Template For ICPC

——lovekdl

FFT

```
#include<bits/stdc++.h>
using namespace std;
#define rint register int
#define ll long long
#define rll register long long
#define db long double
const int N=1<<21;
const db pi=acosl(-1);
struct cp{
    db x,y;
    cp operator + (const cp&A)const{return (cp){x+A.x,y+A.y};}
    cp operator - (const cp&A)const{return (cp){x-A.x,y-A.y};}
    cp operator * (const cp&A)const{return (cp){x*A.x-y*A.y,x*A.y+y*A.x};}
    cp operator / (const db&A)const{return (cp){x/A,y/A};}
}w[N],t;
int r[N],pd;
void fft(rint n,vector<cp> &a,rint typ){
    if(pd!=n){
        for(rint i=0;i<n;i++)
            r[i]=(r[i>>1]>>1)|((i&1)?n>>1:0);
        pd=n;
    }
    a.resize(n);
    for(rint i=0;i<n;i++)
        if(i<r[i])swap(a[i],a[r[i]]);
    for(rint mid=1;mid<n;mid<<=1)
    for(rint i=0;i<n;i+=mid<<1)
    for(rint j=0;j<mid;j++)
        t=w[mid+j]*a[i+j+mid],a[i+j+mid]=a[i+j]-t,a[i+j]=a[i+j]+t;
    if(~typ)return ;
    reverse(a.begin()+1,a.end());
    for(rint i=0;i<n;i++)
        a[i]=a[i]/n;
}
void init(){
    w[N/2]=(cp){1.0,0.0};w[N/2+1]=t=(cp){cosl(2*pi/N),sinl(2*pi/N)};
    for(rint i=N/2+2;i<N;i++)w[i]=w[i-1]*t;
    for(rint i=N/2-1;i;i--)w[i]=w[i<<1];
}
vector<cp> Mul(vector<cp> a,vector<cp> b){
    int n=1;
    while(n<=a.size()+b.size())n<<=1;
    fft(n,a,1);fft(n,b,1);
    for(rint i=0;i<n;i++)
```

```
        a[i]=a[i]*b[i];
    fft(n,a,-1);
    return a;
}
inline int read(){
    int x=0,f=1;char ch=getchar();
    while(ch<'0'||ch>'9'){if(ch=='-')f=-1;ch=getchar();}
    while(ch<='9'&&ch>='0'){x=x*10+ch-'0';ch=getchar();}
    return x*f;
}
int main(){
    init();
    rint n=read(),m=read();
    ++n;++m;
    vector<cp> f,g;
    f.resize(n);g.resize(m);
    for(rint i=0;i<n;i++)
        f[i].x=read();
    for(rint i=0;i<m;i++)
        g[i].x=read();
    f=Mul(f,g);
    for(rint i=0;i<n+m-1;i++)
        printf("%d ",int(f[i].x+0.3));
    return 0;
}
```

