

2.4) 20, 21, 28, 33

2.5) 33-36

Proof) NAND is functionally complete

For each of the tables in 18-21, construct (a) a Boolean expression having the given table as its truth table and (b) a circuit having the given table as its input/output table

20)

p	q	r	s
1	1	1	1
1	1	0	0
1	0	1	1
1	0	0	0
0	1	1	0
0	1	0	0
0	0	1	0
0	0	0	1

$(p \quad q \quad r) \quad (p \quad \sim q \quad r) \quad (p \quad \sim q \quad \sim r)$

21)

p	q	r	s
1	1	1	0
1	1	0	1
1	0	1	0
1	0	0	0
0	1	1	1
0	1	0	1
0	0	1	0
0	0	0	0

$(p \quad q \quad \sim r) \quad (\sim p \quad q \quad r) \quad (\sim p \quad q \quad \sim r)$

Use the properties listed in Theorem 2.1.1 to show that each pair of circuits in 26-29 have the same input/output table. (Find the Boolean expressions for the circuits and show that they are logically equivalent when regarded as statement forms.)

28.

$$(P \rightarrow Q) \rightarrow (P \rightarrow \sim Q) \rightarrow (\sim P \rightarrow \sim Q) \rightarrow (P \rightarrow Q) \rightarrow (\sim P \rightarrow \sim Q) \rightarrow (P \rightarrow \sim Q) \rightarrow (P \rightarrow \sim Q)$$

33.

a. Show that for the Sheffer stroke $|$,
 $P \rightarrow Q \equiv (P | Q) | (P | Q)$.

$$P \rightarrow Q \equiv (P | Q)$$

$$\begin{aligned} (P | Q) | (P | Q) &\equiv \sim((P | Q) | (P | Q)) && \text{by definition of } | \\ &\equiv \sim(\sim(P | Q) | \sim(P | Q)) && \text{by definition of } | \\ &\equiv \sim((\sim P | \sim Q) | (\sim P | \sim Q)) && \text{by de morgan's law} \\ &\equiv \sim(\sim P | \sim Q) | \sim(\sim P | \sim Q) && \text{by de morgan's law} \\ &\equiv (\sim\sim P | \sim\sim Q) | (\sim\sim P | \sim\sim Q) && \text{by de morgan's law} \\ &\equiv (P | Q) | (P | Q) && \text{by double negative law} \\ &\equiv P | Q && \text{by idempotent laws} \end{aligned}$$

b. Use the results of Example 2.4.7 and part (a) above to write $P \rightarrow (Q \rightarrow R)$ using only Sheffer strokes.

$$\begin{aligned} P \rightarrow (Q \rightarrow R) &\equiv \sim(\sim(P \rightarrow (Q \rightarrow R))) && \text{by double negative law} \\ &\equiv \sim(\sim P | \sim(Q \rightarrow R)) && \text{by de moregan's laws} \\ &\equiv \sim(\sim P | \sim(Q | \sim R)) && \text{by de moregan's laws} \\ &\equiv \sim(\sim P | (Q | \sim R)) && \text{by double negative law} \\ &\equiv \sim\sim P | \sim(Q | \sim R) && \text{by de morgan's laws} \\ &\equiv \sim\sim P | (Q | \sim R) && \text{by definition of } | \\ &\equiv P | (Q | \sim R) && \text{by by double negative law} \\ &\equiv \sim(\sim(P | (Q | \sim R))) && \text{by double negative law} \\ &\equiv \sim(P | (Q | \sim R)) && \text{by by definition of } | \end{aligned}$$

2.5

Use 8-bit representations to compute the sums in 31-36.

