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BSF-skeleton: user manual

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Content

1. Main specifications and application scope

The BSF-skeleton is designed for creating parallel programs in C++ using the MPI library. The scope of the BSF-skeleton is cluster computing systems and iterative numerical algorithms of high computational complexity. The BSF-skeleton completely encapsulates all aspects that are associated with parallelizing a program on a cluster computing system. The source code of the BSF-skeleton is freely available on Github at https://github.com/leonid-sokolinsky/BSF-skeleton.

2. Theoretical basis

The theoretical basis of the BSF-skeleton is the BSF (Bulk Synchronous Farm) model of parallel computations [1], which allows predicting the scalability boundary* of a parallel algorithm/program at an early stage of its development.

The BSF-skeleton uses the master/worker (master/slave) paradigm to organize interaction between MPI processes (see Fig. 1). This means that worker processes can only exchange messages with the master process.

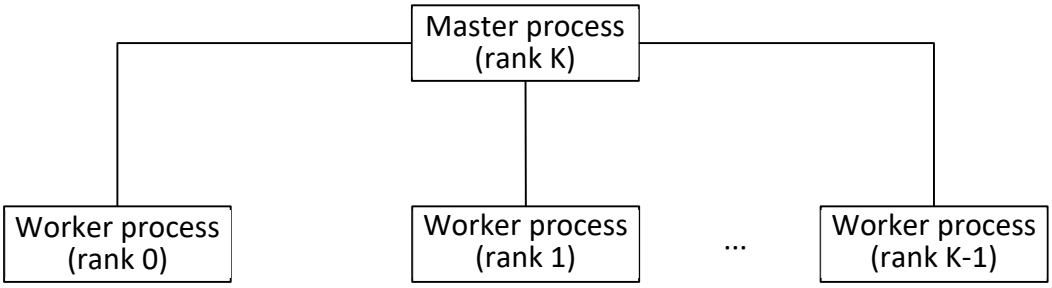


Fig. 1. Interaction of $K+1$ MPI processes in the BSF-skeleton.

To use the BSF-skeleton, you must represent your algorithm in the form of operations on lists using the higher-order functions *Map* and *Reduce* [2]. The higher-order function $Map(f,\mathbb{A})$ applies the function f to each element of list $\mathbb{A}=[a_1,\ldots,a_n]$ converting it to the list $\mathbb{B}=[f(a_1),\ldots,f(a_n)]$. The higher-order function $Reduce(\oplus,\mathbb{B})$ taking an associative binary operation $\oplus$ and a list $\mathbb{B}=[b_1,\ldots,b_n]$ as parameters calculates the element $b=b_1\oplus\ldots\oplus b_n$. One should use the template shown in Fig. 2 to represent an sequential algorithm. Let us comment on Algorithm 1.

Algorithm 1.

1: **input** $A, x^{(0)}$
2: $i := 0$
3: $B := Map\left(F_{x^{(i)}}, A\right)$
4: $s := Reduce(\oplus, B)$
5: $x^{(i+1)} := Compute\left(x^{(i)}, s\right)$
6: $i := i+1$
7: **if** $StopCond\left(x^{(i)}, x^{(i-1)}\right)$ **goto** 9
8: **goto** 3
9: **output** $x^{(i)}$
10: **stop**

Fig. 2. Generic BSF-algorithm template.

* The scalability boundary is the maximum number of processor nodes that speedup increases to.

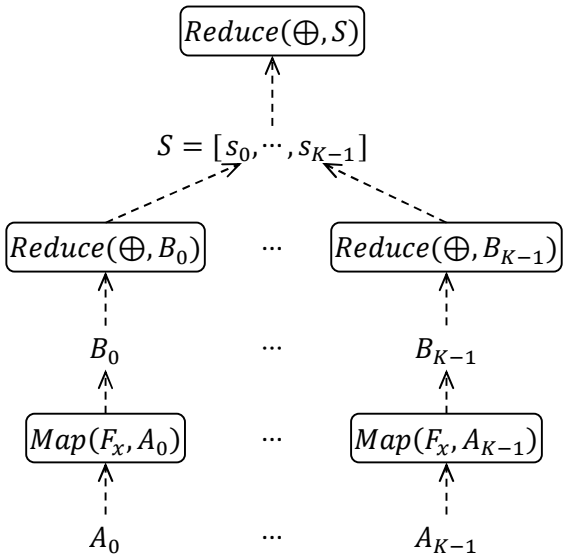


Fig. 3. BSF-skeleton parallelization schema.

The variable i denotes the iteration number; $x^{(0)}$ is an initial approximation; $x^{(i)}$ is the i-th approximation (the approximation can be a number, a vector, or any other data structure); A is the list of elements of a certain set $\mathbb{A}$, which represents the source data of the problem; $F_x : \mathbb{A} \to \mathbb{B}$ is a parameterized user function (the parameter x is the current approximation) that maps the set $\mathbb{A}$ to a set $\mathbb{B}$; B is a list of elements of the set $\mathbb{B}$ calculated by applying the function F_x to each element of the list A; $\oplus$ is an binary associative operation on the set $\mathbb{B}$. Step 1 reads input data of the problem and an initial approximation. Step 2 assigns the zero value to the iteration counter i. Step 3 calculates the list B by applying the higher-order function $Map(F_{x^{(i)}}, A)$. Step 4 assigns the result of the higher-order function $Redice(\oplus, B)$ to the intermediate variable $s \in \mathbb{B}$. Step 5 invocates the user function *Compute* that calculates the next approximation $x^{(i+1)}$ taking two parameters: the current approximation $x^{(i)}$ and the result s of the higher-order function *Reduce*. Step 6 increases the iteration counter i by one. Step 7 checks a termination criteria by invocating the Boolean user function *StopCond*, which takes two parameters: the new approximation $x^{(i)}$ and the previous approximation $x^{(i-1)}$. If *StopCond* returns true, the algorithm outputs $x^{(i)}$ as an approximate problem solution and stops working. Otherwise, the control is passed to Step 3 starting the next iteration.