多项式

```
#include<bits/stdc++.h>
#define int long long
using namespace std;
const int N = 3e5+10;
const int mod = 998244353, g = 3, gi = 332748118;
//const int mod = 4179340454199820289, g = 3, gi = 1393113484733273430;
int n, m;
int a[N], b[N];
int re[N];


int ksm(int a, int b) {
    int ret = 1;
    while(b) {
        if(b & 1) ret = ret * a %mod;
        a = a * a % mod;
        b >>= 1;
    }
    return ret;
}
int inv(int x) {
    return ksm(x, mod - 2);
}

void ntt(int *a, int lim, int opt) {
    for(int i = 0; i < lim; ++i)
        if(i < re[i]) swap(a[i], a[re[i]]);
    for(int len = 1; len < lim; len <<= 1) {
        int wn = ksm(opt == 1 ? g : gi, (mod - 1) / (len << 1));
        for(int i = 0; i < lim; i += (len << 1)) {
            int w = 1;
            for(int j = 0; j < len; ++j) {
                int x = a[i + j], y = w * a[i + j + len] % mod;
                a[i + j] = (x + y) % mod;
                a[i + j + len] = (x - y + mod) % mod;
                w = w * wn % mod;
            }
        }
    }
    if(opt == 1) return;
    int limv = inv(lim);
    for(int i = 0; i < lim; ++i) {
        a[i] = a[i] * limv % mod;
    }
```

```
}

//n次多项式和m次多项式卷积
void mul(int *F,int n, int *G, int m) {
//  if(n+m < 128) {
//      for(int i = 0; i <= n + m; ++i) {
//          H[i] = 0;
//      }
//      for(int i = 0; i <= n; ++i) {
//          for(int j = 0; j <= m; ++j) {
//              H[i+j] = (H[i+j] + F[i] * G[j] % mod) % mod;
//          }
//          F[i] = H[i];
//      }
//      for(int i = n + 1; i <= n + m; ++i) {
//          F[i] = H[i];
//      }
//      return;
//  }
    int lim = 1, ti = 0;
    while(lim <= n + m) {
        lim <<= 1;
        ti++;
    }
    for(int i = n + 1; i < lim; ++i) F[i] = 0;
    for(int j = m + 1; j < lim; ++j) G[j] = 0;
    for(int i = 0; i < lim; ++i) {
        re[i] = (re[i >> 1] >> 1) | ((i & 1) << (ti - 1));
    }
    ntt(F, lim, 1);
    ntt(G, lim, 1);
    for(int i = 0; i < lim; ++i) {
        F[i] = F[i] * G[i] % mod;
    }
    ntt(F, lim, -1);
    for(int i = n + m + 1; i < lim; ++i) assert(F[i] == 0);
}

//n-1次多项式求逆

int H[N];
void inv(int *F, int *G, int n) {
    if(n == 1) {G[0] = inv(F[0]);return;}
    inv(F, G, (n+1)>>1);
    int ti = 0, lim = 1;
    while(lim < n<<1) {
        lim <<= 1;
        ti++;
    }
```

```
    for(int i = 1; i < lim; ++i) {
        re[i] = (re[i >> 1] >> 1) | ((i & 1) << (ti - 1));
    }
    for(int i = 0; i < n; ++i) H[i] = F[i];
    for(int i = n; i < lim; ++i) H[i] = G[i] = 0;
    ntt(H, lim, 1);
    ntt(G, lim, 1);
    for(int i = 0; i < lim; ++i) {
        G[i] = G[i]*(2ll-H[i]*G[i] % mod + mod) % mod;
    }
    ntt(G, lim, -1);
    for(int i = n; i < lim; ++i) G[i] = 0;
}
//求导, F->G
void diff(int *F, int *G, int n) {
    for(int i = 1; i < n; ++i) G[i-1] = F[i] * i % mod;
    G[n-1] = 0;
}
//积分, F->G
void integral(int *F, int *G, int n)  {
    for(int i = 1; i < n; ++i) G[i] = F[i - 1] * inv(i) % mod;
    G[0] = 0;
}

//多项式ln
int Fi[N], Fd[N];
void ln(int *F, int *G, int n) {
    for(int i = 0; i < (n<<2); ++i) G[i] = 0;
    inv(F, Fi, n);
    diff(F, Fd, n);
    mul(Fi, n-1, Fd, n-1);
    integral(Fi, G, n);
}
//多项式exp
int lnG[N];
void exp(int *F, int *G, int n) {
    if(n == 1) {G[0] = 1;return;}
    exp(F, G, n + 1 >> 1);
    assert(G[0] == 1);
    ln(G, lnG, n);
    assert(lnG[0] == 0);
    for(int i = 0; i < n; ++i) lnG[i] = (F[i] - lnG[i] + mod) % mod;

    lnG[0]++;lnG[0] %= mod;
    mul(G, n - 1, lnG, n - 1);
}

int G2[N], GG[N], G2i[N];
void sqrt(int *F, int *G, int n){
```

```
    if(n == 1){
        G[0] = 1;
        return;
    }

    sqrt(F, G, n + 1 >> 1);
    for(int i = 0; i < n; ++i) {
        G2[i] = G[i] * 2 % mod;
        GG[i] = G[i];
        G2i[i] = 0;
    }
    inv(G2, G2i, n);

    mul(G, n - 1, GG, n - 1);
    for(int i = 0; i < n; ++i) {
        G[i] = (G[i] + F[i]) % mod;
    }
    mul(G, n - 1, G2i, n - 1);
    return;
}

void solve() {

    // int x, y;
    // cin>>x>>y;
    // for(int i = 0; i <= x; ++i) {
    //  cin>>a[i];
    // }
    // for(int i = 0; i <= y; ++i) {
    //  cin>>b[i];
    // }
    // mul(a, x, b, y);
    // for(int i = 0; i <= x + y; ++i) {
    //  cout<<a[i]<<" ";
    // }

    // int n;
    // cin>>n;
    // for(int i = 0 ; i < n; ++i) {
    //  cin>>a[i];
    // }
    // inv(a, b, n);
    // for(int i = 0; i < n; ++i) {
    //  cout<<b[i]<<" ";
    // }



    // int n;
```

```
    // cin>>n;
    // for(int i = 0; i < n; ++i) {
    //  cin>>a[i];
    // }
    // ln(a, b, n);
    // for(int i =0; i < n; ++i) {
    //  cout<<b[i]<<" ";
    // }

    // int n;
    // cin>>n;
    // for(int i = 0; i < n; ++i) {
    //  cin>>a[i];
    // }
    // exp(a, b, n);
    // for(int i = 0; i < n; ++i) {
    //  cout<<b[i]<<" ";
    // }

    int n;
    cin>>n;
    for(int i = 0; i < n; ++i) {
        cin>>a[i];
    }
    int sum = 1;
    while(sum <= n) sum <<= 1;
    sqrt(a, b, sum);
    for(int i = 0; i < n; ++i) {
        cout<<b[i]<<" ";
    }
}



signed main() {
    ios::sync_with_stdio(false);
    cin.tie(nullptr);
    solve();
    return 0;
}
```

