

# Implementing a Capability Machine model into Iris

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# Introduction

- ▶ Capability machines allow for fine grained control over pointer permissions
- ▶ Good target for secure compilation
- ▶ In particular: we are interested in enforcing certain higher level abstractions such as *local state encapsulation* as *well-bracketed control flow* at the lowest level of the machine
- ▶ We need tools to reason about these subtle properties in a language that does not enforce them
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# Overview

Capability Machines

Reasoning about Capability Safety

Program Logic

A Unary Logical Relation for Reasoning about Semantic Properties  
of an Untyped Language

The Fundamental Theorem of Logical Relations

Reasoning about Unknown Code

# Capability Machines



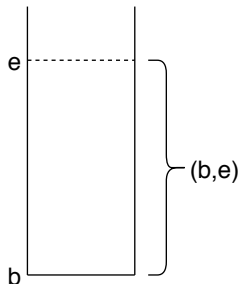
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**Capability:** An unforgeable token of authority



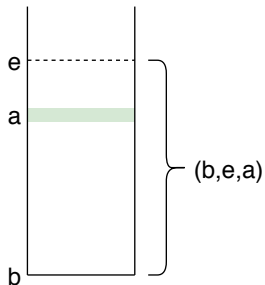
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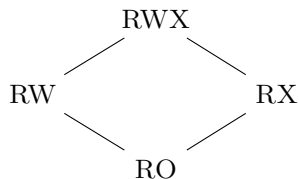
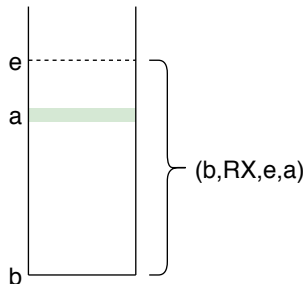
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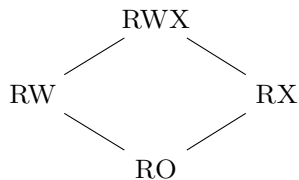
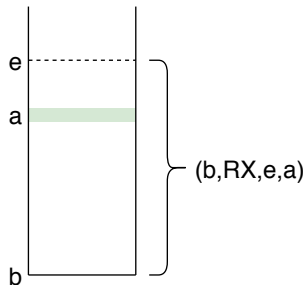
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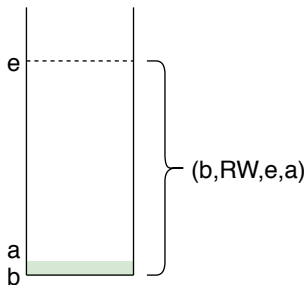
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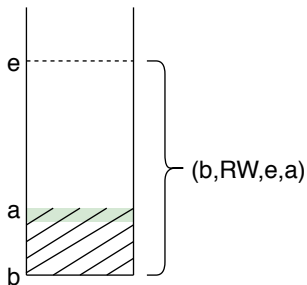
## Enforcing Local Stack Encapsulation using Capabilities

## Local State Encapsulation



```
1 push r_stk 1
2 scall r
3 pop r_stk r_1
4 assert r_1 1
5 halt
```

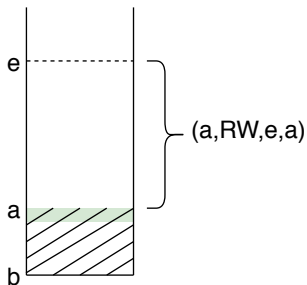
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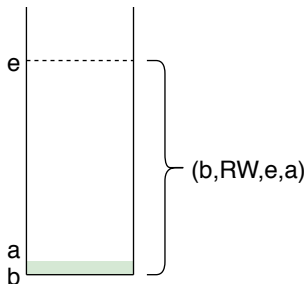
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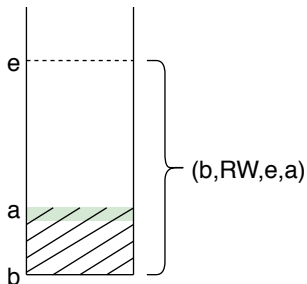
## Enforcing Well Bracketed Control Flow using Capabilities

