Elastic Bands: Connecting Path Planning and Control

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Abstract

Elastic bands are proposed as the basis for a new framework to close the gap between global path planning and real-time sensor-based robot control. An elastic band is a deformable collision-free path. The initial shape of the elastic is the free path generated by a planner. Subjected to artificial forces, the elastic band deforms in real time to a short and smooth path that maintains clearance from the obstacles. The elastic continues to deform as changes in the environment are detected by sensors, enabling the robot to accommodate uncertainties and react to unexpected and moving obstacles. While providing a tight connection between the robot and its environment, the elastic band preserves the global nature of the planned path. This paper outlines the framework and discusses an efficient implementation based on bubbles.

1. Introduction

It is difficult to build a robot system that executes motion tasks autonomously. The problem has generally been approached from two directions: path planning and control.

Path planning uses models of the world and robot to compute a path for the robot to reach its goal. It has been shown that the general problem is computationally expensive although much progress has been made in producing fast planners for practical situations [1]. The output of a path planner is a continuous path along which the robot will not collide with the obstacles. However, any model of the real world will be incomplete and inaccurate, thus collisions may still occur if the robot moves blindly along such a path.

Control theory enables a robot to use sensing to close a feedback loop and interact with the environment in real time. One conventional application is for the robot to track a trajectory. More recently, work has been done on increasing the level of competence of control systems, for example, by including real-time collision avoidance capabilities [2]. Such local or reactive behaviors operate in real time but cannot solve the global problem of moving to an arbitrary goal.

To build a complete system we would like to combine these two approaches. A path planner provides a global solution to move the robot to the goal. A control system then moves the robot along the path while handling disturbance forces, small changes in the environment and unexpected obstacles.

The conventional solution is first to convert the path to a trajectory by time parameterization, then to track the trajectory. Path planners are often designed to find any feasible path, with little attention to its suitability for execution. Even if the time optimal parametrization is used, the path may have abrupt changes in direction or maintain little clearance from obstacles, requiring the robot to move slowly. In addition, if the controller is to implement some sort of real-time obstacle avoidance scheme then it must be able to deviate from the path. Once the robot is off the path, however, the controller has no global information on how to reach the goal.

2. A New Framework

We propose a new framework to close the gap between path planning and control. The idea is to implement local sensor based motions by deforming in real time the path computed by the planner. We call such a deforming collision-free path an *elastic band* [3].

We can view this framework as a three level hierarchy, as depicted in figure 1. At each level there is a closed loop with the environment whose reaction time decreases from slow (at the planning level) to very fast (at the control level). The three levels are:

- Path planning: A world model is used to generate global solutions to specified tasks.
- Elastic bands: The path from the planner is deformed in real time to handle local changes in the environment detected by sensors and to smooth the path.

- Control: A conventional control law is used to move the robot along the elastic band.

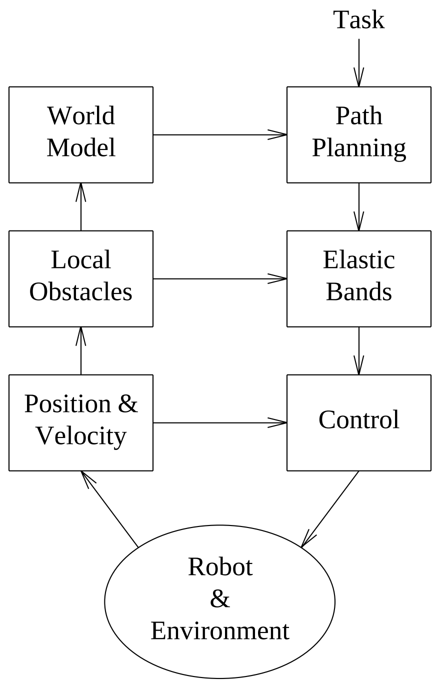

Figure 1. A three level hierarchy for a system that executes motion tasks.

The power of this framework is that the behavior of the robot is not completely determined at the planning level, yet local behavior does not limit the ability to achieve global goals. By deforming the path when changes in the environment are detected, we avoid the expense of recalling the path planner; the robot can react in real time to information obtained by sensors. However, while performing local behaviors we maintain a complete collision-free path to the goal. This property distinguishes the elastic band approach from other attempts to perform local sensor based motions.

3. Elastic Bands

To illustrate the basic behavior of elastic bands, consider a planar robot that can translate, but not rotate, in an environment with obstacles. Suppose a path planner has computed a path for the robot to move between two positions as depicted in figure 2-a. A controller would experience difficulty moving along the path since there are discontinuities in the direction of the path which would require the robot to come to a complete stop.

