Implementing a Capability Machine model into Iris

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Introduction

- Capability machines allow for fine grained control over pointer permissions
- Good target for secure compilation
- In particular: we are interested in enforcing certain higher level abstractions such as *local state encapsulation* as *well-bracketed control flow* at the lowest level of the machine
- We need tools to reason about these subtle properties in a language that does not enforce them
- These tools are elaborate and complex: we want to mechanize them, and facilitate the process of using them

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Overview

Capability Machines

Capability Machine

Capability: An unforgeable token of authority

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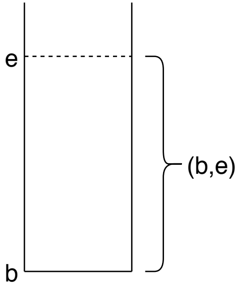
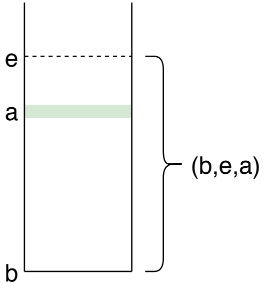


Capability Machine

Capability: An unforgeable token of authority



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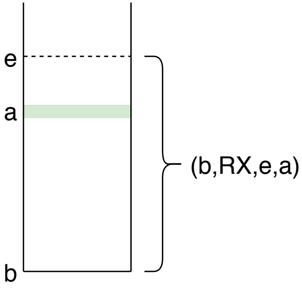

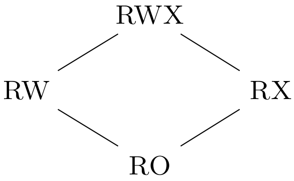

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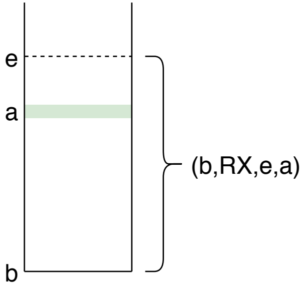
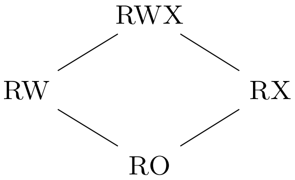
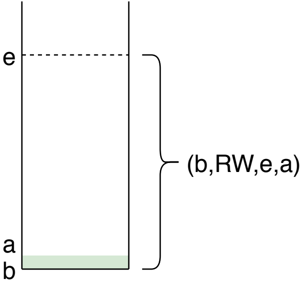




Enforcing Local Stack Encapsulation using Capabilities

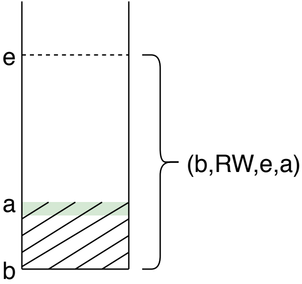
Capability Safety

Local State Encapsulation




```
push r_stk 1
scall r
pop r_stk r_1
assert r_1 1
halt
```

Local State Encapsulation

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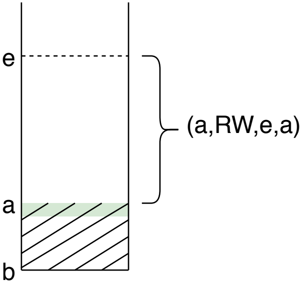


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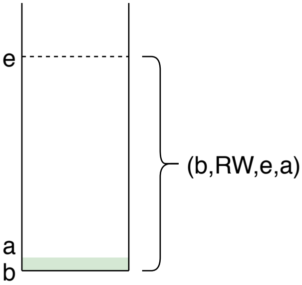
Enforcing Well Bracketed Control Flow using Capabilities

