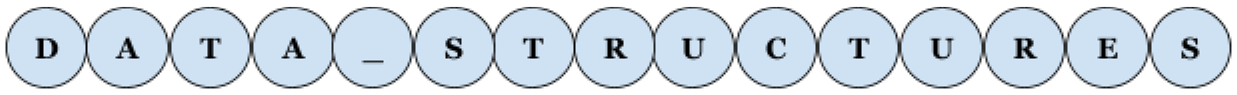
CS261 Data Structures

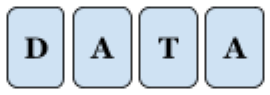
Assignment 2

v 1.09 (revised 3/25/2021)

Your Very Own Dynamic Array (plus Bag, Stack and Queue)

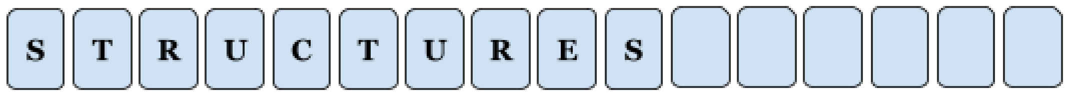


```
da = DynamicArray(list("DATA"))
```



```
self.size = 4
self.capacity = 4
self.data = ['D', 'A', 'T', 'A']
```

```
da = DynamicArray(list("STRUCTURES"))
```



```
self.size = 10
self.capacity = 16
self.data = ['S', 'T', 'R', 'U', 'C', 'T', 'U', 'R', 'E', 'S', None, None, None, None, None, None]
```

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General Instructions

1. Programs in this assignment must be written in Python v3 and submitted to Gradescope before the due date specified in the syllabus. You may resubmit your code as many times as necessary. Gradescope allows you to choose which submission will be graded.

2. In Gradescope, your code will be through several tests. Any failed tests will provide a brief explanation of testing conditions to help you with troubleshooting. Your goal is to pass all tests.

3. We encourage you to create your own test programs and cases even though this work won't have to be submitted and won't be graded. Gradescope tests are limited in scope and may not cover all edge cases. Your submission must work on all valid inputs. We reserve the right to test your submission with more tests than Gradescope.

4. Your code must have an appropriate level of comments. At a minimum, each method should have a descriptive docstring. Additionally, put comments throughout the code to make it easy to follow and understand.

5. You will be provided with a starter "skeleton" code, on which you will build your implementation. Methods defined in the skeleton code must retain their names and input / output parameters. Variables defined in the skeleton code must also retain their names. We will only test your solution by making calls to methods defined in the skeleton code and by checking values of variables defined in the skeleton code. You can add more methods and variables, as needed.

   However, certain classes and methods cannot be changed in any way. Please see the comments in the skeleton code for guidance. In particular, the content of any methods pre-written for you as part of the skeleton code must not be changed.

6. Both the skeleton code and the code examples provided in this document are part of the assignment requirements. They have been carefully selected to demonstrate requirements for each method. Refer to them for a detailed description of expected method behavior, input / output parameters, and the handling of edge cases. Code examples may include assignment requirements not explicitly stated elsewhere.

7. For each method, you can choose to implement a recursive or iterative solution. When using a recursive solution, be aware of maximum recursion depths on large inputs. We will specify the maximum input size that your solution must handle.

Part 1 - Summary and Specific Instructions

1. Implement a Dynamic Array class by completing the skeleton code provided in the file `dynamic_array.py`. The DynamicArray class will use a StaticArray object as its underlying data storage container and will provide many methods similar to those we are used to when working with Python lists. Once completed, your implementation will include the following methods:

   ```
   resize()
   append()
   insert_at_index()
   remove_at_index()
   slice()
   merge()
   map()
   filter()
   reduce()
   ```

   * Several class methods, like is_empty(), length(), get_at_index() and set_at_index() have been pre-written for you.

2. We will test your implementations with different types of objects, not just integers. We guarantee that all such objects will have correct implementation of methods __eq__, __lt__, __gt__, __ge__, __le__, and __str__.

3. The number of objects stored in the array at any given time will be between 0 and 1,000,000 inclusive. An array must allow for the storage of duplicate objects.

4. Variables in the DynamicArray class are not marked as private. For this portion of the assignment, you are allowed to access and change their values directly. You are not required to write getter or setter methods for them.

5. RESTRICTIONS: You are not allowed to use ANY built-in Python data structures and/or their methods (lists, dictionaries, etc.). You must solve this portion of the assignment by importing and using objects of the StaticArray class (prewritten for you) and using class methods to write your solution.

