Exercise involving the Normal distribution and the sample distribution

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In this exercise we will use data from the Normal distribution. It's the main distribution used in Statistics and has important properties such as symmetry around the mean and it's bell shape. This exercise shows this distribution by building upon the exploratory data analysis graphics and commands that we learned previously. Then we will use data to learn properties of the sample distribution and the Central Limit Theorem.

Variables in this data set

name	description
observation	onAn observation from
	the Normal
	distribution
size	The number of
	observations in the
	sample
variance	Variance of the
	Normal distribution
group	The group identifier
-	for a given size and
	variance. There are
	15 groups for each
	size and variance

Normal distribution

First lets learn about the Normal distribution using graphics from Exploratory Data Analysis. Complete the following tasks:

- 1. Load the file normal_2016.dta into Stata.
- 2. Which are the unique variance values?
- 3. How many observations do we have for each unique variance?
- 4. What is the sum of the unique size values?
- 5. Plot the distribution of the observation variable. Does it look bell shaped?
- 6. Plot the distribution of the observation variable separately for each value of the variance variable. Does these plots look different to the previous one? What properties do you observe?
- 7. Compare the distribution of the observation variable against the theoretical Normal distribution. What would you conclude from this quorm plot?
- 8. Make a separate qnorm plot for only the observations with variance equal to 1 (or 5 or 10). Does it look closer to the theoretical Normal distribution?

Solutions

```
library('knitr')
statapath <- '/Applications/Stata/Stata.app/Contents/MacOS/Stata'</pre>
* Load the data
use normal_2016.dta, clear
* Unique variance values
codebook variance
* Observations for each unique variance
codebook variance
* Sum of the unique size values
codebook observation
*5 + 15 + 25 + 100 = 145
* Note 145 * 15 = 2175
* Distribution of observation
histogram observation
graph export "hist1.png", replace
graph box observation
graph export "box1.png", replace
* Separately
histogram observation, by (variance)
graph export "hist2.png", replace
graph box observation, by(variance)
graph export "box2.png", replace
* Global qnorm
qnorm observation
graph export "qnorm1.png", replace
* For each variance
qnorm observation if variance == 1
graph export "qnorm-var1.png", replace
qnorm observation if variance == 5
graph export "qnorm-var5.png", replace
qnorm observation if variance == 10
graph export "qnorm-var10.png", replace
. * Load the data
. use normal_2016.dta, clear
(Written by R.
. * Unique variance values
. codebook variance
```

type: numeric (double)

range: [1,10] units: 1

unique values: 3 missing .: 0/6,525

tabulation: Freq. Value

2,175 1 2,175 5 2,175 10

. * Observations for each unique variance

. codebook variance

variance Variance of the normal distribution for the group

type: numeric (double)

range: [1,10] units: 1

unique values: 3 missing .: 0/6,525

tabulation: Freq. Value

2,175 1 2,175 5 2,175 10

. * Sum of the unique size values

. codebook observation

observation Observed value

type: numeric (double)

range: [-9.9257767,9.8362535] units: 1.000e-12 unique values: 6,525 missing .: 0/6,525

