Atom Physics Review

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Date 3.31

Question 1

If one did the Franck-Hertz experiment on atomic hydrogen vapour, which lines would see if the maximum energy of the electrons were 12.5 eV? Give the λ values.

Solution:

Because

$$\Delta E = E_n - e_1 = \frac{E_1}{n^2} - E1 = \frac{1-n^2}{n^2}E1 \leq 12.5\ eV$$

And

$$n = 1, E_1 = -13.6\ eV$$

Therefore we can get

$$n \leq 3$$

Thus $n = 1, 2, 3$, and there are three lines:

1. from 3rd to 2nd:

$$\lambda_{32} = \frac{hc}{E_3 - E_2} = 657nm$$

2. from 2nd to 1st:

$$\lambda_{21} = \frac{hc}{E_2 - E_1} = 122nm$$

3. from 3rd to 1st:

$$\lambda_{31} = \frac{hc}{E_3 - E_1} = 102nm$$

Question 2

If an electron in hydrogen atom goes from the ground state to the state with $n = 3$

1. Give the energy of the atom absorbed in this transition.
2. Which lines would one see when the atom transmits from the excited state with $n = 3$ to the ground state?

Solution:

1.

$$\Delta E = E_3 - E_1 = \frac{E_1}{3^2} - E_1 = -13.6\ eV(\frac{1}{3^2} - 1) = 12.09\ eV$$

2. There are three lines:

   (a) from 3rd to 2nd:

   $$\lambda_{32} = \frac{hc}{E_3 - E_2} = 657nm$$

   (b) from 2nd to 1st:

   $$\lambda_{21} = \frac{hc}{E_2 - E_1} = 122nm$$

   (c) from 3rd to 1st:

   $$\lambda_{31} = \frac{hc}{E_3 - E_1} = 102nm$$

Date 5.7

Question 1

Explain the symbols for the $3^2D \Rightarrow 3^2P$ transition in sodium. How many lines can be expected in the spectrum?

Solution:

- for 3^2D
$$s = \frac{1}{2}, l = 2$$
so
$$j = \frac{5}{2}, \frac{3}{2} \Rightarrow 3^2D_{\frac{5}{2},\frac{3}{2}}$$

- for 3^2P
$$s = \frac{1}{2}, l = 1$$
so
$$j = \frac{3}{2}, \frac{1}{2} \Rightarrow 3^2P_{\frac{3}{2},\frac{1}{2}}$$

Because of **the selection rule for single electron:**

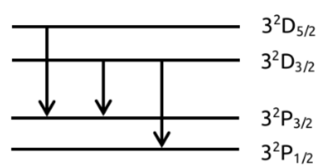
$$\begin{cases} \Delta l = \pm 1 \\ \Delta j = 0, \pm 1 \end{cases}$$

We can get three lines:

$$3^2D_{\frac{5}{2}} \Rightarrow 3^2P_{\frac{3}{2}}$$

$$3^2D_{\frac{3}{2}} \Rightarrow 3^2P_{\frac{3}{2}}$$

$$3^2D_{\frac{3}{2}} \Rightarrow 3^2P_{\frac{1}{2}}$$

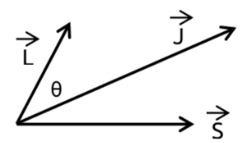


Question 2

Calculate the angle between the total and the orbital angular moments in a $^2F_{\frac{5}{2}}$ state.

Solution:

for $^2F_{\frac{5}{2}}$:



$$j = \tfrac{5}{2}, s = \tfrac{1}{2}, l = 3$$

$$L = \sqrt{l(l+1)}\tfrac{h}{2\pi} = 2\sqrt{3}\hbar$$

$$S = \sqrt{s(s+1)}\tfrac{h}{2\pi} = \tfrac{\sqrt{3}}{2}\hbar$$

$$J = \sqrt{j(j+1)}\tfrac{h}{2\pi} = \tfrac{\sqrt{35}}{2}\hbar$$

because of **the law of cosins:**

$$S^2 = L^2 + J^2 - 2LJ\cos\theta$$

then we can get

$$\begin{aligned}\cos\theta &= \frac{L^2 + J^2 - S^2}{2LJ} \\ &= \frac{12\hbar^2 + \frac{35}{4}\hbar^2 - \frac{3}{4}\hbar^2}{2 \times 2\sqrt{3} \times \frac{\sqrt{35}}{2}\hbar^2} \\ &= \frac{10}{\sqrt{105}}\end{aligned}$$

so

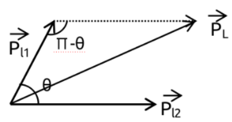
$$\theta = \arccos\frac{10}{\sqrt{105}}$$

Date 5.17

Question 1

In a ^3D state, that deduced from the electronic configuration 2p3d for the case of LS coupling.