   You are also not allowed to directly access any variables of the StaticArray class (like self.data._data[]). All work must be done by using only StaticArray class methods.

resize(self, new_capacity: int) -> None:

This method changes the capacity of the underlying storage for the array elements. It does not change the values or the order of any elements currently stored in the dynamic array.

It is intended to be an "internal" method of the Dynamic Array class, called by other class methods, such as append(), remove_at_index(), or insert_at_index(), to manage the capacity of the underlying storage data structure.

The method should only accept positive integers for `new_capacity`. Additionally, `new_capacity` cannot be smaller than the number of elements currently stored in the dynamic array (which is tracked by the `self.size` variable). If `new_capacity` is not a positive integer or if `new_capacity < self.size`, this method should not do any work and should just exit.

Example #1:
```
da = DynamicArray()
print(da.size, da.capacity, da.data)
da.resize(8)
print(da.size, da.capacity, da.data)
da.resize(2)
print(da.size, da.capacity, da.data)
da.resize(0)
print(da.size, da.capacity, da.data)
```

Output:
```
0 4 STAT_ARR Size: 4 [None, None, None, None]
0 8 STAT_ARR Size: 8 [None, None, None, None, None, None, None, None]
0 2 STAT_ARR Size: 2 [None, None]
0 2 STAT_ARR Size: 2 [None, None]
```

NOTE: Example 2 below will not work properly until after append() method is implemented.

Example #2:
```
da = DynamicArray([1, 2, 3, 4, 5, 6, 7, 8])
print(da)
da.resize(20)
print(da)
da.resize(4)
print(da)
```

Output:
```
DYN_ARR Size/Cap: 8/8 [1, 2, 3, 4, 5, 6, 7, 8]
DYN_ARR Size/Cap: 8/20 [1, 2, 3, 4, 5, 6, 7, 8]
DYN_ARR Size/Cap: 8/20 [1, 2, 3, 4, 5, 6, 7, 8]
```

append(self, value: object) -> None:

This method adds a new value at the end of the dynamic array.

If the internal storage associated with the dynamic array is already full, you need to DOUBLE its capacity before adding a new value.

Example #1:

```
da = DynamicArray()
print(da.size, da.capacity, da.data)
da.append(1)
print(da.size, da.capacity, da.data)
print(da)
```

Output:

```
0 4 STAT_ARR Size: 4 [None, None, None, None]
1 4 STAT_ARR Size: 4 [1, None, None, None]
DYN_ARR Size/Cap: 1/4 [1]
```

Example #2:

```
da = DynamicArray()
for i in range(9):
    da.append(i + 101)
    print(da)
```

Output:

```
DYN_ARR Size/Cap: 1/4 [101]
DYN_ARR Size/Cap: 2/4 [101, 102]
DYN_ARR Size/Cap: 3/4 [101, 102, 103]
DYN_ARR Size/Cap: 4/4 [101, 102, 103, 104]
DYN_ARR Size/Cap: 5/8 [101, 102, 103, 104, 105]
DYN_ARR Size/Cap: 6/8 [101, 102, 103, 104, 105, 106]
DYN_ARR Size/Cap: 7/8 [101, 102, 103, 104, 105, 106, 107]
DYN_ARR Size/Cap: 8/8 [101, 102, 103, 104, 105, 106, 107, 108]
DYN_ARR Size/Cap: 9/16 [101, 102, 103, 104, 105, 106, 107, 108, 109]
```

Example #3:

```
da = DynamicArray()
for i in range(600):
    da.append(i)
print(da.size)
print(da.capacity)
```

Output:

```
600
1024
```

insert_at_index(self, index: int, value: object) -> None:

This method adds a new value at the specified index position in the dynamic array. Index 0 refers to the beginning of the array. If the provided index is invalid, the method raises a custom "DynamicArrayException". Code for the exception is provided in the skeleton file. If the array contains N elements, valid indices for this method are [0, N] inclusive.

If the internal storage associated with the dynamic array is already full, you need to DOUBLE its capacity before adding a new value.