The BSF-skeleton automatically parallelizes Algorithm 1 by splitting the list A into K sublists of equal length (± 1):

$$A = A_0 \mathbin{+\!\!+} \cdots \mathbin{+\!\!+} A_{K-1},$$

where K is the number of worker processes and $+\!\!+$ denotes the operation of list concatenation. This uses the parallelization scheme shown in Fig. 3. The result is the parallel algorithm, shown in Fig. 4. It includes $K+1$ parallel processes: one master process and K worker processes. In Step 2, the master process sends the current approximation $x^{(i)}$ to all worker processes. After that, every j-th worker process independently applies higher-order function *Map* and *Reduce* to its sublist (the steps 3 and 4). In the steps 3 and 4, the master process is idle. In Step 5, all worker processes send the partial foldings $s_0,\ldots,s_{K-1}$ to the master process. In the steps 6-9, the master process performs the following actions: executes the higher-order function *Reduce* over the list of partial foldings $[s_0,\ldots,s_{K-1}]$; invocates the user function *Compute* that calculates the next approximation; checks the termination criteria by using the Boolean user function *StopCond* and assigns its result to the Boolean variable *exit*. In the steps 6-9, the worker processes are idle. In Step 10, the master process sends the *exit* value to all worker processes. If the *exit* value is false, the master process and worker processes go to the next iteration, otherwise the master processes outputs the result and the computation stops. Note that, in the steps 2 and 10, all processes perform the implicit global synchronization.

Algorithm 2.

Master	*j-th worker (j=0,…,K-1)*
1: **input** $x^{(0)}$; $i := 0$	1: **input** A_j
2: $SendToAllWorkers(x^{(i)})$	2: $RecvFromMaster(x^{(i)})$
3:	3: $B_j := Map(F_{x^{(i)}}, A_j)$
4:	4: $s_j := Reduce(\oplus, B_j)$
5: $RecvFromWorkers(s_0, \dots, s_{K-1})$	5: $SendToMaster(s_j)$
6: $s := Reduce(\oplus, [s_0, \dots, s_{K-1}])$	6:
7: $x^{(i+1)} := Compute(x^{(i)}, s)$	7:
8: $i := i + 1$	8:
9: $exit := StopCond(x^{(i)}, x^{(i-1)})$	9:
10: $SendToAllWorkers(exit)$	10: $RecvFromMaster(exit)$
11: **if** $exit$ **goto** 2	11: **if** $exit$ **goto** 2
12: **output** $x^{(i)}$	12:
13: **stop**	13: **stop**

Fig. 4. BSF-skeleton parallelization template.

3. Source code structure of BSF-skeleton

The BSF-skeleton is a compilable but not executable set of files. This set is divided into two groups:

1) files with the "*BSF*" prefix contain problem-independent code and are not subject to changes by the user;
2) files with the "*Problem*" prefix are intended for filling in problem-dependent parts of the program by the user.

Descriptions of all source code files are given in Table 1.

Table 1. Source code files of the BSF-skeleton.

File	Description
Problem-independent code	
BSF-Code.cpp	Implementations of the *main* function and all problem-independent functions
BSF-Data.h	Problem-independent variables and data structures
BSF-Forwards.h	Declarations of the problem-independent functions
BSF-Include.h	The inclusion of problem-independent libraries
BSF-SkeletonVariables.h	Definitions of the skeleton variables (see Section 7)
BSF-ProblemFunctions.h	Declarations of the problem-dependent BSF functions (see Section 8)
BSF-Types.h	Definitions of problem-independent types
Problem-dependent code	
Problem-bsfCode.cpp	Implementations of the problem-dependent BSF functions (see Section 8)
Problem-bsfParameters.h	BSF-skeleton parameters (see Section 4)
Problem-bsfTypes.h	Predefined BSF types (see Section 5)
Problem-Data.h	Problem-dependent variables and data structures
Problem-Forwards.h	Declarations of the problem-dependent functions
Problem-Include.h	Inclusion of problem-dependent libraries
Problem-Parameters.h	Parameters of the problem
Problem-Types.h	Problem types

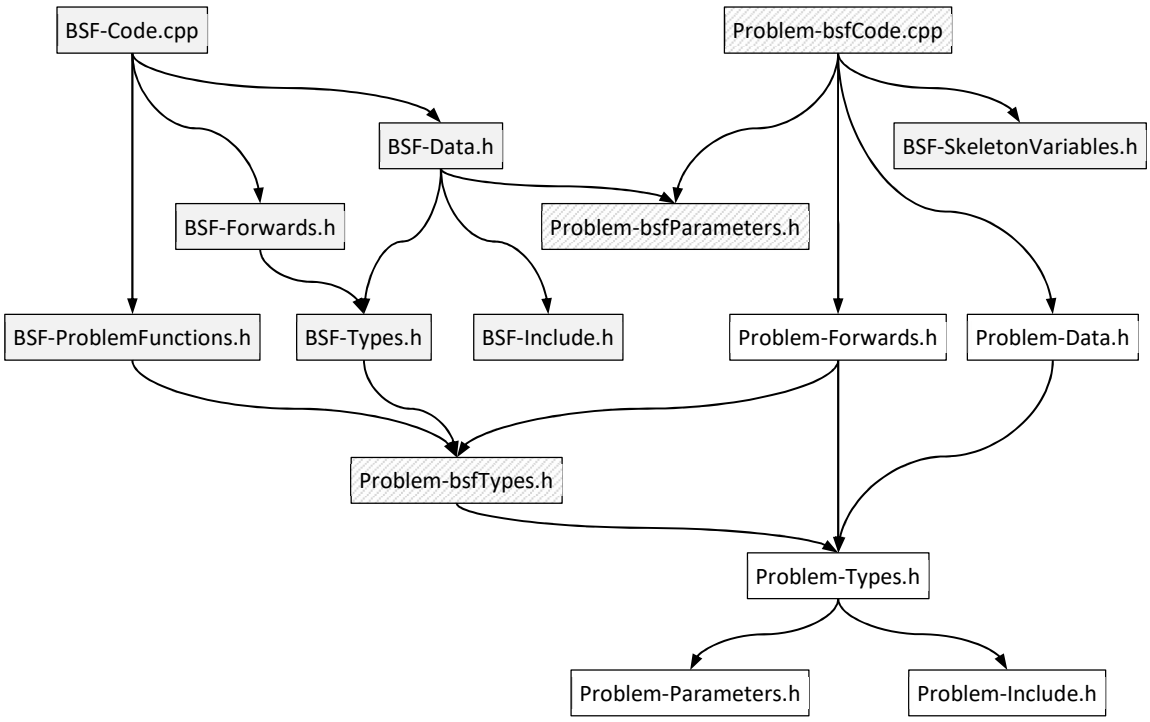


Fig. 5. Dependency graph of the source code files by the directive #include.