33. $(-6) + (-73)$

$$\begin{aligned} -6 \text{ in binary} &= 2^8 - |-6| \\ &= 256 - 6 \\ &= 250 \\ &= (1 \cdot 2^7) + (1 \cdot 2^6) + (1 \cdot 2^5) + (1 \cdot 2^4) + (1 \cdot 2^3) + (0 \cdot 2^2) + (1 \cdot 2^1) + (0 \cdot 2^0) \\ &= 11111010 \end{aligned}$$

$$\begin{aligned}
-73 \text{ in binary} &= 2^8 - |-73| \\
&= 256 - 73 \\
&= 250 \\
&= (1 \cdot 2^7) + (0 \cdot 2^6) + (1 \cdot 2^5) + (1 \cdot 2^4) + (0 \cdot 2^3) + (1 \cdot 2^2) + (1 \cdot 2^1) + (1 \cdot 2^0) \\
&= 10110111
\end{aligned}$$

$$\begin{array}{r}
11111010 \\
+ 10110111 \\
\hline
110110001
\end{array}$$

$$34. \ 89 + (-55)$$

$$\begin{aligned}
89 \text{ in binary} &= (0 \cdot 2^7) + (1 \cdot 2^6) + (0 \cdot 2^5) + (1 \cdot 2^4) + (1 \cdot 2^3) + (0 \cdot 2^2) + (0 \cdot 2^1) + (1 \cdot 2^0) \\
&= 01011001
\end{aligned}$$

$$\begin{aligned}
-55 \text{ in binary} &= 2^8 - |-55| \\
&= 256 - 55 \\
&= 250 \\
&= (1 \cdot 2^7) + (1 \cdot 2^6) + (0 \cdot 2^5) + (0 \cdot 2^4) + (1 \cdot 2^3) + (0 \cdot 2^2) + (0 \cdot 2^1) + (1 \cdot 2^0) \\
&= 11001001
\end{aligned}$$

$$\begin{array}{r}
01011001 \\
+ 11001001 \\
\hline
100100010
\end{array}$$

$$35. \ (-15) + (-46)$$

$$\begin{aligned}
-15 \text{ in binary} &= 2^8 - |-15| \\
&= 256 - 15 \\
&= 250 \\
&= (1 \cdot 2^7) + (1 \cdot 2^6) + (1 \cdot 2^5) + (1 \cdot 2^4) + (0 \cdot 2^3) + (0 \cdot 2^2) + (0 \cdot 2^1) + (1 \cdot 2^0) \\
&= 11110001
\end{aligned}$$

$$\begin{aligned}
-46 \text{ in binary} &= 2^8 - |-46| \\
&= 256 - 46 \\
&= 250 \\
&= (1 \cdot 2^7) + (1 \cdot 2^6) + (0 \cdot 2^5) + (1 \cdot 2^4) + (0 \cdot 2^3) + (0 \cdot 2^2) + (1 \cdot 2^1) + (0 \cdot 2^0) \\
&= 11010010
\end{aligned}$$

$$\begin{array}{r}
11110001 \\
+ 11010010 \\
\hline
\end{array}$$

111000011

36. $123 + (-94)$

$123 \text{ in binary} = (0 \cdot 2^7) + (1 \cdot 2^6) + (1 \cdot 2^5) + (1 \cdot 2^4) + (1 \cdot 2^3) + (0 \cdot 2^2) + (1 \cdot 2^1) + (1 \cdot 2^0)$
 $= 01111011$

$-94 \text{ in binary} = 2^8 - |-94|$
 $= 256 - 94$
 $= 250$
 $= (1 \cdot 2^7) + (0 \cdot 2^6) + (1 \cdot 2^5) + (0 \cdot 2^4) + (0 \cdot 2^3) + (0 \cdot 2^2) + (1 \cdot 2^1) + (0 \cdot 2^0)$
 $= 10100010$

```
  01111011
+ 10100010
-----
 100011101
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So here is what is needed to prove:

Pre-req

1. What is NAND?

NAND is the logical equivalent of AND + NOT

	IN		OUT	
	p		q	p q
	1	1	1	
	1	1	0	
	1	0	1	
	1	0	0	

2. What is functionally complete? What certifies it?

Attempt:

Knowing the definition of functionally complete and what is needed to certify it, and what NAND is