FCS Week 7 Lecture Note

Notebook:	Fundamentals of Computer Science		
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Cornell Notes	**Topic:** Automata Theory Part 1	Course: BSc Computer Science
		Class: CM1025 Fundamentals of Computer Science[Lecture]
		Date: April 28, 2021

Essential Question:

What is an automata?

Questions/Cues:

- What is an Alphabet $\sum$?
- What is a string or word?
- What are empty strings?
- How do we denote the length of a string?
- How do we denote the set of all strings composed using letters in sigma?
- How do we denote the set of all non-empty strings composed using letters from sigma?
- What is $\sum^k$?
- What is a language?
- What is a finite automaton?
- What are states?
- What are transitions?
- What are accepting states?
- What is a 5-tuple?
- What is the language of a automaton?

Definitions

- An Alphabet, Σ, is a non-empty set of symbols
- For example: Σ = {0, 1} is a binary alphabet. Σ = {a, b, ..., z} is the collection of lowercase letters
- A string or word is a finite sequence of letters drawn from an alphabet
- For example: x = 01110101 is a binary string. "Life is good" is also a string
- Empty strings denoted by ε are strings with zero occurrences of letters. Empty strings can be from any alphabet.

Definitions

- Length of a string, x, is the sum of occurrences of its symbols, denoted by |x|
- For example: if x = 01110101 then |x| = 8, the length of "Life is good" is 12
- The set of all strings composed from letters in Σ is denoted by Σ^*
- For example: if Σ = { 0, 1} then Σ^* = {ε, 0, 1, 00, 01, 10, 11, ...}
- The set of all non-empty strings composed from letters in Σ is denoted by Σ^+.

Definitions

- The set of all strings of length k composed from letters in Σ is denoted by Σ^k
- For example: $\Sigma = \{0, 1\}$ then $\Sigma^2 = \{00, 01, 10, 11\}$
- $\Sigma^3 = \{000, 001, 010, 011, 100, 101, 110, 111\}$
- Size of Σ^k = $|\Sigma|^k$
- A language is a collection of strings over an alphabet
- For example: the language of palindromes over the binary alphabet is {ε, 0, 1, 00, 11, 000, 010, 101, 111, ...}.
    - Note if sigma k is finite, it's size is the size of the alphabet to the power of k.

Examples

- If $\Sigma = \{a, b, c\}$ then what is Σ^2?
- There must be $3^2 = 9$ strings in this set
- $\Sigma^2 = \{aa, ab, ac, ba, bb, bc, ca, cb, cc\}$
- What about Σ^1?
- Σ^1has three strings. $\Sigma^1 = \{a, b, c\}$
- Is Σ^1 the same as Σ?
- The answer is no! Elements in Σ are called symbols and they are letters, whereas the elements in $\Sigma^1$are strings; they just happen to have length 1
- Do not mix types in programming.

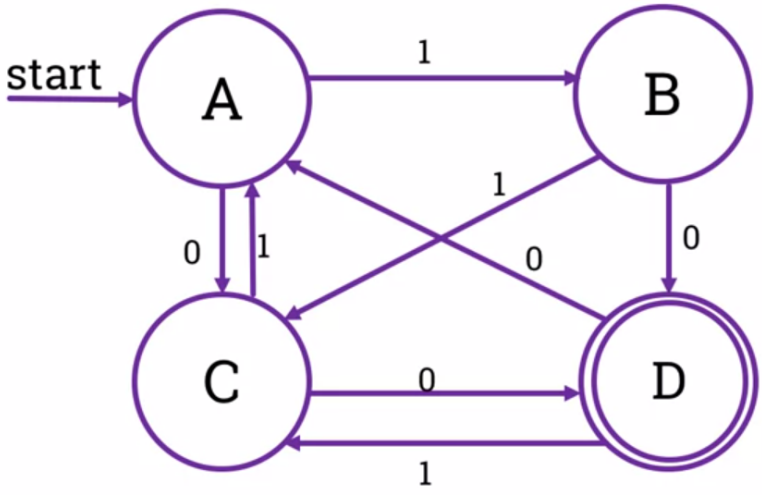
What is a finite automaton?

