

# Implementing a Capability Machine model into Iris

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# Introduction

High-level  
Programming  
Language

Assembly

- ▶ Local state encapsulation
- ▶ Well bracketed control flow

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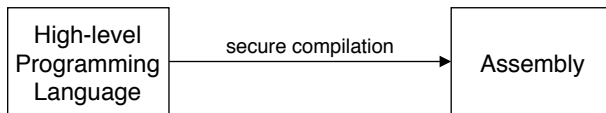
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# Layers of Abstraction

## Programming Languages

- ▶ Local State Encapsulation
- ▶ Well Bracketed Control Flow

## Assembly Language

- ▶ Programs lie in Memory, Program Counter, ...
- ▶ Arbitrary Pointer Manipulation
- ▶ Arbitrary Jumps

## Machine Code

- ▶ Instruction Decoding, Cache, etc.

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# Overview

Capability Machines

Reasoning about Capability Safety

Program Logic

Proving Hoare Triples

A Unary Logical Relation for Reasoning about Semantic Properties  
of an Untyped Language

The Fundamental Theorem of Logical Relations

Reasoning about Unknown Code

# Capability Machines

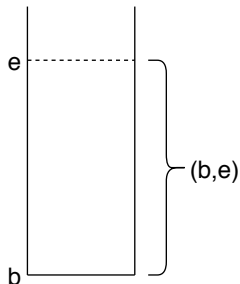
# Capability Machine

**Capability:** An unforgeable token of authority



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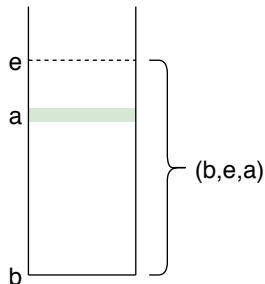
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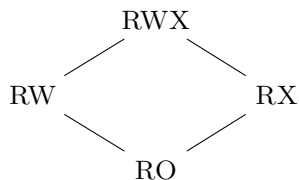
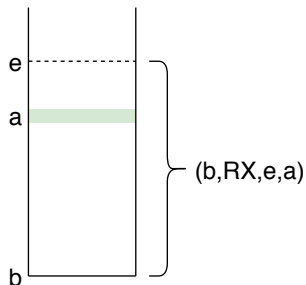
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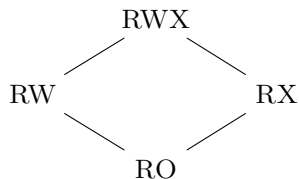
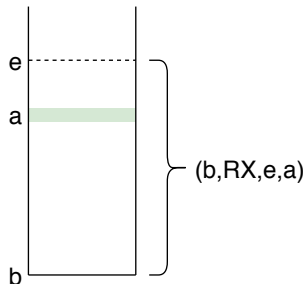
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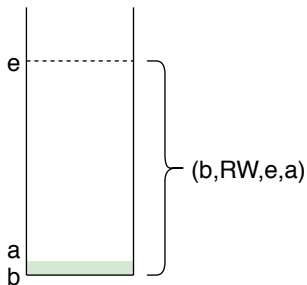
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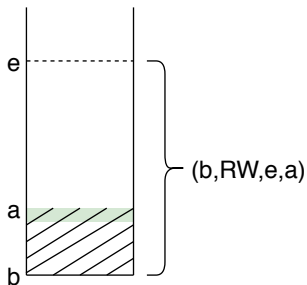
## Enforcing Local Stack Encapsulation using Capabilities

## Local State Encapsulation



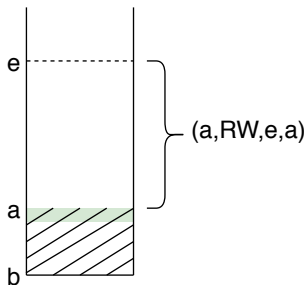
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5 halt
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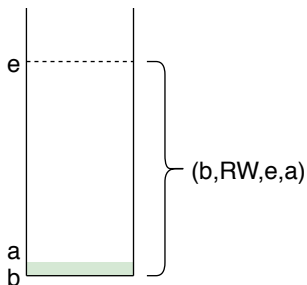


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## Enforcing Well Bracketed Control Flow using Capabilities

