# Implementing a Capability Machine model into Iris

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- Capability machines allow for fine grained control over pointer permissions
- Good target for secure compilation
- In particular: we are interested in enforcing certain higher level abstractions such as local state encapsulation as well-bracketed control flow at the lowest level of the machine
- We need tools to reason about these subtle properties in a language that does not enforce them
- ► These tools are elaborate and complex: we want to mechanize them, and facilitate the process of using them

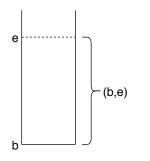
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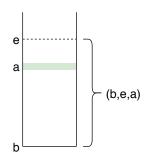
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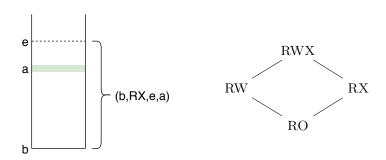


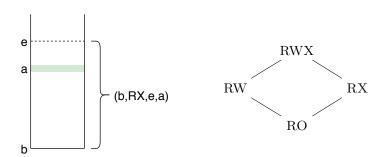




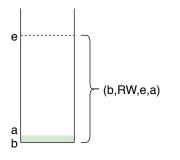




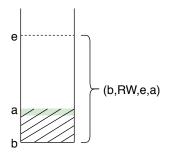




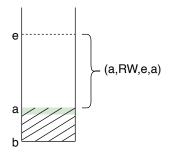
# Enforcing Well Bracketed Control Flow using Capabilities



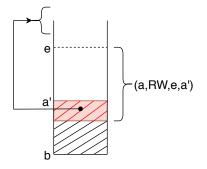
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push r_stk 1
scall r
pop r_stk r_1
assert r_1 1
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```



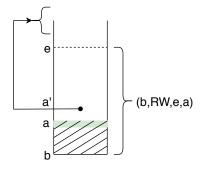
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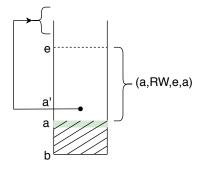
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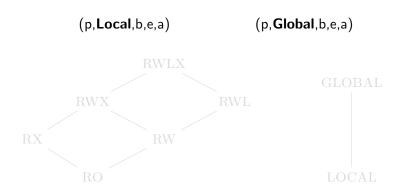
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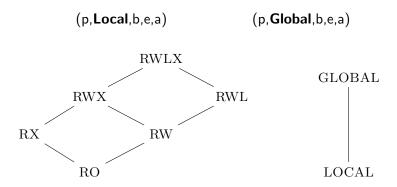
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# Local Capabilities

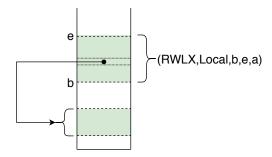
### Local Capabilities



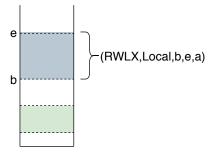
### Local Capabilities



### Calling Convention



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Reasoning about Capability Safety

- using a Program Logic
- using a logical relation to capture invariants on the type system
- using a logical relation on an untyped (or uni-typed)language to capture semantic properties of the language

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$$\mathcal{V}(W) \triangleq \{n, (RW, g, b, e, a) | \cdots \} \cup \cdots$$

- ▶ World-circularity problem
  - Step indexing
- ► The world may evolve: we need future world relation
  - Local capabilities are revoked whereas Global capabilities are not, the relation needs to model this distinction:



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### Iris: Higher-order Concurrent Separation Logic Framework

- Foundational
- Implemented in Coq equipped with an interactive proof mode
- ► Framework embed any language and its operational semantics into Iris
- Comes equipped with:
  - Invariants
  - ► Ghost state
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- ► Future world relation: frame preserving updates and world satisfaction
- Step indexing: later modality

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- Iris abstracts away certain details we want to reason about directly
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- embed the language into Iris
- define a program logic by proving Hoare Triples
- define the logical relation using Iris tools to solve the world circularity problem
- prove the fundamental theorem of logical relations
- use the logical relation to prove examples that rely on local state encapsulation and well-bracketed control flow with calls to unknown adversary

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# Program Logic

$$(\textit{reg}, \textit{mem}) \rightarrow (\textit{reg}', \textit{mem}')$$

- Instr Executable
- ► Instr Halted → HaltedV
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## A Capability Points-to Predicate