The dependency graph of the source code files by the directive *#include* is shown in Fig. 5. The gray rectangles indicate the code files that do not allow changes. The rectangles with striped shading indicate the code files containing predefined declarations that must be defined (filled in) by the user. The white rectangles indicate the code files that should be fully implemented by the user.

4. BSF-skeleton parameters

The BSF-skeleton parameters are declared as macroses in the file *Problem-bsfParameters.h*. They are used in the *BSF-Code.cpp* and should be set by the user. All these parameters are presented in Table 2.

Table 2. Predefined problem-dependent parameters.

ID	Description	Default value
PP_MAX_MPI_SIZE	Defines the maximum possible number of MPI processes (the result returned by the function *MPI_Comm_size* cannot exceed this number).	500
PP_BSF_PRECISION	Sets the decimal precision to be used to format floating-point values on output operations.	4
PP_BSF_ITER_OUTPUT	If this macros is defined, at the end of each *k*-th iteration, the master process will invocate the predefined BSF function *PC_bsf_IterOutput* that outputs intermediate results. The number *k* is defined by the macros *PP_BSF_TRACE_COUNT*.	#undef
PP_BSF_TRACE_COUNT	Defines the number *k* mentioned in the description of the macros *PP_BSF_ITER_OUTPUT*.	1
PP_BSF_MAX_JOB_CASE	Defines the maximum number of activities (jobs) in workflow minus 1. See "Workflow Support" in Section 11.	0
PP_BSF_OMP	If this macros is defined, the worker processes use *#pragma omp parallel for* to perform the higher-order function *Map*.	#undef
PP_BSF_NUM_THREADS	If this macros is defined, *omp parallel for* uses the specified number of threads to perform the higher-order function *Map*. If this macros is not defined, *omp parallel for* uses the maximum possible number of threads.	#undef

5. Predefined problem-depended BSF types

The predefined problem-depended BSF types are declared as data structures in the file *Problem-bsfTypes.h*. They are used in the *BSF-Code.cpp* and should be set by the user. All these types are presented in Table 3.

Table 3. Predefined BSF types (file *Problem-bsfTypes.h*).

Type ID	Data type	Description	Mandatory to fill in
PT_bsf_parameter_T	struct	Defines the structure (set of data elements) that is transferred by the master process to all the worker processes in Step 2 of Algorithm 2 (see Fig. 4) and includes the order parameters (usually the current approximation).	Yes
PT_bsf_mapElem_T	struct	Defines the record that represents an element in the map-list (list *A* in Algorithm 1).	Yes
PT_bsf_reduceElem_T	struct	Defines the record that represents an element in the reduce-list† (list *B* in Algorithm 1).	Yes
PT_bsf_reduceElem_T_1, PT_bsf_reduceElem_T_2, PT_bsf_reduceElem_T_3	struct	Alternative types of the reduce-list elements that are used to organize the workflow (see section 11).	No

6. Extended reduce-list

The BSF-skeleton appends to each element of the reduce-list the additional integer field called *reduceCounter*. This extended reduce-list is presented by the pointer *BD_extendedReduceList* declared in the *BSF-Data.h*. When performing the *Reduce* function (see *BC_ProcessExtendedReduceList* in Section 8.1), the elements that have this field equal to zero are ignored. For elements where *reduceCounter* is not zero, the values of the *reduceCounter* are added together. By default, the function *BC_WorkerMap* (see Section 8.1) sets the *reduceCounter* to 1. The user can set the value of this field to 0 by setting the parameter **success* of the function *PC_bsf_MapF* (see Section 8.2.3) to 0.