mean: -.039081 std. dev: 2.35833

percentiles: 10% 25% 50% 75% 90%

-2.81559 -1.28552 -.04104 1.17471 2.80592

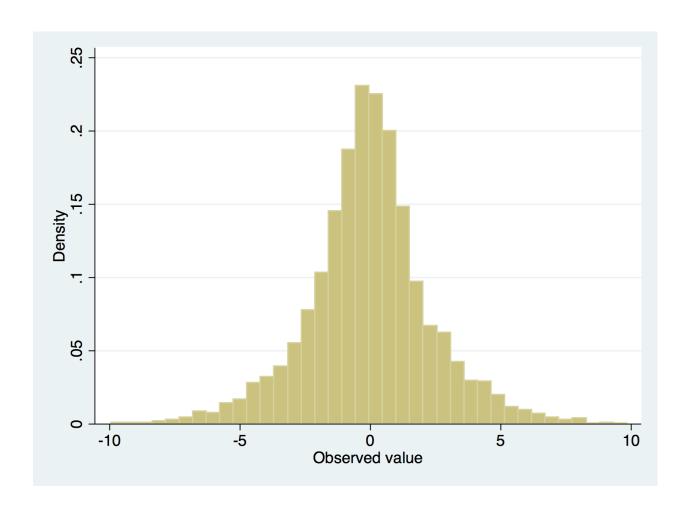
. * 5 + 15 + 25 + 100 = 145

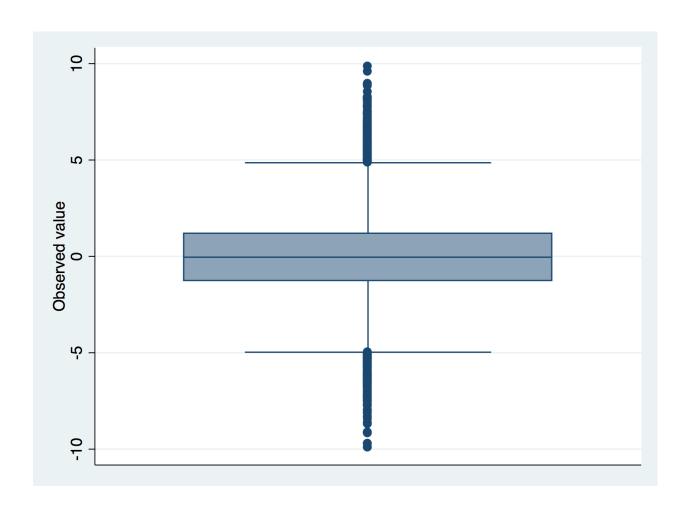
. * Note 145 * 15 = 2175

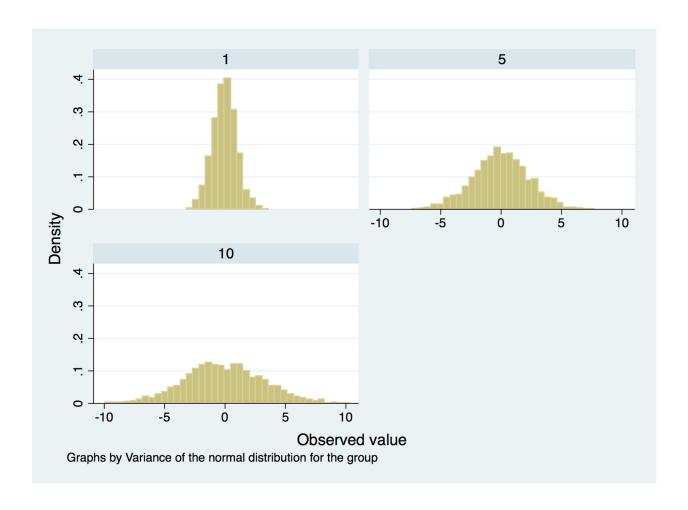
. * Distribution of observation

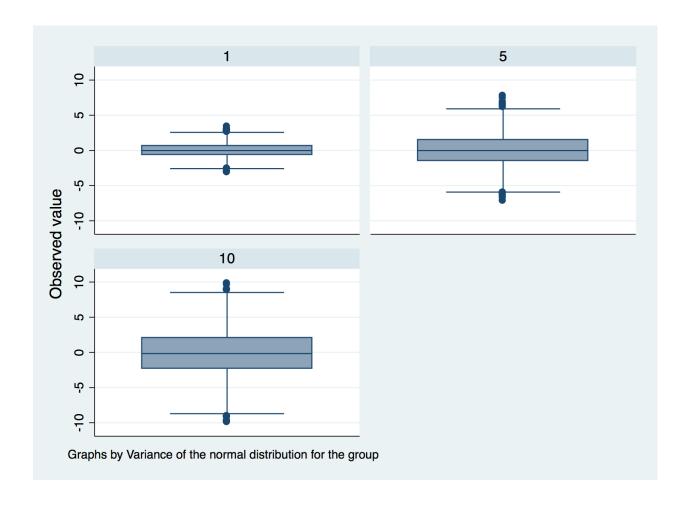
. histogram observation (bin=38, start=-9.9257767, width=.52005342) . graph export "hist1.png", replace (file hist1.png written in PNG format) . graph box observation . graph export "box1.png", replace (file box1.png written in PNG format) . * Separately . histogram observation, by(variance) . graph export "hist2.png", replace (file hist2.png written in PNG format) . graph box observation, by(variance) . graph export "box2.png", replace (file box2.png written in PNG format) . * Global qnorm . qnorm observation . graph export "qnorm1.png", replace (file qnorm1.png written in PNG format) . * For each variance . qnorm observation if variance == 1 . graph export "qnorm-var1.png", replace (file qnorm-var1.png written in PNG format) . qnorm observation if variance == 5 . graph export "qnorm-var5.png", replace (file qnorm-var5.png written in PNG format) . qnorm observation if variance == 10

. graph export "qnorm-var10.png", replace
(file qnorm-var10.png written in PNG format)