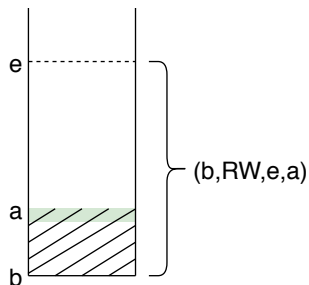


## Well Bracketed Control Flow



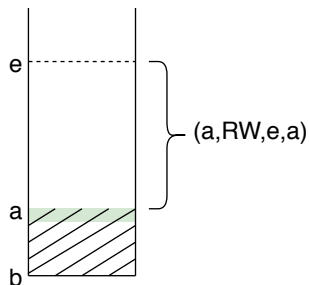
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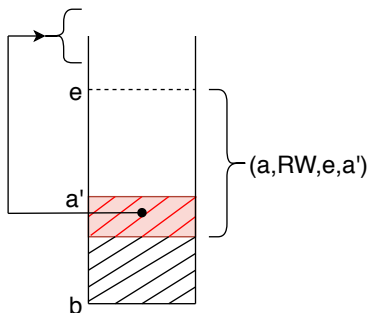
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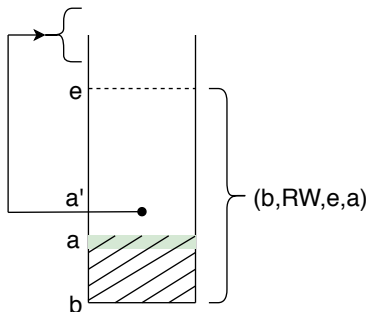
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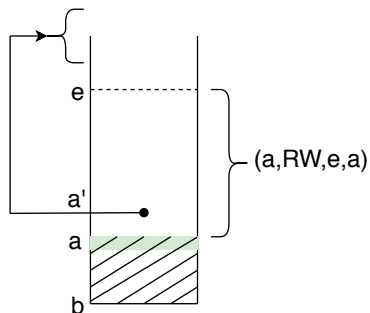
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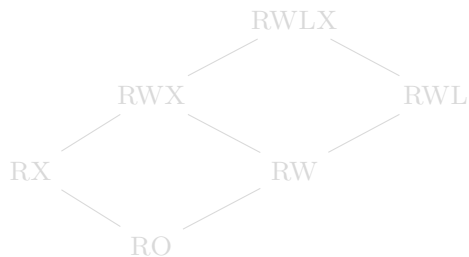


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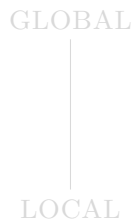
# Local Capabilities

(p,**Local**,b,e,a)



well-bracketed

(p,**Global**,b,e,a)

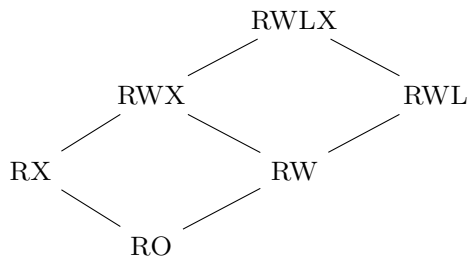


not well-bracketed



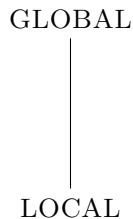
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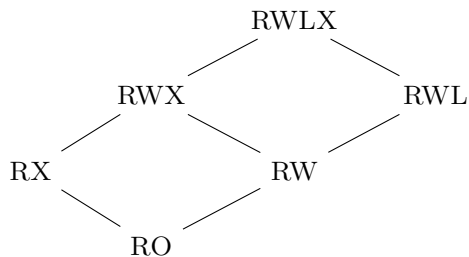
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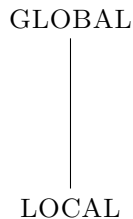
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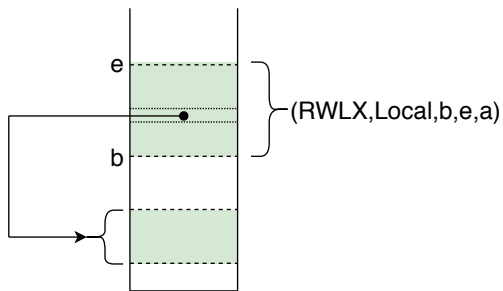
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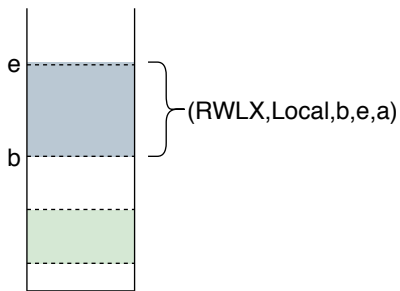
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# Calling Convention



r_stk	$(RWLX, Local, b, e, a)$
-------	--------------------------

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## Reasoning about Capability Safety