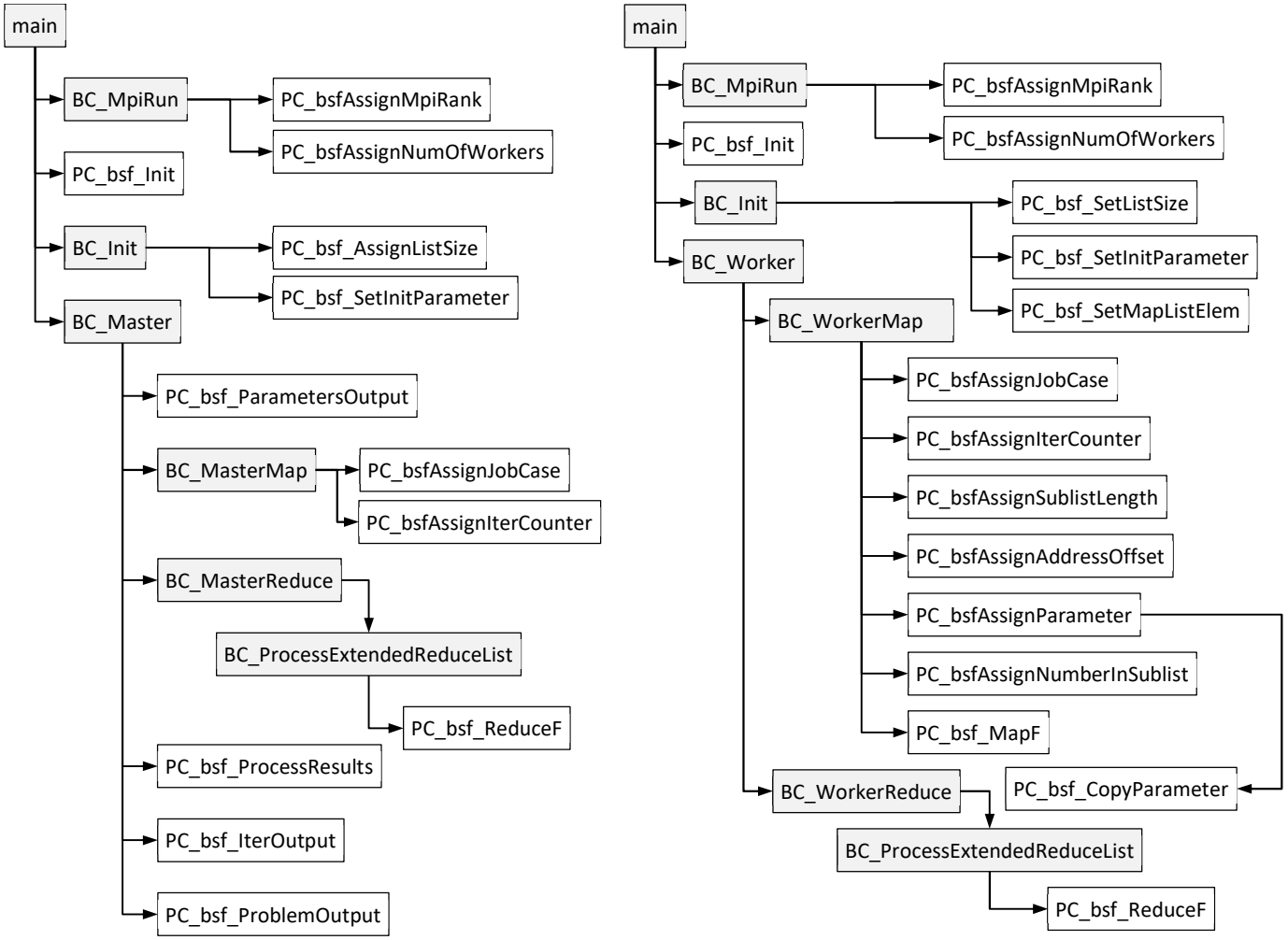
7. Skeleton variables

The skeleton variables are declared in the file *BSF-SkeletonVariables.h*. The user can exploit these variables for the sake of debugging, tracing, and non-standard implementing (see, for example, Section 13). The user should not change the values of these variables. All skeleton variables are presented in Table 4.

Table 4. Skeleton variables (file *BSF-SkeletonVariables.h*).

Skeleton variable	Type	Description
BSF_sv_addressOffset	*int*	Contains the number of the first element of the map-sublist appointed to the current worker process.
BSF_sv_iterCounter	*int*	Contains the number of iterations performed so far.
BSF_sv_jobCase	*int*	Contains the number of the current activity (job) in workflow (see Section 11).
BSF_sv_mpiRank	*int*	Contains the rank (number) of current MPI process.
BSF_sv_mpiMaster	*int*	Contains the rank (number) of the master MPI process.
BSF_sv_numberInSublist	*int*	This variable contains the relative number of the element in the map-sublist that the function *Map* is currently applied to.
BSF_sv_numOfWorkers	*int*	Contains the total number of the worker processes.
BSF_sv_parameter	*PT_bsf_parameter_T*	Structure that contains the order parameters.
BSF_sv_sublistLength	*int*	Contains the length of the map-sublist appointed to a worker process.

† *Reduce-list* is the list being the second parameter of the higher-order function *Reduce*.



(*a*) Master process (*b*) Worker process

Fig. 6. Hierarchy of the key function calls.

8. Functions

The skeleton functions are divided into two groups:

1) problem-independent functions with the prefix *BC_* that have implemented in the file *BSF-Code.cpp*;
2) problem-dependent functions (*predefined BSF functions*) with the prefix *PC_bsf_* that have declared in the file *Problem-Code.cpp*.

The user cannot change the headers and bodies of the functions with the prefix *BC_*. The user also cannot change function headers with the prefix *PC_bsf_* but must write an implementation of these functions. The body of a predefined BSF function cannot include calls of problem-independent functions with the prefix *BC_*. The hierarchy of the key function calls is presented in Fig. 6.

8.1 Key problem-independent functions (prefix *BC_*)

The implementations of all problem-independent functions can be found in the file *BSF-Code.cpp*. Descriptions of some key problem-independent functions are presented in Table 5.

Table 5. Key problem-independent functions (file *BSF-Code.cpp*).

Function	Description
BC_Init	Performs the memory allocation and the initialization of the skeleton data structures and variables.
BC_Master	The head function of the master process.
BC_MasterMap	Forms an order and sends it to the worker processes to perform the *Map* function in the current iteration.
BC_MasterReduce	Receives the results produced by the worker processes, collects them in a list, and performs the function *Reduce* on this list.
BC_MpiRun	Executes the MPI initialization. After it, the number of worker processes is accessible by the skeleton variable *BSF_sv_numOfWorkers*; total number of MPI processes (*MPI_Comm_size*) is equal to $(BSF\_sv\_numOfWorkers + 1)$; the rank of the current MPI process (*MPI_Comm_rank*) is accessible by the skeleton variable *BSF_sv_mpiRank*; the rank of the master MPI process is accessible by the skeleton variable *BSF_sv_mpiMaster* (is equal to *MPI_Comm_size*-1). The MPI ranks of the worker processes have values from 0 to $(BSF\_sv\_numOfWorkers - 1)$. The MPI rank of the worker process is equal to *BSF_sv_numOfWorkers*.
BC_ProcessExtendedReduceList	This function finds the first element in the extended reduce-list with the *reduceCounter* not equal to zero and adds to it all other elements that have the *reduceCounter* not equal to zero. For pairwise addition of elements of the original reduce-list, the function *PC_bsf_ReduceF* (see Section 8.2.8) is used.
BC_Worker	The head function of a worker process.
BC_WorkerMap	Receives the order from the master process, assigns the skeleton variables (see Section 7), and applies the function *PC_bsf_MapF* to the appointed map-sublist to produce the corresponding part of the reduce-list.
BC_WorkerReduce	Sends to the master process the element that is the sum of all reduce-sublist elements.

8.2 Predefined problem-dependent BSF functions (prefix *PC_bsf_*)

This section contains detailed descriptions of the predefined problem-dependent BSF functions with the prefix *PC_bsf_* declared in *Problem-bsfCode.cpp*. The user must implement all these functions. Step-by-step instruction is presented in Section 8.2.9. An example is presented in Section 10.

8.2.1 PC_bsf_CopyParameter

Copies all order parameters from the in-structure to the out-structure. The order parameters are declared in the predefined problem-depended BSF type *PT_bsf_parameter_T* (see Section 5).

Syntax

```
void PC_bsf_CopyParameter(
    PT_bsf_parameter_T parameterIn,
    PT_bsf_parameter_T* parameterOutP
);
```

In parameters

parameterIn

The structure from which parameters are copied.

Out parameters

parameterOutP

The pointer to the structure to which parameters are copied.

8.2.2 PC_bsf_Init

Initializes the problem-depended variables and data structures defined in *Problem-Data.h*.

Syntax

```
void PC_bsf_Init(
     bool* success
);
```

Out parameters

**success*

Must be set to *false* if the initialization failed. The default value is *true*.

8.2.3 PC_bsf_IterOutput

Outputs intermediate results of the current iteration.