- A finite automaton is a simple mathematical machine; it is a representation of how computations are performed with limited memory space

- It is a model of computation, which consists of a set of states that are connected by transitions

- It has input. We will see later on
- It has output: Reject or Accept.

Example



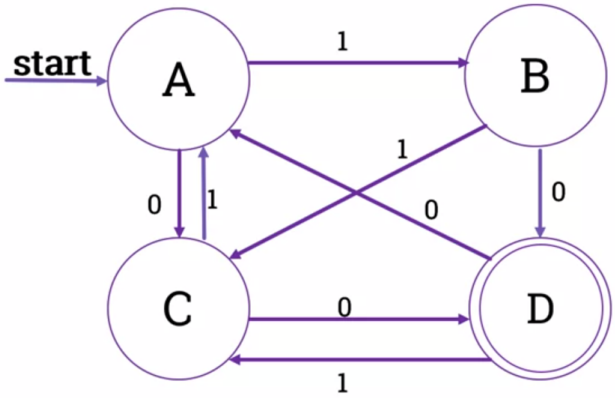
- The circles are the states of the automaton. The one with an arrow coming from nowhere or labeled start is called the initial state or the beginning of the computation.
- The labels or "numbers" on the arrows are called transitions and the labels are the alphabet dictating what to do next
- The arrow labeled "1" or transition "1" from A to B tells us that if we are at state A and we read "1" or one, we go to state B. On the other hand, if we are at A and we read zero or "0", we go the state C
- Double circled states are called accepting states. If the computation ends at any of the accepting states, the machine outputs accept, otherwise it outputs reject.

- Five-tuple = is a sequence of five elements

An automaton M is 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where:

- Q is a finite set called the states
- Σ is a finite set called the alphabet
- δ: Q x $\Sigma \rightarrow$ Q is the transition function
- $q_0 \in Q$ is the start state
- $F \subseteq Q$ is the set of accept states

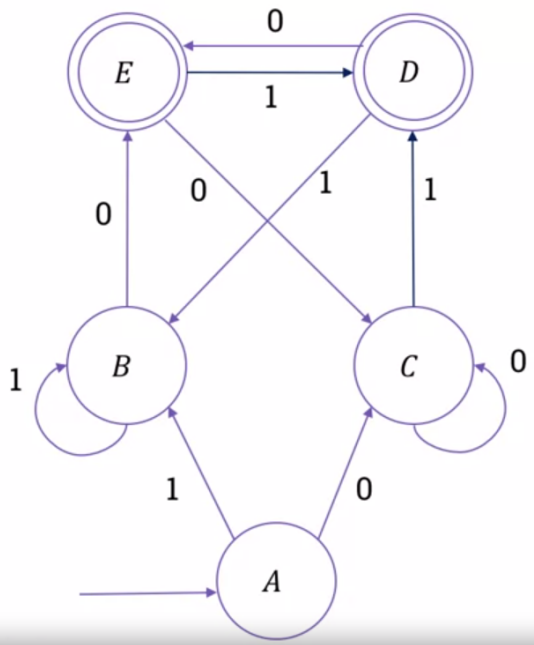
Example



- $Q = \{A, B, C, D\}$
- $\Sigma = \{0, 1\}$
- $q_0 = A$
- $F = \{D\}$
- $\delta = ?$