Example #1:

```
da = DynamicArray([100])
print(da)
da.insert_at_index(0, 200)
da.insert_at_index(0, 300)
da.insert_at_index(0, 400)
print(da)
da.insert_at_index(3, 500)
print(da)
da.insert_at_index(1, 600)
print(da)
```

Output:

```
DYN_ARR Size/Cap: 1/4 [100]
DYN_ARR Size/Cap: 4/4 [400, 300, 200, 100]
DYN_ARR Size/Cap: 5/8 [400, 300, 200, 500, 100]
DYN_ARR Size/Cap: 6/8 [400, 600, 300, 200, 500, 100]
```

Example #2:

```
da = DynamicArray()
try:
    da.insert_at_index(-1, 100)
except Exception as e:
    print("Exception raised:", type(e))
da.insert_at_index(0, 200)
try:
    da.insert_at_index(2, 300)
except Exception as e:
    print("Exception raised:", type(e))
print(da)
```

Output:

```
Exception raised: <class '__main__.DynamicArrayException'>
Exception raised: <class '__main__.DynamicArrayException'>
DYN_ARR Size/Cap: 1/4 [200]
```

Example #3:

```
da = DynamicArray()
for i in range(1, 10):
    index, value = i - 4, i * 10
    try:
        da.insert_at_index(index, value)
    except Exception as e:
        print("Cannot insert value", value, "at index", index)
print(da)
```

Output:

```
Cannot insert value 10 at index -3
Cannot insert value 20 at index -2
Cannot insert value 30 at index -1
DYN_ARR Size/Cap: 6/8 [40, 50, 60, 70, 80, 90]
```

remove_at_index(self, index: int) -> None:

This method removes the element at the specified index from the dynamic array. Index 0 refers to the beginning of the array. If the provided index is invalid, the method raises a custom "DynamicArrayException". Code for the exception is provided in the skeleton file. If the array contains N elements, valid indices for this method are [0, N - 1] inclusive.

When the number of elements stored in the array (before removal) is STRICTLY LESS than ¼ of its current capacity, the capacity must be reduced to TWICE the number of current elements. This check / capacity adjustment must happen BEFORE removal of the element.

If the current capacity (before reduction) is 10 elements or less, reduction should not happen at all. If the current capacity (before reduction) is greater than 10 elements, the reduced capacity cannot become less than 10 elements. Please see the examples below, especially example #3, for clarifications.

Example #1:
```
da = DynamicArray([10, 20, 30, 40, 50, 60, 70, 80])
print(da)
da.remove_at_index(0)
print(da)
da.remove_at_index(6)
print(da)
da.remove_at_index(2)
print(da)
```

Output:
```
DYN_ARR Size/Cap: 8/8 [10, 20, 30, 40, 50, 60, 70, 80]
DYN_ARR Size/Cap: 7/8 [20, 30, 40, 50, 60, 70, 80]
DYN_ARR Size/Cap: 6/8 [20, 30, 40, 50, 60, 70]
DYN_ARR Size/Cap: 5/8 [20, 30, 50, 60, 70]
```

Example #2:
```
da = DynamicArray([1024])
print(da)
for i in range(17):
    da.insert_at_index(i, i)
print(da.size, da.capacity)
for i in range(16, -1, -1):
    da.remove_at_index(0)
print(da)
```

Output:
```
DYN_ARR Size/Cap: 1/4 [1024]
18 32
DYN_ARR Size/Cap: 1/10 [1024]
```

Example #3:
```
da = DynamicArray()
print(da.size, da.capacity)
[da.append(1) for i in range(100)]          # step 1 - add 100 elements
print(da.size, da.capacity)
[da.remove_at_index(0) for i in range(68)]  # step 2 - remove 68 elements
print(da.size, da.capacity)
da.remove_at_index(0)                       # step 3 - remove 1 element
print(da.size, da.capacity)
da.remove_at_index(0)                       # step 4 - remove 1 element
print(da.size, da.capacity)
[da.remove_at_index(0) for i in range(14)]  # step 5 - remove 14 elements
print(da.size, da.capacity)
da.remove_at_index(0)                       # step 6 - remove 1 element
print(da.size, da.capacity)
da.remove_at_index(0)                       # step 7 - remove 1 element
print(da.size, da.capacity)

for i in range(14):
    print("Before remove_at_index(): ", da.size, da.capacity, end="")
    da.remove_at_index(0)
    print(" After remove_at_index(): ", da.size, da.capacity)
```

Output:
```
0 4
100 128
32 128
31 128
30 62
16 62
15 62
14 30
Before remove_at_index():  14 30 After remove_at_index():  13 30
Before remove_at_index():  13 30 After remove_at_index():  12 30
Before remove_at_index():  12 30 After remove_at_index():  11 30
Before remove_at_index():  11 30 After remove_at_index():  10 30
Before remove_at_index():  10 30 After remove_at_index():  9 30
Before remove_at_index():  9 30 After remove_at_index():  8 30
Before remove_at_index():  8 30 After remove_at_index():  7 30
Before remove_at_index():  7 30 After remove_at_index():  6 14
Before remove_at_index():  6 14 After remove_at_index():  5 14
Before remove_at_index():  5 14 After remove_at_index():  4 14
Before remove_at_index():  4 14 After remove_at_index():  3 14
Before remove_at_index():  3 14 After remove_at_index():  2 10
Before remove_at_index():  2 10 After remove_at_index():  1 10
Before remove_at_index():  1 10 After remove_at_index():  0 10
```