分治NTT

```
/*
分治ntt
计算f[i] = sum(f[i-j] * g[j])
*/

#include <bits/stdc++.h>
using namespace std;
#define int long long
const int maxn = 2e6+10;
const int mod = 998244353,G = 3,Gi = (mod+1)/3;
int r[maxn], a[maxn], b[maxn], f[maxn], g[maxn],n;
int qpow(int a, int b) {
    int ret = 1;
    while(b) {
        if(b & 1) ret = ret * a % mod;
        a = a * a % mod;
        b >>= 1;
    }
    return ret;
}
void NTT(int *a,int limit,int type) {
    for(int i = 0; i < limit; i++)
        if(i < r[i]) swap(a[i], a[r[i]]);
    for(int mid = 1; mid < limit; mid <<= 1) {
        int wn = qpow((type == 1 ? G : Gi), (mod - 1) / (mid << 1));
        for(int R = mid << 1, i = 0; i < limit; i += R)
        for(int k = 0, w = 1; k < mid; k++, w = w * wn % mod) {
            int x = a[i+k], y = a[i+k+mid] * w % mod;
            a[i+k] = (x + y) % mod, a[i+k+mid] = (x - y + mod) % mod;
        }
    }
    if(type == 1)   return;
    int inv = qpow(limit, mod-2);
    for(int i = 0;i < limit; i++)   a[i] = a[i] * inv % mod;
}
void mul(int *a, int *b, int limit) {
    for(int i = 0; i < limit; i++)
        r[i] = (r[i >> 1] >> 1) | ((i & 1) ? limit >> 1 : 0 );
    NTT(a, limit, 1); NTT(b, limit, 1);
    for(int i = 0; i < limit; i++)  a[i] = a[i] * b[i] % mod;
    NTT(a, limit, -1);
}
void solve(int l, int r) {
    if(l == r) return;
    int mid = l + r >> 1;
    solve(l, mid);
    int limit = 1;
```

```
    while(limit <= mid - l + r - l) limit <<= 1;
    for(int i = 0; i < limit; i++)  a[i] = b[i] = 0;
    for(int i = l; i <= mid; i++)   a[i - l] = f[i];
    for(int i = 1; i <= r - l; i++) b[i] = g[i];
    mul(a, b, limit);
    for(int i = mid + 1; i <= r; i++)   f[i] = (f[i] + a[i - l])%mod;
    solve(mid+1 , r);
}
signed main() {
    cin >> n;
    for(int i = 1; i < n; i++)  scanf("%lld", &g[i]);
    f[0] = 1;
    solve(0, n-1);
    for(int i = 0; i < n; i++)  printf("%lld ", f[i]);
}
```

拉格朗日插值

```cpp
#include<bits/stdc++.h>
#define int long long
using namespace std;
const int N = 1e4+10;
const int mod = 998244353;
int T;
int n, k;
int x[N], y[N];

int qpow(int a, int b) {
    int ret = 1;
    while(b) {
        if(b & 1) ret = ret * a % mod;
        a = a * a % mod;
        b >>= 1;
    }
    return ret;
}

int lagrange(int k) {
    int ans = 0;
    for(int i = 1; i <= n; ++i) {
        int now = y[i];
        for(int j = 1; j <= n; ++j) {
            if(j == i) continue;
            now = now * (k - x[j]) % mod * qpow(x[i] - x[j], mod - 2) %mod;
        }
        ans = (ans + now) % mod;
    }
    if(ans < 0) ans += mod;
    return ans;
}
```

exgcd

```cpp
int exgcd(int a, int b, int &x, int &y) {
    if (b == 0) {
        x = 1;
        y = 0;
        return a;
    }
    int d = exgcd(b, a % b, y, x);
    y -= (a / b) * x;
    return d;
}
```

合并两个同余方程(CRT)

```cpp
//x = a mod b
//x = c mod d
void merge(ll &a, ll &b, ll c, ll d) {
    if (a == -1 && b == -1) return;
    ll x, y;
    ll g = exgcd(b, d, x, y);
    if ((c - a) % g != 0) {
        a = b = -1;
        return;
    }
    d /= g;
    ll t0 = ((c - a) / g) % d * x % d;
    if (t0 < 0) t0 += d;
    a = b * t0 + a;
    b = b * d;
}
```

