# Homework of 10.12
# Differential equations and dynamic systems.
# Solutions.

Глеб Минаев @ 204 (20.Б04-мкн)

**Problem 257.**

$$y'^2 - 2xy' = 8x^2$$
$$y'^2 - 2xy' - 8x^2 = 0$$
$$(y' - 4x)(y' + 2x) = 0$$
$$\left[\begin{array}{l} y' = 4x \\ y' = -2x \end{array}\right.$$
$$\left[\begin{array}{l} y = 2x^2 + C \\ y = -x^2 + C \end{array}\right.$$

So $2x^2 + C$ and $-x^2 + C$ are the only types of solutions of the equation.

Singular solution are are solutions of equation

$$2y' - 2x = 0$$
$$y' = x.$$

No solution of previous equation satisfy the new equation. Hence there is no singular solutions.

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**Problem 269.**

$$x = y'\sqrt{y'^2 + 1}$$

Let $p := y'$. Then $dy = pdx$. So

$$x = p\sqrt{p^2 + 1}$$
$$y = \int dy = \int pdx = px - \int xdp = p^2\sqrt{p^2 + 1} - \int p\sqrt{p^2 + 1}dp$$
$$= p^2\sqrt{p^2 + 1} - \frac{1}{3}(p^2 + 1)\sqrt{p^2 + 1} + C = \sqrt{p^2 + 1}\frac{1}{3}(2p^2 - 1) + C$$

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**Problem 289.**

$$y = 2xy' - 4y'^3$$

Let $p := y'$. Then $dy = pdx$. So

$$y = 2xp - 4p^3$$
$$pdx = dy = 2xdp + 2pdx - 12p^2dp$$
$$0 = 2x + px'_p - 12p^2$$
$$\frac{d}{dp}(p^2x) = 2px + p^2x'_p = 12p^3 = \frac{d}{dp}(3p^4)$$
$$p^2x = 3p^4 + C$$
$$x = 3p^2 + \frac{C}{p^2}$$
$$y = 2xp - 4p^3 = 6p^3 + \frac{2C}{p} - 4p^3 = 2p^3 + \frac{2C}{p}.$$

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**Problem 293.**

$$xy' - y = \ln(y')$$

Let $p := y'$. Then $dy = pdx$. So

$$xp - y = \ln(p)$$
$$pdx + xdp - pdx = \frac{dp}{p}$$
$$xdp = \frac{dp}{p}$$
$$x = \frac{1}{p}$$
$$y = xp - \ln(p) = 1 - \ln(p)$$

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**Problem 421.**

$$x^2y'' = y'^2.$$

Let $p := y'$. Then

$$x^2p' = p^2$$

$x = e^t$, $p = e^tz(t)$. Then $p' = 2z + z'$. So

$$e^{2t}(2z + z') = e^{2t}z^2$$
$$2z + z' = z^2$$
$$\frac{2}{z} - \left(\frac{1}{z}\right)' = 1$$
$$e^{-2t}\left(\frac{1}{z}\right)' + \frac{(e^{-2t})'}{z} = -e^{-2t}$$
$$\left(\frac{e^{-2t}}{z}\right)' = -e^{-2t}$$
$$\frac{e^{-2t}}{z} = \frac{1}{2}e^{-2t} + C$$
$$z = \frac{e^{-2t}}{\frac{1}{2}e^{-2t} + C} = \frac{1}{\frac{1}{2} + Ce^{2t}} = \frac{2}{Ce^{2t} + 1}$$

$$p = \frac{2e^t}{Ce^{2t}+1} = \frac{2x}{Cx^2+1}$$
$$y = \int \frac{2xdx}{Cx^2+1} = \frac{\ln(Cx^2+1)}{C} + B$$

In case $C = 0$

$$y = \int \frac{2xdx}{0x^2+1} = \int 2xdx = x^2 + B$$

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