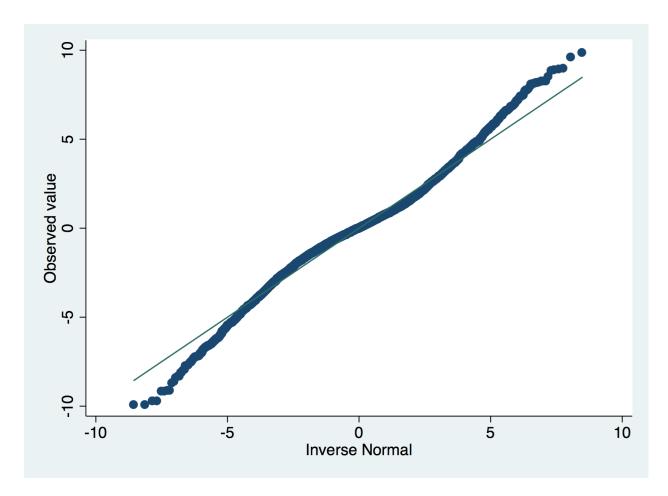
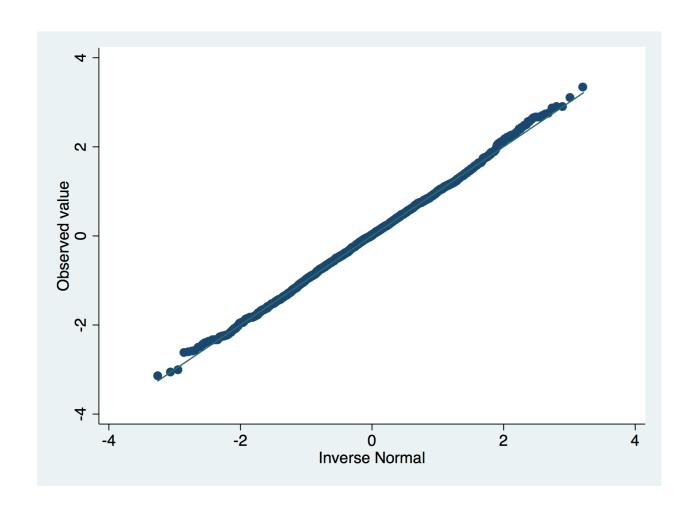
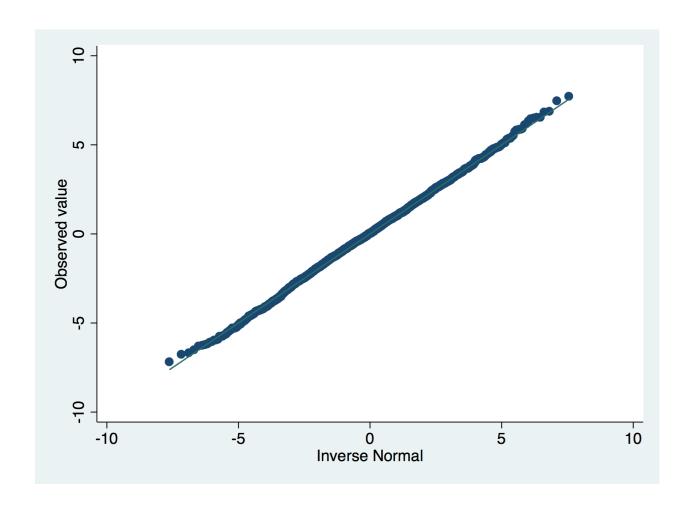
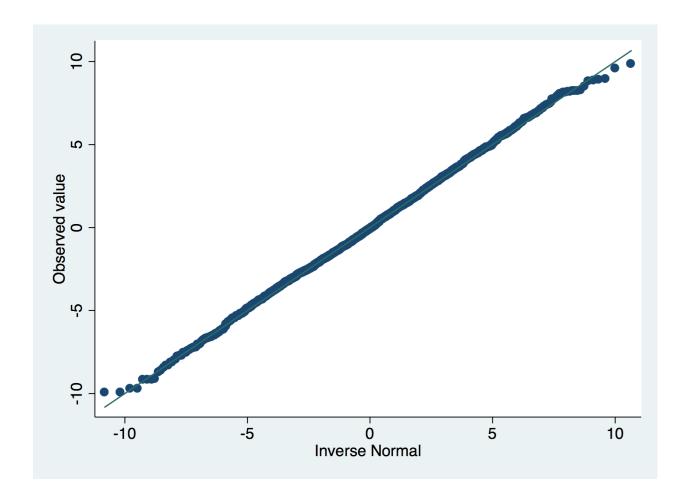


Figure 1: qnorm1







Sampling

- 1. Load the file normal_2016.dta into Stata.
- 2. What is the standard deviation of the observation variable?
- 3. What is the standard deviation of the observation variable when variance is equal to 1? Is it equal to 1? If not, how close is it to 1?
- 4. Repeat the same process for each unique value of the group variable when variance is equal to 1. Are the means equal to 0? Are the standard deviations equal to 1? What differences do you notice with respect to the previous bullet point?

To differentiate the properties of the sample distribution and the theoretical distribution, we say that:

- μ is the mean of the theoretical distribution
- σ is the standard deviation of the theoretical distribution
- \bar{x} is the mean of the sample distribution, also known as the sample mean
- $s = \sqrt{\frac{\sum (x_i \bar{x})^2}{n-1}}$ is the standard deviation of the sample distribution

Solutions

* Load the data use normal_2016.dta, clear

```
* SD of observation
summarize observation
* SD of observation when variance is equal to 1
summarize observation if variance == 1
* SD of observation when variance is equal to 1 for each group
bys group: summarize observation if variance == 1
. * Load the data
. use normal_2016.dta, clear
(Written by R.
. * SD of observation
. summarize observation
   Variable | Obs Mean Std. Dev. Min Max
-----
observation |
              6,525 -.0390809
                              2.358333 -9.925777 9.836253
. * SD of observation when variance is equal to 1
. summarize observation if variance == 1
   Variable | Obs Mean Std. Dev. Min Max
observation |
              2,175 -.0213126 .9758493 -3.145141 3.332773
. \star SD of observation when variance is equal to 1 for each group
. bys group: summarize observation if variance == 1
-> group = 1
   Variable | Obs Mean Std. Dev. Min Max
______
                145 -.0035318
                              .9033008 -2.488066 2.395979
\rightarrow group = 2
   Variable | Obs Mean Std. Dev. Min Max
                145 .0815653
                              .9707447 -2.246286 2.489026
observation |
-> group = 3
  Variable | Obs Mean Std. Dev. Min Max
```

