Implementing a Capability Machine model into Iris

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Introduction

- Capability machines allow for fine grained control over pointer permissions
- Good target for secure compilation
- In particular: we are interested in enforcing certain higher level abstractions such as *local state encapsulation* as *well-bracketed control flow* at the lowest level of the machine
- We need tools to reason about these subtle properties in a language that does not enforce them
- These tools are elaborate and complex: we want to mechanize them, and facilitate the process of using them

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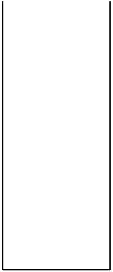
Capability Machines

Capability Machine

Capability: An unforgeable token of authority



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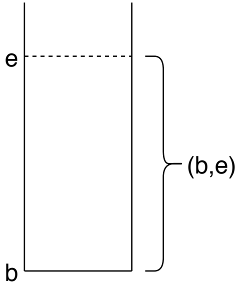

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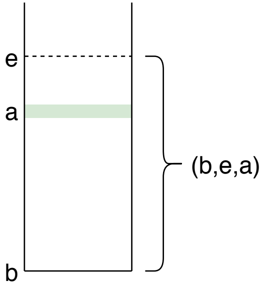

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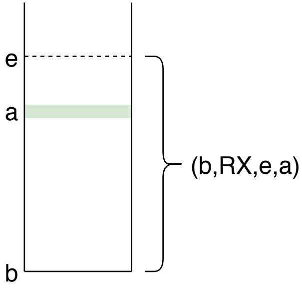

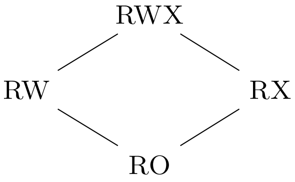

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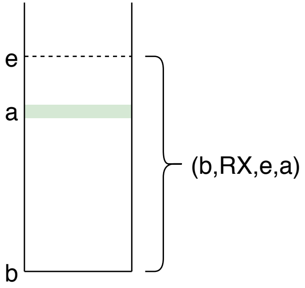
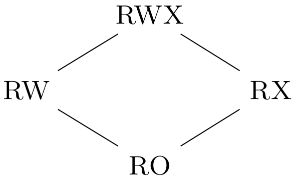
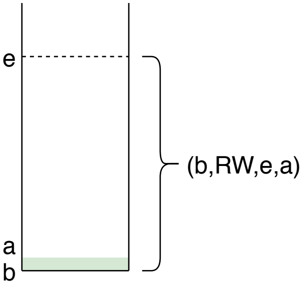