To improve the shape of the path we apply two forces: an internal contraction force and an external repulsive force. The contraction force simulates the tension in a stretched elastic band and removes any slack in the path. To counter the contraction force and to give the robot clearance around the obstacles, the elastic band is repelled from the obstacles. The two forces deform the elastic until equilibrium is reached as shown in figure 2-b.

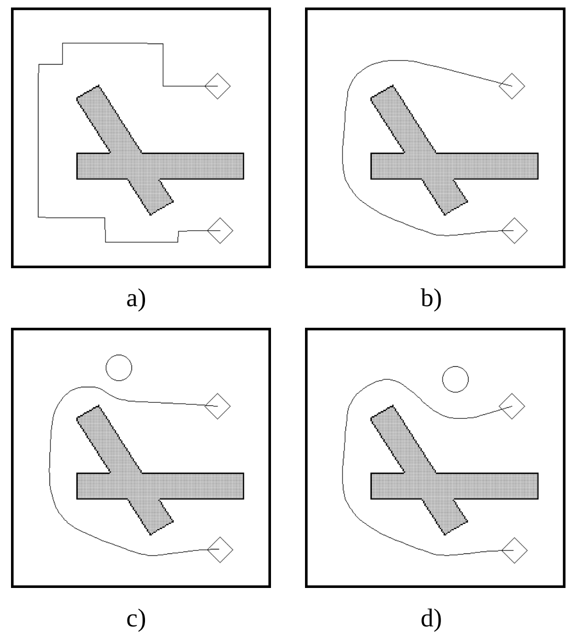

Figure 2. a) A path generated by a planner. **b)** Applying both an internal contraction force and a external repulsion force. **c, d)** A new moving obstacle deforms the path.

These two forces also enable the elastic band to handle changes in the environment. The appearance of new obstacles or the detection of uncertainties in the environment change the forces on the elastic, causing it to deform to a new equilibrium position. In our example, figures 2-c and 2-d depict the deformation of the path in the presence of new moving obstacle.

Obviously, if the changes in the environment are large, the elastic band could fail to deform to a collision-free path even if one exists. An example of this would be closing the door through which a robot had planned to move. A different path, say though a different door, may exist but to find such a path may require global search. This problem is typical of local collision avoidance methods and is the primary reason that path planning is needed. In such a situation, the failure can be detected and a new path found by replanning. However, for small changes, the elastic band is expected to deform to a good path that reflects the new state of the obstacles.

For the elastic band framework to operate in real time, we need an efficient implementation. We desire to update the path at a rate that is only one or perhaps two orders of magnitude slower that the control system. Of course, the actual rate depends on the particular application but a reasonable goal might be ten Hertz.

In many respects, elastic bands are similar to *snakes* [4]. A snake, as used in computer vision, is a deformable curve guided by artificial forces designed to pull it towards features in an image. The major difference is that an elastic band must represent a collision free path for the robot. The repulsive force mentioned above tends to push the elastic away from possible collisions but this alone is not enough. The problem is that elastic bands, and snakes as well, are represented by a finite series of points; the continuous curve is generated from these points. We must be able to restrict the position of the points so that the continuous curve is a collision free path.

To check that a curve is collision free, we must examine the configuration space, or c-space, of the robot [5]. The free space of a robot is the area of the c-space in which the robot does not touch any obstacles. The determination of whether a path lies in the free space is difficult for two reasons. First, the free space is computationly expensive to generate and difficult to represent. For even the simple case of a planar polygonal robot and polygonal obstacles, the boundary of the free space is a complex curved three dimensional manifold. Second, we desire a smooth path for the robot and hence are required to check that curves, rather than line segments, lie in the free space. The complexity of such computations is exemplified by the number of path planners that output paths as a series of line segments.

4. Bubbles: The Key to Implementation

The *bubble* concept offers an efficient method of maintaining a collision-free path even in high dimensional c-spaces. Instead of trying to compute and represent the entire free space, we use a model of the environment and robot to generate, on the fly, local subsets of the free space. Each subset, called a bubble, is computed by examining the local freedom of the robot at a given configuration.

To make the concept of a bubble more concrete, consider the example described in the previous section of a planar robot that cannot rotate. The configuration of such a robot is defined by the position of some point on the robot. We can define the function $\rho(\mathbf{b})$ that gives the minimum distance between the robot at configuration $\mathbf{b}$ and the obstacles in the environment. Obviously, the robot can move up to a distance $\rho(\mathbf{b})$ in any direction and still be guaranteed not to collide with an obstacle. From this observation we can define a subset of the free space around the configuration $\mathbf{b}$ using the equation

$$B(\mathbf{b}) = \{\mathbf{q} : \|\mathbf{b}-\mathbf{q}\| < \rho(\mathbf{b})\}.$$

The subset $B(\mathbf{b})$ is labeled the bubble at $\mathbf{b}$.