Well Bracketed Control Flow

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push r_stk 1
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push r_stk 2
scall r
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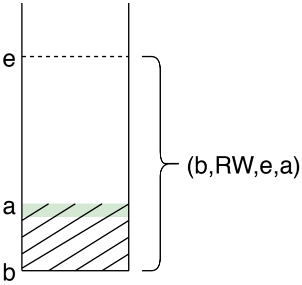
```

Well Bracketed Control Flow

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Capability Safety

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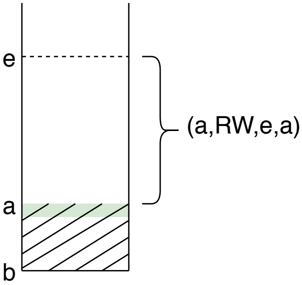


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Capability Safety

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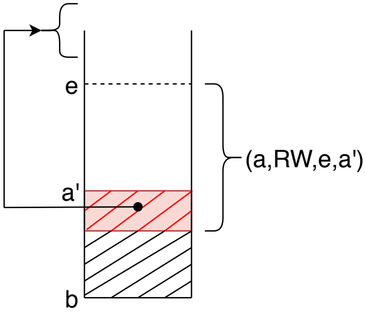


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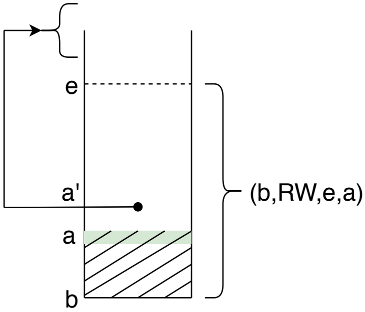


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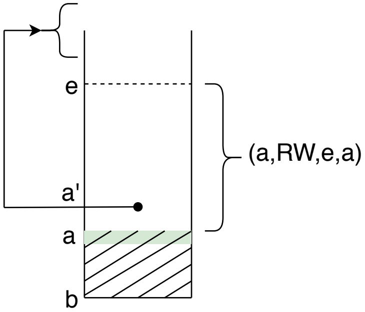


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(p,**Global**,b,e,a)

well-bracketed

not well-bracketed

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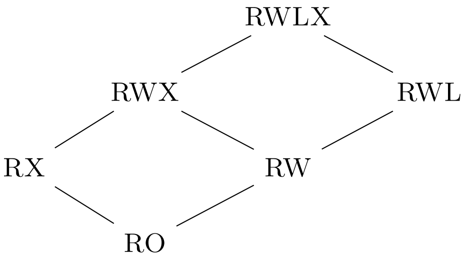

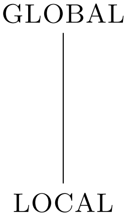

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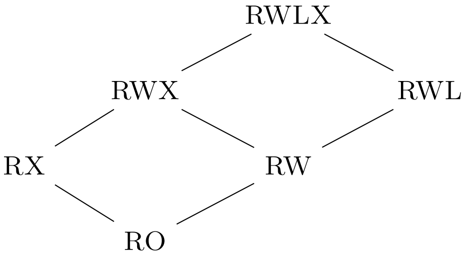

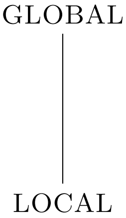

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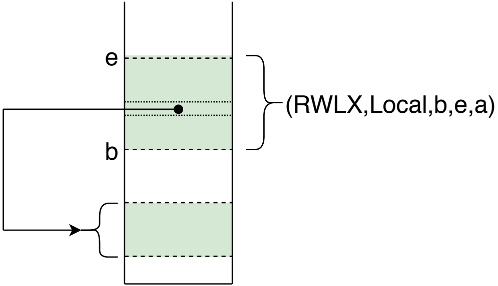
Calling Convention



`r_stk`	$(RWLX, Local, b, e, a)$

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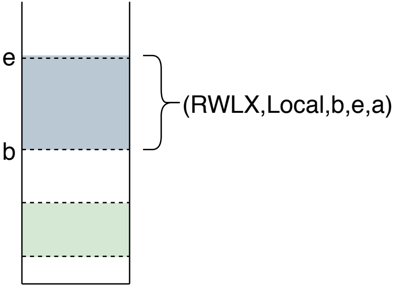

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Reasoning about Capability Safety

Expressing Capability Safety

- using a Program Logic
- using a logical relation to capture invariants on the type system
- **using a logical relation on an untyped (or uni-typed) language to capture semantic properties of the language**

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Step-indexed Kripke Logical Relation

$$\mathcal{V}(W) \triangleq \{n, (RW, g, b, e, a) | \cdots\} \cup \cdots$$

- World-circularity problem
  - Step indexing
- The world may evolve: we need future world relation
  - Local capabilities are revoked whereas Global capabilities are not, *the relation needs to model this distinction:*

$$\sqsupseteq_{pub} \qquad \textit{and} \qquad \sqsupseteq_{priv}$$

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Expressing Capability Safety in Iris - an Iris primer

Iris: Higher-order Concurrent Separation Logic Framework

- Foundational
- Implemented in Coq – equipped with an interactive proof mode
- Framework – embed any language and its operational semantics into Iris
- Comes equipped with:
  - Invariants
  - Ghost state
  - Always and Later Modalities

We can take advantage of Iris' step-indexed model and invariants to mechanize step-indexed Kripke logical relations with recursive worlds in a succinct and elegant way.

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Expressing Capability Safety in Iris - Challenges

- Region invariants: Iris invariants
- Future world relation: frame preserving updates and world satisfaction
- Step indexing: later modality

Challenges

- Iris was designed with more high level languages in mind, how do we embed a low level machine language into Iris
- Iris abstracts away certain details we want to reason about directly
- There is only one frame preserving update, we need to distinguish between two future world relations

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Roadmap

- embed the language into Iris
- define a program logic by proving Hoare Triples
- define the logical relation – using Iris tools to solve the world circularity problem
- prove the fundamental theorem of logical relations
- use the logical relation to prove examples that rely on local state encapsulation and well-bracketed control flow with calls to unknown adversary

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Program Logic

Abstract Instructions

$$(reg, mem) \to (reg', mem')$$

- Instr Executable
- Instr Halted $\to$ HaltedV