Syntax

```
void PC_bsf_IterOutput(
     PT_bsf_reduceElem_T* reduceResult,
     int reduceCounter,
     PT_bsf_parameter_T parameter,
     double elapsedTime,
     int newJobCase
);
void PC_bsf_IterOutput_1(
     PT_bsf_reduceElem_T_1* reduceResult,
     int reduceCounter,
     PT_bsf_parameter_T parameter,
     double elapsedTime,
     int newJobCase
);
void PC_bsf_IterOutput_2(
     PT_bsf_reduceElem_T_2* reduceResult,
     int reduceCounter,
     PT_bsf_parameter_T parameter,
     double elapsedTime,
     int newJobCase
);
void PC_bsf_IterOutput_3(
     PT_bsf_reduceElem_T_3* reduceResult,
     int reduceCounter,
     PT_bsf_parameter_T parameter,
     double elapsedTime,
     int newJobCase
);
```

In parameters

reduceResult

Pointer to the structure that contains the result of executing the *Reduce* function.

reduceCounter

The number of summed (by $\oplus$) elements in the reduce-list. This number matches the number of extended reduce-list elements that have the value 1 in the field *reduceCounter* (see Section 6).

Remarks

The functions *PC_bsf_IterOutput_1*, *PC_bsf_IterOutput_2* and *PC_bsf_IterOutput_3* are used to organize a workflow (optional filling).

8.2.4 PC_bsf_MapF

Implements the function that is applied to the map-list elements when performing the higher-order function *Map*. To implement the *PC_bsf_MapF* function, we can use the problem-dependent variables and data structures defined in the file *Problem-Data.h*, and the structure *BSF_sv_parameter* of the type *PT_bsf_parameter_T* defined in *Problem-bsfTypes.h*.

Syntax

```
void PC_bsf_MapF(
    PT_bsf_mapElem_T* mapElem,
    PT_bsf_reduceElem_T* reduceElem,
    int* success
);
void PC_bsf_MapF_1(
    PT_bsf_mapElem_T* mapElem,
    PT_bsf_reduceElem_T_1* reduceElem,
    int* success
);
void PC_bsf_MapF_2(
    PT_bsf_mapElem_T* mapElem,
    PT_bsf_reduceElem_T_2* reduceElem,
    int* success
);
void PC_bsf_MapF_3(
    PT_bsf_mapElem_T* mapElem,
    PT_bsf_reduceElem_T_3* reduceElem,
    int* success
);
```

In parameters

mapElem

The pointer to the structure that is the current element of the map-list.

Out parameters

reduceElem

The pointer to the structure that is the corresponding reduce-list element to be calculated.

**success*

Must be set to *false* if the corresponding reduce-list element must be ignored when the *Reduce* function will be executed. The default value is *true*.

Remarks

The functions *PC_bsf_MapF_1*, *PC_bsf_MapF_2* and *PC_bsf_MapF_3* are used to organize a workflow (optional filling).

8.2.5 PC_bsf_ParametersOutput

Outputs parameters of the problem before starting the iterative process.

Syntax

```
void PC_bsf_ParametersOutput(
    PT_bsf_parameter_T parameter
);
```

In parameters

parameter

The structure containing the parameters of the problem.

8.2.6 PC_bsf_ProblemOutput

Outputs the results of solving the problem.

Syntax

```
void PC_bsf_ProblemOutput(
     PT_bsf_reduceElem_T* reduceResult,
     int reduceCounter,
     PT_bsf_parameter_T parameter,
     double t
);
void PC_bsf_ProblemOutput_1(
     PT_bsf_reduceElem_T_1* reduceResult,
     int reduceCounter,
     PT_bsf_parameter_T parameter,
     double t
);
void PC_bsf_ProblemOutput_2(
     PT_bsf_reduceElem_T_2* reduceResult,
     int reduceCounter,
     PT_bsf_parameter_T parameter,
     double t
);
void PC_bsf_ProblemOutput_3(
     PT_bsf_reduceElem_T_3* reduceResult,
     int reduceCounter,
     PT_bsf_parameter_T parameter,
     double t
);
```

In parameters

reduceResult

The pointer to the structure that is the result of executing the higher-order function *Reduce*.

parameter

The structure containing the parameters of the final iteration.

Remarks

The functions PC_bsf_ProblemOutput_1, PC_bsf_ProblemOutput_2 and PC_bsf_ProblemOutput_3 are used to organize a workflow (optional filling).

8.2.7 PC_bsf_ProcessResults

Processes the results of the current iteration: computes the order parameters for the next iteration and checks the stop condition.

Syntax

```
void PC_bsf_ProcessResults(
     PT_bsf_reduceElem_T* reduceResult,
     int reduceCounter,
     PT_bsf_parameter_T* parameter,
     int* newJobCase,
     bool* exit
);
```

```
void PC_bsf_ProcessResults_1(
    PT_bsf_reduceElem_T_1* reduceResult,
    int reduceCounter,
    PT_bsf_parameter_T* parameter,
    int* newJobCase,
    bool* exit
);
void PC_bsf_ProcessResults_2(
    PT_bsf_reduceElem_T_2* reduceResult,
    int reduceCounter,
    PT_bsf_parameter_T* parameter,
    int* newJobCase,
    bool* exit
);
void PC_bsf_ProcessResults_3(
    PT_bsf_reduceElem_T_3* reduceResult,
    int reduceCounter,
    PT_bsf_parameter_T* parameter,
    int* newJobCase,
    bool* exit
);
```

In parameters

reduceResult

The pointer to the structure that is the result of executing the higher-order function *Reduce*.

reduceCounter

The number of summed (by $\oplus$) elements in the reduce-list. This number matches the number of extended reduce-list elements that have the value 1 in the field *reduceCounter* (see Section 6).

In|out parameters

parameter

The pointer to the structure containing the parameters of the current iteration. This structure must be modified by setting new values of the parameters for the next iteration.

Out parameters

**nextJob*

If a workflow is used (see Section 11), then this variable must be assigned the number of the next action (job). Otherwise, this parameter is not used.

**exit*

If the stop condition holds, then this variable must be assigned *true*. The default value is *false*.

Remarks

Important: The use of the structure *BSF_sv_parameter* is not allowed in the implementations of these functions.

The functions *PC_bsf_ProcessResults_1*, *PC_bsf_ProcessResults_2* and *PC_bsf_ProcessResults_3* are used to organize a workflow (optional filling).

8.2.8 PC_bsf_ReduceF

Implements the operation $z = x \oplus y$ (see Section 2).