δ	0	1
A	C	B
B	D	C
C	D	A
*D	A	C

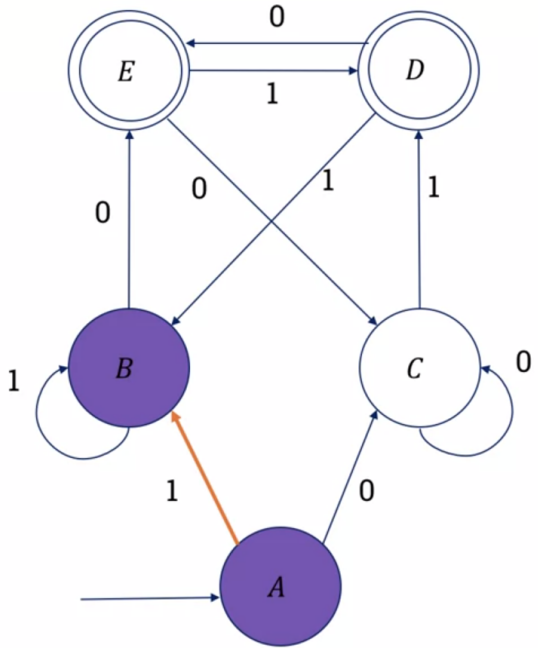
Example



- $Q = \{A, B, C, D, E\}$
- $\Sigma = \{0, 1\}$
- $q_0 = A$
- $F = \{D, E\}$
- $\delta =$

	0	1
A	C	B
B	E	B
C	C	D
$*D$	E	B
$*E$	C	D

- 10011



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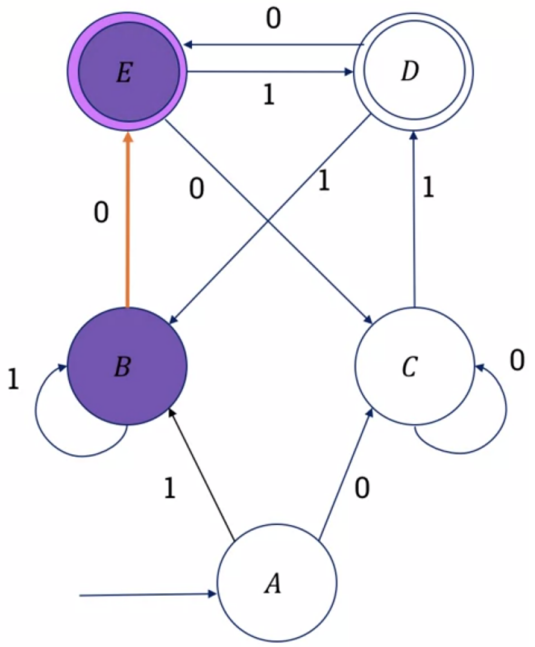

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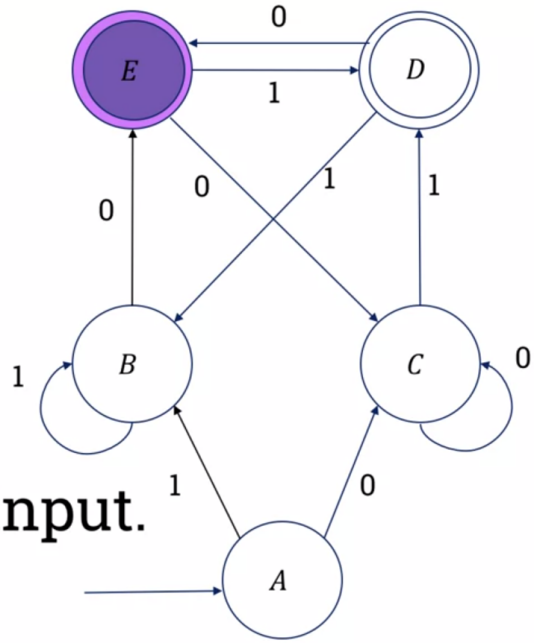
•Do we accept at E?

•No, not the end of the input.



