

# Homework of 10.05

## Differential equations and dynamic systems.

### Solutions.

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#### Problem 115.

$$\begin{aligned}x - y - 1 + (y - x + 2)y' &= 0 \\y' &= \frac{y - x + 1}{y - x + 2} \\y' &= 1 + \frac{-1}{y - x + 2}\end{aligned}$$

Substitute  $z = y - x + 2$ . Hence

$$z' = y' - 1 = \frac{-1}{y - x + 2} = \frac{-1}{z}.$$

Obtained equation with separated variables for  $m(x) = 1$ ,  $n(z) = -1/z$ . So a map

$$U(x, y) := \int -z dz - \int 1 dx = -\frac{1}{2}z^2 - x + C.$$

Hence

$$\begin{aligned}z^2 &= C - 2x \\z &= \pm \sqrt{C - 2x} \\y &= z + x - 2 = \pm \sqrt{C - 2x} + x - 2\end{aligned}$$

#### Problem 205.

$$\begin{aligned}(x^2 + 2x + y)dx &= (x - 3x^2y)dy \\(x^2 + 2x)dx + ydx - xdy + 3x^2ydy &= 0\end{aligned}$$

(divide by  $x^2$ ; will lose solution  $x \equiv 0$ )

$$\begin{aligned}\left(1 + 2\frac{1}{x}\right)dx - d\left(\frac{y}{x}\right) + 3ydy &= 0 \\dx + 2d\ln(x) - d\left(\frac{y}{x}\right) + \frac{3}{2}d(y^2) &= 0 \\d\left(x + 2\ln(x) - \frac{y}{x} + \frac{3}{2}y^2\right) &= 0 \\x + 2\ln(x) - \frac{y}{x} + \frac{3}{2}y^2 &= C \\3xy^2 - 2y + 2x^2 + 4x\ln(x) - 2xC &= 0 \\y = \frac{1 \pm \sqrt{1 - 3x(2x^2 + 4x\ln(x) - C)}}{3x} &= \frac{1 \pm \sqrt{1 + 6Cx^2 - 6x^3 - 12x^2\ln(x)}}{3x}\end{aligned}$$

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**Problem 191.**

$$2x \left(1 + \sqrt{x^2 - y}\right) dx - \sqrt{x^2 - y} dy = 0$$

$$d(x^2) + \frac{2}{3}d((x^2 - y)^{3/2}) = 0$$

$$d\left(x^2 + \frac{2}{3}(x^2 - y)^{3/2}\right) = 0$$

$$\frac{3}{2}x^2 + (x^2 - y)^{3/2} = C$$

$$x^2 - y = \left(C - \frac{3}{2}x^2\right)^{2/3}$$

$$y = x^2 - \left(C - \frac{3}{2}x^2\right)^{2/3}$$

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