Syntax

```
void PC_bsf_ReduceF(
    PT_bsf_reduceElem_T* x,
    PT_bsf_reduceElem_T* y,
    PT_bsf_reduceElem_T* z
);
```

```
void PC_bsf_ReduceF_1(
    PT_bsf_reduceElem_T_1* x,
    PT_bsf_reduceElem_T_1* y,
    PT_bsf_reduceElem_T_1* z
);
void PC_bsf_ReduceF_2(
    PT_bsf_reduceElem_T_2* x,
    PT_bsf_reduceElem_T_2* y,
    PT_bsf_reduceElem_T_2* z
);
void PC_bsf_ReduceF_3(
    PT_bsf_reduceElem_T_3* x,
    PT_bsf_reduceElem_T_3* y,
    PT_bsf_reduceElem_T_3* z
);
```

In parameters

x

The pointer to the structure that presents the first term.

y

The pointer to the structure that presents the second term.

Out parameters

z

The pointer to the structure that presents the result of the operation.

Remarks

The functions *PC_bsf_ReduceF_1*, *PC_bsf_ReduceF_2* and *PC_bsf_ReduceF_3* are used to organize a workflow (optional filling).

8.2.9 PC_bsf_SetInitParameter

Sets initial order parameters for the workers in the first iteration. These order parameters are declared in the predefined problem-depended BSF type *PT_bsf_parameter_T* (see Section 5).

Syntax

```
void PC_bsf_SetInitParameter(
    PT_bsf_parameter_T* parameter
);
```

Out parameters

parameter

The pointer to the structure that the initial parameters should be assigned to.

8.2.10 PC_bsf_SetListSize

Sets the length of the list.

Syntax

```
void PC_bsf_SetListSize(
    int* listSize
);
```

Out parameters

**listSize*

Must be assigned a positive integer that specifies the length of the list.

Remarks

The list size should be greater than or equal to the number of workers.

8.2.11 PC_bsf_SetMapListElem

Initializes the map-list element with the number *i*.

Syntax

```
void PC_bsf_SetMapListElem(
    PT_bsf_mapElem_T* elem,
    int i
);
```

In parameters

elem

The pointer to the map-list element.

i

The ordinal number of the specified element.

Remarks

Important: The numbering of elements in the list begins from zero.

8.2.12 PC_bsfAssignAddressOffset

Assigns the number of the first element of the map-sublist to the skeleton variables *BSF_sv_addressOffset* (see Section 7).

Syntax

```
void PC_bsfAssignAddressOffset(int value);
```

In parameters

value

Non-negative integer value.

Remarks

Important: The user should not use this function.

8.2.13 PC_bsfAssignIterCounter

Assigns the number of the first element of the map-sublist to the skeleton variables *BSF_sv_iterCounter* (see Section 7).

Syntax

```
void PC_bsfAssignIterCounter(int value);
```

In parameters

value

Non-negative integer value.

Remarks

Important: The user should not use this function.

8.2.14 PC_bsfAssignJobCase

Assigns the number of the current activity (job) in workflow to the skeleton variables *BSF_sv_jobCase* (see Section 7).

Syntax

```
void PC_bsfAssignJobCase(int value);
```

In parameters

value

Non-negative integer value.

Remarks

Important: The user should not use this function.

8.2.15 PC_bsfAssignMpiMaster

Assigns the rank of the master MPI process to the skeleton variables *BSF_sv_mpiMaster* (see Section 7).

Syntax

```
void PC_bsfAssignMpiMaster(int value);
```

In parameters

value

Non-negative integer value.

Remarks

Important: The user should not use this function.

8.2.16 PC_bsfAssignMpiRank

Assigns the rank of current MPI process to the skeleton variables *BSF_sv_mpiRank* (see Section 7).

Syntax

```
void PC_bsfAssignMpiRank(int value);
```

In parameters

value

Non-negative integer value.

Remarks

Important: The user should not use this function.

8.2.17 PC_bsfAssignNumberInSublist

Assigns the number of the current element in the map-sublist to the skeleton variables *BSF_sv_numberInSublist* (see Section 7).

Syntax

```
void PC_bsfAssignNumberInSublist(int value);
```

In parameters

value

Non-negative integer value.

Remarks

Important: The user should not use this function.

8.2.18 PC_bsfAssignNumOfWorkers

Assigns the total number of the worker processes to the skeleton variables *BSF_sv_numOfWorkers* (see Section 7).

Syntax

```
void PC_bsfAssignNumberInSublist(int value);
```

In parameters

value

Non-negative integer value.

Remarks

Important: The user should not use this function.

8.2.19 PC_bsfAssignParameter

Assigns the order parameters to the structure *BSF_sv_parameter* (see Section 7).

Syntax

```
void PC_bsfAssignParameter(PT_bsf_parameter_T parameter);
```

In parameters

parameter

The structure from which the order parameters are taken.

Remarks

Important: The user should not use this function.

8.2.20 PC_bsfAssignSublistLength

Assigns the length of the current map-sublist to the skeleton variables *BSF_sv_sublistLength* (see Section 7).

Syntax

```
void PC_bsfAssignSublistLength(int value);
```

In parameters

value

Non-negative integer value.

Remarks

Important: The user should not use this function.

9. Step-by-step instruction

This section contains step-by-step instructions on how to use the BSF-skeleton to quickly create a parallel program. Starting from Step 2, we strongly recommend compiling the program after adding each language construction.

Step 1. First of all, we must represent our algorithm in the form of operations on lists using the higher-order functions *Map* and *Reduce* (see the generic BSF-algorithm template shown in Fig. 2). An example is presented in Section 10.

Step 2. In the file *Problem-Parameters.h*, define problem parameters. For example:

```
#define PP_N 3 // Dimension of space
```

Step 3. In the file *Problem-Types.h*, declare problem types (optional). For example:

```
typedef double PT_point_T[PP_N]; // Point in n-Dimensional Space
```

Step 4. In the file *Problem-bsfTypes.h*, implement the predefined BSF types. If we do not use a workflow then we do not have to implement the types *PT_bsf_reduceElem_T_1*, *PT_bsf_reduceElem_T_2*, *PT_bsf_reduceElem_T_3*, but we can't delete these empty structures. For example:

```
struct PT_bsf_parameter_T {
    PT_point_T approximation;  // Current approximation
};
struct PT_bsf_mapElem_T {
    int columnNo;              // Column number in matrix Alpha
};
struct PT_bsf_reduceElem_T {
```

```
    double column[PP_N];        // Column of intermediate matrix
};
struct PT_bsf_reduceElem_T_1 { };
struct PT_bsf_reduceElem_T_2 { };
struct PT_bsf_reduceElem_T_3 { };
```

Step 5. In the file *Problem-Data.h*, define the problem-dependent variables and data structures. For example:

```
static double PD_A[PP_N][PP_N]; // Coefficients of equations
```

Step 6. In the file *Problem-bsfCode.cpp*, implement the predefined problem-dependent BSF functions (see Section 8.2) in the suggested order. To implement these functions, the user can write additional *problem (user) functions* in the *Problem-bsfCode.cpp*. The prototypes of these problem functions must be included in the *Problem-Forwards.h*.

