Review of Chapter 1

Wave Nature of Light

EXAMPLE: Group and phase velocity

Consider a light traveling in a pure SiO_2 glass medium. If the wavelenth of light is 1um and the refractive index at this wavelength is 1.450, what is the phase velocity, group index (Ng) and group velocity (Vg)?

Solution:

The phase velocity is given by:

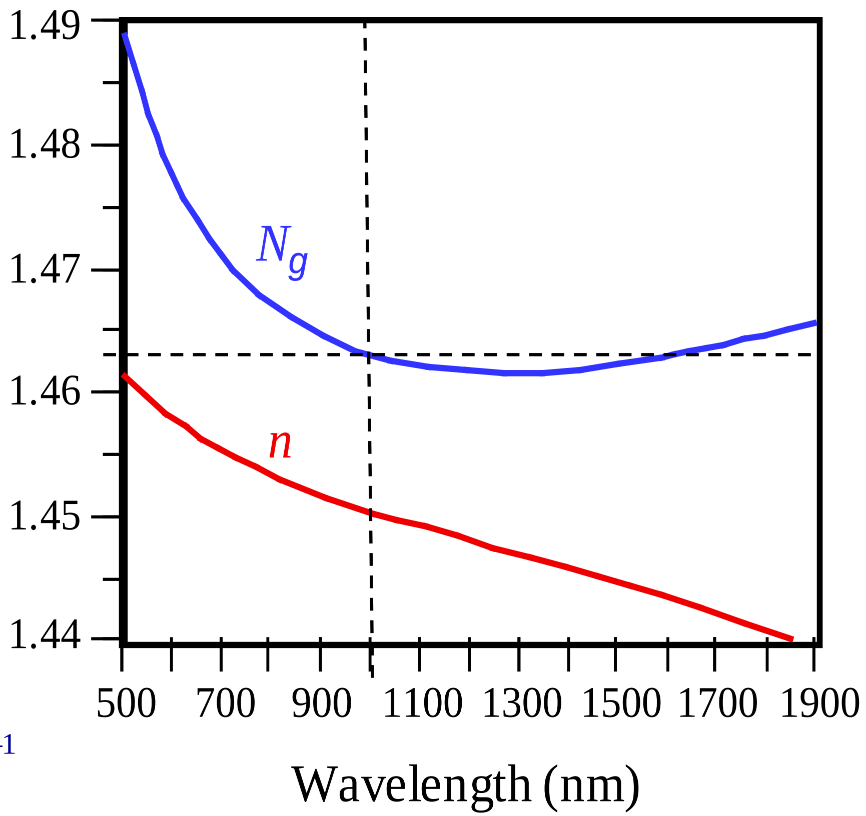
$$v = \frac{c}{n} = \frac{3\times10^8}{1.450} = 2.069\times10^8 m/s$$

From Fig.:

