

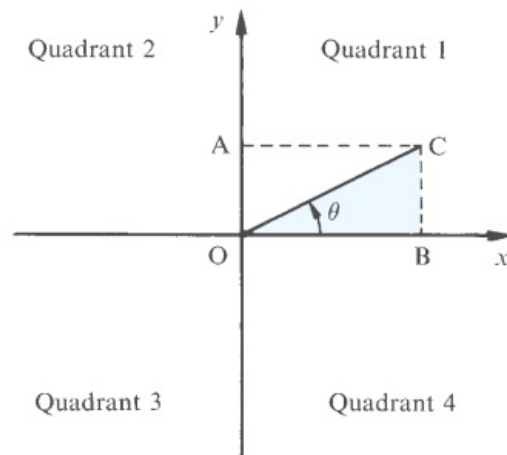
Cheatsheet - Trigonometrical Functions & Identities

Fabio Lama – fabio.lama@pm.me

1. Quadrants & Projections

The x and y axes divide the plane into four quadrants. The angle θ is of the following degree based where the arm is located:

- First quadrant, then $0^\circ \leq \theta \leq 90^\circ$
- Second quadrant, then $90^\circ \leq \theta \leq 180^\circ$
- Third quadrant, then $180^\circ \leq \theta \leq 270^\circ$
- Fourth quadrant, then $270^\circ \leq \theta \leq 360^\circ$



Consider the projection of the arm to be OC and label the x projection OB and the y projection OA . The trigonometrical ratios are defined as:

$$\sin \theta = \frac{OA}{OC} = \frac{BC}{OC}$$

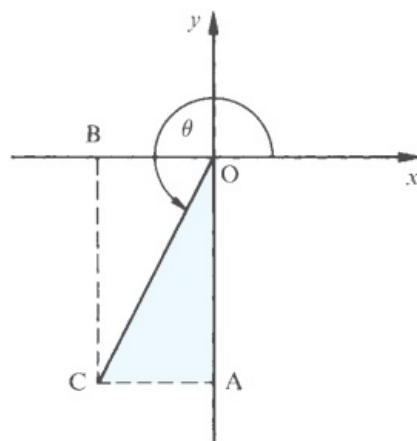
$$\cos \theta = \frac{OB}{OC} = \frac{AC}{OC}$$

$$\tan \theta = \frac{OA}{OB} = \frac{BC}{AC}$$

(Note that in the image above, $OA=BC$ and $OB=AC$. The rules for those trigonometrical functions are not any different than what's already covered in 'Cheatsheet - Trigonometry')

Given that x and y can be negative, this also means that $\sin \theta$, $\cos \theta$ and/or $\tan \theta$ can either be positive or negative, depending on in which quadrant the arm is located and which projection is used for the calculation.

For example, in this graph:



$\sin \theta$ and $\cos \theta$ are negative and $\tan \theta$ is positive, because x (OB) and y (OA) are negative, but OC is positive. Respectively:

$$\sin \theta = \frac{AC}{OC} = \frac{OB}{OC}$$

$$\cos \theta = \frac{OA}{OC} = \frac{BC}{OC}$$

$$\tan \theta = \frac{AC}{OA} = \frac{AC}{BC} = \frac{OB}{OA} = \frac{OB}{BC}$$

Given that $OB=AC$ and $BC=OA$.

2. Adding/Subtracting 360°

Adding or subtracting 360° to/from the arm does not alter the ratios, since it's a full rotation. Hence:

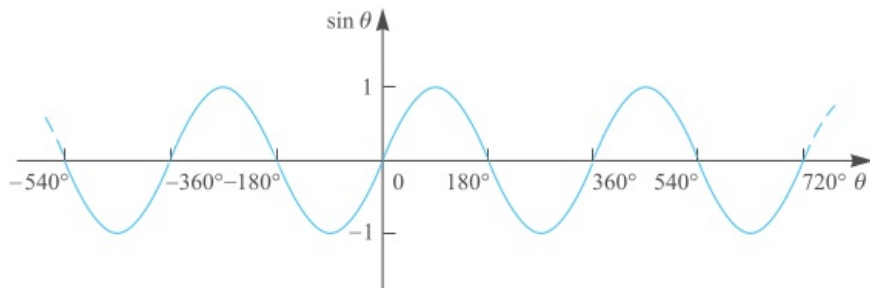
$$\sin \theta = \sin(\theta + 360^\circ) = \sin(\theta - 360^\circ)$$