1. Calculate the angle between $\overrightarrow{P_{l_1}}$ and $\overrightarrow{P_{l_2}}$
2. Calculate the angle between $\overrightarrow{P_{s_1}}$ and $\overrightarrow{P_{s_2}}$



1. for ^3D (Note: you don't need to calculate J):

$$S = 1, L = 2$$

for 2p:

$$s_1 = \frac{1}{2}, l_1 = 1$$

for 3d:

$$s_2 = \frac{1}{2}, l_2 = 2$$

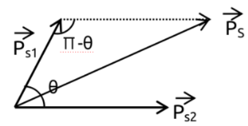
$$P_L = \sqrt{L(L+1)} = \sqrt{6}\hbar$$
$$P_{l_1} = \sqrt{l_1(l_1+1)}\hbar = \sqrt{2}\hbar$$
$$P_{l_2} = \sqrt{l_2(l_2+1)}\hbar = \sqrt{6}\hbar$$

$$\begin{aligned} P_L^2 &= P_{l_1}^2 + P_{l_2}^2 - 2P_{l_1}P_{l_2}\cos(\pi-\theta) \\ &= P_{l_1}^2 + P_{l_2} + 2P_{l_1}P_{l_2}\cos\theta \end{aligned}$$

therefore

$$\cos\theta = \frac{P_L^2 - P_{l_1}^2 - P_{l_2}^2}{2P_{l_1}P_{l_2}} = -\frac{\sqrt{3}}{6}$$

$$\theta = \arccos(-\frac{\sqrt{3}}{6})$$



2.

$$
\begin{aligned}
P_{s_1} &= \sqrt{s_1(s_1+1)}\hbar = \tfrac{\sqrt{3}}{2}\hbar \\
P_{s_2} &= \sqrt{s_2(s_2+1)}\hbar = \tfrac{\sqrt{3}}{2}\hbar \\
P_S &= \sqrt{S(S+1)}\hbar = \sqrt{2}\hbar
\end{aligned}
$$

$$
\begin{aligned}
P_S^2 &= P_{s_1}^2 + P_{s_2}^2 - 2P_{s_1}P_{s_2}\cos(\pi-\theta) \\
&= P_{s_1}^2 + P_{s_2}^2 + 2P_{s_1}P_{s_2}\cos\theta
\end{aligned}
$$

therefore

$$\cos\theta = \frac{P_S^2 - P_{s_1}^2 - P_{s_2}^2}{2P_{s_1}P_{s_2}} = \frac{1}{3}$$

$$\theta = \arccos\frac{1}{3}$$

Question 2

In LS coupling, determine the number of possible terms of an excited carbon atom with the electronic configuration 2p3d? Which possible term is the ground state?

Solution:

$$\begin{aligned} &for\ 2p : s_1 = \tfrac{1}{2}, l_1 = 1 \\ &for\ 3d : s_2 = \tfrac{1}{2}, l_2 = 2 \end{aligned} \Rightarrow \begin{cases} S = 1, 0 \\ L = 3, 2, 1 \end{cases}$$

$$\begin{cases} S = 0 \\ L = 3, 2, 1 \end{cases} \Rightarrow J = 3, 2, 1 \Rightarrow \begin{cases} {}^1F_3 \\ {}^1D_2 \\ {}^1P_1 \end{cases}$$

$$\begin{cases} S = 1 \\ L = 3 \end{cases} \Rightarrow J = 4, 3, 2 \Rightarrow {}^3F_{4,3,2}$$

$$\begin{cases} S = 1 \\ L = 2 \end{cases} \Rightarrow J = 3, 2, 1 \Rightarrow {}^3D_{3,2,1}$$

$$\begin{cases} S = 1 \\ L = 1 \end{cases} \Rightarrow J = 2, 1, 0 \Rightarrow {}^3P_{2,1,0}$$

So there are 12 possible terms.

According to **the Hund's rules**, ${}^3F_2$ is the ground state.

Date 5.22

Question 1

Explain the symbols for the $4^3F \Rightarrow 4^3D$ transition in sodium. How many lines can be expected in the spectrum?