observation						
-> group = 4						
Variable						
observation	145	0059926	.9485665	-2.37961	2.107302	
-> group = 5						
Variable						
observation						
-> group = 6						
Variable						
observation	145	0448871	.9687357	-2.587038	3.332773	
Variable				Min	Max	
observation	145	.0147168	.9272922			
-> group = 8						
Variable		Mean	Std. Dev.	Min	Max	
observation		.1171013	.994055	-2.2517	2.859635	
-> group = 9						
Variable			Std. Dev.			
observation	145	0288215	1.034813	-3.061025	2.659089	
-> group = 10						
Variable			Std. Dev.			
observation						
-> group = 11						

Variable						
observation	145	1366	1.035556	-2.611554	2.467293	
-> group = 12						
Variable						
observation	145	0781324	.9741902	-2.253071	2.317058	
-> group = 13						
Variable						
observation	145	.0359654	.9722664	-2.275057	2.478941	
-> group = 14						
Variable	Obs	Mean	Std. Dev.	Min	Max	
observation	145	.1516833	.9738071	-2.513651	3.099097	
-> group = 15						
Variable						
observation end of do-file						

Central Limit Theorem

In many applications of biostatistics we study the *sample mean* and the Central Limit Theorem (a theoretical result) provides us with a incredibly powerful tool. The CLT tells us that as the sample size n increases, the distribution of the sample mean barx gets closer to the Normal distribution with mean μ and variance $\frac{s}{\sqrt{n}}$.

This sounds great, but lets see it in action. Complete the following tasks:

- 1. Load the file normal_2016.dta into Stata.
- 2. For each group of observations that have sample size equal to 5 and variance 10, calculate the mean. Open a text editor or Excel and write down the 15 sample means in column 1 (name it mean) and write 5 as the sample size in the second column (name it size).
- 3. Repeat this process for each group of observations that have sample size equal to 15 and variance 10. Add the means to the same table from the previous step.
- 4. Repeat this process for each group of observations that have sample size equal to 25 and variance 10.
- 5. Repeat this process for each group of observations that have sample size equal to 100 and variance 10.

That is fill out the following table.

nean	size
	5

mean	siz
	5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5

mean	size
	100
	100
	100
	100
	100
	100
	100

- 6. Save your new table and import it with Stata. Alternatively use sample_means_2016.txt.
- 7. Plot the distribution of the sample mean separately for each unique sample size (*size* variable). What happens to the distribution of the sample mean as you increase the sample size?
- 8. You can optionally repeat this process for the observations drawn from a Normal distribution with variance 1 or 5 (don't mix them!). Can be useful too if you have a large class and want to divide them into 3.

Solutions

```
* Load the data
use normal_2016.dta, clear
* Calculate the means but then register them manually
bys group: summarize observation if variance == 10 & size == 5
bys group: summarize observation if variance == 10 & size == 15
bys group: summarize observation if variance == 10 & size == 25
bys group: summarize observation if variance == 10 & size == 100
* Calculate the means with programatic code (this is called looping)
* and save the results to the sample means 2016.txt file.
* For moe details to understand this code check
* http://www.ats.ucla.edu/stat/stata/faq/filewrite.htm
file open myresults using "sample_means_2016.txt", write replace
file write myresults "mean" _tab "size" _n
set more off
foreach samplesize of numlist 5 15 25 100 {
   foreach groupid of numlist 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 {
        summarize observation if variance == 10 & size == `samplesize' & group == `groupid'
        file write myresults (r(mean)) _tab "`samplesize'" _n
   }
}
file close myresults
set more on
* Load the sample means table
import delimited "sample_means_2016.txt", clear
* Make the graphics
histogram mean, by(size)
graph export "hist3.png", replace
graph box mean, by(size)
graph export "box3.png", replace
```

```
* Optional qnorm plots
qnorm mean if size == 5
graph export "qnorm_mean_size5.png", replace
qnorm mean if size == 15
graph export "qnorm_mean_size15.png", replace
qnorm mean if size == 25
graph export "qnorm_mean_size25.png", replace
qnorm mean if size == 100
graph export "qnorm_mean_size100.png", replace
. * Load the data
. use normal_2016.dta, clear
(Written by R.
. * Calculate the means but then register them manually
. bys group: summarize observation if variance == 10 & size == 5
-> group = 1
  Variable | Obs Mean Std. Dev. Min Max
______
observation |
            5 .2589072 1.425755 -1.648349 1.861652
-> group = 2
  Variable | Obs Mean Std. Dev. Min Max
             5 -1.026144 3.231694 -5.056384 2.442126
------
\rightarrow group = 3
  Variable | Obs Mean Std. Dev. Min Max
______
             5 -.1037298
                          2.94163 -4.149247 3.451327
______
\rightarrow group = 4
  Variable | Obs Mean Std. Dev. Min Max
             5 .9424025 3.402822 -4.129304 4.411133
observation |
-> group = 5
  Variable | Obs Mean Std. Dev. Min Max
______
```

