Use of Hawkes processes in a Cramér-Lundberg type model

Poisson processes project

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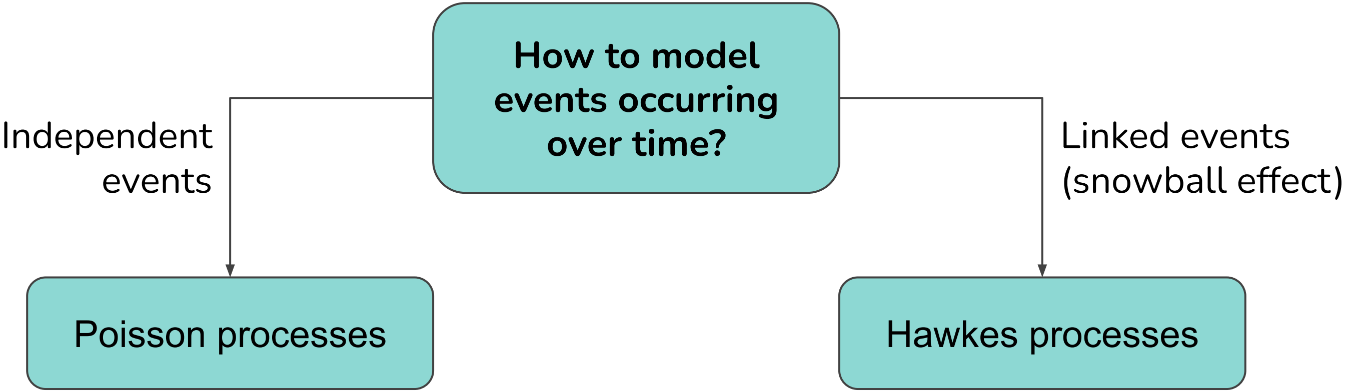
Introduction

Main aim of the project

Modelling events that occur in chain

Seismic events, retweets on a post,
drops in financial market...



Outline

I. Hawkes processes: mathematical aspects
II. Comparison of Poisson and Hawkes processes for the Cramér-Lundberg model
III. Fitting a Hawkes process on real data

I. Hawkes processes: mathematical aspects

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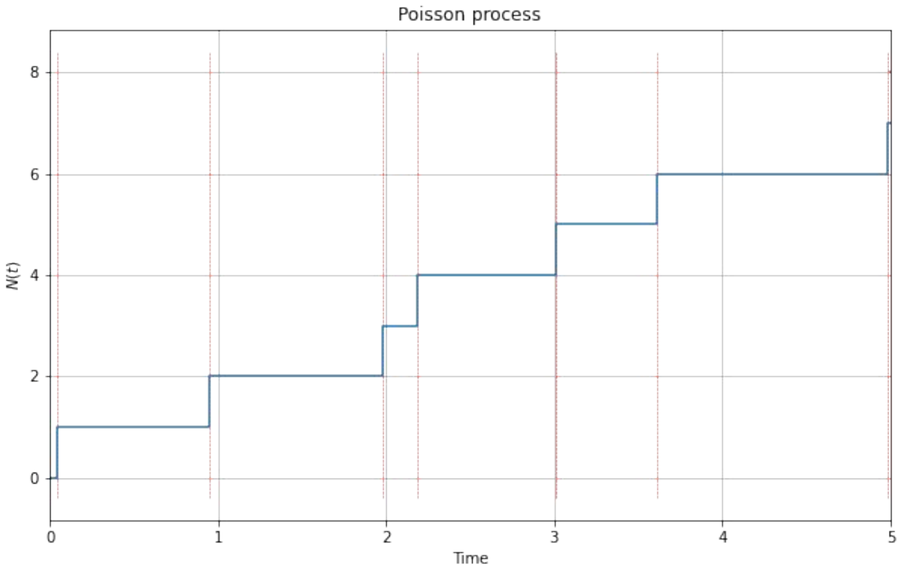
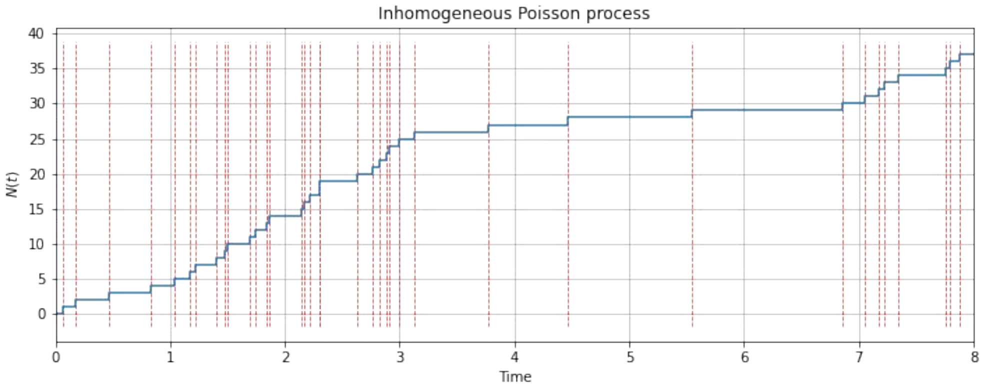
Counting process: models events occurring over time