## Well Bracketed Control Flow



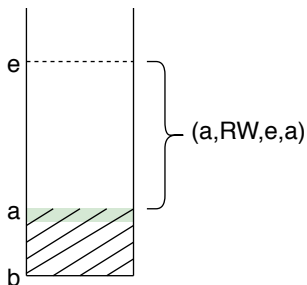
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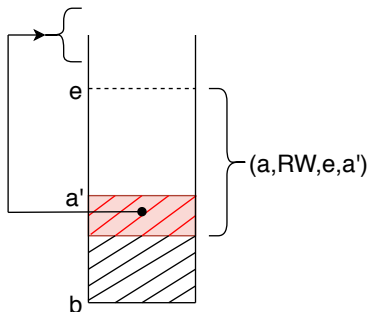
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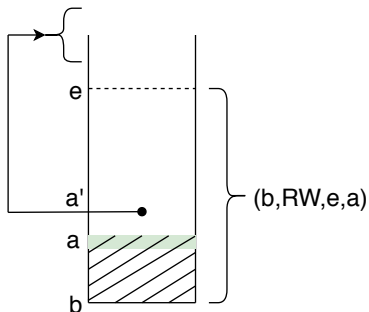
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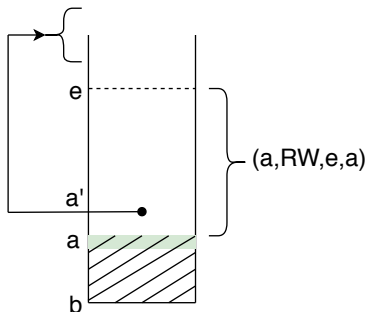
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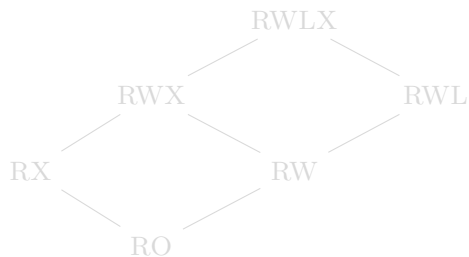
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## Local Capabilities

# Local Capabilities

(p, **Local**, b, e, a)



well-bracketed

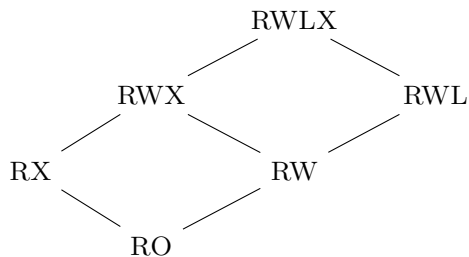
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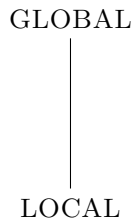
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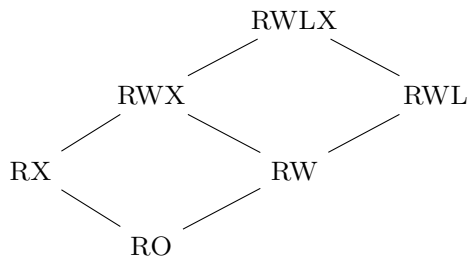
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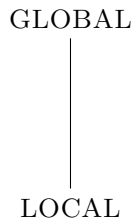
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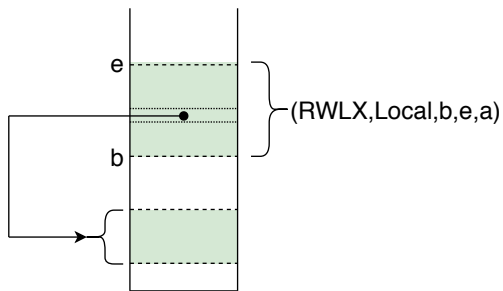
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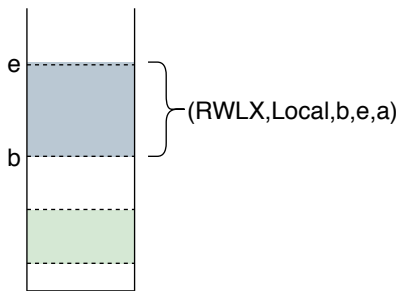


