\} \subset E$$

 $\{1,2,3,4\} \not\subset E$
 $\{1,2,3,4\} \subseteq E$

Do note that any set also contains an empty subset:

$$(\{\} = \phi) \subseteq E$$

4. Special Sets

Commonly defined and used sets in mathematics:

$$\mathbb{N} = \{1,2,3,4,...\} \quad \text{natural numbers}$$

$$\mathbb{W} = \{0,1,2,3,...\} \quad \text{whole numbers}$$

$$\mathbb{Z} = \{...,\, -1,0,1,2,...\} \quad \text{integers (no fractions)}$$

$$\mathbb{Q} = \left\{...,\, -1,\, -\left(\frac{1}{2}\right),0,\frac{2}{3},1,...\right\} \quad \text{rational numbers}$$

$$\mathbb{R} = \quad \text{real numbers (non complex-numbers)}$$

Note that infinite numbers are $irrational\ numbers$ and are not part of $\mathbb Q$, such as $\pi \notin \mathbb Q$.

Additionally:

 $\mathbb{N}\subset\mathbb{Z}\subset\mathbb{Q}\subset\mathbb{R}$

5. Set Builder

We define:

$$E = \{1, 2, 3, 4\}$$

and use the set builder to create a new set, for example:

$$V = \{2n \mid n \in E\}$$
 $V = \{2, 4, 6, 8\}$

Read as "each element set V is two times n for each n in set E".

We now define the set J:

$$J = \{2n \mid n \in E \text{ and } n < 3\}$$

$$J = \{2, 4\}$$

6. Powerset

Given a set S, the powerset of that set is denoted as P(S) and contains all subsets of S:

$$S = \{a, b\}$$

$$P(S) = \{\phi, \{a\}, \{b\}, \{a, b\}\}$$

since

$$\phi \subseteq S$$
 $\{a\} \subseteq S$ $\{b\} \subseteq S$ $\{a,b\} \subseteq S$

6.1. Cardinality of a powerset

We define:

$$|P(S)|=2^{|S|}$$

For example:

$$S = \{1,2,3\}$$

$$|S| = 3$$

$$P(S) = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$$

$$|P(S)| = 2^3 = 8$$

7. Set Operations

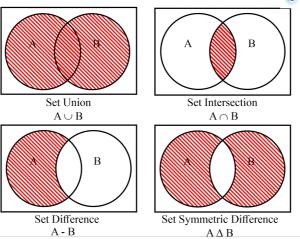


Figure 1. Venn Diagram, source: https://www.embibe.com/exams/set-theoretic-approach/

7.1. Union

Given two sets A and B, the union of A and B, $A \cup B$, contains all the element in **either** A and B.

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

For example:

$$A = \{1, 2, 3\}$$
 $B = \{4, 5, 6\}$ $A \cup B = \{1, 2, 3, 4, 5, 6\}$

and

$$A = \{1, 2, 3\}$$

$$C = \{2, 3, 4\}$$

$$A \cup C = \{1, 2, 3, 4\}$$

7.2. Intersection

Given two sets A and B, the intersection of A and B, $A \cap B$, contains all the elements in both A and B.

$$A\cap B=\{x\mid x\in A\ \text{ and }\ x\in B\}$$

For example:

$$A = \{1, 2, 3\}$$
$$B = \{2, 3, 4\}$$
$$A \cap B = \{2, 3\}$$

7.3. Set Difference

Given two sets A and B, the set difference, A-B, contains the elements that are in A but not in B.

$$A - B = \{x \mid x \in A \text{ and } x \notin B\}$$

For example:

$$A = \{1, 2, 3\}$$

 $B = \{2, 3, 4\}$
 $A - B = \{1\}$

NOTE
$$A-B=A\backslash B$$

7.4. Symmetric Difference

Given two sets A and B, the symmetric difference, $A \oplus B$, contains the elements that are in A or in B but **not in both**.

$$A \oplus B = \{x \mid (x \in A \text{ or } x \in B) \text{ and } x \notin A \cap B\}$$

For example

$$A = \{1, 2, 3\}$$

$$B = \{2, 3, 4\}$$

$$A \oplus B = \{1, 4\}$$

NOTE
$$A \oplus B = A\Delta B$$

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