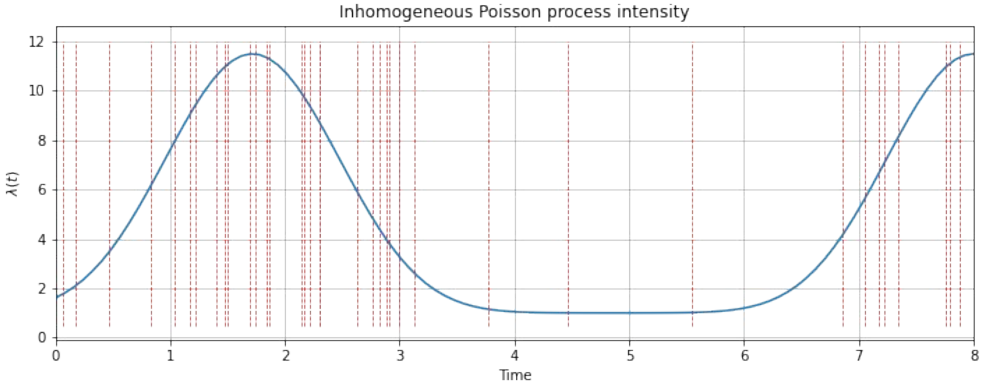


Homogeneous Poisson process :

- Well suited for **independent** events
- **Intensity** of the process: **λ** → linked to the frequency of events
- Intensity **constant**: frequency always the same over time