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# Calling Convention



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<code>r_stk</code>	$(RWLX, Local, b, e, a)$
--------------------	--------------------------

## Reasoning about Capability Safety

# Expressing Capability Safety

- ▶ using a Program Logic
- ▶ using a logical relation to capture invariants on the type system
- ▶ using a logical relation on an untyped (or uni-typed) language to capture semantic properties of the language



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# Step-indexed Kripke Logical Relation

$$\mathcal{V}(W) \triangleq \{n, (RW, g, b, e, a) | \dots\} \cup \dots$$

- ▶ World-circularity problem
  - ▶ Step indexing
- ▶ The world may evolve: we need future world relation
  - ▶ Local capabilities are revoked whereas Global capabilities are not, *the relation needs to model this distinction:*

$\sqsubseteq_{pub}$       *and*       $\sqsubseteq_{priv}$

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## **Iris:** Higher-order Concurrent Separation Logic Framework

- ▶ Foundational
- ▶ Implemented in Coq – equipped with an interactive proof mode
- ▶ Framework – embed any language and its operational semantics into Iris
- ▶ Comes equipped with:
  - ▶ Invariants
  - ▶ Ghost state

We can take advantage of Iris' step-indexed model and invariants to mechanize step-indexed Kripke logical relations with recursive worlds in a succinct and elegant way.

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## Roadmap

- ▶ embed the language into Iris
- ▶ define a program logic by proving Hoare Triples
- ▶ define the logical relation – using Iris tools to solve the world circularity problem
- ▶ prove the fundamental theorem of logical relations
- ▶ use the logical relation to prove examples that rely on local state encapsulation and well-bracketed control flow with calls to unknown adversary

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## Program Logic

# Abstract Instructions

$$(reg, mem) \rightarrow (reg', mem')$$

- ▶ Instr Executable
- ▶ Instr Halted  $\rightarrow$  HaltedV
- ▶ Instr Failed  $\rightarrow$  FailedV



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## A Capability Points-to Predicate

# Points-to Predicate with Permissions

$$a \mapsto_a [RWL]w$$

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$$a \mapsto_a [RWL]_w \Longrightarrow^* a \mapsto_a [RWL]((p, Local), b, e, l)$$

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$$\begin{aligned} a \mapsto_a [RWL]_w &\Longrightarrow^* a \mapsto_a [RWL]((p, Local), b, e, l) \\ &\Longrightarrow^* a \mapsto_a [RW]((p, Local), b, e, l) \\ &\not\Longrightarrow^* a \mapsto_a [RW]((p', Local), b', e', l') \end{aligned}$$

# A Unary Logical Relation for Reasoning about Semantic Properties of an Untyped Language

## The Value Relation

## A unary logical relation of an un-typed language

$$\mathcal{V} : \text{Word} \rightarrow iProp \Sigma$$

**Challenge:** distinguish between Local and Global capabilities:

- ▶ At the level of the value relation
- ▶ Model revocation

**STS:** A collection of state transition systems

$$\mathcal{V}((\text{RW}, g), b, e, a) \triangleq \bigstar_{a \in [b, e]} \boxed{\exists w, a \mapsto_a [RW]w \multimap \mathcal{V}(w)}$$

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**STS:** A collection of state transition systems

$$\mathcal{V}(\textcolor{red}{W})((\textcolor{blue}{r}_W, g), b, e, a) \triangleq \bigstar_{a \in [b, e]} \boxed{\exists w, a \mapsto_a [RW]w \multimap \mathcal{V}(\textcolor{red}{W})(w)}$$