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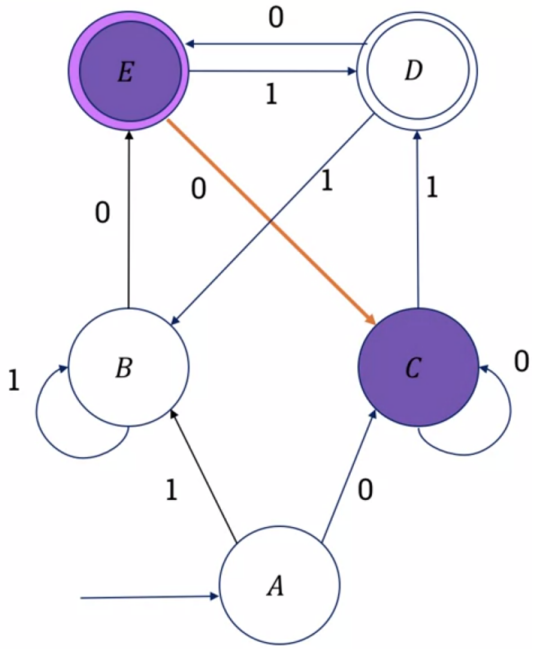

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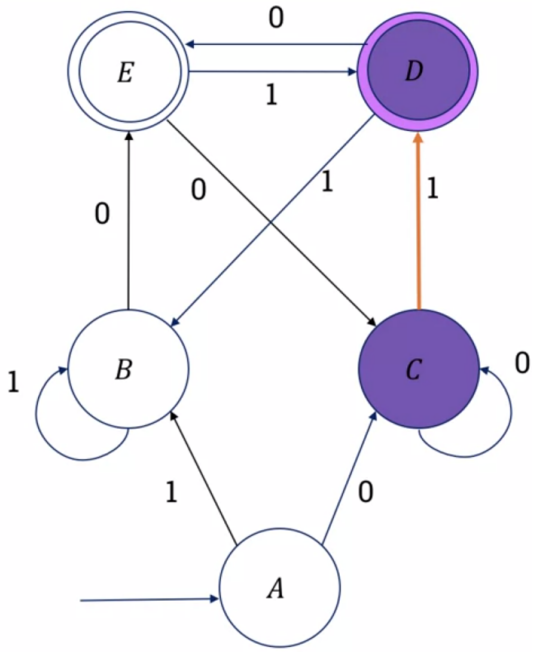

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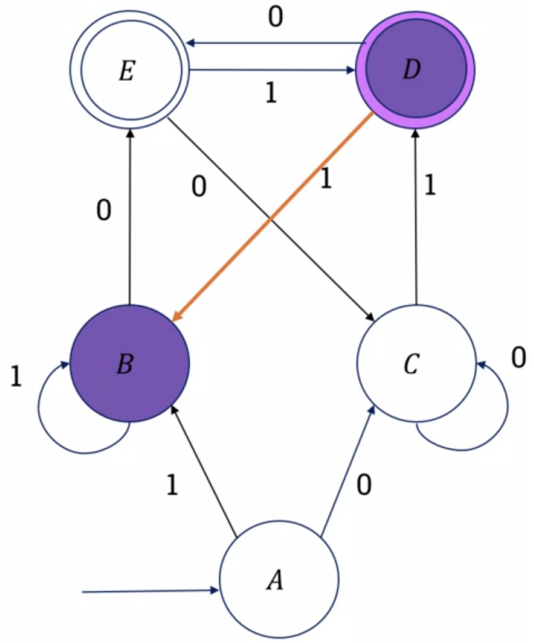
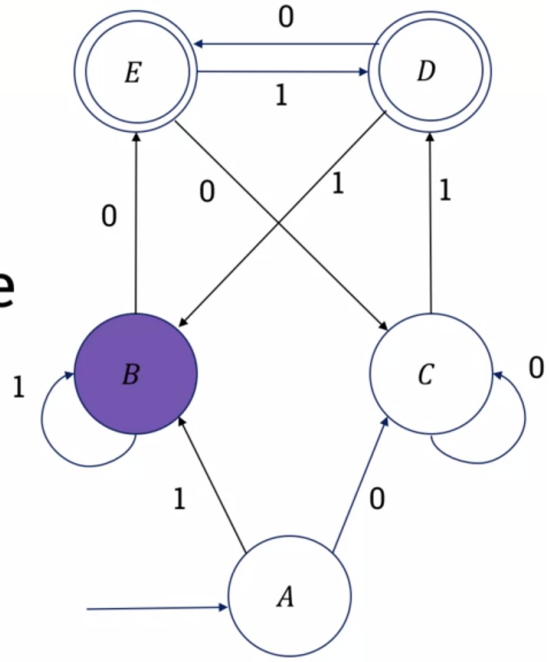