I. Hawkes processes: mathematical aspects



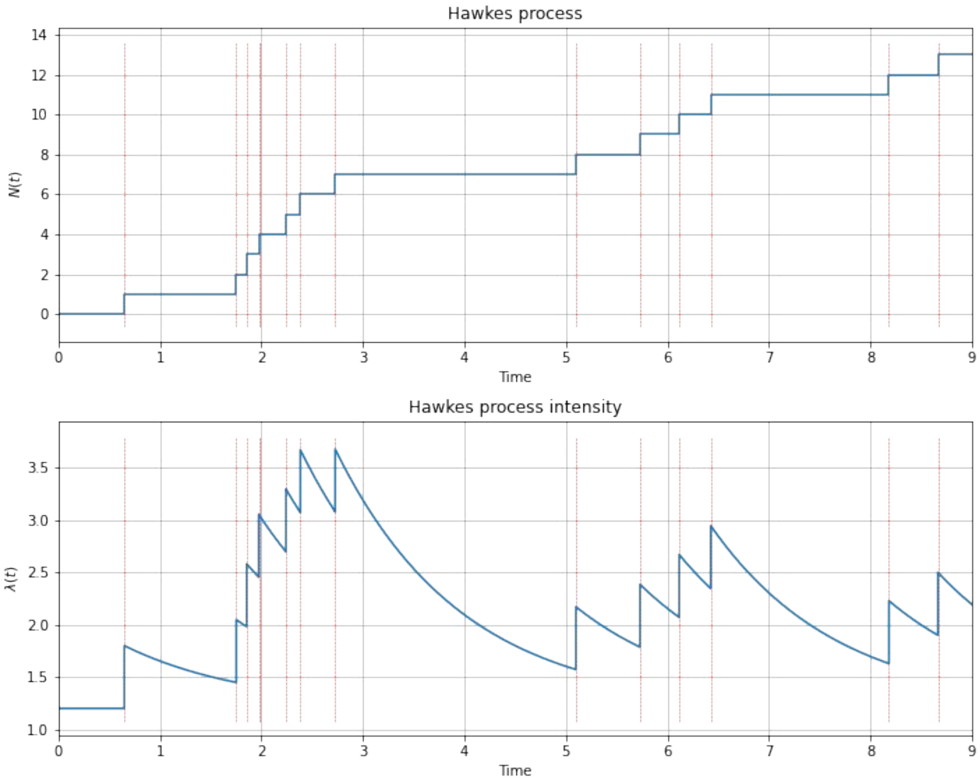


Inhomogeneous Poisson process :

- **Intensity** of the process **not** constant
- Frequency of events varying over time
- **Intensity function fixed and known**

$$\lambda(t) = \left[0.8 \times \cos\left(t - 8\right) + 1\right]^4 + 1$$

I. Hawkes processes: mathematical aspects



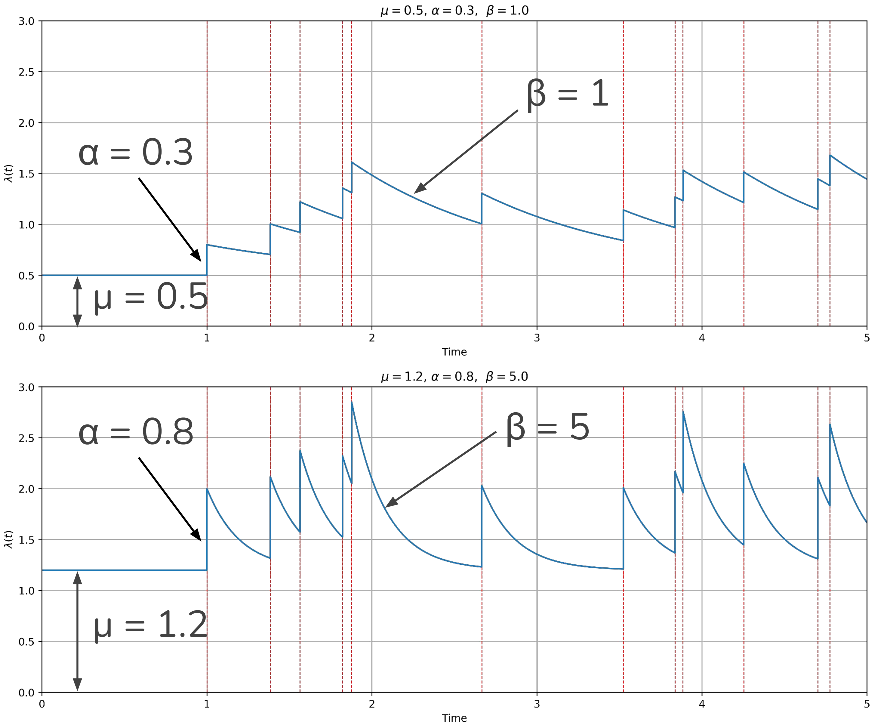
Hawkes process :

- **Intensity** of the process not constant
  → Frequency of events varying
- **Intensity function unknown**
- **Increment** in intensity at each event
  → **self exciting process**
- Progressive **exponential decay** after some time without event

$$\lambda(t) = \mu + \sum_{\{k|t_k<t\}} \alpha e^{-\beta(t-t_k)}$$

$$\mu > 0, \quad 0 < \alpha < \beta$$

I. Hawkes processes: mathematical aspects



Influence of parameters

$$\lambda(t) = \mu + \sum_{\{k|t_k<t\}} \alpha e^{-\beta(t-t_k)}$$

- **μ : baseline intensity**
  - Minimum intensity the process can have
- **α : increment size**
  - Reinforcement of the chain effect
- **β : decay rate**
  - Speed at which the process “forgets” events

