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Te Whare Wānanga o Waitaha
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Department of Mathematics and Statistics

CSE Exercises – Week 11

1. Let X be an exponential random variable with mean 2. Use Monte Carlo integration to estimate $E(\log X)$, with n = 10^2, 10^4 and 10^6. For each n, compute the estimate, the 0.95 confidence bound and approximate 0.95 confidence interval.

2. Let X be a Laplacian(0,1) random variable. Use Monte Carlo integration to estimate $P(X > 2)$, with n = 10^3, 10^4, 10^5 and 10^6. For each n, compute the estimate, the 0.95 confidence bound and approximate 0.95 confidence interval.

3. Consider the integral,

$$\int_0^1 [\cos(50x) + \sin(20x)]^2 \, dx .$$

(a) Find the exact answer analytically.

(b) Estimate the integral using Monte Carlo integration with n = 10^2, 10^4 and 10^6. For each n, compute the estimate, the 0.95 confidence bound and approximate 0.95 confidence interval.

4. Estimate the following integrals using Monte Carlo integration with n = 10^2, 10^4 and 10^6. For each n, compute the estimate, the 0.95 confidence bound and approximate 0.95 confidence interval.

(a) $\displaystyle\int_0^1 \frac{\ln x}{1-x} \, dx .$

(b) $\displaystyle\int_0^\infty \frac{1}{x^2+25} \, dx .$

(c) $\displaystyle\int_{-\infty}^\infty \frac{\cos 2x}{\cosh 3x} \, dx .$

5. Let X be a Cauchy random variable whose density is

$$f(x) = \frac{1}{\pi(1+x^2)}.$$

We wish to estimate

$$P(X > 2) = \int_2^\infty f(x)dx, \tag{1}$$

which can be shown to be equivalent to

$$P(X > 2) = \int_0^{1/2} \frac{f(1/y)}{y^2} dy. \tag{2}$$

Compare the estimates of $P(X > 2)$ using

(a) direct Monte Carlo integration with equation (1),

(b) Monte Carlo integration with equation (2);

for = 10^2, 10^4 and 10^6. For each n, compute the estimate, the 0.95 confidence bound and approximate 0.95 confidence interval.

6. Let X be an exponential random variable with mean 1, i.e. the density of x is

$$f(x) = e^{-x},$$

for $x \geq 0$. We wish to estimate

$$P(X > 4) = \int_4^\infty f(x)dx. \tag{3}$$

(a) Show that

$$P(X > 4) = \int_0^{1/4} \frac{f(1/y)}{y^2} dy. \tag{4}$$

Hint: Use the substitution, $y = 1/x$.

(b) Compare the estimates of $P(X > 4)$ using

(i) direct Monte Carlo integration with equation (3),

(ii) Monte Carlo integration with equation (4),

(iii) importance sampling with a Cauchy distribution (defined in exercise 5) that is left-truncated at 4 as the importance distribution;

for = 10^2, 10^4 and 10^6. For each n, compute the estimate, the 0.95 confidence bound and approximate 0.95 confidence interval.