

Atelier LATEX



Objectives for today:

- Quick revision of quadratic functions
- Factorising Quadratics
- Proving Vieta's formulas
- Carrying out gained knowledge by working out some word problems

Quick Revision

Forms of Quadratic Function

- $f(x) = ax^2 + bx + c$ is called the **standard form**.
- $f(x) = a(x - x_1)(x - x_2)$ is called the **factored form**, where x_1 and x_2 are the roots of a quadratic function.
- $f(x) = a(x - h)^2 + k$ is called the **vertex form**.

Delta Δ

Δ determines tells us how many solutions quadratic equation have

| | |
|-----------------------|---------------------|
| number of solutions = | 2 when $\Delta > 0$ |
| | 1 when $\Delta = 0$ |
| | 0 when $\Delta < 0$ |

The Quadratic Formula

$$x = \frac{-b \pm \sqrt{\Delta}}{2a}$$

Graph of Quadratic Function

Candix & DAO

28/02 Jeudi S4

Inscriptions obligatoires sur louvainlinux.org

Factorising a Quadratic

Factorising a quadratic means putting it into two brackets, this is useful if you're trying to draw a graph of a quadratic solve a quadratic equation. It's pretty easy if $a = 1$ (in $ax^2 + bx + c$ form), but can be a real pain otherwise

In order to factorise a quadratic you should follow steps outlined below

1. Rearrange the equation into the standard form $ax^2 + bx + c = 0$
2. Write down two brackets: $(x \quad)(x \quad)$
3. Find two numbers that multiply to give 'c' and add or subtract to give 'b' (ignoring signs).
4. Put the numbers in brackets and check their signs.

Factorising

1. Factorise $x^2 - x - 12$.

2. Solve $x^2 - 8 = 2x$ by factorising

Myth of Delta Δ

It is commonly believed that in order to work out roots of a quadratic function you need to use the quadratic formula. However this is untrue since factorising in many cases is even better than simply counting Δ .

Example of Factorisation

Solve $x^2 + 4x - 21 = 0$ by factorising.

$$x^2 + 4x - 21 = (x \quad)(x \quad)$$

1. 1 and 21 multiply to give 21 - and add or subtract to give 4
2. 3 and 7 multiply to give 21 - and add or subtract to give 4

$$x^2 + 4x + 21 = (x + 7)(x - 3)$$

And solving the equation $(x + 7)(x - 3) = 0$ we get

$$x = -7, \quad x = 3$$

Proof of Vieta's Formulas

Let's prove that:

$$x_1 + x_2 = -\frac{b}{a}$$

When Δ is positive we have two roots:

$$x_1 = \frac{-b - \sqrt{\Delta}}{2a}, \quad x_2 = \frac{-b + \sqrt{\Delta}}{2a}$$

Substituting for x_1 and x_2 respectively, we receive

$$\begin{aligned} x_1 + x_2 &= \frac{-b - \sqrt{\Delta}}{2a} + \frac{-b + \sqrt{\Delta}}{2a} = \\ &= \frac{(-b - \sqrt{\Delta}) + (-b + \sqrt{\Delta})}{2a} = \frac{-2b}{2a} = -\frac{b}{a} \end{aligned}$$

The same we could do with another pattern, which state that $x_1 x_2 = \frac{c}{a}$, but proving this is going to be in the next section.

Some Necessary and Useful Vocabulary

- (n.) sign $\rightarrow +$ or $-$
- (n.) equation \rightarrow something $= 0$
- (n.) factor \rightarrow two multiplied factors give result
- (v.) factorise \rightarrow putting into brackets
- (n.) coefficient \rightarrow a constant number i.e. a, b, c in a pattern $ax^2 + bx + c$
- (n.) quadratic function $\rightarrow f(x) = ax^2 + bx + c$
- (n.) root \rightarrow sth or solution of equation
- (n.) formula $=$ pattern