II. Application in the context of the Cramér-Lundberg model

II. Comparison of Poisson and Hawkes processes for the Cramér-Lundberg model

- What is ruin theory ?
  - Mathematical model that reflects the risk incurred by an insurer to experience ruin, in other words bankruptcy.
  - The model to evaluate the risk R(t) is known as Cramér-Lundberg model.
  - It is defined as follow :

$$R(t) = u + pt - \sum_{i=1}^{N_t} X_i \quad \text{for } t \geq 0$$

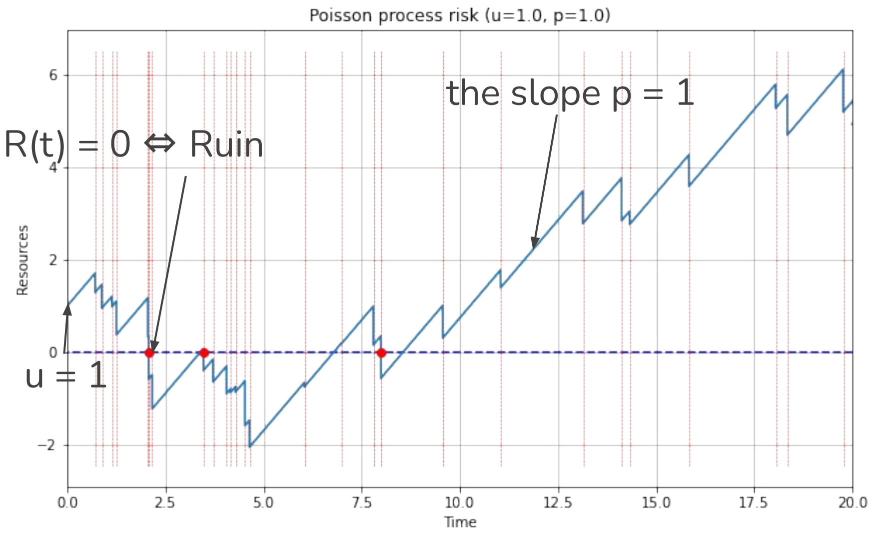
II. Comparison of Poisson and Hawkes processes for the Cramér-Lundberg model

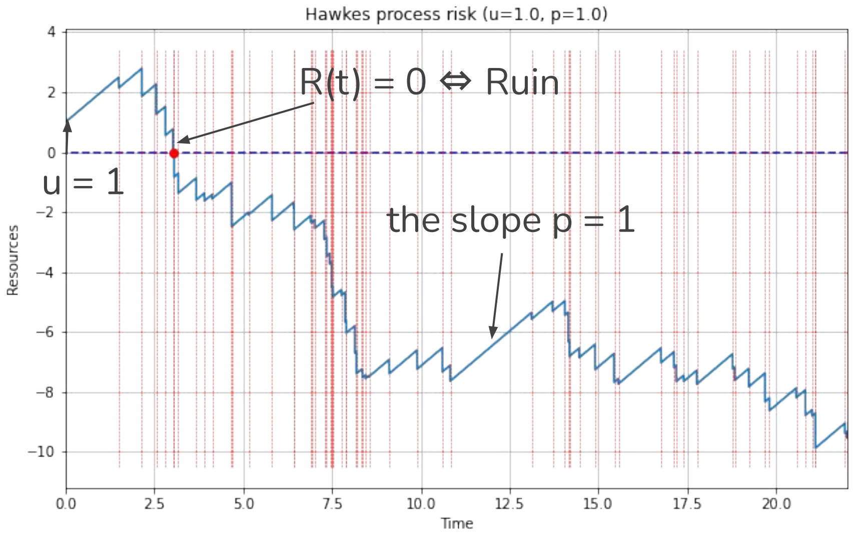
$$R(t) = u + pt - \sum_{i=1}^{N_t} X_i \quad \text{for } t \geq 0$$

- What defines the Cramér-Lundberg model ?
  - $u \geq 0$: initial capital at time t=0 ;
  - $p > 0$: constant premium rate ;
  - X_i : the amount of the i-th claim (real random variable) ;
  - N_t: the counting process (Poisson, Hawkes...)