Step 7. In the file *Problem-bsfCode.cpp*, we can configure the BSF-skeleton parameters (see Section 4).

Step 8. Build and run the solution in the MPI environment.

10. Example of using the BSF-skeleton

In this section, we show how to use the BSF-skeleton to implement the iterative Jacobi method as an example. The *Jacobi method* [3] is a simple iterative method for solving a system of linear equations. Let us give a brief description of the Jacobi method. Let a joint square system of linear equations in a matrix form be given in Euclidean space $\mathbb{R}^n$:

$$Ax = b, \qquad (1)$$

where

$$A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{pmatrix};$$
$$x = (x_1, \ldots, x_n);$$
$$b = (b_1, \ldots, b_n).$$

It is assumed that $a_{ii} \neq 0$ for all $i = 1, \ldots, n$. Let us define the matrix

$$C = \begin{pmatrix} c_{11} & \cdots & c_{1n} \\ \vdots & \ddots & \vdots \\ c_{n1} & \cdots & c_{nn} \end{pmatrix}$$

in the following way:

$$c_{ij} = \begin{cases} -\dfrac{a_{ij}}{a_{ii}}, \forall j \neq i; \\ 0, \forall j = i. \end{cases}$$

Let us define the vector $d = (d_1,\ldots,d_n)$ as follows: $d_i = b_i/a_{ii}$. The Jacobi method of finding an approximate solution of system (1) consists of the following steps:

Step 1. $k := 0$; $x^{(0)} := d$.
Step 2. $x^{(k+1)} := Cx^{(k)} + d$.
Step 3. If $\left\|x^{(k+1)} - x^{(k)}\right\|^2 < \varepsilon$, go to Step 5.
Step 4. $k := k + 1$; go to Step 2.
Step 5. Stop.

In the Jacobi method, an arbitrary vector $x^{(0)}$ can be taken as the initial approximation. In Step 1, the initial approximation $x^{(0)}$ is assigned by the vector d. In Step 3, the Euclidean norm $\|\cdot\|$ is used in the termination criteria. The *diagonal dominance* of the matrix A is a sufficient condition for the convergence of the Jacobi method:

$$|a_{ii}| \geq \left(\sum_{j=1}^{n} |a_{ij}|\right) - |a_{ii}|$$

for all $i = 1,\ldots,n$, and at least one inequality is strict. In this case, the system (1) has a unique solution for any right-hand side.

Let us represent the Jacobi method in the form of algorithm on lists. Let c_j denotes the *j*-th column of matrix C:

$$c_j = \begin{pmatrix} c_{1j} \\ \vdots \\ c_{nj} \end{pmatrix}.$$

Let $G = [1,\ldots,n]$ be the list of natural numbers from 1 to *n*. For any vector $x = (x_1,\ldots,x_n) \in \mathbb{R}^n$, let us define the function $F_x : \{1,\ldots,n\} \to \mathbb{R}^n$ as follows:

$$F_x(j) = x_j c_j = \begin{pmatrix} x_j c_{1j} \\ \vdots \\ x_j c_{nj} \end{pmatrix},$$

i.e. the function $F_x(j)$ multiplies the *j*-th column of the matrix C by the *j*-th coordinate of the vector x. The BSF-implementation of the Jacobi method shown in Fig. 7 can be easily obtained from the generic BSF-algorithm template shown in Fig. 2. In the algorithm 3, $\dot{+}$ and $\dot{-}$ denote the operations of vector addition and subtraction, respectively. Note that the matrix C entered in line 1 is implicitly used to calculate the values of the function $F_{x^{(k)}}$ in line 3.

Algorithm 3.

1: **input** C, d
2: $k := 0; x^{(0)} := d; G := [1, \dots, n]$
3: $B := Map\left(F_{x^{(k)}}, G\right)$
4: $s := Reduce\left(\vec{+}, B\right)$
5: $x^{(k+1)} := s\vec{+}d$
6: $k := k + 1$
7: **if** $\left\|x^{(k)}\vec{-}x^{(k-1)}\right\|^2 < \varepsilon$ **goto** 9
8: **goto** 3
9: **output** $x^{(i)}$
10: **stop**

Fig. 7. BSF-Jacobi algorithm that uses Map and Reduce.

The source code of the BSF-Jacobi algorithm, implemented by using the BSF-skeleton, is freely available on Github at https://github.com/leonid-sokolinsky/BSF-Jacobi. Another implementation of the Jacobi method using a BSF-skeleton is discussed in Section 13. This implementation uses only the higher-order function *Map* without the higher-order function *Reduce*. An information about other solutions using the BSF-skeleton are presented in Section 14.

11. Workflow support

The BSF-skeleton supports workflows. A workflow consists of orchestrated and repeatable activities (jobs). The BSF-skeleton supports up to four different jobs. The starting job is always numbered 0 (omitted in the source codes). The other jobs have sequential numbers 1, ..., 3. Each job has its own type of reduce-list elements defined in the file *Problem-bsfTypes.h*. All jobs have the same type of map list elements. To organize the workflow, we need to follow these steps:

1. In the file *Problem-bsfParameters.h*, redefine the macros *PP_BSF_MAX_JOB_CASE* specifying the largest number of a job. For example, if the total job quantity is 3, the number to be assigned to *PP_BSF_MAX_JOB_CASE* must be 2.
2. In the file *Problem-bsfTypes.h*, define the types of reduce-list elements for all jobs whose sequential numbers are less than or equal to *PP_BSF_MAX_JOB_CASE*.
3. In the file *Problem-bsfCode.cpp*, implement the functions *PC_bsf_MapF[_*]*, *PC_bsf_ReduceF[_*]*, *PC_bsf_ProcessResults[_*]*, *PC_bsf_ProblemOutput[_*]* and *PC_bsf_IterOutput[_*]* for all jobs whose sequential numbers are less than or equal to *PP_BSF_MAX_JOB_CASE*. The functions *PC_bsf_ProblemOutput[_*]* should assign the parameter **nextJob* a sequential number of the next job (possibly the same).

