## 1. Intro

(NOTE: Reading the Postulates of Boolean Algebra cheatsheet is recommended here)

Logic gates are basic elements of circuits implementing Boolean operations. The most basic circuits are **OR** gates, **AND** gates and invertors (**NOT** gates). All boolean functions can be written in terms of these three logic operations.

- AND operation is represented as f = x. y or f = xy.
- **OR** operation is represented as f = x + y.
- **NOT** operation is represented as  $f = \bar{x}$ .

Other gates:

- XOR operation is *true* only when the value of the inputs differ.
- NAND operations is equivalent to "not AND".
- NOR operation is equivalent to "not OR".
- XNOR operation is equivalent to a "not XOR".

AND, OR, XOR and XNOR are **commutative** (e.g. a+b=b+a) and **associative** (e.g. a+(b+c)=(a+b)+c). NAND and NOR are commutative but not associative.

Out Out BUF NOT (Buffer) (Inverter) 0 0 0 1 0 1 1 AND NAND 0 0 0 (NOT AND) 0 0 0 (In1) Out 0 0 Out) 0 (In2) (In2) OR NOR 0 0 0 0 0 (NOT OR) 0 (In1) (In1 Out Out 0 0 0 ln2 Out ln2 In1 In1 XOR XNOR 0 0 0 0 0 (Exclusive Or) (NOT XOR) 0 0 0 (In1) (In1 0 0 0 Ω

Logic Gates - Symbols and Truth Tables

A circle behind a symbol indicates that the output signal is inverted.

Figure 1. Source: http://www.exclusivearchitecture.com/?page\_id=2425

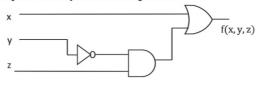
## 2. Circuits

We describe the combination of logic gates as a circuit.

• Let's consider the Boolean function f defined as:

$$f(x, y, z) = x + y'z$$

f can be represented by the following circuit:



A circuit that's used for the **addition** of inputs is called an **adder**. A **half adder** takes two inputs and generates a **carry** and a **sum**. A **full adder** takes three inputs and generates a carry and a sum.

For example, an half adder:

$$\mathrm{sum} = xy' + x'y = x \oplus y$$

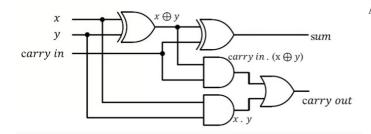


Figure 3. An half adder

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## And a **full adder**:

$$\mathrm{sum} = x \oplus y \ \text{ carry in}$$
 
$$\mathrm{carry \ out} = xy + \mathrm{carry \ in.} \ (x \oplus y)$$

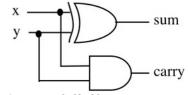


Figure 2. An half adder