II. Comparison of Poisson and Hawkes processes for the Cramér-Lundberg model

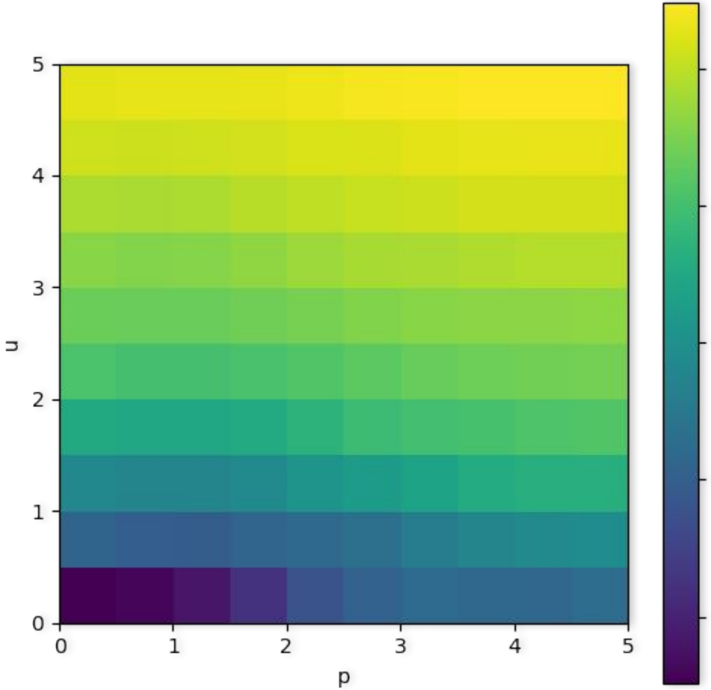
- And now what does the risk look like graphically ?





II. Comparison of Poisson and Hawkes processes for the Cramér-Lundberg model

- What is the influence of the parameters *u* (initial resources) and *p* (premium) on the arrival time of the ruin?



Evolution of the arrival time of the ruin

III. Fitting a Hawkes process on real data

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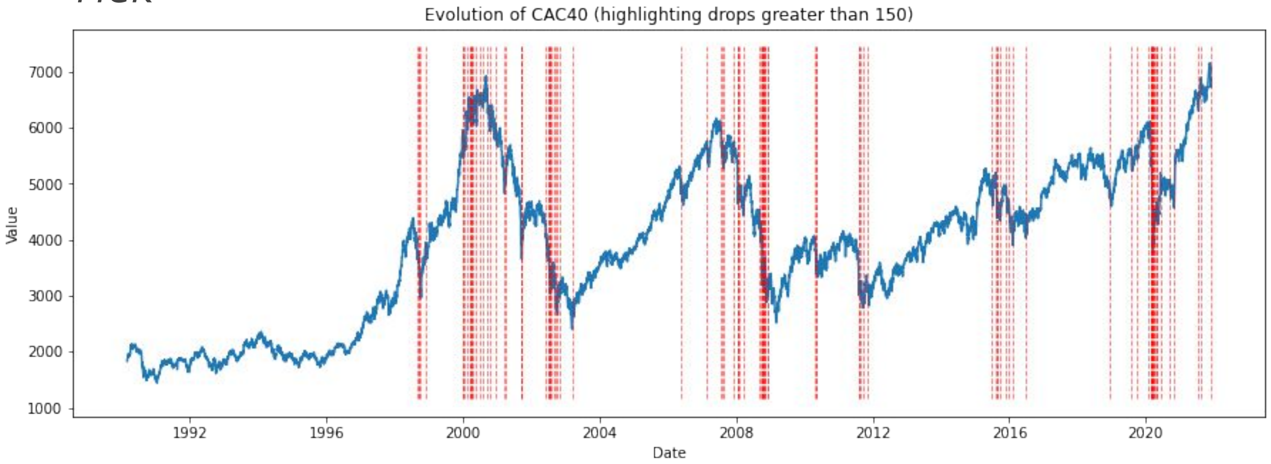
Aim : Fit a Hawkes processes on financial data (CAC 40).