 $a\mapsto_a [RWL]w$ 

$$a \mapsto_a [RWL]w \Rightarrow a \mapsto_a [RWL]((p, Local), b, e, l)$$

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A Unary Logical Relation for Reasoning about Semantic Properties of an Untyped Language

### A unary logical relation of an un-typed language

$$\mathcal{V}: \mathit{Word} \to \mathit{iProp}\ \Sigma$$

Challenge: distinguish between Local and Global capabilities:

- At the level of the value relation
- ► Model revocation

$$\mathcal{V}((\mathsf{RW},g),b,e,a) \triangleq \underset{a \in [b,e]}{\bigstar} \boxed{\exists w,a \mapsto_a [RW]w * \mathcal{V}(w)}$$

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#### The Value Relation

#### A unary logical relation of an un-typed language

$$\mathcal{V}: STS \rightarrow Word \rightarrow iProp \Sigma$$

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STS: A collection of state transition systems

$$\mathcal{V}(\Sigma)((\mathsf{RW},g),b,e,a) \triangleq \underset{a \in [b,e]}{\bigstar} \left[ \exists w, a \mapsto_a [RW]w * \mathcal{V}(\Sigma)(w) \right]$$

# From World to state transition system collection

On paper:

$$\begin{array}{ll} \mathsf{Region} = & \{\mathit{Revoked}\} \ \uplus \\ & \{\mathit{Temporary}\} \times \mathsf{State} \times \mathsf{Rels} \\ & \times (\mathsf{State} \to (\mathit{Wor} \xrightarrow{\mathit{mon,ne}} \mathsf{UPred}(\mathsf{MemSeg}))) \ \uplus \\ & \{\mathit{Permanent}\} \times \mathsf{State} \times \mathsf{Rels} \\ & \times (\mathsf{State} \to (\mathit{Wor} \xrightarrow{\mathit{mon,ne}} \mathsf{UPred}(\mathsf{MemSeg}))) \\ & \mathsf{World} = & \mathbb{N} \to \mathsf{Region} \end{array}$$

In the Iris mechanization, we use a collection of state transition systems:

$$\Sigma : \mathbb{N} \longrightarrow States \times \mathbb{N} \longrightarrow Rels$$

The world circularity problem is now handled using Iris invariants and saved predicates.

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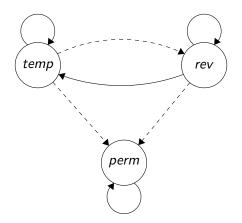
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## Standard STS

 $\Sigma : \mathbb{N} \longrightarrow States \times \mathbb{N} \longrightarrow Rels$ 



#### What's new

#### What's new: capability machine viewpoint

- ightharpoonup Mechanized formalization: currently  $\sim$  25000 lines of Iris code
- ► At a higher level of abstraction
  - Step index → later modality
  - lacktriangle World ightarrow collection of state transition systems

#### What's new: Iris formalization viewpoint

- ► Formalization of a machine level language, with no distinction between program and memory
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#### Conclusion

- Embed a capability machine into Iris
- ► Define its program logic
- Mechanize a unary logical relation for an untyped capability machine language
- Prove the fundamental theorem of logical relations
- Reason about examples that rely on Local Stack Encapsulation and Well-Bracketed Control Flow with calls to an unknown adversary

### References



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## The Execute Condition

#### The Execute Condition

$$\mathsf{exec\_cond}(\Sigma)(\mathsf{p},\mathsf{g},\mathsf{b},\mathsf{e}) \triangleq \begin{cases} \forall \mathsf{a} \in [b\ e], \Sigma' \sqsupseteq_{\mathsf{pub}} \Sigma. \\ \rhd \ \mathcal{E}(\Sigma')(((\mathsf{p},\mathsf{g}),\mathsf{b},\mathsf{e},\mathsf{a})) \quad \mathsf{g} = \mathsf{Local} \end{cases}$$