Example #4:

```
da = DynamicArray([1, 2, 3, 4, 5])
print(da)
for _ in range(5):
    da.remove_at_index(0)
    print(da)
```

Output:

```
DYN_ARR Size/Cap: 5/8 [1, 2, 3, 4, 5]
DYN_ARR Size/Cap: 4/8 [2, 3, 4, 5]
DYN_ARR Size/Cap: 3/8 [3, 4, 5]
DYN_ARR Size/Cap: 2/8 [4, 5]
DYN_ARR Size/Cap: 1/8 [5]
DYN_ARR Size/Cap: 0/8 []
```

slice(self, start_index: int, size: int) -> object:

This method returns a new Dynamic Array object that contains the requested number of elements from the original array starting with the element located at the requested start index. If the array contains N elements, a valid `start_index` is in range [0, N - 1] inclusive. A valid size is a non-negative integer.

If the provided start index or size is invalid, or if there are not enough elements between the start index and the end of the array to make the slice of the requested size, this method raises a custom "DynamicArrayException". Code for the exception is provided in the skeleton file.

Example #1:

```
da = DynamicArray([1, 2, 3, 4, 5, 6, 7, 8, 9])
da_slice = da.slice(1, 3)
print(da, da_slice, sep="\n")
da_slice.remove_at_index(0)
print(da, da_slice, sep="\n")
```

Output:

```
DYN_ARR Size/Cap: 9/16 [1, 2, 3, 4, 5, 6, 7, 8, 9]
DYN_ARR Size/Cap: 3/4 [2, 3, 4]
DYN_ARR Size/Cap: 9/16 [1, 2, 3, 4, 5, 6, 7, 8, 9]
DYN_ARR Size/Cap: 2/4 [3, 4]
```

Example #2:

```
da = DynamicArray([10, 11, 12, 13, 14, 15, 16])
print("SOURCE:", da)
slices = [(0, 7), (-1, 7), (0, 8), (2, 3), (5, 0), (5, 3), (6, 1), (6, -1)]
for i, cnt in slices:
    print("Slice", i, "/", cnt, end="")
    try:
        print(" --- OK: ", da.slice(i, cnt))
    except:
        print(" --- exception occurred.")
```

Output:

```
SOURCE: DYN_ARR Size/Cap: 7/8 [10, 11, 12, 13, 14, 15, 16]
Slice 0 / 7 --- OK:  DYN_ARR Size/Cap: 7/8 [10, 11, 12, 13, 14, 15, 16]
Slice -1 / 7 --- exception occurred.
Slice 0 / 8 --- exception occurred.
Slice 2 / 3 --- OK:  DYN_ARR Size/Cap: 3/4 [12, 13, 14]
Slice 5 / 0 --- OK:  DYN_ARR Size/Cap: 0/4 []
Slice 5 / 3 --- exception occurred.
Slice 6 / 1 --- OK:  DYN_ARR Size/Cap: 1/4 [16]
Slice 6 / -1 --- exception occurred.
```

merge(self, second_da: object) -> None:

This method takes another Dynamic Array object as a parameter, and appends all elements from this second array to the current one, in the same order as they are stored in the second array.

Example #1:

```
da = DynamicArray([1, 2, 3, 4, 5])
da2 = DynamicArray([10, 11, 12, 13])
print(da)
da.merge(da2)
print(da)
```

Output:

```
DYN_ARR Size/Cap: 5/8 [1, 2, 3, 4, 5]
DYN_ARR Size/Cap: 9/16 [1, 2, 3, 4, 5, 10, 11, 12, 13]
```

Example #2:

```
da = DynamicArray([1, 2, 3])
da2 = DynamicArray()
da3 = DynamicArray()
da.merge(da2)
print(da)
da2.merge(da3)
print(da2)
da3.merge(da)
print(da3)
```

Output:

```
DYN_ARR Size/Cap: 3/4 [1, 2, 3]
DYN_ARR Size/Cap: 0/4 []
DYN_ARR Size/Cap: 3/4 [1, 2, 3]
```

map(self, map_func) ->object:

This method creates a new Dynamic Array where the value of each element is derived by applying a given map_func to the corresponding value from the original array.