An example of a solution using the BSF-skeleton with the workflow support is freely available on Github at https://github.com/leonid-sokolinsky/Apex-method.

12. OpenMP support

The BSF-skeleton supports a parallelization of the map-list processing cycle in the worker processes (the function *BC_WorkerMap*) using the `#pragma omp parallel for`. This support is disabled by default. To enable this support, we must define the macros *PP_SF_OMP* in the file *Problem-bsfParameters.h*. Using the macros *PP_BSF_NUM_THREADS*, we can specify the number of threads to use in the `parallel for`. By default, all available threads are used.

13. Using Map without Reduce

Some numerical algorithms can be implemented naturally using the function *Map* without the function *Reduce*. In this section, we will show how to use the BSF-skeleton in this case. As an example, we use Jacobi method discussed in Section 10.

Let $G=[1,\ldots,n]$ be the list of natural numbers from 1 to n. For any vector $x=(x_1,\ldots,x_n)\in\mathbb{R}^n$, let us define the function $\Phi_x:\{1,\ldots,n\}\to\mathbb{R}$ as follows:

$$\Phi_x(i)=d_i+\sum_{j=1}^{n}c_{ij}x_j, \qquad (2)$$

i.e. the function $\Phi_x(i)$ calculates the i-th coordinate of the next approximation. An implementation of the Jacobi method that uses only a higher-order function *Map* is shown in Fig. 8. In this case, the reduce-list consists of coordinates of the next approximation and does not require performing *Reduce*.

An implementation of Algorithm 4 using the BSF-skeleton is freely available on Github at https://github.com/leonid-sokolinsky/BSF-Jacobi-Map. In the implementation of the function *PC_bsf_MapF*, we had to apply a couple of tricks that use the skeleton variables *BSF_sv_numberInSublist*, *BSF_sv_addressOffset* and *BSF_sv_sublistLength* (see Section 7).

Algorithm 4.

1: **input** C, d
2: $k := 0; x^{(0)} := d; G := [1, \ldots, n]$
3: $x^{(k+1)} := Map\left(\Phi_{x^{(k)}}, G\right)$
4: $k := k + 1$
5: **if** $\left\|x^{(k)}\overrightarrow{-}x^{(k-1)}\right\|^2 < \varepsilon$ **goto** 7
6: **goto** 3
7: **output** $x^{(i)}$
8: **stop**

Fig. 8. BSF-Jacobi algorithm that uses only *Map*.

14. Appendix: Solutions using the BSF-skeleton

Title	Description	URL on GitHub	References
Jacobi Algo-rithm with *Map* & *Reduce*	An iterative algorithm for solving a system of linear equations. This al-gorithm uses the higher-order functions *Map* and *Reduce*.	https://github.com/leonid-sokolinsky/BSF-Jacobi	1. L.B. Sokolinsky, BSF: a parallel computation model for scalability estimation of iterative numerical algorithms on cluster computing systems, Chelyabinsk, Russia, 2020. http://arxiv.org/abs/2008.03485. 2. N.A. Ezhova, L.B. Sokolinsky, Scalability Evaluation of Iterative Algorithms Used for Su-percomputer Simulation of Physical processes, in: Proc. - 2018 Glob. Smart Ind. Conf. GloSIC 2018, Art. No. 8570131, IEEE, 2018: p. 10. https://doi.org/10.1109/Glo-SIC.2018.8570131.

Jacobi Algorithm with *Map*	An iterative algorithm for solving a system of linear equations. This algorithm uses the higher-order function *Map* only.	https://github.com/leonid-sokolinsky/BSF-Jacobi-Map	1. N.A. Ezhova, L.B. Sokolinsky, Scalability Evaluation of Iterative Algorithms Used for Supercomputer Simulation of Physical processes, in: Proc. - 2018 Glob. Smart Ind. Conf. GloSIC 2018, Art. No. 8570131, IEEE, 2018: p. 10. https://doi.org/10.1109/GloSIC.2018.8570131.
BSF-Gravity	An iterative algorithm solving a simplified n-body problem, which describes how a small body will move under the influence of gravitational force among large motionless bodies.	https://github.com/leonid-sokolinsky/BSF-gravity	1. L.B. Sokolinsky, BSF: a parallel computation model for scalability estimation of iterative numerical algorithms on cluster computing systems, Chelyabinsk, Russia, 2020. http://arxiv.org/abs/2008.03485.
ModAPL	A scalable iterative projection-type algorithm for solving non-stationary systems of linear inequalities.	https://github.com/leonid-sokolinsky/NSLP-Quest	1. L.B. Sokolinsky, I.M. Sokolinskaya, Scalable parallel algorithm for solving non-stationary systems of linear inequalities, Lobachevskii J. Math. 41 (2020) 1571–1580. https://doi.org/10.1134/S1995080220080181.
Apex-method	A new algorithm for solving large-scale LP problems. This algorithm uses the workflow technique.	https://github.com/leonid-sokolinsky/Apex-method	1. L.B. Sokolinsky, I.M. Sokolinskaya, Scalable Method for Linear Optimization of Industrial Processes, Chelyabinsk, Russia, 2020. http://arxiv.org/abs/2006.14921.
Cimmino Algorithm	An iterative algorithm of projection type that can be used to solve the linear equation systems and some type of linear inequality systems	https://github.com/leonid-sokolinsky/BSF-Cimmino	[1] I.M. Sokolinskaya, L.B. Sokolinsky, Scalability Evaluation of Cimmino Algorithm for Solving Linear Inequality Systems on Multiprocessors with Distributed Memory, Supercomput. Front. Innov. 5 (2018) 11–22. https://doi.org/10.14529/jsfi180202.

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[1] L.B. Sokolinsky, BSF: a parallel computation model for scalability estimation of iterative numerical algorithms on cluster computing systems, Chelyabinsk, Russia, 2020. http://arxiv.org/abs/2008.03485.

[2] R.S. Bird, Lectures on Constructive Functional Programming, in: M. Broy (Ed.), Constr. Methods Comput. Sci. NATO ASI Ser. F Comput. Syst. Sci. Vol. 55, Springer, Berlin, Heidlberg, 1988: pp. 151–216.

[3] H. Rutishauser, The Jacobi Method for Real Symmetric Matrices, in: Bauer F.L. (Ed.), Handb. Autom. Comput. Vol 2. Linear Algebr., Springer Berlin Heidelberg, Berlin, Heidelberg, 1971: pp. 202–211. https://doi.org/10.1007/978-3-662-39778-7_12.