$$\cos \theta = \cos(\theta + 360^\circ) = \cos(\theta - 360^\circ)$$

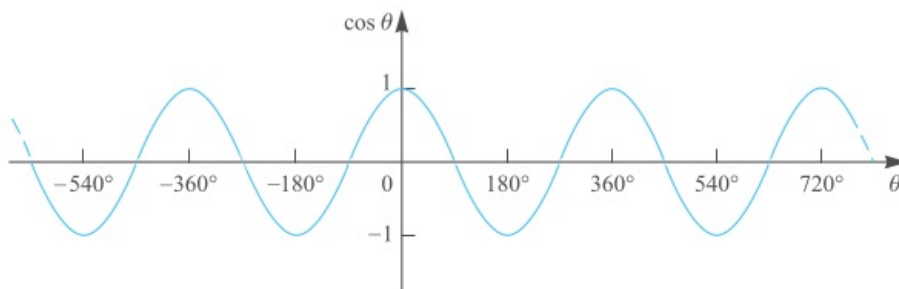
$$\tan \theta = \tan(\theta + 360^\circ) = \tan(\theta - 360^\circ)$$

3. The Sine, Cosine and Tans Functions

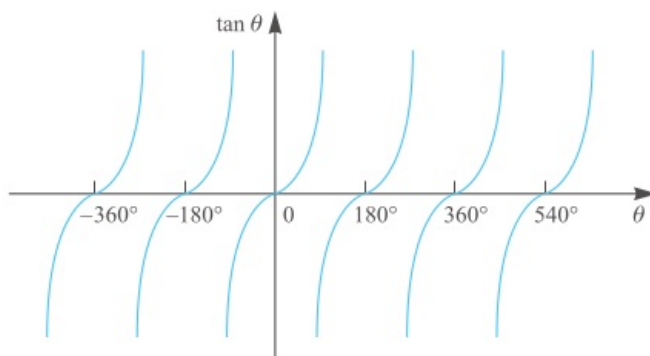
The sine function is $y = \sin \theta$, which repeats every 360° :



This cosine function is $y = \cos \theta$, which repeats every 360° :



The tangent function is $y = \tan \theta$, which repeats every 180° :



4. Identities

Identities imply equations that are true for every value of the involved variables. Important ones are:

$$\frac{\sin \theta}{\cos \theta} = \tan \theta$$

and:

$$\begin{aligned} &(\sin A)^2 + (\cos A)^2 \\ &= \sin^2 A + \cos^2 A = 1 \end{aligned}$$

Additionally:

$$\sin \theta = -\sin(\theta - 180^\circ)$$

$$\sin \theta = \sin(180^\circ - \theta)$$

$$\sin \theta = -\sin(360^\circ - \theta)$$

$$\cos \theta = -\cos(\theta - 180^\circ)$$

$$\cos \theta = -\cos(180^\circ - \theta)$$

$$\cos \theta = \cos(360^\circ - \theta)$$

$$\tan \theta = -\tan(180^\circ - \theta)$$

$$\tan \theta = \tan(\theta - 180^\circ)$$

$$\tan \theta = -\tan(360^\circ - \theta)$$

IMPORTANT | This means that trigonometric functions can have more than one solution.

4.1. Common Identities

$$\frac{\sin A}{\cos A} = \tan A$$

$$\sin(A + B) = \sin A \cos B + \sin B \cos A$$

$$\sin(A - B) = \sin A \cos B - \sin B \cos A$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\cos 2A = (\cos A)^2 - (\sin A)^2 = \cos^2 A - \sin^2 A$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\sin A + \sin B = 2 \sin\left(\frac{A + B}{2}\right) \cos\left(\frac{A - B}{2}\right)$$

$$\sin A - \sin B = 2 \sin\left(\frac{A - B}{2}\right) \cos\left(\frac{A + B}{2}\right)$$

$$\cos A + \cos B = 2 \cos\left(\frac{A + B}{2}\right) \cos\left(\frac{A - B}{2}\right)$$

$$\cos A - \cos B = -2 \sin\left(\frac{A - B}{2}\right) \sin\left(\frac{A + B}{2}\right)$$

$$\sin A = -\sin(-A)$$

$$\cos A = \cos(-A)$$

$$\tan A = -\tan(-A)$$