when $\lambda = 1\mu m$

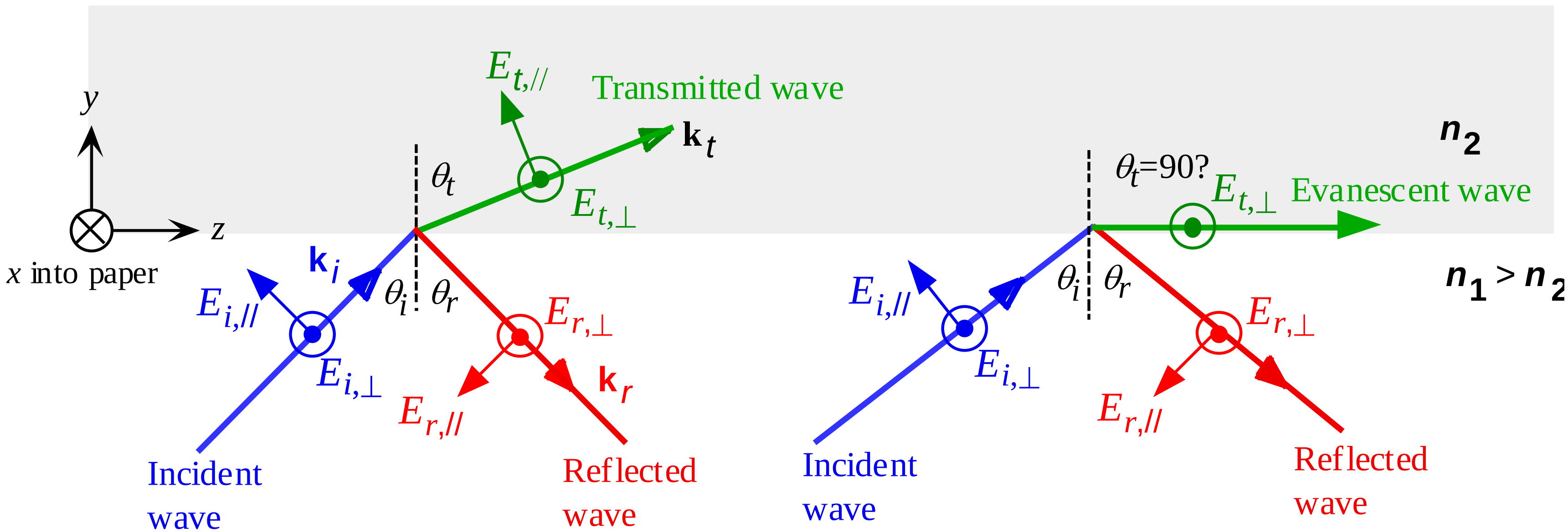
$$N_g = 1.463$$

thus $v_g = \frac{c}{N_g} = \frac{3\times10^8}{1.463} = 2.051\times10^8 ms^{-1}$



Refractive index n and the group index N_g of pure SiO_2 (silica) glass as a function of wavelength.

Fresnel's Equations



(a) $\theta_i < \theta_c$ then some of the wave is transmitted into the less dense medium. Some of the wave is reflected.

(b) $\theta_i > \theta_c$ then the incident wave suffers total internal reflection. However, there is an evanescent wave at the surface of the medium.

Light wave travelling in a more dense medium strikes a less dense medium. The plane of incidence is the plane of the paper and is perpendicular to the flat interface between the two media. The electric field is normal to the direction of propagation . It can be resolved into perpendicular (⊥) and parallel (//) components

at normal incidence: $\theta_i = 0$ $\quad r_{//} = r_{\perp} = \dfrac{n_1 - n_2}{n_1 + n_2}$

Brewster's angle

(Polarization angle)

$$\tan\theta_p = \frac{n_2}{n_1}$$

the phase change

of reflected wave

$$\begin{cases} \tan(\frac{1}{2}\phi_{\perp}) = \dfrac{\sqrt{\sin^2\theta_i - n^2}}{\cos\theta_i} \\ \tan(\frac{1}{2}\phi_{//} + \frac{1}{2}\pi) = \dfrac{\sqrt{\sin^2\theta_i - n^2}}{n^2\cos\theta_i} \end{cases}$$

EXAMPLE: Reflection at interface

Consider the reflection of light at normal incidence on a boundary between a glass medium of refractive index 1.5 and air of refractive index 1.

(1). If light is traveling from air to glass, what is the reflection coefficient and the intensity of the reflected light with respects to that of the incident light?

(2). If light is traveling from glass to air, what is the reflection coefficient and the intensity of the reflected light with respect to that of the incident light?

(1)

$$r_{//} = r_{\perp} = \frac{n_1 - n_2}{n_1 + n_2} = \frac{1 - 1.5}{1 + 1.5} = -0.2 \qquad R_{//} = R_{\perp} = \left| r_{\perp} \right|^2 = 0.04$$

(2)

$$r_{//} = r_{\perp} = \frac{n_1 - n_2}{n_1 + n_2} = \frac{1.5 - 1}{1.5 + 1} = 0.2 \qquad R_{//} = R_{\perp} = \left| r_{//} \right|^2 = 0.04$$

Review of Chapter 2

Dielectric Waveguides and Optical Fiber

Single and Multimode Waveguides

Maximum number of mode being allowed in the waveguide

$$m \leq \frac{2V - \phi}{\pi}$$

V-number

$$V = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2}$$

EXAMPLE V-Number and the number of mode

Estimate the number of modes that can be supported in a planar dielectric waveguide that is $100\mu m$ **wide and has** $n_1 = 1.490$ **and** $n_2 = 1.470$ **at the free-space source wavelength(** $\lambda = 1\mu m$ **) .**

Solution:

$$m \le \frac{2V - \phi}{\pi} = \frac{2V}{\pi} - \frac{\phi}{\pi}$$

$$V = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2} = \frac{2\times 3.14\times 50\times 10^{-6}}{1\times 10^{-6}}\sqrt{1.49^2 - 1.47^2}$$

$$= 76.44$$

$$\because \quad \frac{\phi}{\pi} < 1 \qquad \therefore \quad m \le \frac{2V}{\pi} = \frac{2\times 76.44}{3.14} = 48$$