An elastic band is represented by a finite series of bubbles, constructed from a series of configurations or via points for the robot. To insure that a collision free path can be generated between the via points, we impose the condition that the bubbles at consecutive via points overlap. As long as the path remains inside the bubbles, it will be collision free. Obviously, for the circular bubbles a straight line between the via points will satisfy this requirement, however, by selecting more complex curves we can greatly improve the path from a control point of view.

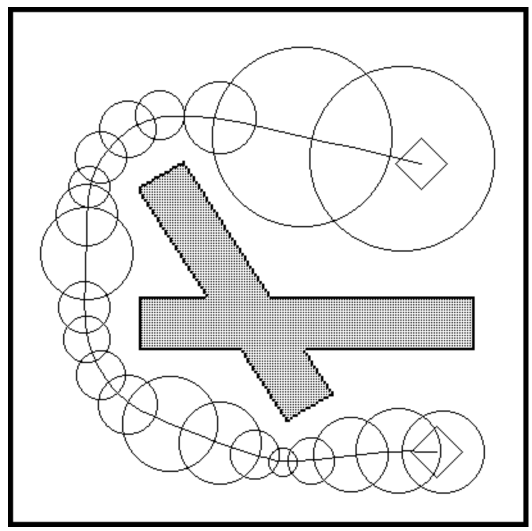

Figure 3. The bubbles along a path.

Although the details are not give in this paper, we have developed a closed form solution for determining minimum strain energy splines that can be used to construct a path for the robot that is contained by the bubbles of an elastic band. The resulting path has first order continuity which is required if the robot is to track the path without stopping. Figure 3 depicts a series of bubbles and a path for the robot.

The bubble representation of an elastic band has the desirable property that the complexity of the representation is related to the complexity of the situation. When the robot is far from obstacles, the bubbles will tend to be large and can hence be spaced far apart. In contrast, if the robot is maneuvering close to an obstacle then the bubbles would be smaller and more bubbles are required to describe the elastic band.

In the example above, the shape of the defined bubbles is a circle. As bubbles lie in the free space of the robot, which has a complex shape, a circular bubble can only coarsely represent the free space around the robot. More complex shapes, such as ellipses, could be used to better represent the local free space but there is a tradeoff. Fewer bubbles may be needed to describe the elastic band, but each bubble is more difficult to compute. The effect of this tradeoff might be interesting to investigate.

5. Bubbles for Higher Dimensions

The bubble representation enables elastic bands to be implemented for complex robots in changing environments. The dimension of the c-space of a robot typically equals the number of degrees of freedom of the mechanism. For complex robots in changing environments it is currently infeasible to compute and represent the global free space in real time. Using bubbles, we only need information about the free space around the path, a one dimensional manifold in the higher dimensional c-space. Also, the bubbles are computed from local information such as the function ρ in the above example. Such a function has completity determined by the representation used to model the robot and environment and not by the number of degrees of freedom.

Consider the case where our planar robot can rotate. In this situation the free space is three dimensional and quite complex to generate compared to the non rotating case. In contrast, we can compute three dimensional bubbles of free space with almost no extra effort. Assume that the configuration of the robot is described by a three dimensional vector $\mathbf{b} = (x, y, \theta)$ for some origin on the robot and we have calculated a constant $r_{\max}$ that represents the maximum distance from the origin of the robot to any other point on the robot. If the robot is moved from configuration $\mathbf{b}$ to $\mathbf{b}'$ then the maximum distance any point on the robot will travel can be bounded by the function $D(\mathbf{b} - \mathbf{b}')$ given by

$$D(\Delta\mathbf{b}) = \sqrt{\Delta\mathbf{b}_x^2 + \Delta\mathbf{b}_y^2} + r_{\max}|\Delta\mathbf{b}_\theta|.$$

Thus, a three dimensional bubble of free space for the robot can be defined by

$$\mathbf{B}(\mathbf{b}) = \{\mathbf{q} : D(\mathbf{b} - \mathbf{q}) < \rho(\mathbf{b})\}.$$

Notice that only information about the environment needed is given by the function ρ just as in the non rotating case. Similar extensions can be defined for manipulators operating in the plane. A current area of our research is extending this idea to robots in three dimensional environments.