•10011

•B is not an accept state

•Reject 10011



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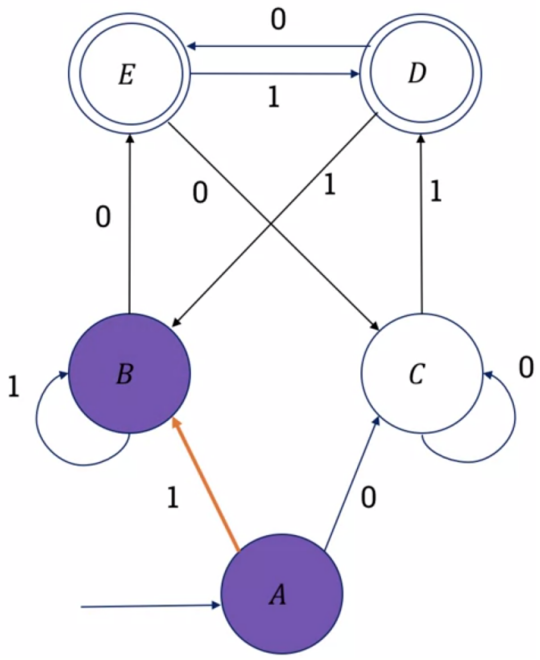

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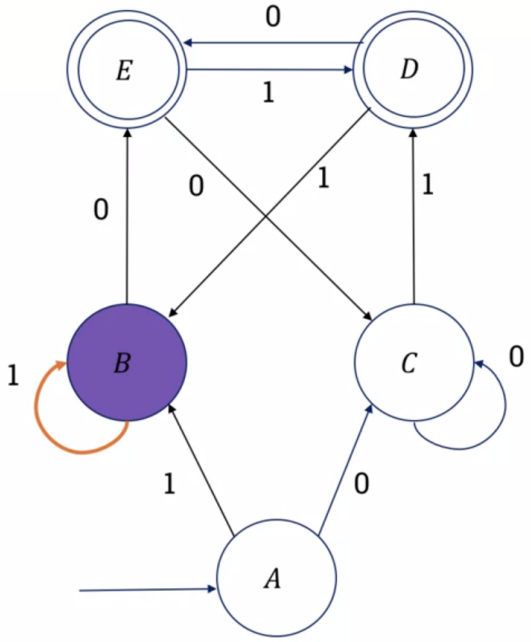

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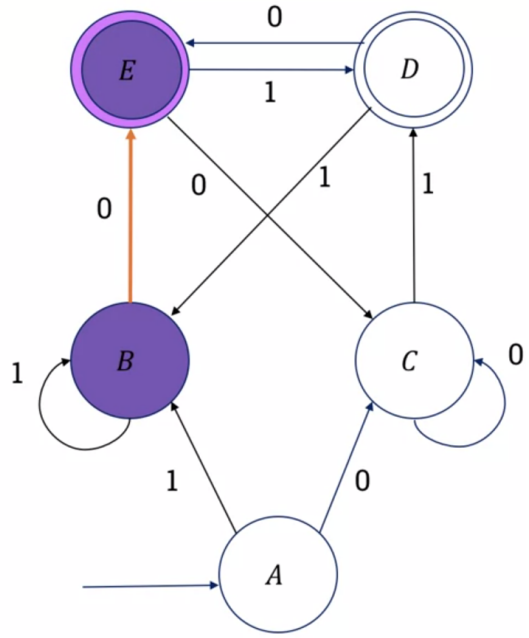