observation						
-> group = 6						
Variable						
observation	5	0983589	3.102556	-2.976197	3.934591	
-> group = 7						
Variable						
observation						
group = 8						
Variable						
observation	5	-3.633132	3.384315	-9.172001	0727315	
group = 9						
Variable				Min	Max	
observation	5	.6918162	3.217179			
group = 10						
Variable		Mean		Min	Max	
observation				-6.270437	4.448832	
group = 11						
Variable						
observation	5	-1.625534	.9935249	-2.809964	0869543	
-> group = 12						
Variable						
observation	5	3757984	1.047461	-1.804874	.5804903	
-> group = 13						

Variable	l Obs						
observation	-						
-> group = 14							
Variable	Obs						
observation	5	.5663508	2.005641	-1.64762	3.094349		
-> group = 15							
Variable	0bs						
observation	5	.8156639	3.47456	-2.588775	6.61. bys	s group: summarize	observation :
-> group = 1							
Variable	Obs						
	15						
-> group = 2							
Variable	Obs	Mean	Std. Dev.	Min	Max		
observation	15	.3319717	3.830569	-9.122918	5.582776		
group = 3							
Variable	Obs						
observation	15	1.167319	3.629757	-6.140796			
group = 4							
	Obs						
observation	15	.0111664	2.893321	-4.243984	6.554693		
group = 5							
	0bs						
observation	•						

-> group = 6						
Variable	Obs	Mean	Std. Dev.	Min	Max	
observation	15	.0140114	2.937627	-7.251023	4.63186	
-> group = 7						
Variable						
observation	15	2918965	2.854115	-6.517677	4.405031	
-> group = 8						
Variable	Obs	Mean	Std. Dev.	Min	Max	
observation						
-> group = 9						
Variable						
observation	15	1541863	2.896933	-4.935158	4.383737	
-> group = 10						
Variable	Obs	Mean	Std. Dev.	Min	Max	
observation						
-> group = 11						
Variable				Min	Max	
observation				-7.201359	6.894349	
-> group = 12						
Variable						
observation						
Variable						
observation						

-> group = 14					
Variable		Mean			
observation					
		Mean			
observation					
. bys group: su					
group = 1					
Variable	Obs	Mean	Std. Dev.	Min	Max
observation	25	-1.156035	3.230117	-7.048663	6.613952
		Mean			
observation		5435422			
		Mean			
observation		.3093568			
Variable	Obs	Mean	Std. Dev.	Min	Max
observation	25	. 4389188	3.088024	-4.856911	7.297013
Variable	Obs	Mean	Std. Dev.	Min	Max
observation	25	-1.166001	3.340921	-9.188413	5.386401

-> group = 6					
Variable					
observation	25	.5979134	3.313374	-6.395189	8.264582
-> group = 7					
Variable					
observation	25	.5478882	3.072026	-5.974334	6.27983
-> group = 8					
Variable		Mean		Min	Max
observation	25	1541815	4.551343		
-> group = 9					
Variable					
observation					
-> group = 10					
Variable		Mean	Std. Dev.	Min	Max
observation		9920868	2.826544	-6.322566	3.63698
group = 11					
Variable		Mean	Std. Dev.	Min	Max
observation		718968	3.872108	-9.71908	6.369649
-> group = 12					
Variable	Obs	Mean	Std. Dev.	Min	Max
observation					
-> group = 13					
Variable					
observation					

-> group = 14					
	Obs				
	25				
5 -	Obs	Moan	Std Dow	Min	Мач
	+				
observation	l 25	2420888	3.75885	-7.558431	7.056714
. bys group:	summarize obs	servation if	variance ==	10 & size =	= 100
-> group = 1		.,	G. 1. D		
	Obs +				
	100				
-> group = 2					
	l Obs				
observation	100				
-> group = 3					
	Obs				
observation	100	1207133	3.082861	-7.415754	8.970666
Variable	l Obs	Mean	Std. Dev.	Min	Max
observation	+ 100	.1647736	2.960644	-7.550699	8.227775
-> group = 5					
Variable	Obs +	Mean	Std. Dev.	Min	Max
observation	100	1129649	3.181103	-7.347415	6.563672