Solution:

1. for 4^3F :

$$s_1 = 1, l_1 = 3, j_1 = 4, 3, 2$$

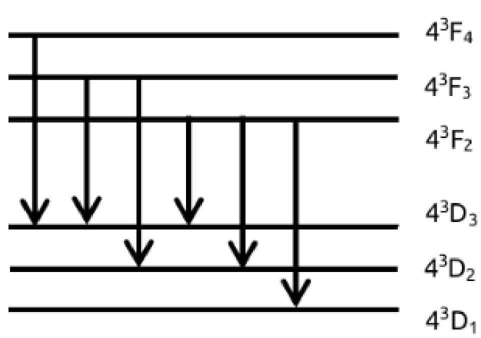
2. for 4^3D :

$$s_2 = 1, l_2 = 3, j_2 = 3, 2, 1$$

According to **the selection rules for two-electron atoms:**

$$\begin{cases} \Delta S = 0 \\ \Delta L = 0, \pm 1 \\ \Delta J = 0, \pm 1 (J = 0 \nRightarrow J = 0) \end{cases}$$

We can get 6 lines:



Question 2

1. Discuss a two-electron system with a 2p and a 3d electron for the case of **LS coupling** and show the number of possible state are the same as in **jj coupling.**
2. Calculate the ground state of the electron configuration 2p3d (in LS coupling).

Solution:

1. in LS coupling:

   - for 2p
   $$s_1 = \frac{1}{2},\ l_1 = 1,\ j_1 = \frac{3}{2}, \frac{1}{2}$$
   - for 3d
   $$s_2 = \frac{1}{2},\ l_2 = 2,\ j_2 = \frac{5}{2}, \frac{3}{2}$$

   $$\Rightarrow \begin{cases} S = 1, 0 \\ L = 3, 2, 1 \end{cases} \Rightarrow \begin{cases} {}^1F_3,\ {}^1D_2,\ {}^1P_1 \\ {}^3F_{4,3,2} \\ {}^3D_{3,2,1} \\ {}^3P_{2,1,0} \end{cases}$$

   So there are 12 states in LS coupling.

   In jj coupling:

   $$\begin{cases} s_1 = s_2 = \frac{1}{2} \\ l_1 = 1 \\ l_2 = 2 \end{cases} \Rightarrow \begin{cases} j_1 = \frac{1}{2}, \frac{3}{2} \\ \\ j_2 = \frac{3}{2}, \frac{5}{2} \end{cases} \Rightarrow \begin{cases} (\frac{1}{2}, \frac{3}{2})_{1,2} \\ \\ (\frac{1}{2}, \frac{5}{2})_{2,3} \\ \\ (\frac{3}{2}, \frac{3}{2})_{0,1,2,3} \\ \\ (\frac{3}{2}, \frac{5}{2})_{1,2,3,4} \end{cases}$$

   So there are also 12 states in jj coupling.
2. According to **the Hund's rules**, the ground state is ${}^3F_2$.

Question 3

Ignoring spin-orbit coupling, determine the number of possible terms of an excited carbon atom with the electronic configuration $1S^2 2S^2 2P 2P$? Which possible term is the ground state?

Solution:

2p2p are **Equivalent Electrons**, we can calculate 2p3p first

$$\begin{array}{l} s_1 = s_2 = \frac{1}{2}, \rightarrow S = 1, 0 \\ l_1 = l_2 = 1, \rightarrow L = 2, 1, 0 \end{array} \Rightarrow \begin{cases} {}^1D_2, {}^1P_1, {}^1S_0 \\ {}^3D_{3,2,1} \\ {}^3P_{2,1,0} \end{cases}$$

As for 2p2p, because of **the Paul Principle**: only tje atomic states which satisfied

$$L + S = 2n(n = 0, 1, 2, \dots)$$

can exist. They are the following states:

$$\begin{cases} {}^1D_2 \\ {}^1S_0 \\ {}^3P_{2,1,0} \end{cases}$$

Finally, according to **the Hund's Rules**, ground state is ${}^3P_0$

Question 4

Calculate the effective magnetic moments of the states ${}^1P_1$, ${}^2P_{\frac{3}{2}}$, ${}^4D_{\frac{1}{2}}$ and ${}^2S_{\frac{1}{2}}$.