It works similarly to the built-in Python map() function. If you would like to review how Python's map() works, the following are good resources:
https://docs.python.org/3/library/functions.html#map
https://www.geeksforgeeks.org/python-map-function/

Example #1:

```
da = DynamicArray([1, 5, 10, 15, 20, 25])
print(da)
print(da.map(lambda x: (x ** 2)))
```

Output:

```
DYN_ARR Size/Cap: 6/8 [1, 5, 10, 15, 20, 25]
DYN_ARR Size/Cap: 6/8 [1, 25, 100, 225, 400, 625]
```

Example #2:

```
def double(value):
    return value * 2

def square(value):
    return value ** 2

def cube(value):
    return value ** 3

def plus_one(value):
    return value + 1

da = DynamicArray([plus_one, double, square, cube])
for value in [1, 10, 20]:
    print(da.map(lambda x: x(value)))
```

Output:

```
DYN_ARR Size/Cap: 4/4 [2, 2, 1, 1]
DYN_ARR Size/Cap: 4/4 [11, 20, 100, 1000]
DYN_ARR Size/Cap: 4/4 [21, 40, 400, 8000]
```

filter(self, filter_func) ->object:

This method creates a new Dynamic Array populated only with those elements from the original array for which filter_func returns True.

It works similarly to the built-in Python filter() function. If you would like to review how Python's filter() works, the following are good resources:
https://docs.python.org/3/library/functions.html#filter
https://www.geeksforgeeks.org/filter-in-python/

Example #1:
```
def filter_a(e):
    return e > 10

da = DynamicArray([1, 5, 10, 15, 20, 25])
print(da)
result = da.filter(filter_a)
print(result)
print(da.filter(lambda x: (10 <= x <= 20)))
```

Output:
```
DYN_ARR Size/Cap: 6/8 [1, 5, 10, 15, 20, 25]
DYN_ARR Size/Cap: 3/4 [15, 20, 25]
DYN_ARR Size/Cap: 3/4 [10, 15, 20]
```

Example #2:
```
def is_long_word(word, length):
    return len(word) > length

da = DynamicArray("This is a sentence with some long words".split())
print(da)
for length in [3, 4, 7]:
    print(da.filter(lambda word: is_long_word(word, length)))
```

Output:
```
DYN_ARR Size/Cap: 8/8 [This, is, a, sentence, with, some, long, words]
DYN_ARR Size/Cap: 6/8 [This, sentence, with, some, long, words]
DYN_ARR Size/Cap: 2/4 [sentence, words]
DYN_ARR Size/Cap: 1/4 [sentence]
```

reduce(self, reduce_func, initializer=None) ->object:

This method sequentially applies the reduce_func to all elements of the Dynamic Array and returns the resulting value. The method takes an optional initializer parameter. If this parameter is not provided, the first value in the array is used as the initializer. If the Dynamic Array is empty, the method returns the value of the initializer (or None, if it was not provided).

This method works similarly to the Python reduce() function. If you would like to review how Python's reduce() works, the following is a good starting point:
https://www.geeksforgeeks.org/reduce-in-python/

Example #1:

```
values = [100, 5, 10, 15, 20, 25]
da = DynamicArray(values)
print(da)
print(da.reduce(lambda x, y: x + y ** 2))
print(da.reduce(lambda x, y: x + y ** 2, -1))
```

Output:

```
DYN_ARR Size/Cap: 6/8 [100, 5, 10, 15, 20, 25]
1475
11374
```

Explanation:

$1475 = 100 + 5^2 + 10^2 + 15^2 + 20^2 + 25^2$
First value is not squared because it is used as an initializer.
$11374 = -1 + 100^2 + 5^2 + 10^2 + 15^2 + 20^2 + 25^2$

Example #2:

```
da = DynamicArray([100])
print(da.reduce(lambda x, y: x + y ** 2))
print(da.reduce(lambda x, y: x + y ** 2, -1))
da.remove_at_index(0)
print(da.reduce(lambda x, y: x + y ** 2))
print(da.reduce(lambda x, y: x + y ** 2, -1))
```

Output:

```
100
9999
None
-1
```

Part 2 - Summary and Specific Instructions

1. Implement a Bag ADT class by completing skeleton code provided in the file `bag_da.py`. You will use the Dynamic Array data structure that you implemented in part 1 of this assignment as the underlying data storage for your Bag ADT.

2. Once completed, your implementation will include the following methods:

   ```
   add()
   remove()
   count()
   clear()
   equal()
   ```

3. We will test your implementation with different types of objects, not just integers. We guarantee that all such objects will have correct implementation of methods __eq__, __lt__, __gt__, __ge__, __le__, and __str__.