-> group = 6					
Variable				Min	Max
observation				-9.151102	9.836253
-> group = 7					
Variable				Min	Max
observation				-8.431155	8.177943
-> group = 8					
Variable				Min	Max
observation				-6.662359	8.513903
-> group = 9					
Variable					
observation					
-> group = 10					
Variable		Mean	Std. Dev.	Min	Max
observation		.4189958	3.202346	-9.925777	6.724799
-> group = 11					
Variable		Mean	Std. Dev.	Min	Max
observation		. 1742163	3.468608	-7.731831	8.916285
-> group = 12					
Variable					
observation	100	.4067078	3.203779	-5.795245	8.824605
-> group = 13					
Variable					
observation					

```
\rightarrow group = 14
   Variable | Obs Mean Std. Dev. Min Max
  ______
               100 -.1594001 3.16435 -8.615282 7.327055
-> group = 15
   Variable | Obs Mean Std. Dev. Min Max
              100 .3011528 3.201704 -7.442526 7.15992
observation |
. * Calculate the means with programatic code (this is called looping)
. * and save the results to the sample_means_2016.txt file.
. * For moe details to understand this code check
. * http://www.ats.ucla.edu/stat/stata/faq/filewrite.htm
. file open myresults using "sample_means_2016.txt", write replace
. file write myresults "mean" _tab "size" _n
. set more off
. for
each samplesize of numlist 5 15 25 100 \{
 2. foreach groupid of numlist 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 {
         summarize observation if variance == 10 & size == `samplesize' & group == `g
 3.
> roupid'
 4.
        file write myresults (r(mean)) _tab "`samplesize'" _n
 5.
 6. }
  Variable | Obs Mean Std. Dev. Min Max
observation | 5 .2589072 1.425755 -1.648349 1.861652
   Variable | Obs Mean Std. Dev. Min Max
                5 -1.026144 3.231694 -5.056384 2.442126
observation |
   Variable | Obs Mean Std. Dev. Min Max
observation | 5 -.1037298 2.94163 -4.149247 3.451327
   Variable | Obs Mean Std. Dev. Min Max
                 5 .9424025
                               3.402822 -4.129304 4.411133
observation |
   Variable | Obs Mean Std. Dev. Min Max
```

5 -.3513 2.728499 -2.991597 4.036663

observation |

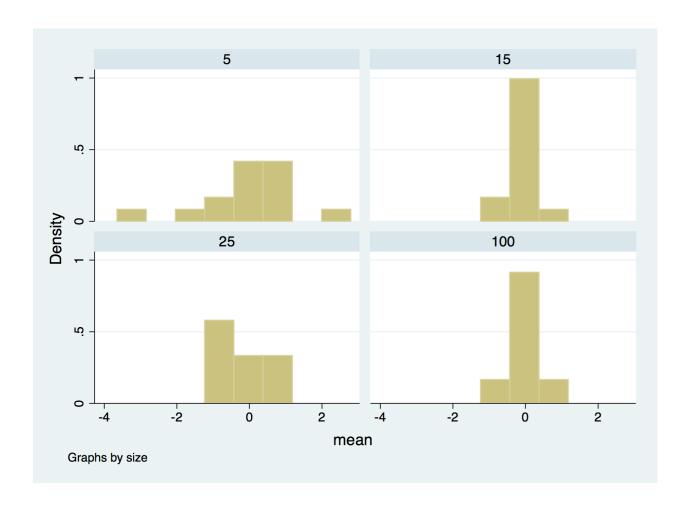
Variable	Obs	Mean	Std. Dev.	Min	Max
observation	5	0983589	3.102556	-2.976197	3.934591
Variable	Obs	Mean	Std. Dev.	Min	Max
observation	5	-1.05307	1.599494	-2.660201	1.078001
Variable	Obs	Mean	Std. Dev.	Min	Max
observation	5	-3.633132	3.384315	-9.172001	0727315
Variable	Obs	Mean	Std. Dev.	Min	Max
observation	5	.6918162	3.217179	-2.668756	5.420033
Variable	Obs	Mean	Std. Dev.	Min	Max
observation	5	. 4025495	4.19843	-6.270437	4.448832
Variable	Obs	Mean	Std. Dev.	Min	Max
observation	5	-1.625534	. 9935249	-2.809964	0869543
Variable	Obs	Mean	Std. Dev.	Min	Max
observation	5	3757984	1.047461	-1.804874	.5804903
Variable	Obs	Mean	Std. Dev.	Min	Max
observation	5	1.999273	2.863914	-1.778234	6.0721
Variable	Obs	Mean	Std. Dev.	Min	Max
observation	5	. 5663508	2.005641	-1.64762	3.094349
Variable	0bs	Mean	Std. Dev.	Min	Max
observation		.8156639	3.47456	-2.588775	6.61666
Variable			Std. Dev.		
observation	•				
			Std. Dev.		
observation	•				
			Std. Dev.		
observation	•				
Variable	Obs	Mean	Std. Dev.	Min	Max