莫比乌斯反演

$$f[n]=\sum_{d|n}g(d)\text{求}g(n)$$

```cpp
#include<bits/stdc++.h>
#define uint unsigned int
using namespace std;
typedef long long ll;
const int N = 1e6+101;
uint f[N];
int n;
int p[N], pr[N], pe[N];
int cnt;
uint mu[N];
uint g[N];
unsigned int A,B,C;
void solve() {
    scanf("%d", &n);
    for (int i = 1; i <= n; i++)
        cin>>f[i];
    p[1] = 1;mu[1] = 1;
    for(int i = 2; i <= n; ++i) {
        if(!p[i]) {
            p[i] = i;
            pr[++cnt] = i;
            mu[i] = (uint)-1;
        }
        for(int j = 1; j <= cnt && i * pr[j] <= n; ++j) {
            p[i * pr[j]] = pr[j];
            if(p[i] == pr[j]) {
                mu[i * pr[j]] = 0;
                break;
            }
            else mu[i * pr[j]] = (uint) -mu[i];
        }
    }
    for(int d1 = 1; d1 <= n; ++d1)
        for(int d2 = 1; d2 * d1 <= n; ++d2)
            g[d1 * d2] += f[d1] * mu[d2];
}
int main() {
    solve();
    return 0;
}
```

求积性函数

```
void compute(function<void(int)> calc) {
    f[1] = 1;
    uint ans = 0;
    for(int i = 2; i <= n; ++i) {
        if(i == pe[i]) calc(i);
        else f[i] = f[i / pe[i]] * f[pe[i]];
        ans = ans ^ (a * i * f[i] + b);
    }
    ans = ans ^ (a + b);
    printf("%u\n", ans);
}

void solve() {
    scanf("%d%u%u", &n, &a, &b);
    for(int i = 2; i <= n; ++i) {
        if(!p[i]) {
            p[i] = i;
            pe[i] = i;
            pr[++cnt] = i;
        }
        for(int j = 1; j <= cnt && i * pr[j] <= n; ++j) {
            p[i * pr[j]] = pr[j];
            if(p[i] == pr[j]) {
                pe[i * pr[j]] = pe[i] * pr[j];
                break;
            }
            else {
                pe[i * pr[j]] = pr[j];
            }
        }
}
```

O($\sqrt{n}$)求$\sum_{i=1}^{n} gcd(i,n)$

积性函数，$g(p^a) = (a+1)p^a - ap^{a-1}$

```
#include<bits/stdc++.h>
#define int long long
using namespace std;
int g(int n){
    int ans = 1;
    for(int i=2;i*i<=n;++i){
        if(n%i) continue;
        int a=0;
        int p=1;
        while(n%i==0) {
            p *= i;
            ++a;
            n /= i;
        }
        ans *= (a+1)*p - p/i*a;
    }
    if(n>1){
        ans *= n+n-1;
    }
    return ans;
}

signed main(){
    int n;
    cin >> n;
    cout << g(n) << endl;
    return 0;
}
```

tarjan缩点

```
#include<bits/stdc++.h>
#define int long long
using namespace std;
const int maxn = 1e5+100;
int n, m;
int num, ans;
int w[maxn];
int dfn[maxn], low[maxn], vis[maxn];
int belong[maxn], in[maxn], f[maxn];
struct Edge {
    int next[maxn], to[maxn], from[maxn];
    int head[maxn], cnt;
    void con(int u, int v) {
        next[++cnt] = head[u];
        to[cnt] = v;
        from[cnt] = u;
        head[u] = cnt;
    }
}ED1, ED2;
stack <int> stu;
void tarjan(int x) {
    low[x] = dfn[x] = ++num;
    vis[x] = 1;
    stu.push(x);
    for(int i = ED1.head[x]; i; i = ED1.next[i]) {
        int v = ED1.to[i];
        if(!dfn[v]) {
            tarjan(v);
            low[x] = min(low[x], low[v]);
        }
        if(vis[v])
            low[x] = min(low[x], low[v]);
    }
    if(dfn[x] == low[x]) {
        while(stu.top() != x) {
            int y = stu.top();
            stu.pop();
            belong[y] = x;
            w[x] += w[y];
            vis[y] = 0;
        }
        belong[x] = x;
        vis[x] = 0;
        stu.pop();
    }
}
signed main() {
```

```
    cin>>n>>m;
    for(int i = 1; i <= n; ++i)
        scanf("%lld", &w[i]);
    for(int i = 1; i <= m; ++i) {
        int u, v;
        scanf("%lld%lld", &u, &v);
        ED1.con(u, v);
    }
    for(int i = 1; i <= n; ++i) {
        if(!dfn[i]) tarjan(i);
    }
    for(int i = 1; i <= m; ++i) {
        int u = belong[ED1.from[i]], v = belong[ED1.to[i]];
        if(u == v) continue;
        ++in[v];
        ED2.con(u, v);
    }
    //topo
    return 0;
}
```