Enforcing Well Bracketed Control Flow using Capabilities

Well Bracketed Control Flow

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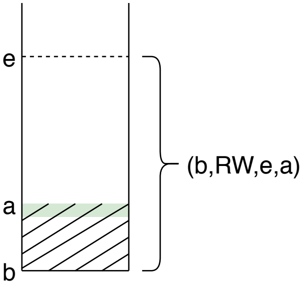
push r_stk 1
scall r
pop r_stk r_1
assert r_1 1
push r_stk 2
scall r
halt
```

Well Bracketed Control Flow

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Capability Safety

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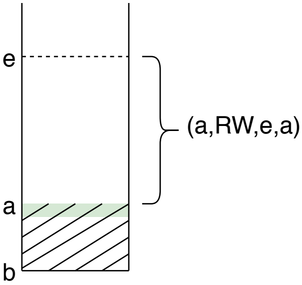


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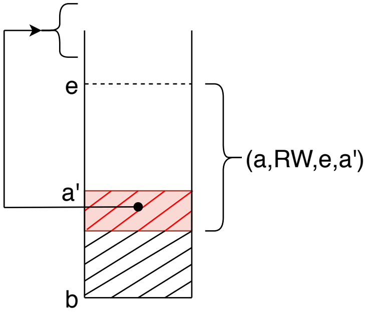


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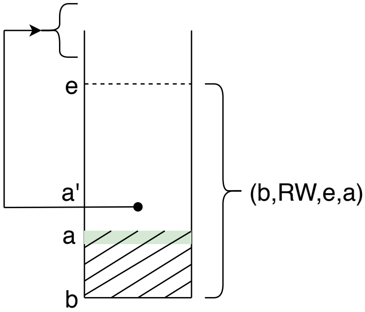


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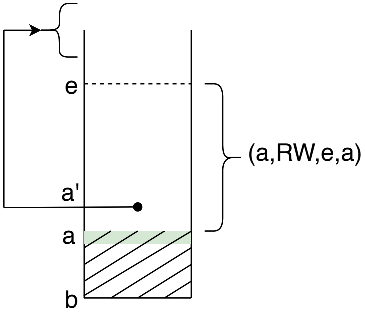


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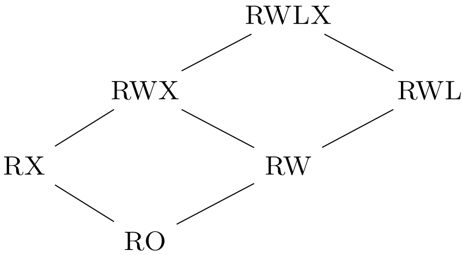
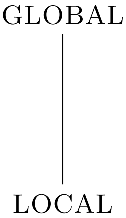
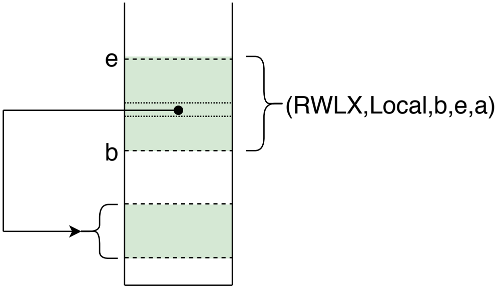




Calling Convention



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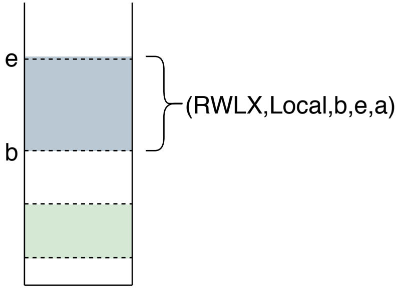

Reasoning about Capability Safety

Expressing Capability Safety

- using a Program Logic
- using a logical relation to capture invariants on the type system
- **using a logical relation on an untyped (or uni-typed) language to capture semantic properties of the language**

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Step-indexed Kripke Logical Relation

$$\mathcal{V}(W) \triangleq \{n, (RW, g, b, e, a) | \cdots\} \cup \cdots$$

- World-circularity problem
  - Step indexing
- The world may evolve: we need future world relation
  - Local capabilities are revoked whereas Global capabilities are not, *the relation needs to model this distinction:*

$$\sqsupseteq_{pub} \qquad \textit{and} \qquad \sqsupseteq_{priv}$$

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Iris: Higher-order Concurrent Separation Logic Framework

- Foundational
- Implemented in Coq – equipped with an interactive proof mode
- Framework – embed any language and its operational semantics into Iris
- Comes equipped with:
  - Invariants
  - Ghost state
  - Always and Later Modalities

We can take advantage of Iris' step-indexed model and invariants to mechanize step-indexed Kripke logical relations with recursive worlds in a succinct and elegant way.

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Expressing Capability Safety in Iris - Challenges

- Region invariants: Iris invariants
- Future world relation: frame preserving updates and world satisfaction
- Step indexing: later modality

Challenges

- Iris was designed with more high level languages in mind, how do we embed a low level machine language into Iris
- Iris abstracts away certain details we want to reason about directly
- There is only one frame preserving update, we need to distinguish between two future world relations

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Roadmap

- embed the language into Iris
- define a program logic by proving Hoare Triples
- define the logical relation – using Iris tools to solve the world circularity problem