# Expressing Capability Safety

- ▶ using a Program Logic
- ▶ using a logical relation to capture invariants on the type system
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  3. define the logical relation
  4. prove the fundamental theorem of logical relations
  5. use the logical relation to prove examples that rely on local state encapsulation and well-bracketed control flow with calls to unknown adversary

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## Program Logic



# Abstract Instructions

$$(reg, mem) \rightarrow (reg', mem')$$

- ▶ Instr Executable
- ▶ Instr Halted  $\rightarrow$  HaltedV
- ▶ Instr Failed  $\rightarrow$  FailedV

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## A Capability Points-to Predicate

# Points-to Predicate with Permissions

$$a \mapsto_a [RWL]w$$

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$$a \mapsto_a [RWL]_w \Longrightarrow^* a \mapsto_a [RWL]((p, Local), b, e, l)$$



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$$\begin{aligned} a \mapsto_a [RWL]_w &\Longrightarrow^* a \mapsto_a [RWL]((p, Local), b, e, l) \\ &\Longrightarrow^* a \mapsto_a [RW]((p, Local), b, e, l) \end{aligned}$$

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## Proving Hoare Triples

Successful Execution

# Hoare Triples of the Program Logic: Success

$$\begin{aligned} & \text{decode}(w) = \text{Load dst src} \\ & \wedge \text{isCorrectPC } ((p_{pc}, g_{pc}), b_{pc}, e_{pc}, a_{pc}) \\ & \wedge \text{readAllowed } p_{src} \wedge \text{withinBounds } (b_{src}, e_{src}, a_{src}) \end{aligned}$$
$$\begin{aligned} & \{ \{ \{ \text{PC} \mapsto_r ((p_{pc}, g_{pc}), b_{pc}, e_{pc}, a_{pc}) * a_{pc} \mapsto_a [p_{pc}]w \\ & \quad * \text{dst} \mapsto_r w_{dst} * \text{src} \mapsto_r ((p_{src}, g_{src}), b_{src}, e_{src}, a_{src}) \\ & \quad * a_{src} \mapsto_a [p_{src}]w_{src} \} \} \} \end{aligned}$$

Instr Executable

$$\begin{aligned} & \{ \{ \{ \text{PC} \mapsto_r ((p_{pc}, g_{pc}), b_{pc}, e_{pc}, a_{pc} + 1) * a_{pc} \mapsto_a [p_{pc}]w \\ & \quad * \text{dst} \mapsto_r w_{src} * \text{src} \mapsto_r ((p_{src}, g_{src}), b_{src}, e_{src}, a_{src}) \\ & \quad * a_{src} \mapsto_a [p_{src}]w_{src} \} \} \} \end{aligned}$$

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## Failed Execution



# Hoare Triples of the Program Logic: Failure

$$\begin{aligned} & \text{decode}(w) = \text{Load dst src} \\ & \wedge \text{isCorrectPC } ((p_{pc}, g_{pc}), b_{pc}, e_{pc}, a_{pc}) \\ & \wedge \neg \text{readAllowed } p_{src} \vee \neg \text{withinBounds } (b_{src}, e_{src}, a_{src}) \\ & \{ \{ \{ \text{PC} \mapsto_r ((p_{pc}, g_{pc}), b_{pc}, e_{pc}, a_{pc}) * a_{pc} \mapsto_a [p_{pc}]w \\ & \quad * \text{src} \mapsto_r ((p_{src}, g_{src}), b_{src}, e_{src}, a_{src}) \} \} \} \\ & \text{Instr Executable} \\ & \{ \{ \{ \text{FailedV}, \top \} \} \} \end{aligned}$$