线性基

```cpp
#include<bits/stdc++.h>
using namespace std;
#define int long long
const int maxn = 2e5 + 10;

const int B = 60;



struct LinearBasis {
    vector<int> a = vector<int>(B, 0);


    bool insert(int x) {
        for(int i = B - 1; i >= 0; i--) {
            if(x & (1LL << i)) {
                if(a[i] == 0) { a[i] = x; return true; }
                x ^= a[i];
            }
        }
        return false;
    }


    int queryMin(int x) {
        for(int i = B - 1; i >= 0; i--) {
            x = min(x, x ^ a[i]);
        }
        return x;
    }
    int queryMax(int x) {
        for(int i = B - 1; i >= 0; i--) {
            x = max(x, x ^ a[i]);
        }
        return x;
    }
};

void work() {
    int n, k;
    cin >> n >> k;

    vector<int> a(n + 1);
    for(int i = 1; i <= n; i++) cin >> a[i];

    LinearBasis b;
    int cnt0 = 0;
```

```cpp
    for(int i = 1; i <= n; i++) {
        if(!b.insert(a[i])) {
            cnt0++;
        }
    }

    // 为什么是向下取整呢？因为有第0小的数，所以是向下取整
    k = k / (1LL << cnt0);
    // k >>= cnt0;

    int ans = 0;
    int cnt = 0;
    for(int i = 0; i < B; i++) {
        if(b.a[i] == 0) continue;
        cnt++;
    }
    cnt--;
    for(int i = B - 1; i >= 0; i--) {
        if(b.a[i] == 0) continue;
        if(k >= (1LL << cnt)) {
            k -= 1LL << cnt;
            ans = max(ans, ans ^ b.a[i]);
        } else {
            ans = min(ans, ans ^ b.a[i]);
        }
        cnt--;
    }

    cout << ans << endl;

}

signed main() {
    ios::sync_with_stdio(0);
    cin.tie(0);

    int t = 1;
    // cin >> t;
    while(t--) {
        work();
    }

    return 0;
}
```

虚树

```cpp
#include<bits/stdc++.h>
#define int long long
#define PII pair<int, int>
using namespace std;
const int N = 3e5 + 1010;
const int inf = 1ll<< 60;
int n, m, k;
int x[N],tot;
int w[N], va[N], dep[N], minp[N], dfn[N], cnt;
int lc[N][19];
vector<PII> e[N];
vector<int> e1[N];

void dfs1(int x, int fa) {
    dfn[x] = ++tot;
    dep[x] = dep[fa] + 1;
    lc[x][0] = fa;
    for(auto t : e[x]) {
        int v = t.first;
        if(v == fa) continue;
        minp[v] = min(minp[x], t.second);
        dfs1(v, x);
    }
}
bool cmp(int a, int b) {
    return dfn[a] < dfn[b];
}
int lca(int x, int y) {
    int flag = 0;
    if(x == 8 && y == 3) {
        flag = 1;
    }
    if(dep[x] < dep[y]) swap(x, y);
    for(int j = 18; j >= 0; --j) {
        if(dep[lc[x][j]] >= dep[y]) {
            x = lc[x][j];
        }
    }

    if(x == y) return x;
    for(int j = 18; j >= 0; --j) {
        if(lc[x][j] == lc[y][j]) continue;
        x = lc[x][j];
        y = lc[y][j];
    }
    return lc[x][0];
}
```

```cpp
void con(int x, int y) {
    e1[x].push_back(y);
    e1[y].push_back(x);
}
int stac[N], top = 0;
//建树
void build() {
    sort(w + 1, w + 1 + k, cmp);
    top = 0;
    stac[++top] = w[1];
    for(int i = 2; i <= k; ++i) {
        int lc = lca(stac[top], w[i]);
        while(dep[stac[top - 1]] >= dep[lc])      {
            con(stac[top], stac[top - 1]);
            top--;
        }
        if(stac[top] != lc) {
            con(lc, stac[top]);
            stac[top] = lc;
        }
        stac[++top] = w[i];
    }
    for(int i = top; i >= 2; --i) {
        con(stac[i], stac[i - 1]);
    }
}
//dp
int dfs(int x, int fa) {
    int now = 0;
    for(auto v : e1[x]) {
        if(v == fa) continue;
        now += dfs(v, x);
    }
    //记得清除虚树
    e1[x].clear();
    if(va[x]) {
        va[x] = 0;
        return minp[x];
    }
    else return min(now, minp[x]);
}
void solve() {
    cin>>n;
    for(int i = 1, u, v, d; i < n; ++i) {
        cin>>u>>v>>d;
        e[u].push_back({v, d});
        e[v].push_back({u, d});
    }

```

```
    for(int i = 1; i <= n; ++i) minp[i] = inf;
    minp[1] = inf;
    dfs1(1, 0);
    for(int j = 1; j <= 18; ++j) {
        for(int i = 1; i <= n; ++i) {
            lc[i][j] = lc[lc[i][j-1]][j-1];
        }
    }
    cin>>m;
    for(int i = 1; i <= m; ++i) {
        cin>>k;
        for(int i = 1; i <= k; ++i) {
            cin>>w[i];
            va[w[i]] = 1;
        }
        build();
        cout<<dfs(stac[1], 0)<<endl;
    }
}

signed main() {
    ios::sync_with_stdio(false);
    cin.tie(nullptr);

    solve();
    return 0;
}
```

LCT维护最大生成树(边的消失时间)