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A Capability Points-to Predicate

Points-to Predicate with Permissions

$$a \mapsto_a [RWL] w$$

Points-to Predicate with Permissions

$$a \mapsto_a [RWL] w \Longrightarrow\!\!\!\!* \; a \mapsto_a [RWL]((p, Local), b, e, l)$$

Points-to Predicate with Permissions

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Points-to Predicate with Permissions

$$
\begin{aligned}
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&\Longrightarrow\!\!\!\ast\; a \mapsto_a [RW]((p, Local), b, e, l) \\
&\not\Longrightarrow\!\!\!\ast\; a \mapsto_a [RW]((p', Local), b', e', l')
\end{aligned}
$$

A Unary Logical Relation for Reasoning about Semantic Properties of an Untyped Language

The Value Relation

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A unary logical relation of an un-typed language

$$\mathcal{V} : \mathit{Word} \rightarrow \mathit{iProp}\ \Sigma$$

Challenge: distinguish between Local and Global capabilities:

- At the level of the value relation
- Model revocation

STS: A collection of state transition systems

$$\mathcal{V}((\mathrm{RW}, g), b, e, a) \triangleq \mathop{\huge *}_{a \in [b,e]} \boxed{\exists w, a \mapsto_a [RW] w \twoheadrightarrow* \mathcal{V}(w)}$$

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The Value Relation

A unary logical relation of an un-typed language

$$\mathcal{V} : STS \rightarrow \; Word \rightarrow iProp\ \Sigma$$

Challenge: distinguish between Local and Global capabilities:

- At the level of the value relation
- Model revocation

STS: A collection of state transition systems

$$\mathcal{V}(W)((\textsc{rw}, g), b, e, a) \triangleq \mathop{\scalebox{2}{$\ast$}}_{a \in [b,e]} \boxed{\exists w, a \mapsto_a [RW]w \mathrel{-\!\!*} \mathcal{V}(W)(w)}$$

The Execute Condition

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$$\text{exec\_cond(W)(p,g,b,e)} \triangleq \begin{cases} \forall a \in [b\ e], W' \sqsupseteq_{pub} W. & \\ \quad \triangleright\ \mathcal{E}(W')((((p,g),b,e,a)) & g = Local \\ & \\ \forall a \in [b\ e], W' \sqsupseteq_{priv} W. & \\ \quad \triangleright\ \mathcal{E}(W')((((p,g),b,e,a)) & g = Global \end{cases}$$

The Expression Relation

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$$\mathcal{E}(W)(pc) \triangleq \forall r, \mathcal{R}(W)(r) \ * \ \text{context}(W)(r[\text{PC} := pc])$$
$$-\!\!* \ \mathrm{WP}\ \text{Seq (Instr Executable)}$$
$$\{v, v = HaltedV \implies \exists W' r', W' \sqsubseteq_{priv} W$$
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$$\text{context}(W)(r) = \ ?$$

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The Fundamental Theorem of Logical Relations

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If we can read a region, and every word in that region is safe, then we can safely execute it

- "If we can read a region" : $p = \mathrm{RX} \vee p = \mathrm{RWX} \vee p = \mathrm{RWLX}$
- "and every word in that region is safe": read_write_cond (p, b, e)
- "then we can safely execute it": $\mathcal{E}(W)(((p, g), b, e, a))$

$$(p = \mathrm{RX} \vee p = \mathrm{RWX} \vee p = \mathrm{RWLX}) \implies$$
$$\text{read\_write\_cond } (p, b, e) \implies \mathcal{E}(W)(((p, g), b, e, a))$$

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Reasoning about Unknown Code

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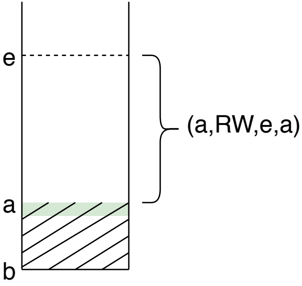
We use the fundamental theorem to reason about calls to an unknown adversary




```
push r_stk 1
scall r
pop r_stk r_1
assert r_1 1
halt
```

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Conclusion

- Embed a capability machine into Iris
- Define its program logic
- Mechanize a unary logical relation for an untyped capability machine language
- Prove the fundamental theorem of logical relations
- Reason about examples that rely on Local Stack Encapsulation and Well-Bracketed Control Flow with calls to an unknown adversary

References

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