Including $m = 0$ **:** $\therefore \quad \mathrm{M} = 48 + 1 = 49$

V-number of step index fiber

$$V = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2}$$

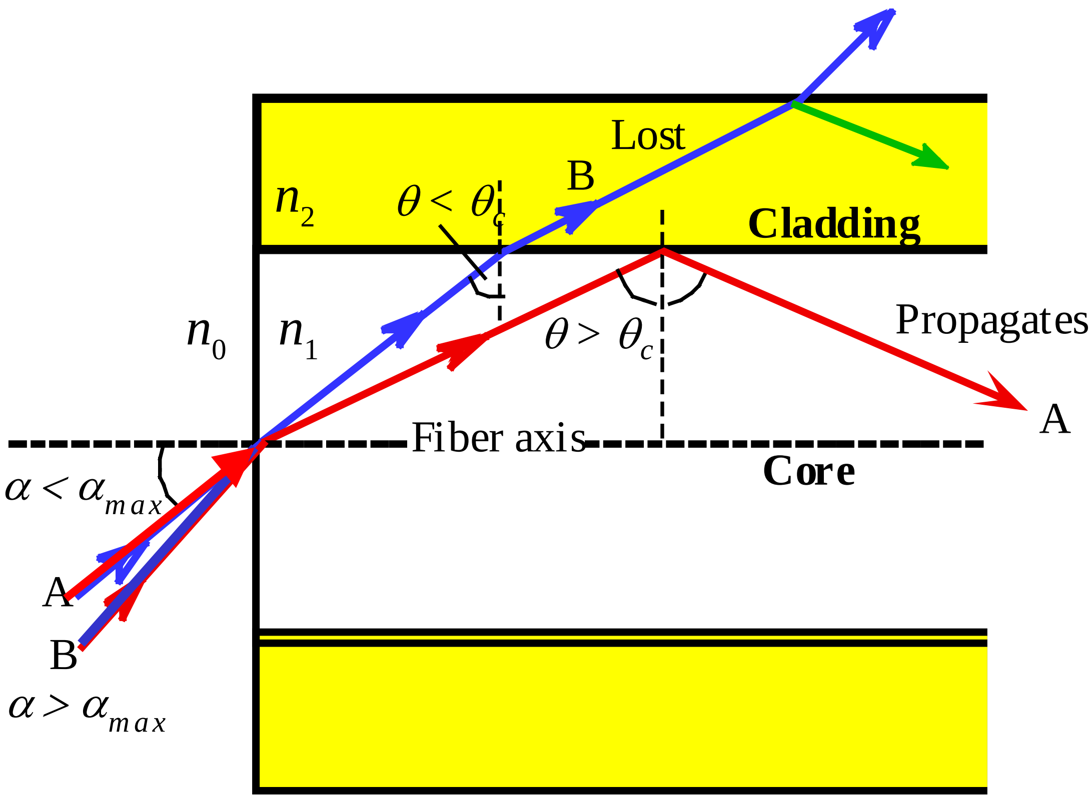
Cut-off wavelength of step index fiber

$$V_{cut-off} = \frac{2\pi a}{\lambda_c}\sqrt{n_1^2 - n_2^2} = 2.045$$

Normalized propagation constant

$$b = \frac{(\beta/k)^2 - n_2^2}{n_1^2 - n_1^2}$$

NUMERIAL APERTURE



Maximum acceptance angle α_{max} is that which just gives total internal reflection at the core-cladding interface, i.e. when $\alpha = \alpha_{max}$ then $\theta = \theta_c$. Rays with $\alpha > \alpha_{max}$ (e.g. ray B) become refracted and penetrate the cladding and ar eventually lost.

maximum acceptance angle $\alpha_{\max}$

Snell's law: $$\frac{\sin\alpha_{\max}}{\sin(90^0-\theta_c)}=\frac{n_1}{n_2}$$

$$\theta_c=\frac{n_2}{n_1} \qquad \sin\alpha_{\max}=\frac{\sqrt{n_1^2-n_2^2}}{n_0}$$