4. The number of objects stored in the Bag at any given time will be between 0 and 1,000,000 inclusive. The bag must allow for the storage of duplicate objects.

5. RESTRICTIONS: You are not allowed to use ANY built-in Python data structures and/or their methods. You must solve this portion of the assignment by importing the DynamicArray class that you wrote in part 1 and using class methods to write your solution.

   You are also not allowed to directly access any variables of the DynamicArray class (like self.size, self.capacity and self.data in part 1). All work must be done by using only class methods.

add(self, value: object) -> None:

This method adds a new element to the bag. It must be implemented with O(1) amortized runtime complexity.

Example #1:
```
bag = Bag()
print(bag)
values = [10, 20, 30, 10, 20, 30]
for value in values:
    bag.add(value)
print(bag)
```

Output:
```
BAG: 0 elements. []
BAG: 6 elements. [10, 20, 30, 10, 20, 30]
```

remove(self, value: object) -> bool:

This method removes any one element from the bag that matches the provided "value" object. The method returns True if some object was actually removed from the bag. Otherwise it returns False. It must be implemented with O(N) runtime complexity.

Example #1:
```
bag = Bag([1, 2, 3, 1, 2, 3, 1, 2, 3])
print(bag)
print(bag.remove(7), bag)
print(bag.remove(3), bag)
print(bag.remove(3), bag)
print(bag.remove(3), bag)
print(bag.remove(3), bag)
```

Output:
```
BAG: 9 elements. [1, 2, 3, 1, 2, 3, 1, 2, 3]
False BAG: 9 elements. [1, 2, 3, 1, 2, 3, 1, 2, 3]
True BAG: 8 elements. [1, 2, 1, 2, 3, 1, 2, 3]
True BAG: 7 elements. [1, 2, 1, 2, 1, 2, 3]
True BAG: 6 elements. [1, 2, 1, 2, 1, 2]
False BAG: 6 elements. [1, 2, 1, 2, 1, 2]
```

count(self, value: object) -> int:

This method counts the number of elements in the bag that match the provided "value" object. It must be implemented with O(N) runtime complexity.

Example #1:
```
bag = Bag([1, 2, 3, 1, 2, 2])
print(bag, bag.count(1), bag.count(2), bag.count(3), bag.count(4))
```

Output:
```
BAG: 6 elements. [1, 2, 3, 1, 2, 2] 2 3 1 0
```

clear(self) -> None:

This method clears the contents of the bag. It must be implemented with O(1) runtime complexity.

Example #1:
```
bag = Bag([1, 2, 3, 1, 2, 3])
print(bag)
bag.clear()
print(bag)
```

Output:
```
BAG: 6 elements. [1, 2, 3, 1, 2, 3]
BAG: 0 elements. []
```

equal(self, second_bag: object) -> bool:

This method compares the contents of a bag with the content of a second bag provided as a parameter. The method returns True if the bags are equal (have the same number of elements and contain the same elements without regards to the order of elements). Otherwise, it returns False. An empty bag is only considered equal to another empty bag. This method should not change the contents of either bag.

The runtime complexity of this implementation should be no worse than O(N^2). The maximum test case size for this method will be limited to bags with 1,000 items in each.

Example #1:
```
bag1 = Bag([10, 20, 30, 40, 50, 60])
bag2 = Bag([60, 50, 40, 30, 20, 10])
bag3 = Bag([10, 20, 30, 40, 50])
bag_empty = Bag()

print(bag1, bag2, bag3, bag_empty, sep="\n")
print(bag1.equal(bag2), bag2.equal(bag1))
print(bag1.equal(bag3), bag3.equal(bag1))
print(bag2.equal(bag3), bag3.equal(bag2))
print(bag1.equal(bag_empty), bag_empty.equal(bag1))
print(bag_empty.equal(bag_empty))
print(bag1, bag2, bag3, bag_empty, sep="\n")

bag1 = Bag([100, 200, 300, 200])
bag2 = Bag([100, 200, 30, 100])
print(bag1.equal(bag2))
```

Output:
```
BAG: 6 elements. [10, 20, 30, 40, 50, 60]
BAG: 6 elements. [60, 50, 40, 30, 20, 10]
BAG: 5 elements. [10, 20, 30, 40, 50]
BAG: 0 elements. []
True True
False False
False False
False False
True
BAG: 6 elements. [10, 20, 30, 40, 50, 60]
BAG: 6 elements. [60, 50, 40, 30, 20, 10]
BAG: 5 elements. [10, 20, 30, 40, 50]
BAG: 0 elements. []
False
```

Part 3 - Summary and Specific Instructions

1. Implement a Queue ADT class by completing the skeleton code provided in the file `queue_da.py`. You will use the Dynamic Array data structure that you implemented in part 1 of this assignment as the underlying data storage for your Queue ADT.