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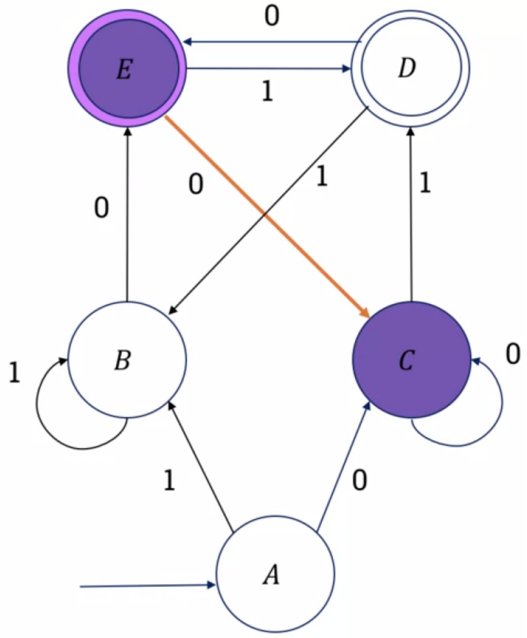

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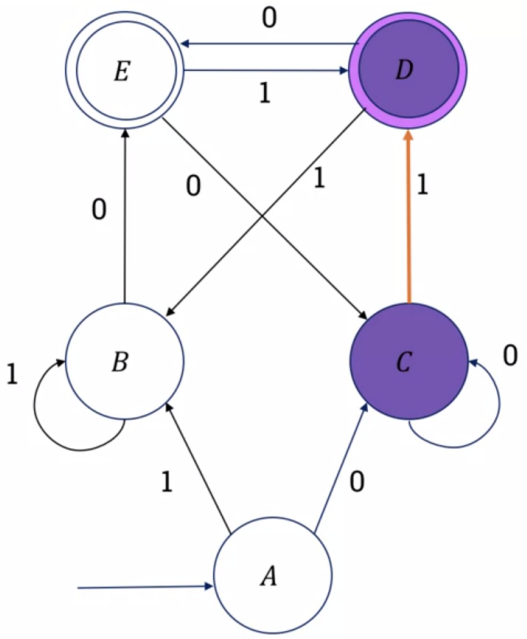
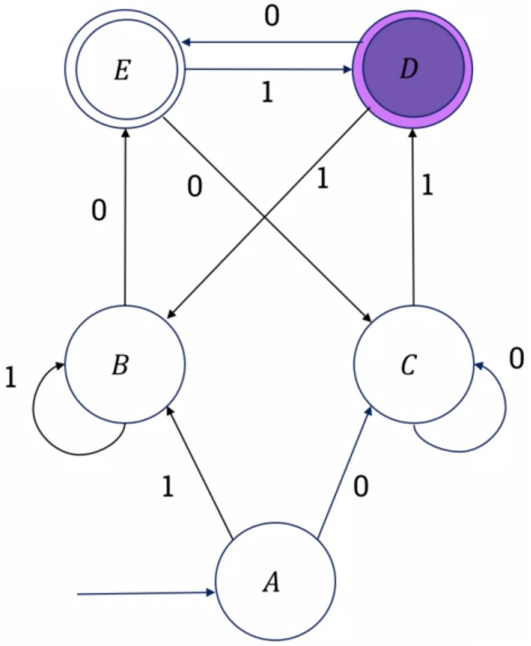


•11001

•Do we accept?

•Yes!

• End of the input at an accepting state.