- prove the fundamental theorem of logical relations
- use the logical relation to prove examples that rely on local state encapsulation and well-bracketed control flow with calls to unknown adversary

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Program Logic

Abstract Instructions

$$(reg, mem) \rightarrow (reg', mem')$$

- Instr Executable
- Instr Halted $\rightarrow$ HaltedV
- Instr Failed $\rightarrow$ FailedV

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A Capability Points-to Predicate

Points-to Predicate with Permissions

$$a \mapsto_a [RWL]w$$

Points-to Predicate with Permissions

$$a \mapsto_a [RWL]w \Rrightarrow a \mapsto_a [RWL]((p, Local), b, e, l)$$

Points-to Predicate with Permissions

$$
\begin{aligned}
a \mapsto_a [RWL]w &\Rrightarrow a \mapsto_a [RWL]((p, Local), b, e, l) \\
&\Rrightarrow a \mapsto_a [RW]((p, Local), b, e, l)
\end{aligned}
$$

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&\not\Rrightarrow a \mapsto_a [RW]((p', Local), b', e', l')
\end{aligned}
$$

A Unary Logical Relation for Reasoning about Semantic Properties of an Untyped Language

The Value Relation

A unary logical relation of an un-typed language

$$\mathcal{V} : \mathit{Word} \rightarrow \mathit{iProp}\ \Sigma$$

Challenge: distinguish between Local and Global capabilities:

- At the level of the value relation
- Model revocation

STS: A collection of state transition systems

$$\mathcal{V}((\textsc{rw}, g), b, e, a) \triangleq \mathop{\ast}_{a \in [b,e]} \boxed{\exists w, a \mapsto_a [RW]w \ast \mathcal{V}(w)}$$

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From World to state transition system collection

On paper:

$$
\begin{aligned}
\text{Region} = \;& \{\mathit{Revoked}\} \uplus \\
& \{\mathit{Temporary}\} \times {\color{red}\text{State} \times \text{Rels}} \\
& \times (\text{State} \to (\mathit{Wor} \xrightarrow[\sqsupseteq_{pub}]{mon,ne} \text{UPred}(\text{MemSeg}))) \uplus \\
& \{\mathit{Permanent}\} \times {\color{red}\text{State} \times \text{Rels}} \\
& \times (\text{State} \to (\mathit{Wor} \xrightarrow[\sqsupseteq_{priv}]{mon,ne} \text{UPred}(\text{MemSeg}))) \\
\text{World} = \;& \mathbb{N} \rightharpoonup \text{Region}
\end{aligned}
$$

In the Iris mechanization, we use a collection of state transition systems:

$$\Sigma : \mathbb{N} \rightharpoonup \mathit{States} \times \mathbb{N} \rightharpoonup \mathit{Rels}$$

The world circularity problem is now handled using Iris invariants and saved predicates.

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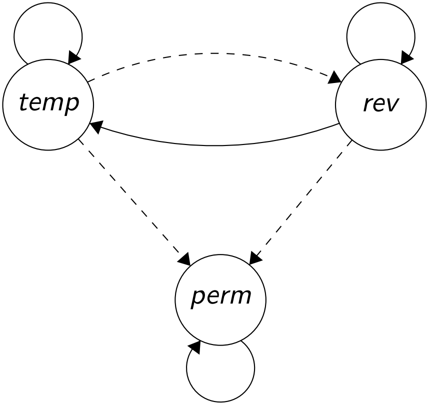
In the Iris mechanization, we use a collection of state transition systems:

$$\Sigma : \mathbb{N} \rightharpoonup \mathit{States} \times \mathbb{N} \rightharpoonup \mathit{Rels}$$

The world circularity problem is now handled using Iris invariants and saved predicates.

Standard STS

$$\Sigma : \mathbb{N} \rightharpoonup \mathit{States} \times \mathbb{N} \rightharpoonup \mathit{Rels}$$



What's new

What's new: capability machine viewpoint

- Mechanized formalization: currently $\sim$ 25000 lines of Iris code