## The Execute Condition

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$$\text{exec\_cond}(W)(p,g,b,e) \triangleq \begin{cases} \forall a \in [b \ e], W' \sqsubseteq_{\text{pub}} W. \\ \quad \triangleright \mathcal{E}(W')(((p, g), b, e, a)) \quad g = \text{Local} \\ \\ \forall a \in [b \ e], W' \sqsubseteq_{\text{priv}} W. \\ \quad \triangleright \mathcal{E}(W')(((p, g), b, e, a)) \quad g = \text{Global} \end{cases}$$

## The Expression Relation

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$$\begin{aligned}\mathcal{E}(W)(pc) &\triangleq \forall r, \mathcal{R}(W)(r) * \text{context}(W)(r[\text{PC} := pc]) \\ &\quad \rightarrow * \text{WP Seq (Instr Executable)} \\ &\quad \{v, v = \text{HaltedV} \implies \exists W' r', W' \sqsubseteq_{\text{priv}} W \\ &\quad * \text{context}(W')(r')\}\end{aligned}$$

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$$\text{context}(W)(r) = ?$$

# The Expression Relation

$$\begin{aligned}\mathcal{E}(W)(pc) &\triangleq \forall r, \mathcal{R}(W)(r) * \text{context}(W)(r[\text{PC} := pc]) \\ &\quad * \text{WP Seq (Instr Executable)} \\ &\quad \{v, v = \text{HaltedV} \implies \exists W' r', W' \sqsubseteq_{\text{priv}} W \\ &\quad * \text{context}(W')(r')\}\end{aligned}$$

$$\text{context}(W)(r) = ( *_{r_i \mapsto w \in r} r_i \mapsto_r w) \wedge \text{full\_map } r$$

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$$\begin{aligned}\text{context}(W)(r) = & \left( \bigstar_{r_i \mapsto w \in r} r_i \mapsto_r w \right) \wedge \text{full\_map } r \\ & * \text{na\_inv } \gamma_{na} \top\end{aligned}$$



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# The Fundamental Theorem of Logical Relations

# The Fundamental Theorem of logical relations

If we can read a region, and every word in that region is safe, then  
we can safely execute it

- ▶ "If we can read a region" :  $p = \text{RX} \vee p = \text{RWX} \vee p = \text{RWLX}$
- ▶ "and every word in that region is safe":  
 $\text{read\_write\_cond}(p, b, e)$
- ▶ "then we can safely execute it":  $\mathcal{E}(W)((p, g), b, e, a)$

$$(p = \text{RX} \vee p = \text{RWX} \vee p = \text{RWLX}) \implies$$

$$\text{read\_write\_cond}(p, b, e) \implies \mathcal{E}(W)((p, g), b, e, a)$$

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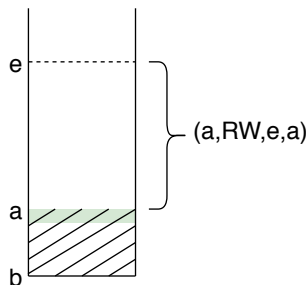
$$\text{read\_write\_cond}(p, b, e) \implies \mathcal{E}(W)((p, g), b, e, a)$$



## Reasoning about Unknown Code

# Reasoning about Unknown Code

**We use the fundamental theorem to reason about calls to an unknown adversary**



```
1 push r_stk 1
2 scall r
3 pop r_stk r_1
4 assert r_1 1
5 halt
```

$$\mathcal{E}(W)(pc) \triangleq \forall r, \mathcal{R}(W)(r) * \text{context}(W)(r[\text{PC} := pc])$$

\* WP Seq (Instr Executable)

$$\{v, v = \text{HaltedV} \implies \exists W' r', W' \sqsubseteq_{\text{priv}} W$$

\* context( $W'$ )( $r'$ )\}

# Conclusion

- ▶ Embed a capability machine into Iris
- ▶ Define its program logic
- ▶ Mechanize a unary logical relation for an untyped capability machine language
- ▶ Prove the fundamental theorem of logical relations
- ▶ Reason about examples that rely on Local Stack Encapsulation and Well-Bracketed Control Flow with calls to an unknown adversary

# References



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