2. Once completed, your implementation will include the following methods:

   ```
   enqueue()
   dequeue()
   ```

3. We will test your implementation with different types of objects, not just integers. We guarantee that all such objects will have correct implementation of methods __eq__, __lt__, __gt__, __ge__, __le__, and __str__.

4. The number of objects stored in the Queue at any given time will be between 0 and 1,000,000 inclusive. The queue must allow for the storage of duplicate elements.

5. RESTRICTIONS: You are not allowed to use ANY built-in Python data structures and/or their methods. You must solve this portion of the assignment by importing the DynamicArray class that you wrote in part 1 and using class methods to write your solution.

   You are also not allowed to directly access any variables of the DynamicArray class (like self.size, self.capacity and self.data in part 1). All work must be done by using only class methods.

enqueue(self, value: object) -> None:

This method adds a new value to the end of the queue. It must be implemented with O(1) amortized runtime complexity.

Example #1:

```
q = Queue()
print(q)
for value in [1, 2, 3, 4, 5]:
    q.enqueue(value)
print(q)
```

Output:

```
QUEUE: 0 elements. []
QUEUE: 5 elements. [1, 2, 3, 4, 5]
```

dequeue(self) -> object:

This method removes and returns the value at the beginning of the queue. It must be implemented with O(N) runtime complexity. If the queue is empty, the method raises a custom “QueueException”. Code for the exception is provided in the skeleton file.

Example #1:

```
q = Queue()
for value in [1, 2, 3, 4, 5]:
    q.enqueue(value)
print(q)
for i in range(6):
    try:
        print(q.dequeue())
    except Exception as e:
        print("No elements in queue", type(e))
```

Output:

```
QUEUE: 5 elements. [1, 2, 3, 4, 5]
1
2
3
4
5
No elements in queue <class '__main__.QueueException'>
```

Part 4 - Summary and Specific Instructions

1. Implement a MaxStack ADT class by completing the skeleton code provided in the file `max_stack_da.py`. You will use the Dynamic Array data structure you implemented in part 1 of this assignment as the underlying data storage for your MaxStack ADT.

2. Your MaxStack ADT implementation will include standard Stack methods and also one new method - get_max():

   ```
   push()
   pop()
   top()
   get_max()
   ```

3. We will test your implementation with different types of objects, not just integers. We guarantee that all such objects will have correct implementation of methods __eq__, __lt__, __gt__, __ge__, __le__, and __str__.

4. The number of objects stored in the Stack at any given time will be between 0 and 1,000,000 inclusive. The stack must allow for the storage of duplicate objects.

5. RESTRICTIONS: You are not allowed to use ANY built-in Python data structures and/or their methods. You must solve this portion of the assignment by importing the DynamicArray class that you wrote in part 1 and using class methods to write your solution.

   You are also not allowed to directly access any variables of the DynamicArray class (like self.size, self.capacity and self.data in part 1). All work must be done by using only class methods.

push(self, value: object) -> None:

This method adds a new element to the top of the stack. It must be implemented with O(1) amortized runtime complexity.

Example #1:
```
s = MaxStack()
print(s)
for value in [1, 2, 3, 4, 5]:
    s.push(value)
print(s)
```

Output:
```
MAX STACK: 0 elements. []
MAX STACK: 5 elements. [1, 2, 3, 4, 5]
```

pop(self) -> object:

This method removes the top element from the stack and returns its value. It must be implemented with O(1) amortized runtime complexity. If the stack is empty, the method raises a custom “StackException”. Code for the exception is provided in the skeleton file.

Example #1:
```
s = MaxStack()
try:
    print(s.pop())
except Exception as e:
    print("Exception:", type(e))
for value in [1, 2, 3, 4, 5]:
    s.push(value)
for i in range(6):
    try:
        print(s.pop())
    except Exception as e:
        print("Exception:", type(e))
```

Output:
```
Exception: <class '__main__.StackException'>
5
4
3
2
1
Exception: <class '__main__.StackException'>
```

top(self) -> object:

This method returns the value of the top element of the stack without removing it. It must be implemented with O(1) runtime complexity. If the stack is empty, the method raises a custom "StackException". Code for the exception is provided in the skeleton file.