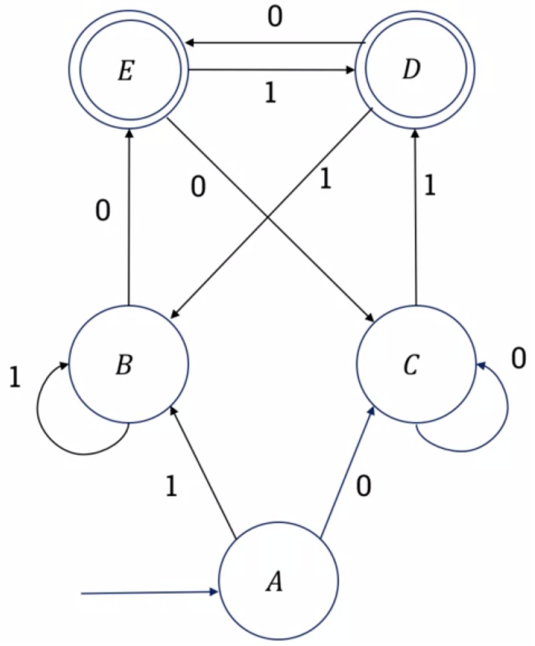


- We do ***not*** accept when passing through an accepting state.
- Only accept if the input ***ends*** in an accepting state.

What else is accepted by this automaton?

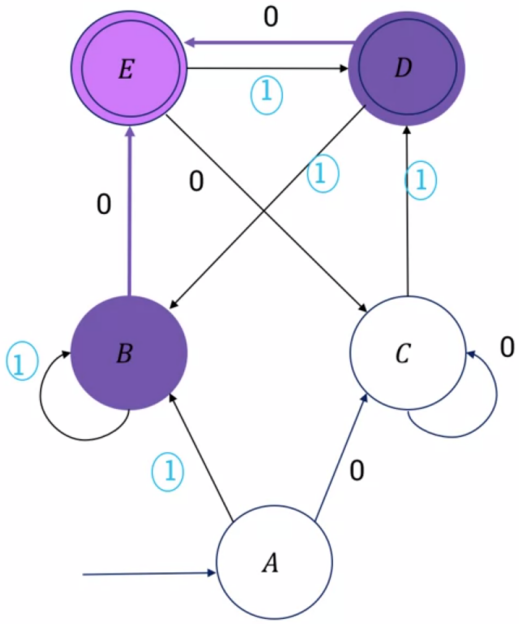
- 10
- 110
- 1110
- 1010
- 10110

- Do you see a pattern?



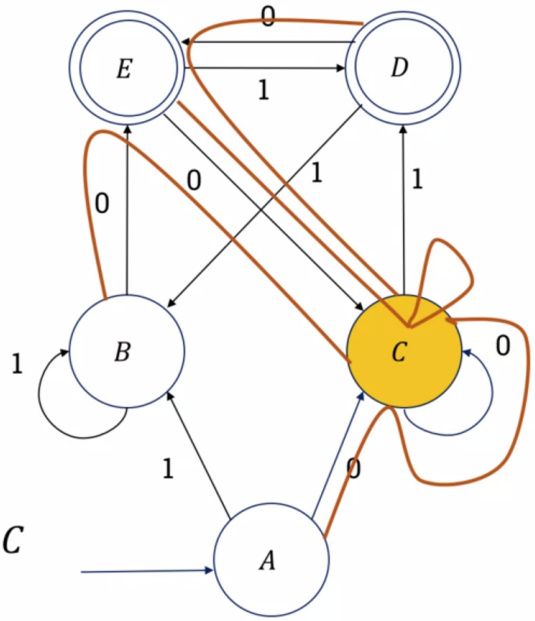
Backward computation

- Choose an accept state, E
- Incoming transitions to E
- Input ends with 0
- Penultimate states: B, D
- Incoming transitions to B, D
- Input ends with 10