Solution:

We need the following formulas:

$$g = 1 + \frac{j(j+1) - l(l+1) + s(s+1)}{2j(j+1)}$$

$$P_J = \sqrt{J(J+1)\hbar}$$

$$M_J = g\frac{e}{2m}P_J$$

1. for ${}^1P_1$:

$$s = 0, l = 1, j = 1$$

$$\Rightarrow g = 1, P_J = \sqrt{2}\hbar$$

$$\Rightarrow M_J = \frac{e\sqrt{2}}{2m}\hbar$$

2. for ${}^2P_{\frac{3}{2}}$:

$$s = \frac{1}{2}, l = 1, j = \frac{3}{2}$$

$$\Rightarrow g = \frac{4}{3}, P_J = \frac{\sqrt{15}}{2}\hbar$$

$$\Rightarrow M_J = \frac{\sqrt{15}e}{3m}\hbar$$

3. for $^4D_{\frac{1}{2}}$:

$$s = \frac{3}{2}, l = 2, j = \frac{1}{2}$$

$$\Rightarrow g = 0$$

$$\Rightarrow M_J = 0$$

4. for $^2S_{\frac{1}{2}}$:

$$s = \frac{1}{2}, l = 0, j = \frac{1}{2}$$

$$\Rightarrow g = 2, P_J = \frac{\sqrt{3}}{2}\hbar$$

$$\Rightarrow M_J = \frac{\sqrt{3}e}{2m}\hbar$$

Date 6.17

Question 1

1. Calculate the maximum components of the magnetic moments in the direction of the magnetic field for the vanadium (^4F), manganese(^6S) and iron(^5D), if beams of these atoms are split into $6, 6, 9$ parts in a Stern-Gerlach experiment.
2. What is the term symbol of the siglet state with a total splitting of $\bar{\nu} = 1.4\ cm^{-1}$ in a magnetic field $B_0 = 0.75$ tesla?

1. - for ^4F:

$$S = \frac{3}{2}, L = 3$$

$$2J + 1 = 6 \Rightarrow J = \frac{5}{2}$$

$$g = 1 + \frac{J(J+1) - L(L+1) + S(S+1)}{2J(J+1)} = \frac{36}{35}$$

$$M_J = \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}$$

$$M_{Jz} = M_J g \mu_B = \pm\frac{18}{35}\mu_B, \pm\frac{54}{35}\mu_B, \pm\frac{18}{7}\mu_B$$

$$M_{Jz}|_{max} = \frac{18}{7}\mu_B$$

- for ^6S:

$$S = \frac{5}{2}, L = 0$$

$$2J + 1 = 6 \Rightarrow J = \frac{5}{2}$$

$$g = 2$$

$$M_J = \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}$$

$$M_{Jz} = M_J g \mu_B = \pm\mu_B, \pm 3\mu_B, \pm 5\mu_B$$

$$M_{Jz}|_{max} = 5\mu_B$$

- for ^5D:

$$S = 2, L = 2$$

$$2J + 1 = 9 \Rightarrow J = 4$$

$$g = \frac{3}{2}$$

$$M_J = 0, \pm 1, \pm 2, \pm 3, \pm 4$$

$$M_{Jz} = 0, \pm\frac{3}{2}\mu_B, \pm 3\mu_B, \pm\frac{9}{2}\mu_B, \pm 6\mu_B$$

$$M_{Jz}|_{max} = 6\mu_B$$

2. Because of the **siglet state**, $2S + 1 = 1 \Rightarrow S = 0$
   Therefore $L = J$

$$g = 1 + \frac{J(J+1) - L(L+1) + S(S+1)}{2J(J+1)} = 1$$

As for total splitting:

$$\Delta E|_{max} = M_J|_{max} \mu_B B_0 = J\mu_B B_0$$

$$\Delta E|_{min} = M_J|_{min} \mu_B B_0 = -J\mu_B B_0$$

Before splitting, the energy is E_0
After splitting:

$$(E_0 + \Delta E|_{max}) - (E_0 + \Delta E|_{min}) = 2J\mu_B B_0$$

Because

$$\Delta E = hc\bar{\nu}$$

Then

$$2J\mu_B B_0 = hc\bar{\nu}$$

$$J = L = 2$$

(Sorry I don't know how to calculate this $\uparrow$)

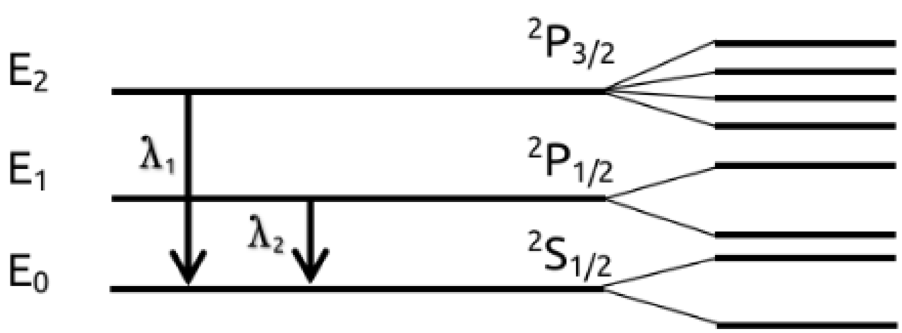
The state is ${}^1D_2$

Question 2

The special lines corresponding to the $3p3s$ transition in sodium have the wavelengths $\lambda_2 = 5895.9\ \mathring{A}$ and $\lambda_1 = 5889.6\ \mathring{A}$