- CAC 40 :
  - The main stock market index of the Paris stock exchange.
  - A bucket of 40 French companies selected from the 100 French companies with the highest trading volume.
- High variations during certain periods of time.
  - ➢ Can lead to loss of money
- Need of a model representing it's evolution.

III. Fitting a Hawkes model on real data

How did we fit a Hawkes processes on the CAC 40 ?

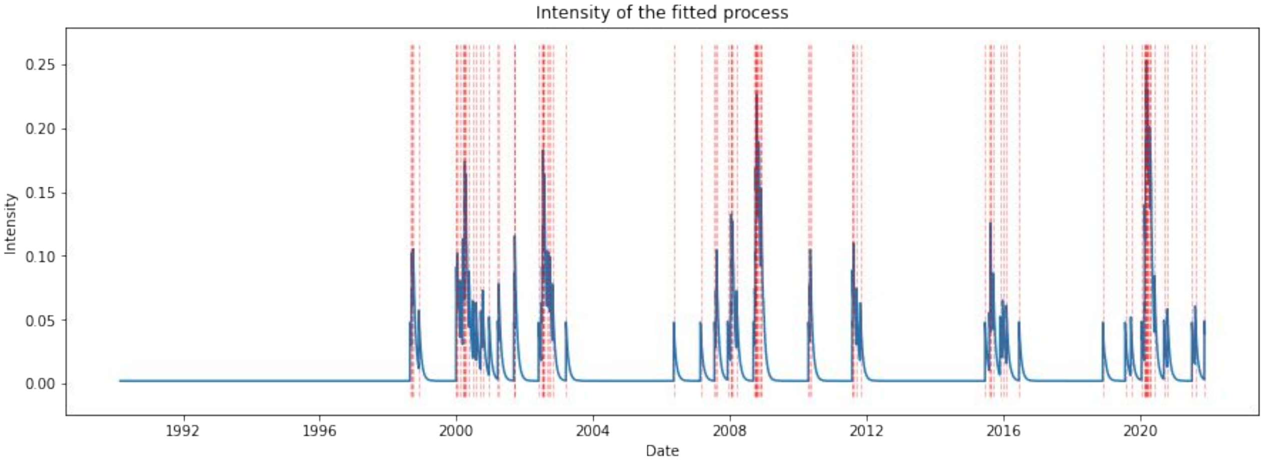
- **<u>Define the Events</u>**: When CAC40 drop by 150 in values
- **<u>Use *Tick*</u>**: Python library to be able to fit the Hawks process
- **<u>Tuning</u>**: Find the best parameters to the exponential kernel of *Tick*