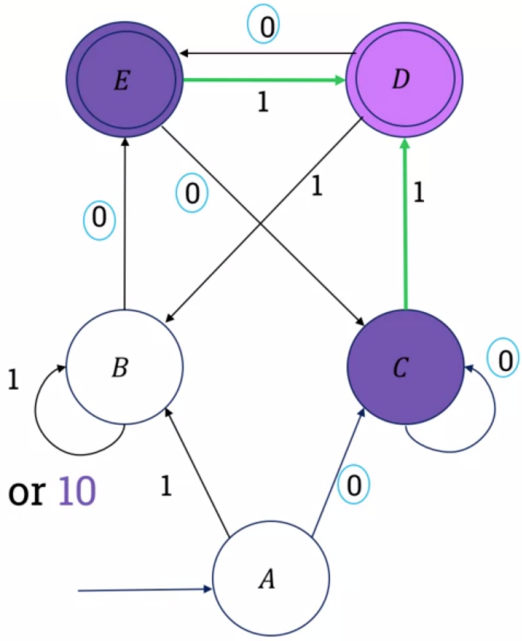
Backward computation

- Choose an accept state, E
- Incoming transitions to E
- Input ends with 0
- Penultimate states: B, D
- Incoming transitions to B, D
- Input ends with 10
- Check states with incoming 0, C
- Paths to C: ACC, ECC, DEC, BEC
- No input ending with 10 ends at C



Backward computation

- Other accepting state: D
- This automaton is symmetrical!
- Input ends with 1
- Penultimate states: C, E
- Incoming transitions to C, E
- Input ends with 01



- Any string ending with either 01 or 10

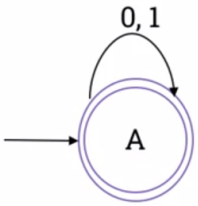
- The set of all strings accepted by an automaton is called the *language* of that automaton

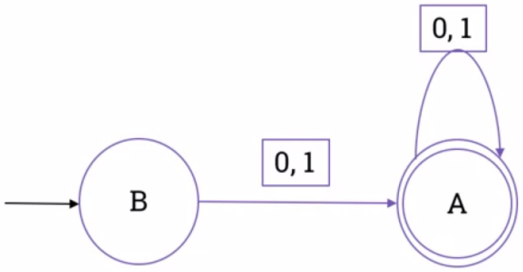
- If M is an automaton on alphabet Σ, then $\mathcal{L}(M)$ is the language of M:

$$\mathcal{L}(M) = \{x \in \Sigma^* | M \text{ accept } x\}$$

Automaton accepting every binary

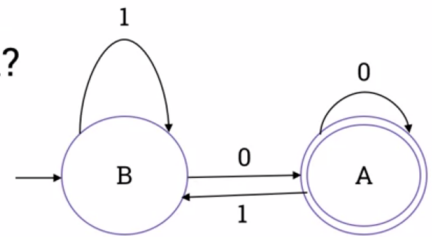
- There is a final state, A
- The incoming arrows have no restriction
- The outgoing arrows from A end at A
- Initial state, B
- 0 and 1 must be accepted





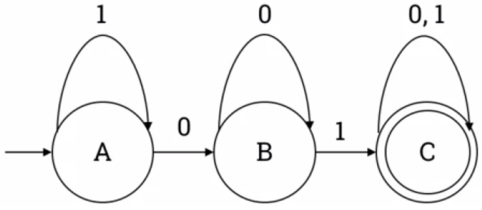
Automaton accepting strings ending in 0

- There is a final state, A
- The incoming arrow is labelled 0
- The outgoing arrow labelled 0 ends at A
- String 0 is accepted, initial state B
- What happens to arrows labelled 1?
- They cannot end at A
- End at B



Automaton accepting strings with 01

- String 01 is accepted, so we have at least 3 states
- If we read 01, anything after that is accepted
- Loop over state C with 0, 1
- If we read 0 at state B, we stay put
- What if we read 1 at state A?
- We stay put.



Summary

In this week, we learned about finite state automatons and automata theory.