Homework #1

Problem

Use the SOR method to solve the linear system $Ax = b$, where the entries of A and b are

$$a_{i,j} = \begin{cases} 2i, & \text{when } j = i \text{ and } i = 1,2,...,N, \\ 0.5i, & \text{when } \begin{cases} j = i+2 \text{ and } i = 1,2,...,N-2, \\ j = i-2 \text{ and } i = 3,4,...,N, \end{cases} \\ 0.25i, & \text{when } \begin{cases} j = i+4 \text{ and } i = 1,2,...,N-4, \\ j = i-4 \text{ and } i = 5,6,...,N, \end{cases} \\ 0, & \text{otherwise,} \end{cases}$$

and $b_i = \pi$, for each $i = 1,...,N$.

- Set $N = 160$ (or larger) and run SOR for different $\omega = 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4$.
- Set the tolerance $TOL = 10^{-6}$ and using the norm $\left\| \mathbf{x}^{(k)} - \mathbf{x}^{(k-1)} \right\|_\infty$ as the stopping criterion.
- Output a table which shows the number of iterations and $\left\| \mathbf{x}^{(k)} - \mathbf{x}^{(k-1)} \right\|_\infty$ of final approximate solutions for different values of ω.