> ## 输入格式
>
> 第一行三个整数 n, m, k。
>
> 接下来 m 行，每行四个整数 x, y, l, r，表示有一条连接 x, y 的边在 l 时刻出现 r 时刻消失。
>
> ## 输出格式
>
> k 行，第 i 行一个字符串 `Yes` 或 `No`，表示在第 i 时间段内这个图是否是二分图。

```
#include<bits/stdc++.h>
#define int long long
using namespace std;

const int N = 5e5 + 1010;
const int inf = 1ll << 60;
int n, m, k;
int a[N];
int stac[N];

vector<int> in[N];
vector<int> out[N];
int mark[N];
struct ED {
    int u, v, l, r;
}edge[N];

struct node{
    int son[2], fa, val, sum;
    int st;
    int mint, minid;
    int flag;
}t[N];

void update(int x) {
    //t[x].sum = t[t[x].son[0]].sum ^ t[t[x].son[1]].sum ^ t[x].val;
    t[x].sum = t[t[x].son[0]].sum + t[t[x].son[1]].sum + t[x].val;
    t[x].mint = t[x].st;
    t[x].minid = x;
    if(t[x].son[0] && t[t[x].son[0]].mint < t[x].mint) {
        t[x].mint = t[t[x].son[0]].mint;
        t[x].minid = t[t[x].son[0]].minid;
    }
    if(t[x].son[1] && t[t[x].son[1]].mint < t[x].mint) {
        t[x].mint = t[t[x].son[1]].mint;
        t[x].minid = t[t[x].son[1]].minid;
```

```
    }
}

void lazy(int x) {
    swap(t[x].son[0], t[x].son[1]);
    t[x].flag ^= 1;
}

void pushdown(int x) {
    if(!t[x].flag) return;
    lazy(t[x].son[0]);
    lazy(t[x].son[1]);
    t[x].flag = 0;
}

bool isroot(int x) {
    return (t[t[x].fa].son[0] != x && t[t[x].fa].son[1] != x);
}

void rotate(int x){
    int y = t[x].fa, z = t[y].fa;
    int tag = (t[y].son[1] == x);
    if(!isroot(y)) t[z].son[t[z].son[1]==y] = x;
    t[x].fa = z;
    t[y].son[tag] = t[x].son[tag^1];
    t[t[x].son[tag^1]].fa = y;
    t[x].son[tag^1] = y;
    t[y].fa = x;
    update(y);update(x);
}

void splay(int x) {
    int ptr = 0, y = x;
    stac[ptr++] = y;
    while(!isroot(y)) {
        stac[ptr++] = t[y].fa;
        y = t[y].fa;
    }
    while(ptr--) pushdown(stac[ptr]);
    while(!isroot(x)) {
        int y = t[x].fa, z = t[y].fa;
        if(!isroot(y)) {
            (t[y].son[0] == x) ^ (t[z].son[0] == y) ? rotate(x) : rotate(y);
        }
        rotate(x);
    }
    update(x);
}
```

```
void access(int x) {
    int tp = x, y = 0;
    while(x) {
        splay(x);
        t[x].son[1] = y;
        update(x);
        y = x;
        x = t[x].fa;
    }
    splay(tp);
}

void makeroot(int x) {
    access(x);
    lazy(x);
}

int findroot(int x) {
    access(x);
    while(t[x].son[0]) {
        pushdown(x);
        x = t[x].son[0];
    }
    splay(x);
    return x;
}

void split(int x, int y) {
    makeroot(x);
    access(y);
}

void link(int x, int y) {
    makeroot(x);
    if(findroot(y) != x) {
        t[x].fa = y;
    }

}

void cut(int x, int y) {
    if(findroot(x) != findroot(y)) return;
    split(x, y);
    if(t[y].son[0] == x && t[x].son[1] == 0) {
        t[y].son[0] = 0;
        t[x].fa = 0;
        update(y);
```

```
    }
}



void solve() {
    cin>>n>>m>>k;

    for(int i = 1; i <= n; ++i) {
        t[i].st = t[i].mint = inf;
        t[i].minid = i;
        t[i].val = t[i].sum = 0;
    }
    for(int i = 1, x, y, l, r; i <= m; ++i) {
        cin>>x>>y>>l>>r;
        edge[i].u = x;
        edge[i].v = y;
        edge[i].l = l;
        edge[i].r = r;
        in[l].push_back(i);
        out[r].push_back(i);
        t[i + n].mint = r;
        t[i + n].st = r;
        t[i + n].minid = i + n;
        t[i + n].val = t[i + n].sum = 1;
    }
    int ans = 0;
    for(int i = 0; i < k; ++i) {

        for(auto j : in[i]) {
            int u = edge[j].u, v = edge[j].v;
            if(findroot(u) == findroot(v)) {
                split(u, v);
                int mid = t[v].minid, mint = t[v].mint;
                int sum = t[v].sum;
                if(edge[j].r <= mint) {
                    if(sum % 2 == 0) {ans++;mark[j] = 1;}
                    continue;
                    //mark
                }
                if(sum%2 == 0) {
                    mark[mid - n] = 1;
                    ans++;
                }
                cut(edge[mid - n].u, mid);
                cut(edge[mid - n].v, mid);
            }
            link(u, j + n);
            link(v, j + n);
```

```
        }
        for(auto j : out[i]) {
            int u = edge[j].u, v = edge[j].v;
            cut(u, j + n);
            cut(v, j + n);
            if(mark[j]) ans--;
        }
        if(ans) {
            cout<<"No\n";
        }
        else cout<<"Yes\n";
    }
    return;
}

signed main() {
    ios::sync_with_stdio(false);
    cin.tie(nullptr);

    int t = 1;
    //cin>>t;
    while(t--) {
        solve();
    }

    return 0;
}
```