1. Calculate the magnetic field strength at which the lowest Zeeman level of the $^2P_{\frac{3}{2}}$ term would coincide with the highest level of the $^2P_{\frac{1}{2}}$ term. If the conditions for the anomalous Zeeman Effect were still fulfile.
2. How large are the frequency differences between the outer two components of the two lines in a magnetic of the 1 tesla.

Solution:



1.
- for $^2P_{\frac{3}{2}}$:
$$J_2 = \frac{3}{2}, S_2 = \frac{1}{2}, L_2 = 1, g_2 = \frac{4}{3}$$
- for $^2P_{\frac{1}{2}}$:
$$J_1 = \frac{1}{2}, S_1 = \frac{1}{2}, L_1 = 1, g_1 = \frac{2}{3}$$
- for $^2S_{\frac{1}{2}}$
$$J_0 = \frac{1}{2}, S_0 = \frac{1}{2}, L_0 = 0, g_0 = 2$$

The energy of the lowest Zeeman Level of $^2P_{\frac{3}{2}}$: $E_2 + \Delta E_2|_{min}$

The energy of the highest Zeeman Level of $^2P_{\frac{1}{2}}$: $E_1 + \Delta E_1|_{max}$

According to the question:

$$E_2 + \Delta E_2|_{min} = E_1 + \Delta E_1|_{max}$$

- On the one hand
$$\begin{aligned} E_2 - E_1 &= \Delta E_1|_{max} - \Delta E_2|_{min} \\ &= M_{J_1} g_1 \mu_B B - M_{J_2} g_2 \mu_B B \\ &= [J_1 g_1 - (-J_2 g_2)]\mu_B B \\ &= \frac{7}{3}\mu_B \end{aligned}$$

- On the other hand
$$E_2 - E_1 = \frac{hc}{\lambda_1} - \frac{hc}{\lambda_2}$$

$$\Rightarrow B = \frac{3hc}{7\mu_B}(\frac{1}{\lambda_1} - \frac{1}{\lambda_2})$$

2. (a) $^2P_{\frac{3}{2}} \to^2 S_{\frac{1}{2}}$

$$\Delta\tilde{\nu}_1 = [M_{J_2} \cdot g_2 - M_{J_0} \cdot g_0]L$$
$$= [\pm\frac{1}{3}, \pm\frac{3}{3}, \pm\frac{5}{3}]L \quad (\textit{Rule } \Delta M_J = 0, \pm 1 \textit{ on page } 64)$$

The outer two components:

$$\frac{1}{\lambda_1'} - \frac{1}{\lambda_1} = \frac{5}{3}L$$

$$\frac{1}{\lambda_2'} - \frac{1}{\lambda_1} = -\frac{5}{3}L$$

The frequency difference:

$$\begin{aligned}\frac{c}{\lambda_1'} - \frac{c}{\lambda_2'} &= \frac{10c}{3}L \\ &= \frac{10c}{3} \cdot \frac{eB}{4\pi mc} \\ &= \frac{10B}{3} \cdot \frac{eh}{4\pi m} \cdot \frac{1}{h} \\ &= \frac{10B\mu_B}{3h} \\ &= 9\end{aligned}$$

(b) $^2P_{\frac{1}{2}} \to^2 S_{\frac{1}{2}}$

$$\Delta\tilde{\nu}_2 = [M_{J_1} \cdot g_1 - M_{J_0} \cdot g_0]L$$
$$= [\pm\frac{2}{3}, \pm\frac{4}{3}]L$$

The outer two components:

$$\frac{1}{\lambda_1'} - \frac{1}{\lambda_2} = \frac{4}{3}L$$

$$\frac{1}{\lambda_2'} - \frac{1}{\lambda_2} = -\frac{4}{3}L$$

The frequency difference:

$$\frac{c}{\lambda_1'} - \frac{c}{\lambda_2'} = \frac{8c}{3}L = \frac{8B\mu_B}{3h} = 9$$