$$\forall \mathsf{a} \in [b\ e], \Sigma' \sqsupseteq_{\mathsf{priv}} \Sigma. \\ \rhd \ \mathcal{E}(\Sigma')(((\mathsf{p},\mathsf{g}),\mathsf{b},\mathsf{e},\mathsf{a})) \quad \mathsf{g} = \mathsf{Global} \end{cases}$$

```
 \mathcal{E}(\Sigma)(pc) \triangleq \forall r, \mathcal{R}(\Sigma)(r) * \operatorname{context}(\Sigma)(r[\operatorname{PC} := pc]) \\ - * \operatorname{WP} \operatorname{Seq} (\operatorname{Instr} \operatorname{Executable}) \\ \{v, v = \operatorname{Halted}V \implies \exists \Sigma'r', \Sigma' \supseteq_{\operatorname{priv}} \Sigma \\ * \operatorname{context}(\Sigma')(r')\}
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 $context(\Sigma)(r) = ?$ 

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$$* \operatorname{context}(\Sigma')(r')\}$$

$$context(\Sigma)(r) = (\underset{r_i \mapsto w \in r}{\bigstar} r_i \mapsto_r w) \land full\_map r$$

$$\begin{split} \mathcal{E}(\Sigma)(pc) &\triangleq \forall r, \mathcal{R}(\Sigma)(r) \ * \ \mathsf{context}(\Sigma)(r[\mathsf{PC} := pc]) \\ & \twoheadrightarrow \ \mathsf{WP} \ \mathsf{Seq} \ (\mathsf{Instr} \ \mathsf{Executable}) \\ & \qquad \qquad \{v, v = \mathsf{Halted}V \implies \exists \Sigma'r', \Sigma' \sqsupseteq_{\mathit{priv}} \Sigma \\ & \qquad \qquad * \ \mathsf{context}(\Sigma')(r') \} \end{split}$$

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$$\begin{aligned} \mathsf{context}(\Sigma)(r) &= \big( \underset{r_i \mapsto w \in r}{\bigstar} r_i \mapsto_r w \big) \land \mathsf{full\_map} \ r \\ &* \mathsf{na\_inv} \ \gamma_{na} \top \\ &* \mathsf{sts\_full} \ \Sigma \\ &* \mathsf{region} \ \Sigma \end{aligned}$$

The Fundamental Theorem of Logical Relations

# The Fundamental Theorem of logical relations

If we can read a region, and every word in that region is safe, then we can safely execute it

- ▶ "If we can read a region" :  $p = RX \lor p = RWX \lor p = RWLX$
- "and every word in that region is safe": read\_write\_cond (p, b, e)
- ▶ "then we can safely execute it":  $\mathcal{E}(\Sigma)(((p,g),b,e,a))$

$$(p = \text{RX} \lor p = \text{RWX} \lor p = \text{RWLX}) \implies$$
 $\text{read\_write\_cond} (p, b, e) \implies \mathcal{E}(\Sigma)(((p, g), b, e, a))$ 

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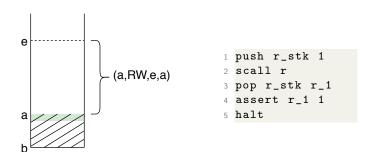
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# Reasoning about Unknown Code

## Reasoning about Unknown Code

We use the fundamental theorem to reason about calls to an unknown adversary



$$\mathcal{E}(\Sigma)(pc) \triangleq \forall r, \mathcal{R}(\Sigma)(r) * \operatorname{context}(\Sigma)(r[\operatorname{PC} := pc])$$

$$-* \operatorname{WP} \operatorname{Seq} (\operatorname{Instr} \operatorname{Executable})$$

$$\{v, v = \operatorname{\textit{HaltedV}} \implies \exists \Sigma' r', \Sigma' \sqsupseteq_{priv} \Sigma$$

$$* \operatorname{context}(\Sigma')(r') \}$$

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WP Seq (Instr Executable) 
$$\{v, v = HaltedV \implies \exists \Sigma' r', \Sigma' \supseteq_{priv} \Sigma * context(\Sigma')(r')\}$$

$$\begin{split} \mathrm{WP} \ \mathsf{Seq} \ \mathsf{(Instr} \ \mathsf{Executable)} \ \{ v, & v = \mathit{HaltedV} \implies \\ \exists \Sigma' r', \Sigma' \ \exists_{\mathit{priv}} \ \Sigma * \mathsf{context}(\Sigma')(r') \} \end{split}$$

WP Seq (Instr Executable) 
$$\{v, v = HaltedV \implies \exists \Sigma' r', \Sigma' \supseteq_{priv} \Sigma * \text{context}(\Sigma')(r')\}$$

$$\exists \Sigma' r', \Sigma' \supseteq_{priv} \Sigma * \mathsf{context}(\Sigma')(r') \}$$

WP Seq (Instr Executable)  $\{v,v = HaltedV \implies$