Homework of 09.21
Differential equations and dynamic systems.
Solutions.

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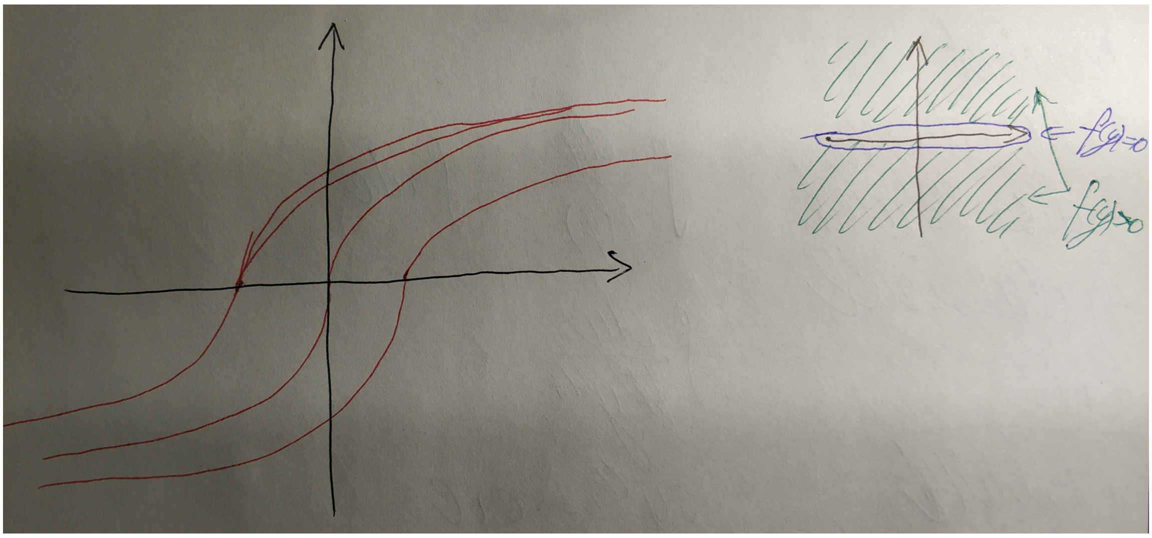
Problem 227. As we already know a map

$$U(x, y) = \int \frac{dy}{f(y)} - x = \int \frac{dy}{y^{2/3}} - x = 3y^{1/3} - x$$

is an integral of the equation. So as far as $\lim_{y \to 0} y^{2/3}$ converges (to 0) there will be 4 different types of solutions: for some constant $c \in \mathbb{R}$ $y(c) = 0$ and to both right and left there will be 2 options $(x - c)^3$ and 0. I.e. every solution is one of the four kinds:

$$y_1(x) = (x-c)^3, \qquad y_2(x) = 0, \qquad y_3(x) = \begin{cases} (x-c)^3 & \text{if } x \geqslant c, \\ 0 & \text{if } x \leqslant c \end{cases} \qquad y_3(x) = \begin{cases} (x-c)^3 & \text{if } x \leqslant c, \\ 0 & \text{if } x \geqslant c. \end{cases}$$



Problem 138.

$$y' + y \tan(x) = \frac{1}{\cos(x)}$$

Introduce a map

$$\varphi(x) := e^{\int_0^x \tan(t)dt} = e^{-\ln(\cos(x))} = \frac{1}{\cos(x)}.$$

Then

$$
\begin{aligned}
\varphi y' + \varphi y \tan(x) &= \frac{\varphi}{\cos(x)} \\
\varphi y' + \varphi' y &= \frac{\varphi}{\cos(x)} \\
(\varphi y)' &= \frac{\varphi}{\cos(x)} \\
\varphi y = \int \frac{\varphi(t)}{\cos(t)} dt &= \int \frac{dt}{\cos(t)^2} = \tan(x) + C \\
y &= \sin(x) + C\cos(x).
\end{aligned}
$$

Problem 142.

$$
\begin{aligned}
2x(x^2 + y)dx &= dy \\
2x^3 + 2xy &= y' \\
y' - 2xy &= 2x^3
\end{aligned}
$$

Introduce a map

$$\varphi(x) := e^{\int_0^x -2t dt} = e^{-x^2}.$$

Then

$$
\begin{aligned}
\varphi y' - \varphi 2xy &= 2x^3\varphi \\
\varphi y' + \varphi' y &= 2x^3\varphi \\
(\varphi y)' &= 2x^3\varphi \\
\varphi y = \int 2t^3\varphi(t)dt &= \int 2t^3 e^{-t^2} = -(x^2+1)e^{-x^2} + C \\
y &= -(x^2+1) + Ce^{x^2}.
\end{aligned}
$$

Problem 149.

$$
\begin{aligned}
y' &= \frac{y}{3x - y^2} \\
\frac{dy}{dx} &= \frac{y}{3x - y^2} \\
\frac{dx}{dy} &= \frac{3x - y^2}{y} \\
x' &= \frac{3}{y}x - y \\
x' - \frac{3}{y}x &= -y
\end{aligned}
$$

Introduce a map

$$\varphi(y) := e^{\int_1^y \frac{-3}{t} dt} = e^{-3\ln(y)} = \frac{1}{y^3}.$$

Then

$$\varphi x' - \varphi \frac{3}{y} x = -y\varphi$$
$$\varphi x' + \varphi' x = -y\varphi$$
$$(\varphi x)' = -y\varphi$$
$$\varphi x = \int -t\varphi(t)dt = \int \frac{-dt}{t^2} = \frac{1}{y} + C$$
$$x = y^2 + Cy^3.$$