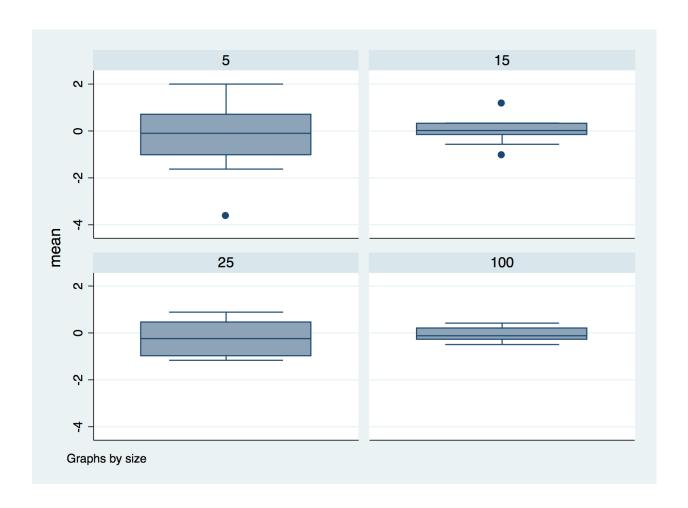
6.554693	-4.243984	2.893321	.0111664	15	observation
Max	Min	Std. Dev.	Mean	Obs	Variable
4.823821	-9.709027	3.63934	-1.030764	15	observation
Max	Min	Std. Dev.	Mean	Obs	Variable
4.63186	-7.251023	2.937627	.0140114	15	observation
Max	Min	Std. Dev.	Mean	Obs	Variable
4.405031	-6.517677	2.854115	2918965	15	observation
Max	Min	Std. Dev.	Mean	Obs	Variable
4.62143	-5.193361	3.038959	.111334	15	observation
Max	Min	Std. Dev.	Mean	Obs	Variable
4.383737	-4.935158	2.896933	1541863	15	observation
Max	Min	Std. Dev.	Mean	Obs	Variable
4.482293	-4.874509	3.327492	5667987	15	observation
Max	Min	Std. Dev.	Mean	Obs	Variable
6.894349	-7.201359	3.666581	.1455095	15	observation
Max	Min	Std. Dev.	Mean	Obs	Variable
7.140138	-5.488784	3.304709	.0024407	15	observation
Max	Min	Std. Dev.	Mean	Obs	Variable
5.038344	-6.626971	3.929371	.306066	15	observation
				Obs	Variable
					observation
					Variable
					observation
				Obs	Variable
					observation
				Obs	Variable
					observation

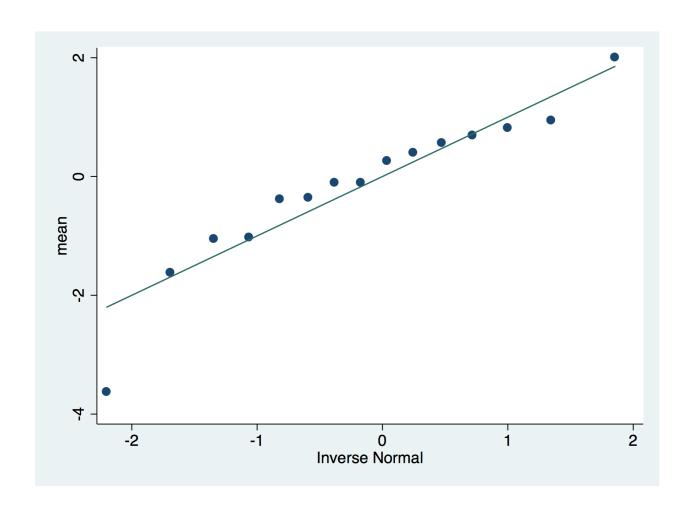
Variable	Obs	Mean	Std. Dev.	Min	Max
observation	25	.3093568	3.280309	-6.810264	7.966505
Variable	l Obs	Mean	Std. Dev.	Min	Max
observation	25	. 4389188	3.088024	-4.856911	7.297013
Variable	Obs	Mean	Std. Dev.	Min	Max
observation	25	-1.166001	3.340921	-9.188413	5.386401
Variable	Obs	Mean	Std. Dev.	Min	Max
observation	25	.5979134	3.313374	-6.395189	8.264582
Variable	Obs	Mean	Std. Dev.	Min	Max
observation	25	.5478882	3.072026	-5.974334	6.27983
Variable	Obs	Mean	Std. Dev.	Min	Max
observation	l 25	1541815	4.551343	-8.713041	9.584764
Variable	Obs	Mean	Std. Dev.	Min	Max
observation	I 25	.886669	3.869363	-7.518301	6.874853
Variable	0bs	Mean	Std. Dev.	Min	Max
observation	I 25	9920868	2.826544	-6.322566	3.63698
Variable	0bs	Mean	Std. Dev.	Min	Max
observation	l 25	718968	3.872108	-9.71908	6.369649
Variable	Obs	Mean		Min	
observation	•				
	0bs				
observation	25	-1.015911	3.490921	-6.750055	5.913665
	Obs				
observation	-	.3622803			
	Obs				
	l 25				
Variable	l Obs	Mean	Std. Dev.	Min	Max