- At a higher level of abstraction
  - Step index $\rightarrow$ later modality
  - World $\rightarrow$ collection of state transition systems

What's new: Iris formalization viewpoint

- Formalization of a machine language, with no distinction between program and memory
- Distinction between well-bracketed and non well-bracketed calls: using public/private transitions

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Conclusion

- Embed a capability machine into Iris
- Define its program logic
- Mechanize a unary logical relation for an untyped capability machine language
- Prove the fundamental theorem of logical relations
- Reason about examples that rely on Local Stack Encapsulation and Well-Bracketed Control Flow with calls to an unknown adversary

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Logical Step-Indexed Logical Relations
LMCS 7(2:16).

The Execute Condition

The Execute Condition

$$\text{exec\_cond}(\Sigma)(\text{p,g,b,e}) \triangleq \begin{cases} \forall a \in [b\ \ e], \Sigma' \sqsupseteq_{pub} \Sigma. & \\ \quad \triangleright\ \ \mathcal{E}(\Sigma')(((p,g),b,e,a)) & g = Local \\ & \\ \forall a \in [b\ \ e], \Sigma' \sqsupseteq_{priv} \Sigma. & \\ \quad \triangleright\ \ \mathcal{E}(\Sigma')(((p,g),b,e,a)) & g = Global \end{cases}$$

The Expression Relation

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$$\mathcal{E}(\Sigma)(pc) \triangleq \forall r, \mathcal{R}(\Sigma)(r) \; * \; \mathsf{context}(\Sigma)(r[\mathsf{PC} := pc])$$
$$\mathrel{-\!\!*} \; \mathrm{WP}\ \mathsf{Seq}\ (\mathsf{Instr}\ \mathsf{Executable})$$
$$\{v, v = \mathit{HaltedV} \implies \exists \Sigma' r', \Sigma' \sqsupseteq_{priv} \Sigma$$
$$* \, \mathsf{context}(\Sigma')(r')\}$$

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$$\text{context}(\Sigma)(r) = \ ?$$

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$$\text{context}(\Sigma)(r) = \Big(\mathop{\Large *}_{r_i \mapsto w \in r} r_i \mapsto_r w \Big) \wedge \mathsf{full\_map}\ r$$

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$$* \; \text{na\_inv } \gamma_{na} \top$$

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$$* \ \mathsf{sts\_full}\ \Sigma$$
$$* \ \mathsf{region}\ \Sigma$$

The Fundamental Theorem of Logical Relations

The Fundamental Theorem of logical relations

If we can read a region, and every word in that region is safe, then we can safely execute it

- "If we can read a region" : $p = \mathrm{RX} \vee p = \mathrm{RWX} \vee p = \mathrm{RWLX}$
- "and every word in that region is safe": read_write_cond (p, b, e)
- "then we can safely execute it": $\mathcal{E}(\Sigma)(((p, g), b, e, a))$

$$(p = \mathrm{RX} \vee p = \mathrm{RWX} \vee p = \mathrm{RWLX}) \implies$$
$$\text{read\_write\_cond } (p, b, e) \implies \mathcal{E}(\Sigma)(((p, g), b, e, a))$$

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Reasoning about Unknown Code

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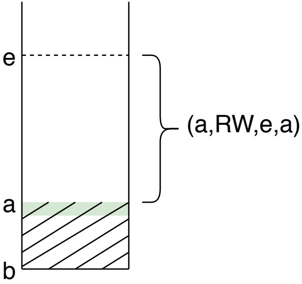
We use the fundamental theorem to reason about calls to an unknown adversary