III. Fitting a Hawkes model on real data

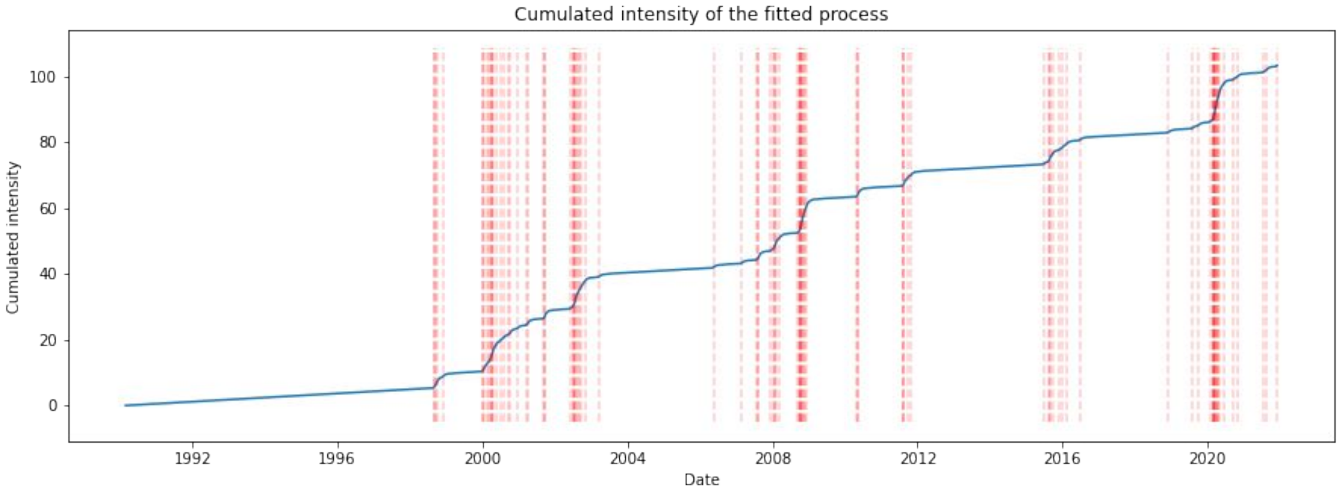
Results

- **Two major peak**: 2008 and in 2020.
  - Corresponds to the subprime and covid 19 crisis
- **Self exciting property**: Before reaching these peak
  - Reflects the snowball effect and the history of the process



III. Fitting a Hawkes model on real data

Visualization of the snowball effect.

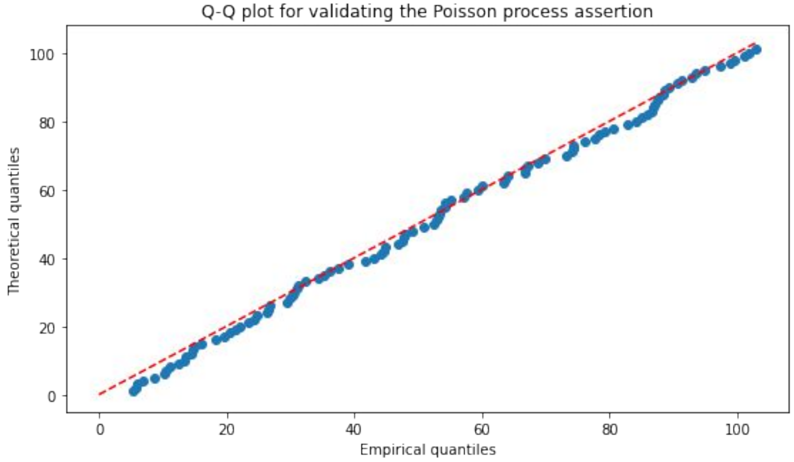


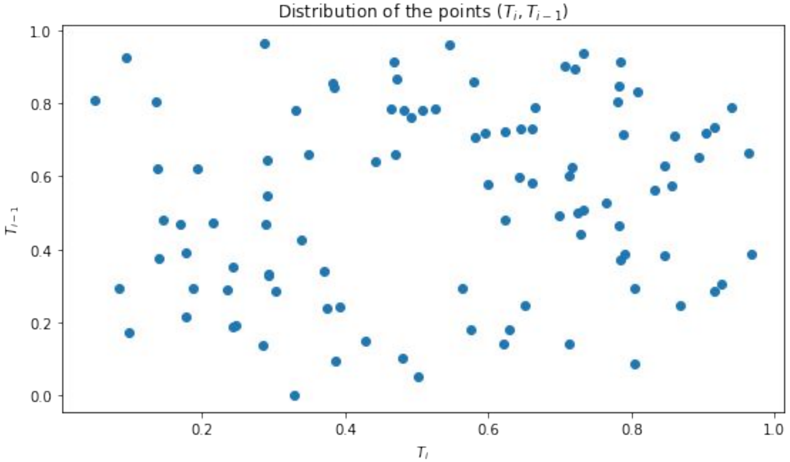
- Increase drastically when there is posteriors events
- For example : the year 2009 and 2020

III. Fitting a Hawkes model on real data

Validation on the Hawkes process property.

- Transformed realizations of Hawkes processes follow Poisson process.
- <u>The aim is to check:</u>
  - The law of the transformed events.
  - The independence of the points.





Conclusion

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Hawkes processes :

- Represent phenomena where history matters.
- Models the snowball effect :
  - Geological field: earthquakes.
  - Epidemiology: propagation of a virus.
  - Finance: big drops in market stocks
- Decrease the time of ruin in the Cramér-Lundberg model.

References

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Appendix: Thinning method

Numerical simulations of Hawkes processes: **thinning algorithm**