Example #1:

```
s = MaxStack()
try:
    s.top()
except Exception as e:
    print("No elements in stack", type(e))
s.push(10)
s.push(20)
print(s)
print(s.top())
print(s.top())
print(s)
```

Output:

```
No elements in stack <class '__main__.StackException'>
MAX STACK: 2 elements. [10, 20]
20
20
MAX STACK: 2 elements. [10, 20]
```

get_max(self) -> object:

This method returns the maximum value currently stored in the stack. It must be implemented with O(1) runtime complexity. If the stack is empty, the method raises a custom "StackException". Code for the exception is provided in the skeleton file.

Example #1:

```
s = MaxStack()
for value in [1, -20, 15, 21, 21, 40, 50]:
    print(s, ' ', end='')
    try:
        print(s.get_max())
    except Exception as e:
        print(type(e))
    s.push(value)
while not s.is_empty():
    print(s.size(), end='')
    print(' Pop value:', s.pop(), ' get_max after: ', end='')
    try:
        print(s.get_max())
    except Exception as e:
        print(type(e))
```

Output:

```
MAX STACK: 0 elements. []  <class '__main__.StackException'>
MAX STACK: 1 elements. [1]  1
MAX STACK: 2 elements. [1, -20]  1
MAX STACK: 3 elements. [1, -20, 15]  15
MAX STACK: 4 elements. [1, -20, 15, 21]  21
MAX STACK: 5 elements. [1, -20, 15, 21, 21]  21
MAX STACK: 6 elements. [1, -20, 15, 21, 21, 40]  40
7 Pop value: 50  get_max after: 40
6 Pop value: 40  get_max after: 21
5 Pop value: 21  get_max after: 21
4 Pop value: 21  get_max after: 15
3 Pop value: 15  get_max after: 1
2 Pop value: -20  get_max after: 1
1 Pop value: 1  get_max after: <class '__main__.StackException'>
```

Part 5 - Summary and Specific Instructions

1. Implement a Queue ADT class by completing the skeleton code provided in the file `queue_from_stacks.py`. You will use the MaxStack ADT that you implemented in part 4 of this assignment as the underlying data storage for your Queue ADT.

2. Your Queue ADT implementation will include the following methods:

   ```
   enqueue()
   dequeue()
   ```

3. We will test your implementation with different types of objects, not just integers. We guarantee that all such objects will have correct implementations of methods __eq__, __lt__, __gt__, __ge__, __le__, and __str__.

4. The number of objects stored in the Queue at any given time will be between 0 and 1,000,000 inclusive. The Queue must allow for the storage of duplicate elements.

5. RESTRICTIONS: You are not allowed to use ANY built-in Python data structures and/or their methods. You must solve this portion of the assignment by importing the MaxStack class that you wrote in part 4 and using class methods to write your solution.

   You are also not allowed to directly access any variables of the MaxStack class. All work must be done by using only class methods (push(), pop(), top(), size(), and is_empty()). You don't need to use the method get_max() in your implementation.

enqueue(self, value: object) -> None:

This method adds a new value to the end of the queue. It must be implemented with O(1) amortized runtime complexity.

Example #1:
```
q = Queue()
print(q)
for value in [1, 2, 3, 4, 5]:
    q.enqueue(value)
print(q)
```

Output:
```
QUEUE: 0 elements. MAX STACK: 0 elements. []
QUEUE: 5 elements. MAX STACK: 5 elements. [1, 2, 3, 4, 5]
```

dequeue(self) -> object:

This method removes and returns the value at the beginning of the queue. It must be implemented with O(N) runtime complexity.

If the queue is empty, the method raises a custom "QueueException". Code for the exception is provided in the skeleton file.

Example #1:
```
q = Queue()
for value in [1, 2, 3, 4, 5]:
    q.enqueue(value)
print(q)
for i in range(6):
    try:
        print(q.dequeue(), q)
    except Exception as e:
        print("No elements in queue", type(e))
```

Output:
```
QUEUE: 5 elements. MAX STACK: 5 elements. [1, 2, 3, 4, 5]
1 QUEUE: 4 elements. MAX STACK: 4 elements. [2, 3, 4, 5]
2 QUEUE: 3 elements. MAX STACK: 3 elements. [3, 4, 5]
3 QUEUE: 2 elements. MAX STACK: 2 elements. [4, 5]
4 QUEUE: 1 elements. MAX STACK: 1 elements. [5]
5 QUEUE: 0 elements. MAX STACK: 0 elements. []
No elements in queue <class '__main__.QueueException'>
```