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# A Unary Logical Relation for Reasoning about Semantic Properties of an Untyped Language

## The Value Relation

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## A unary logical relation of an un-typed language

$$\mathcal{V} : \text{Word} \rightarrow iProp \Sigma$$

- **World**: A collection of state transition systems to reason about *local state*

$$\mathcal{V}(W)(z) \triangleq \exists z' \in \mathbb{Z}. z = z'$$

$$\mathcal{V}(W)((\text{RO}, g), b, e, a) \triangleq \text{read\_write\_cond}(\text{RO}, b, e)$$

$$\begin{aligned} \mathcal{V}(W)((\text{RX}, g), b, e, a) &\triangleq \text{read\_write\_cond}(\text{RX}, b, e) \\ &* \Box \text{exec\_cond}(W)(\text{RX}, g, b, e) \end{aligned}$$

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## The Execute Condition



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$$\text{exec\_cond}(W)(p,g,b,e) \triangleq \begin{cases} \forall a \in [b \ e], W' \sqsubseteq_{\text{pub}} W. \\ \quad \triangleright \mathcal{E}(W')(((p, g), b, e, a)) \quad g = \text{Local} \\ \\ \forall a \in [b \ e], W' \sqsubseteq_{\text{priv}} W. \\ \quad \triangleright \mathcal{E}(W')(((p, g), b, e, a)) \quad g = \text{Global} \end{cases}$$

## The Expression Relation

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$$\begin{aligned}\mathcal{E}(W)(pc) &\triangleq \forall r, \mathcal{R}(W)(r) * \text{context}(W)(r[\text{PC} := pc]) \\ &\quad \rightarrow * \text{WP Seq (Instr Executable)} \\ &\quad \{v, v = \text{HaltedV} \implies \exists W' r', W' \sqsubseteq_{\text{priv}} W \\ &\quad * \text{context}(W')(r')\}\end{aligned}$$

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$$\text{context}(W)(r) = ?$$

# The Expression Relation

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$$\text{context}(W)(r) = \left( \bigstar_{r_i \mapsto w \in r} r_i \mapsto_r w \right) \wedge \text{full\_map } r$$

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# The Fundamental Theorem of Logical Relations

# The Fundamental Theorem of logical relations

If we can read a region, and every word in that region is safe, then  
we can safely execute it

- ▶ "If we can read a region" :  $p = \text{RX} \vee p = \text{RWX} \vee p = \text{RWLX}$
- ▶ "and every word in that region is safe":  
 $\text{read\_write\_cond}(p, b, e)$
- ▶ "then we can safely execute it":  $\mathcal{E}(W)((p, g), b, e, a)$

$$(p = \text{RX} \vee p = \text{RWX} \vee p = \text{RWLX}) \implies$$

$$\text{read\_write\_cond}(p, b, e) \implies \mathcal{E}(W)((p, g), b, e, a)$$

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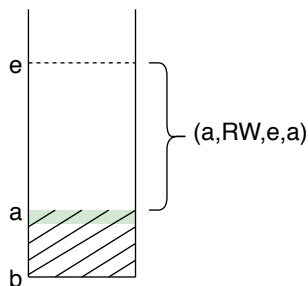
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## Reasoning about Unknown Code

# Reasoning about Unknown Code

**We use the fundamental theorem to reason about calls to an unknown adversary**



```
1 push r_stk 1
2 scall r
3 pop r_stk r_1
4 assert r_1 1
5 halt
```

$$\mathcal{E}(W)(pc) \triangleq \forall r, \mathcal{R}(W)(r) * \text{context}(W)(r[\text{PC} := pc])$$

\* WP Seq (Instr Executable)

$$\{v, v = \text{HaltedV} \implies \exists W' r', W' \sqsubseteq_{\text{priv}} W$$

\* context( $W'$ )( $r'$ )}



# Conclusion

- ▶ Embed a capability machine into Iris
- ▶ Define its program logic
- ▶ Mechanize a unary logical relation for an untyped capability machine language
- ▶ Prove the fundamental theorem of logical relations
- ▶ Reason about examples that rely on Local Stack Encapsulation and Well-Bracketed Control Flow with calls to an unknown adversary

# References



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