Department of Mathematics and Statistics

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Te Whare Wānanga o Waitaha
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CSE Exercises - Week 9

(1) Let $X_1, \ldots, X_n \overset{IID}{\sim} \text{Uniform}(a, b)$, where $a < b$, i.e. the uniform distribution on the interval $[a, b]$.

(a) Find the ML estimators for a and b analytically.

(b) Let $\mu = E(X_1)$, i.e. the mean of the Uniform (a, b) distribution. Find the ML estimator $\hat{\mu}$ for μ.

(c) Suppose that $a = 5$, $b = 8$ and $n = 10$.

(i) Estimate the mean squared error of $\hat{\mu}$ by Monte Carlo simulation.

(ii) Let $\bar{X}$ be the sample mean of $X_1, \ldots, X_n$. Find the mean squared error of $\bar{X}$ analytically.

(iii) Compare and comment on your results in (i) and (ii).

(2) Let $X_1, \ldots, X_n \overset{IID}{\sim} \text{Normal}(\mu, \sigma^2)$. Let q be the 0.95 quantile of the Normal(μ, σ^2) distribution.

(a) Find the ML estimator, $\hat{q}$, analytically.

(b) Suppose that $n = 30$ and the data, $X_1, \ldots, X_n$, are given in the file, normal_data.mat.

(i) Find the ML estimate, $\hat{q}$.

(ii) Estimate the standard error of $\hat{q}$ using the parametric bootstrap.

(iii) Estimate the standard error of $\hat{q}$ using the delta method.

(iv) Compare and comment on your answers in (ii) and (iii).

(3) Recall the definition of the Beta(α, β) distribution in Exercise 5 of week 5.

Let $X_1, \ldots, X_n \overset{IID}{\sim}$ Beta(α, β).

Suppose that $n = 25$ and the data, $X_1, \ldots, X_n$, are given in the file, beta_data.mat.

(a) Use fminsearch to find the ML estimates, $\hat{\alpha}$ and $\hat{\beta}$.

(b) When $\alpha > 1$ and $\beta > 1$, the mode of the Beta(α, β) density is at

$$m = \frac{\alpha - 1}{\alpha + \beta - 2}.$$

Find the ML estimate of m.

(c) Obtain approximate 95% confidence intervals for α, β and m using the parametric bootstrap.