树上背包

```cpp
#include<bits/stdc++.h>
using namespace std;
typedef long long ll;
const int N = 3030;
const ll inf = 1<<29;
int n, q;
int fa[N], a[N], sz[N];
ll f[N][N], tmp[N];
vector<int> e[N];

void dfs(int x) {
    sz[x] = 1;
    for(auto v : e[x])
        dfs(v);
    f[x][1] = a[x];
    for(auto v : e[x]) {
        for(int i = 1; i <= sz[x] + sz[v]; ++i) tmp[i] = -inf;
        for(int i = 1; i <= sz[x]; ++i)
            for(int j = 0; j <= sz[v]; ++j)
                tmp[i + j] = max(tmp[i + j], f[x][i] + f[v][j]);
        for(int i = 1; i <= sz[x] + sz[v]; ++i)
            f[x][i] = tmp[i];
        sz[x] += sz[v];
    }
}
int main() {
    scanf("%d%d", &n, &q);
    for (int i = 2; i <= n; ++i) {
        scanf("%d", &fa[i]);
        e[fa[i]].push_back(i);
    }
    for(int i = 1; i <= n; ++i) scanf("%d", &a[i]);
    dfs(1);
    for(int i = 1, u, m; i <= q; ++i) {
        scanf("%d%d", &u, &m);
        printf("%lld\n", f[u][m]);
    }
    return 0;
}
```

区间合并、区间求交

```cpp
void merge(vector<PII> &segs) {
    if (segs.empty()) return;
    vector<PII> res;
    sort(segs.begin(), segs.end());
    int st = segs[0].l, ed = segs[0].r;
    for (auto seg : segs) {
        if (seg.l > ed) {
            res.push_back({st, ed});
            st = seg.l, ed = seg.r;
        }
        else ed = max(ed, seg.r);
    }
    res.push_back({st, ed});
    segs = res;
}

vector<PII> intersection(vector<PII> a, vector<PII> b) {
    vector<PII> res;
    int i = 0, j = 0;
    while (i < a.size() && j < b.size()) {
        int l = max(a[i].l, b[j].l);
        int r = min(a[i].r, b[j].r);
        if (l <= r) res.push_back({l, r});
        if (a[i].r < b[j].r) i++;
        else j++;
    }
    return res;
}
```

子集反演

$$g(S)=\sum_{T\subseteq S}f(T)$$

由子集反演可得:

$$f(S)=\sum_{T\subseteq S}(-1)^{|S|-|T|}g(T)$$

```
#include<bits/stdc++.h>
#define int long long
using namespace std;
const int N = 520;
const int mod = 1e9 + 7;
int n;
int a[N][N];
int sum[1 << 20];
vector<pair<int, int>> road[N];
int po[N], cnt;

void clear() {
    for(int i = 1; i <= n + 1; ++i)
        for(int j = 1; j <= n + 1; ++j) a[i][j] = 0;
}

void con(int u, int v) {
    a[u][u]++;
    a[v][v]++;
    a[u][v]--;
    a[v][u]--;
}

void add() {
    for(int i = 1; i <= cnt; ++i) {
        for(auto x : road[po[i]]) {
            con(x.first, x.second);
        }
    }
}

int work() {
    int w = n;
    int ret = 1;/*
    for(int i = 1; i <= w; ++i) {
        for(int j = 1; j <= w; ++j) cout<<a[i][j]<<" ";
```

```
            cout<<endl;
    }
    cout<<endl;*/
    for(int i = 1; i <= w; ++i) {
        for(int j = i + 1; j <= w; ++j) {
            while(a[j][i]) {
                int l = a[i][i] / a[j][i];
                for(int k = i; k <= w; ++k)
                    a[i][k] = (a[i][k] - l * a[j][k] % mod + mod) % mod;
                for(int k = i; k <= w; ++k)
                    swap(a[i][k], a[j][k]);
                ret = ret * -1;
            }
        }
    }
    for(int i = 1; i <= w; ++i) {
        ret = (ret * a[i][i] % mod + mod) % mod;
    }

    return ret;
}

void solve() {
    cin>>n;
    for(int i = 1, x; i < n; ++i) {
        cin>>x;
        for(int j = 1, u, v; j <= x; ++j) {
            cin>>u>>v;
            road[i].push_back({u, v});
        }
    }

    n--;
    int maxn = (1 << n) - 1;
    int ans = 0;
    for(int i = 0; i <= maxn; ++i) {
        sum[i] = sum[i>>1] + (i & 1);
        cnt = 0;
        for(int j = 0; j <= 20; ++j) {
            if(i & (1 << j)) po[++cnt] = j + 1;
        }
        clear();
        add();
        int f = work();
        //cout<<((n - sum[i]) % 2 ? -1 : 1) * f<<endl;
        ans =  (ans + ((n - sum[i]) % 2 ? -1 : 1) * f % mod + mod) % mod;
    }
```

```
    cout<<ans;
}

signed main() {
    ios::sync_with_stdio(false);
    cin.tie(NULL);
    solve();
    return 0;
}
```