6. Manipulating the Bubbles

This section describes how an elastic band, represented by a series of bubbles, is deformed. The overall strategy for deforming the elastic band is to scan up and down the sequence of bubbles, moving each in turn. To maintain the elastic band as a collision free path, we impose the constraint that each bubble overlaps with its two neighbors. This constraint may require new bubbles to be inserted as the elastic band deforms. In addition, it is desirable to remove redundant bubbles to improve efficiency. Figure 4 shows the manipulation of the bubbles of an elastic band as an obstacle moves in the environment.

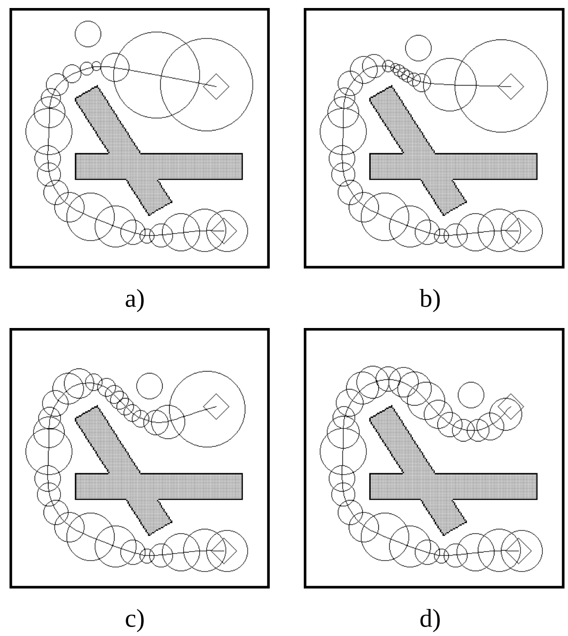

Figure 4. As an obstacle moves, the bubbles also move to minimize the force on the elastic band. If needed, bubbles are inserted and deleted to maintain a collision free path.

The magnitude and direction of the motion of a bubble is determined by computing an artificial force. In the current implementation, the artificial force is made up of an internal contraction force to remove slack in the elastic band and an external repulsion force to move the elastic away from the boundary of the free space.

The internal contraction force models the tension in a physical elastic band. We compute the force for a bubble at $\mathbf{b}_i$ using the following equation

$$\mathbf{f}_c = k_c\left(\frac{\mathbf{b}_{i-1}-\mathbf{b}_i}{\|\mathbf{b}_{i-1}-\mathbf{b}_i\|} + \frac{\mathbf{b}_{i+1}-\mathbf{b}_i}{\|\mathbf{b}_{i+1}-\mathbf{b}_i\|}\right),$$

where k_c is the global contraction gain. The physical interpretation is a series of springs between the the bubbles. The force from each spring is normalized to reflect a uniform tension along the elastic band.

The repulsive force pushes the bubbles away from the obstacles to increase the clearance of the robot. The size of a bubble gives an indication of how far the robot is from collision so we define the repulsive force such that it increases this size. In the case of the circular bubbles, $\rho(\mathbf{b})$ determines the size and one possibility for the repulsion force is

$$\mathbf{f}_r = \begin{cases} k_r(\rho_0 - \rho)\dfrac{\partial\rho}{\partial\mathbf{b}} & \rho < \rho_0 \\ 0 & \rho \ge \rho_0 \end{cases}$$

where k_r is the global repulsion gain and ρ_0 is the distance up to which the force is applied. To approximate $\partial\rho/\partial\mathbf{b}$ we use the finite difference equation

$$\frac{\partial\rho}{\partial\mathbf{b}} = \frac{1}{2h}\begin{bmatrix} \rho(\mathbf{b}-h\mathbf{x})-\rho(\mathbf{b}+h\mathbf{x}) \\ \rho(\mathbf{b}-h\mathbf{y})-\rho(\mathbf{b}+h\mathbf{y}) \end{bmatrix}$$

where h, the step size, is set to $\rho(\mathbf{b})$.

After computing the total force on a bubble, we need a method to determine the new position of the bubble. One simple update equation is

$$\mathbf{b}_{new} = \mathbf{b}_{old} + \alpha\mathbf{f}_{total}.$$

The bubble moves along the force, scaled by some factor α. One possible value for α is $\rho(\mathbf{b}_{old})$; the bubble moves a distance proportional to its size. The idea is that as the bubbles must overlap to form a valid elastic band, small bubbles should move less than large bubbles.

The above update equation implements a form of downhill gradient search to find the equilibrium point for the elastic band. Such methods tend to converge slowly and many other methods are possible; for example, we have implemented a version in which an inertial term is added to the bubbles to simulate a second order control system. However, the details are more numerical in nature and do not greatly affect the overall behavior of the elastic band.