Numerical aperture: $NA=\sqrt{n_1^2-n_2^2}$

maximum acceptance angle: $\sin\alpha_{\max}=\dfrac{NA}{n_0}$

V-number and NA: $V=\dfrac{2\pi a}{\lambda}NA$

EXAMPLE: Multimode fiber

Consider a multimode fiber with a core diameter of $100\mu m$ **, core refractive index of 1.480, and a cladding refractive index of 1.460. Consider operating this fiber at** $\lambda = 850nm$

(1). Calculate the V-number for the fiber.
(2). Calculate the wavelength below which the fiber becomes multimode.
(3). Calculate the numerical aperture.
(4). Calculate the maximum acceptance angle

(1) $$V = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2} = \frac{2\times 3.14\times 50\times 10^{-6}}{850\times 10^{-9}}\sqrt{1.48^2 - 1.46^2} =$$

(2) $$\lambda < \frac{2\pi a}{2.405}\sqrt{n_1^2 - n_2^2} = \frac{2\times 3.14\times 50\times 10^{-6}}{2.405}\sqrt{1.48^2 - 1.46^2} =$$

(3) $$NA = \sqrt{n_1^2 - n_2^2} = \sqrt{1.48^2 - 1.46^2} = 0.2425$$

(4) $$\sin\alpha_{\max} = \frac{NA}{n_0} = \frac{0.2425}{1} = 0.2425 \qquad \alpha_{\max} = 14^0$$

EXAMPLE: Doppler broadened linewidth

For an He-Ne laser, the Doppler broadened linewidth $\Delta\nu_{1/2}$ **of** $632.8nm$ **radiation is about** $1.51GHz$ **,Calculated the Doppler broadened width in the wavelength.**

Solution:

$$\lambda = \frac{c}{\nu} \qquad \nu = \frac{c}{\lambda} = \frac{3\times10^{8}}{632.8\times10^{-9}} = 4.74\times10^{14}$$

$$d\lambda = -\frac{c}{\nu^2}d\nu;$$

$$\frac{\Delta\lambda}{\Delta\nu} = \left|-\frac{c}{\nu^2}\right| = \frac{c}{\nu};$$

$$\Delta\lambda_{1/2} \approx \Delta\nu_{1/2}\frac{\lambda}{\nu} = 1.51\times10^{9}\frac{632.8\times10^{-9}}{4.74\times10^{14}} =$$

EXAMPLE: Modes in a laser

Consider an AlGaAs laser diode which has an optical cavity of length 200 microns. The peak radiation is at 870nm and the refractive index of InGaAsP is 3.7. The optical gain width is 6nm.

(1) What is the mode number *m* value of the peak radiation?

(2) What is the separation $\delta\lambda_m$ between the modes of cavity?

(3) How many modes are there in the cavity?

Modes in an optical cavity: $m\dfrac{\lambda}{2n} = L$; **wavelength:** $\lambda = 870\times10^{-9}\,m$

Cavity length: $L = 200\times10^{-6}\,m$; **index:** 3.7

linewidth: $\Delta\lambda_{1/2} = 6nm$

Solution:

(1) $m = \dfrac{2nL}{\lambda} = \dfrac{2\times3.7\times200\times10^{-6}}{870\times10^{-9}} = 1644\times10^{3} = 1644$

(2) $\delta\lambda_m = \lambda_m - \lambda_{m+1} = \dfrac{2nL}{m} - \dfrac{2nL}{m+1} = \dfrac{2nL}{m(m+1)} \approx \dfrac{2nL}{m^2} = \dfrac{2nL}{(2nL/\lambda)^2}$

$$= \frac{\lambda^2}{2nL} = \frac{(870\times10^{-9})^2}{2\times3.7\times200\times10^{-6}} = 0.547nm$$

(3) $Modes = \dfrac{\Delta\lambda_{1/2}}{\delta\lambda_m} = \dfrac{6nm}{0.547nm} = 10.97$ **So, there is 10 mode in the cavity**

EXAMPLE: laser output wavelength variations

Given that the refractive index n of GsAs has a temperature dependence $dn/dT = 1.5 \times 10^{-4} K^{-1}$ **estimate the change in the emitted wavelength 870nm per degree change in the temperature between mode hops.**

$$m\frac{\lambda}{2n} = L$$

$$\lambda = \frac{2nL}{m}$$

$$d\lambda = \frac{2L}{m} dn$$

$$\frac{d\lambda}{dT} = \frac{2L}{m}\frac{dn}{dT} = \frac{\lambda}{n}\frac{dn}{dT} = \frac{870nm}{3.7}(1.5 \times 10^{-4}) = 0.035nmK^{-1}$$