Cellular Automata for Sandpiles

Markus Frohne
markusfrohne@uni-bonn.de

Pascal Wolf
pwolf@uni-bonn.de

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Outline of talk

- Introduction:
  - Motivation
  - Self-organized criticality (**SOC**)
- Theory and simulation:
  - Cellular automata for sandpile dynamics: The `Bak-Tang-Wiesenfeld` (**BTW**) and custom model
- Analysis
- Results
  - Discussion
  - Comparison of models
- Summary

Introduction

Simulation results and analysis

Sandpile dynamics – Avalanche evolution

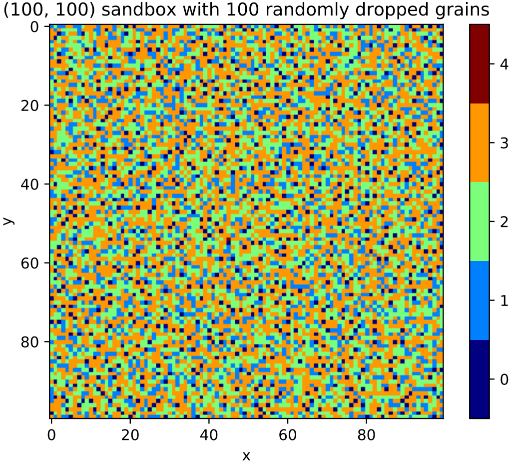
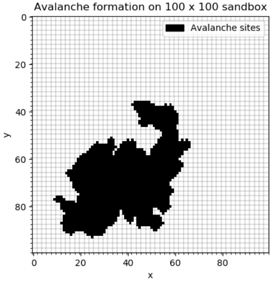
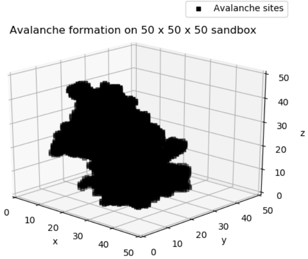


Fig. 1: Avalanche evolution in a 2 dimensional sandbox (BTW)

Sandpile dynamics – Avalanche formation



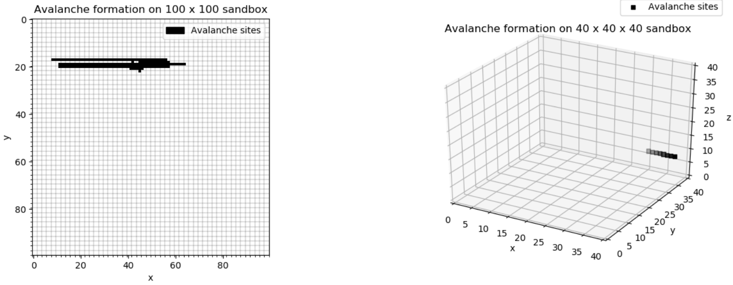
(a) 2 D



(b) 3 D

Fig. 2: Avalanche formation in the BTW model.

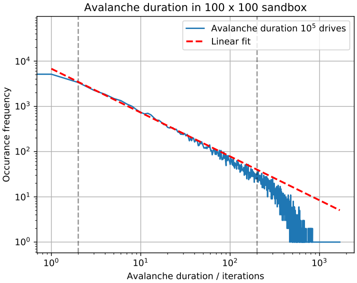
Sandpile dynamics – Avalanche formation



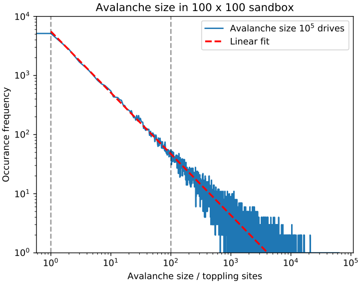
(a) 2 D (b) 3 D

Fig. 2: Avalanche formation in the custom model.

Sandpile dynamics – Power law scaling



(a) Duration



(b) Size

Fig. 3: Power law scaling behavior of avalanche duration and size of the BTW model.

Sandpile dynamics – Moment analysis

- For large lattice sizes L the following relation holds:

$$\left\langle O^n \right\rangle \propto L^{D(1+n-\rho)} \qquad \text{with} \qquad D\,(1+n-\rho) \equiv \sigma_n$$

where O is the PDF of the observable, n is the so-called *moment* and D, ρ are the critical exponents.

- Determination of the exponents of the 1st and 2nd moment σ_1 and σ_2 allows for the calculation of the set of critical exponents D, ρ:

$$D = \sigma_2 - \sigma_1 \quad , \quad \rho = \frac{2\sigma_2 - 3\sigma_1}{\sigma_2 - \sigma_1}$$

⇒ Simulate N samples of O and calculate the critical exponents D, ρ within several *bootstraps* to estimate their numerical value as well as standard deviation from it.