

# Cheatsheet - Exponential & Logarithm Functions

Fabio Lama – fabio.lama@pm.me

## 1. Exponential Expressions

An expression of the form  $a^x$  is called an **exponential expression**, where  $a$  is the **base** and  $x$  is the **exponent, power** or **index**.

### NOTE

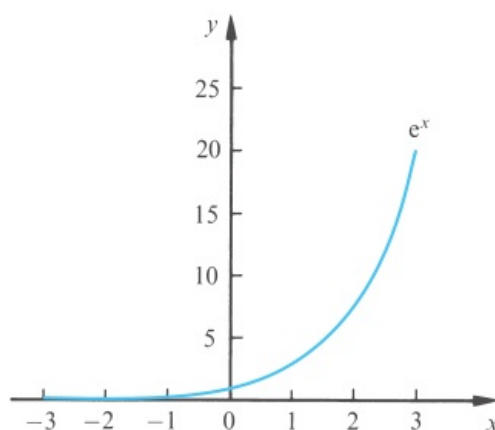
The most common exponential expression is  $e^x$ , where  $e$  is the constant 2.71828.... Also known as "Euler's number". This number is found in many natural phenomena.

## 2. Exponential Functions

An exponential function has the form of  $y = e^x$ . This function has important properties:

- $y$  is never negative.
- When  $x = 0$ , then  $y = 1$
- As  $x$  increases, then  $y$  increases (**exponential growth**).

| $x$   | -3     | -2     | -1     | 0 | 1      | 2      | 3      |
|-------|--------|--------|--------|---|--------|--------|--------|
| $e^x$ | 0.0498 | 0.1353 | 0.3679 | 1 | 2.7183 | 7.3891 | 20.086 |



## 3. Logarithms

The logarithm is the inverse function to the exponentiation. The following two equations are equivalent.

$$y = a^x \Leftrightarrow \log_a(y) = x$$

For example:

$$5^3 = 125 \Leftrightarrow \log_5(125) = 3$$

Also, conventionally we define **natural logarithms** as:

$$\log = \log_{10}$$

$$\ln = \log_e$$

### 3.1. Calculating Logarithms to any Base

$$\log_a X = \frac{\log_{10} X}{\log_{10} a}$$

$$\log_a X = \frac{\ln X}{\ln a}$$

This also means that - for example:

$$\log_5(125) = \frac{\log(125)}{\log(5)}$$

Additionally:

$$\log_a a = 1$$

3.2. Laws

First Law:

$$\log A + \log B = \log AB$$

Second Law:

$$\log A - \log B = \log \left( \frac{A}{B} \right)$$

Also note that:

$$\log 1 = 0$$

Third Law

$$n \log A = \log A^n$$

such as  $3 \log 2 = \log 2^3 = \log 8$ . This applies for all  $n \in \mathbb{R}$ .