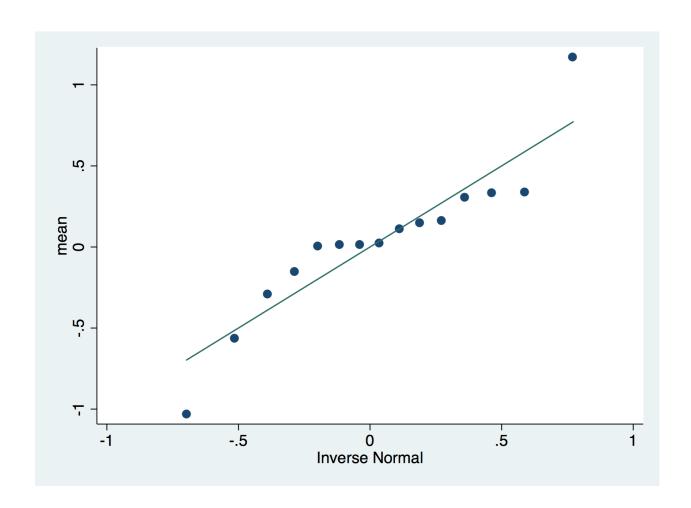
6.735005	-8.32965	3.147817	4383494	100	observation
Max	Min	Std. Dev.	Mean	Obs	Variable
6.329414	-8.11684	2.8653	2713836	100	observation
Max	Min	Std. Dev.	Mean	Obs	Variable
8.970666	-7.415754	3.082861	1207133	100	observation
Max	Min	Std. Dev.	Mean	Obs	Variable
8.227775	-7.550699	2.960644	.1647736	100	observation
Max	Min	Std. Dev.	Mean	Obs	Variable
6.563672	-7.347415	3.181103	1129649	100	observation
Max	Min	Std. Dev.	Mean	Obs	Variable
9.836253	-9.151102	3.691278	4950483	100	observation
Max	Min	Std. Dev.	Mean	Obs	Variable
8.177943	-8.431155	3.43671	.1835885	100	observation
Max	Min	Std. Dev.	Mean	Obs	Variable
8.513903	-6.662359	3.109602	2842181	100	observation
Max	Min	Std. Dev.	Mean	Obs	Variable
7.787577	-6.871425	3.160636	2147722	100	observation
Max	Min	Std. Dev.	Mean	Obs	Variable
6.724799	-9.925777	3.202346	.4189958	100	observation
Max	Min	Std. Dev.	Mean	Obs	Variable
					observation
Max	Min	Std. Dev.	Mean	Obs	Variable
					observation
Max	Min	Std. Dev.	Mean	Obs	Variable
					observation
				Obs	
					observation

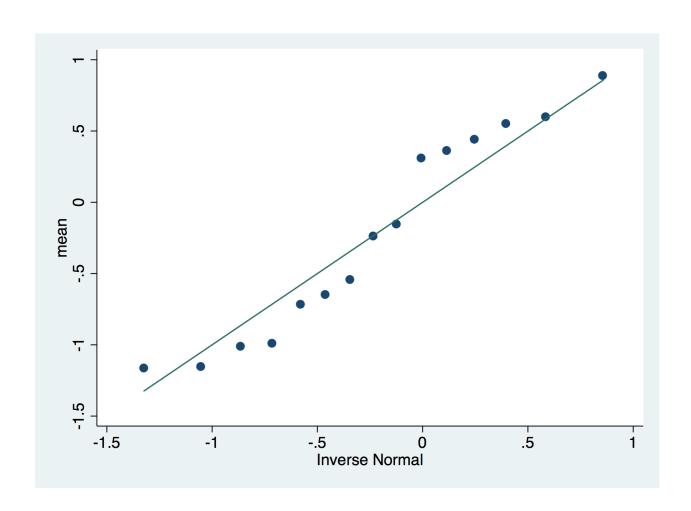
```
Variable | Obs Mean Std. Dev. Min Max
____________
observation |
                  100 .3011528 3.201704 -7.442526 7.15992
. file close myresults
. set more on
. * Load the sample means table
. import delimited "sample_means_2016.txt", clear
(2 vars, 60 obs)
. * Make the graphics
. histogram mean, by(size)
. graph export "hist3.png", replace
(file hist3.png written in PNG format)
. graph box mean, by(size)
. graph export "box3.png", replace
(file box3.png written in PNG format)
. * Optional qnorm plots
. qnorm mean if size == 5
. graph export "qnorm_mean_size5.png", replace
(file qnorm_mean_size5.png written in PNG format)
. qnorm mean if size == 15
. graph export "qnorm_mean_size15.png", replace
(file qnorm_mean_size15.png written in PNG format)
. qnorm mean if size == 25
. graph export "qnorm_mean_size25.png", replace
(file qnorm_mean_size25.png written in PNG format)
. qnorm mean if size == 100
. graph export "qnorm_mean_size100.png", replace
(file qnorm_mean_size100.png written in PNG format)
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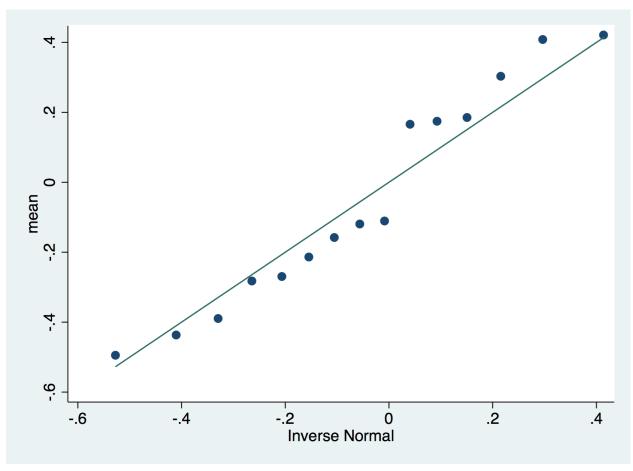












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