After determining the new position of the bubble, we must check that the elastic band is still valid. If the bubble at the new position does not overlap a neighboring bubbles then the elastic band has in effect snapped. In such a situation, we can attempt to reconnect the elastic band by inserting a new bubble between the two. If inserting a single bubble does not reconnect the elastic band then we declare the move a failure and return the bubble to its initial position.

Another modification to the elastic band is to remove redundant bubbles. We can scan the series of bubbles for situations in which a bubble's neighbors overlap each other, allowing the bubble to be removed without breaking the elastic band. Deleting a bubble is desirable because it reduces the number of bubbles that need to be manipulated and thus reduces the computation required to update the elastic band.

The insertion of new bubbles and the removal of redundant bubbles can cause an undesirable side effect. In certain situations, bubbles can be inserted at one point, migrate along the elastic band, and then be removed at another point. The sequence can continue indefinitely and hence the elastic band oscillates in an unstable fashion. One solution to this problem consists of modifying the total force applied to the elastic band such that the tangential component is removed. Such a modified force inhibits the migration of bubbles along the elastic band. Mathematically, the modified force $\mathbf{f}^*$ is given by the formula

$$\mathbf{f}^* = \mathbf{f} - \frac{\mathbf{f}.(\mathbf{b}_{i-1}-\mathbf{b}_{i+1})(\mathbf{b}_{i-1}-\mathbf{b}_{i+1})}{\|\mathbf{b}_{i-1}-\mathbf{b}_{i+1}\|^2}.$$

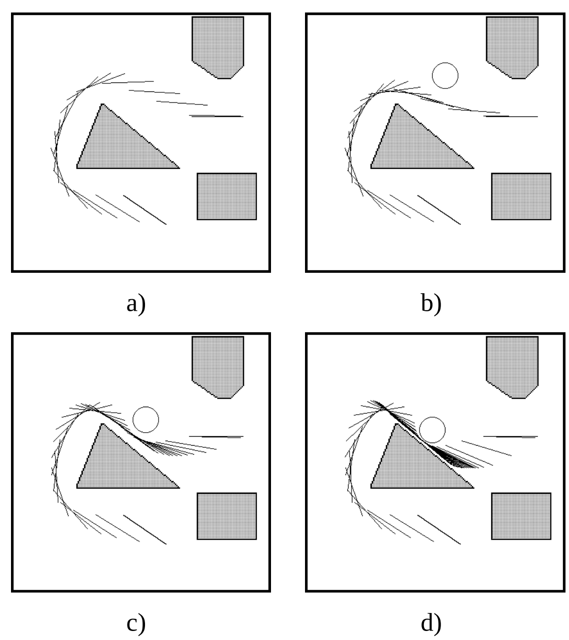

Figure 5. An elastic band for a stick robot. The elastic band deforms in the same fashion as the non rotating case, however the bubbles have three dimensions.

So far, we have discussed the case of two dimensional circular bubbles. The extension to other bubble shapes and dimensions is not difficult. Figure 5 shows an elastic band for a stick robot that can rotate. The bubbles are three dimensional and have the structure described previously. Due to the inability to display three dimensional bubbles we show the configuration of the robot at the center of each bubble. The elastic band deforms as an obstacle moves in the environment.

7. Future Work

Future work will focus on implementing the elastic band framework for a Puma 560 robot moving in a dynamic environment. The challenge is to achieve real-time performance using a model of the environment that is generated by sensor data. To compute a bubble of free space for a given configuration, we need to determine the distance of each link from the obstacles. If an elastic band consists of one hundred bubbles and we desire to modify the elastic at ten Hertz, these distances must be computed in the order of one millisecond. To meet such an objective will require the development of appropriate models for the robot and the environment and the use of high performance multi processor computers.

8. Conclusion

Elastic bands form the basis for an effective framework to deal with real-time collision-free motion control for a robot operating in an evolving environment. A planner provides an initial path that is a solution to the problem of moving a robot between a start and goal configuration. Incremental adjustments to the path are made while maintaining a global path in the free space. These modifications are based on sensory data about the environment and desired criteria concerning the path, such as length, smoothness, and obstacle clearance. Implemented as a real-time servo-loop, an elastic band provides many of the benefits of reactive systems without sacrificing global planning.

Bubbles enable elastic bands to be implemented to operate in real time. A bubble is a local region of free space around a configuration of the robot. The bubbles are in the configuration space of the robot, but we do not compute the global free space. The bubbles are generated by local information obtained directly from a model of the environment and can be implemented efficiently for complex robots.

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