```
push r_stk 1
scall r
pop r_stk r_1
assert r_1 1
halt
```

$$
\begin{aligned}
\mathcal{E}(\Sigma)(pc) \triangleq \forall r, \mathcal{R}(\Sigma)(r) \; * \; & \text{context}(\Sigma)(r[\text{PC} := pc]) \\
\mathrel{-\!\!*} \; & \mathrm{WP}\ \text{Seq (Instr Executable)} \\
& \quad \{v, v = \mathit{HaltedV} \implies \exists \Sigma' r', \Sigma' \sqsupseteq_{priv} \Sigma \\
& \quad * \text{context}(\Sigma')(r')\}
\end{aligned}
$$

Proving the Fundamental Theorem

$$ \tag{1} $$

$$ \mathop{\Large *}_{a \in [b,e]} \text{read\_write\_cond } a \tag{2} $$

$$ * \, \mathcal{R}(r) \tag{3} $$

- -

$$ \mathop{\Large *}_{reg \hookrightarrow w \in r} reg \mapsto_r w \tag{4} $$

$$ \mathrm{WP}\ \text{Seq (Instr Executable)}\ \{v, v = \mathit{HaltedV} \implies \exists \Sigma' r', \Sigma' \sqsupseteq_{priv} \Sigma * \text{context}(\Sigma')(r')\} $$

Proving the Fundamental Theorem

$$(1)$$

$$\mathop{\ast}_{a \in [b,e]} \text{read\_write\_cond } a \tag{2}$$

$$\ast\, \mathcal{R}(r) \tag{3}$$

- -

$$\mathop{\ast}_{reg \hookrightarrow w \in r \setminus \mathrm{PC}} reg \mapsto_r w \tag{4}$$

$$\ast\, \mathrm{PC} \mapsto_r (pc_g, pc_p, pc_b, pc_e, pc_a) \tag{5}$$

$$\mathrm{WP}\ \text{Seq (Instr Executable)}\ \{v, v = HaltedV \implies \exists \Sigma' r', \Sigma' \sqsupseteq_{priv} \Sigma * \text{context}(\Sigma')(r')\}$$

Proving the Fundamental Theorem

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$$\ast\, pc_a \mapsto_a [pc_p]w \tag{6}$$

$$\mathrm{WP}\ \text{Seq (Instr Executable)}\ \{v, v = HaltedV \implies \exists \Sigma' r', \Sigma' \sqsupseteq_{priv} \Sigma * \text{context}(\Sigma')(r')\}$$

Proving the Fundamental Theorem

$$decode(w) = \text{Load } dst\ src \quad (1)$$

$$\ast\ \mathcal{R}(r) \quad (2)$$

- -

$$\mathop{\ast}_{reg \hookrightarrow w \in r \setminus \mathrm{PC}} reg \mapsto_r w \quad (3)$$

$$\ast\ \mathrm{PC} \mapsto_r (pc_g, pc_p, pc_b, pc_e, pc_a) \quad (4)$$

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$$\mathrm{WP}\ \text{Seq (Instr Executable)}\ \{v, v = HaltedV \implies \exists \Sigma' r', \Sigma' \sqsupseteq_{priv} \Sigma * \text{context}(\Sigma')(r')\}$$

Proving the Fundamental Theorem

$$decode(w) = \text{Load } dst\ src \quad (1)$$

$$\ast\ \mathcal{V}(p, g, b, e, a) \quad (2)$$

- -

$$\ast\ dst \mapsto_r w_{dst} \quad (3)$$

$$\ast\ src \mapsto_r (p, g, b, e, a) \quad (4)$$

$$\ast\ \mathrm{PC} \mapsto_r (pc_g, pc_p, pc_b, pc_e, pc_a) \quad (5)$$

$$\ast\ pc_a \mapsto_a [pc_p]w \quad (6)$$

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Proving the Fundamental Theorem

$$decode(w) = \text{Load } dst\ src \tag{1}$$
$$\ast\ \mathcal{V}(p, g, b, e, a) \tag{2}$$
$$\ast\ \mathcal{V}(w_{src}) \tag{3}$$

- -

$$\ast\ dst \mapsto_r w_{dst} \tag{4}$$
$$\ast\ src \mapsto_r (p, g, b, e, a) \tag{5}$$
$$\ast\ \mathrm{PC} \mapsto_r (pc_g, pc_p, pc_b, pc_e, pc_a) \tag{6}$$
$$\ast\ pc_a \mapsto_a [pc_p]w \tag{7}$$
$$\ast\ a \mapsto_a [p]w_{src} \tag{8}$$

$$\mathrm{WP}\ \text{Seq (Instr Executable)}\ \{v, v = HaltedV \implies \exists \Sigma' r', \Sigma' \sqsupseteq_{priv} \Sigma \ast \text{context}(\Sigma')(r')\}$$

Proving the Fundamental Theorem

$$
\begin{array}{ll}
\mathit{decode}(w) = \text{Load } \mathit{dst}\ \mathit{src} & (1)\\
\ast\ \mathcal{V}(p, g, b, e, a) & (2)\\
\ast\ \mathcal{V}(w_{src}) & (3)
\end{array}
$$

- -

$$
\begin{array}{ll}
\ast\ \mathit{dst} \mapsto_r {\color{red} w_{src}} & (4)\\
\ast\ \mathit{src} \mapsto_r (p, g, b, e, a) & (5)\\
\ast\ \mathrm{PC} \mapsto_r (pc_g, pc_p, pc_b, pc_e, pc_a) & (6)\\
\ast\ pc_a \mapsto_a [pc_p] w & (7)\\
\ast\ a \mapsto_a [p] w_{src} & (8)
\end{array}
$$

$$
\mathrm{WP}\ \text{Seq (Instr Executable)}\ \{v, v = \mathit{HaltedV} \implies \exists \Sigma' r', \Sigma' \sqsupseteq_{priv} \Sigma \ast \text{context}(\Sigma')(r')\}
$$