子集dp

```
for(int i=0;i<w;++i)//依次枚举每个维度{
    for(int j=0;j<(1<<w);++j)//求每个维度的前缀和{
        if(j&(1<<i))s[j]+=s[j^(1<<i)];
    }
}
```

矩形面积并

```cpp
#include<bits/stdc++.h>
#define int long long
using namespace std;

const int N = 2e5 + 1010;
const int inf = 1ll << 60;

int n;

vector<array<int, 4>> event;
vector<int> vx;

struct node {
    int mincnt, minv;
    int flag, tag;
}t[N << 4];

void update(int p) {
    if(t[p << 1].minv == t[p << 1 | 1].minv) {
        t[p].mincnt = t[p << 1].mincnt + t[p << 1 | 1].mincnt;
        t[p].minv = t[p << 1].minv;
    }
    else if(t[p << 1].minv < t[p << 1 | 1].minv) {
        t[p].minv = t[p << 1].minv;
        t[p].mincnt = t[p << 1].mincnt;
    }
    else {
        t[p].minv = t[p << 1 | 1].minv;
        t[p].mincnt = t[p << 1 | 1].mincnt;
    }
}
void settag(int p, int val){
    t[p].tag += val;
    t[p].minv += val;
}
void pushdown(int p) {
    int val = t[p].tag;
    settag(p << 1, val);
    settag(p << 1 | 1, val);
    t[p].tag = 0;
}



void build(int p, int l, int r) {
    if(l == r) {
        t[p].minv = t[p].flag = 0;
```

```
        t[p].mincnt = vx[r] - vx[r - 1];
        return;
    }
    int mid = l + r >> 1;
    build(p << 1, l, mid);
    build(p << 1 | 1, mid + 1, r);
    update(p);
}

void insert(int p, int l, int r, int ql, int qr, int val){
    if(ql <= l && r <= qr) {
        settag(p, val);
        return;
    }
    pushdown(p);
    int mid = l + r >> 1;
    if(ql <= mid) insert(p << 1, l, mid, ql, qr, val);
    if(qr > mid) insert(p << 1 | 1, mid + 1, r, ql, qr, val);
    update(p);
}

void solve() {
    cin>>n;
    for(int i = 1, x1, x2, y11, y2; i <= n; ++i) {
        cin>>x1>>x2>>y11>>y2;
        vx.push_back(x1);
        vx.push_back(x2);
        event.push_back({y11, 1, x1, x2});
        event.push_back({y2, -1, x1, x2});
    }
    sort(event.begin(), event.end());
    sort(vx.begin(), vx.end());
    vx.erase(unique(vx.begin(), vx.end()), vx.end());
    int m = vx.size() - 1;
    build(1, 1, m);
    int ans = 0, prey = 0;
    int totlen = t[1].mincnt;//0
    for(auto evt : event) {
        int cov = totlen;
        if(t[1].minv == 0) {
            cov -= t[1].mincnt;
        }
        ans += cov * (evt[0] - prey);
        prey = evt[0];
        int x1 = lower_bound(vx.begin(), vx.end(), evt[2]) - vx.begin() + 1;
        int x2 = lower_bound(vx.begin(), vx.end(), evt[3]) - vx.begin();
        if(x1 > x2) continue;
```

```
        insert(1, 1, m, x1, x2, evt[1]);
    }
    cout<<ans<<"\n";
}

signed main() {
    ios::sync_with_stdio(false);
    cin.tie(nullptr);

    int t = 1;
    //cin>>t;
    while(t--) {
        solve();
    }

    return 0;
}
```

```
#include<bits/stdc++.h>
#define int long long
using namespace std;

const int N = 1e5 + 1010;
const int inf = 1ll << 60;
const int sc = 5e4;
const double eps = 1e-5;
int n;

struct node{
    double k, b;
    int flag = 0;
}t[N<<3];

double calc(node line, int x) {
    return line.k * x + line.b;
}

double cross(node l1, node l2) {
    return (l1.b - l2.b) / (l2.k - l1.k);
}

void insert(int p, int l, int r, int ql, int qr, node line) {
    if(ql <= l && r <= qr) {
        if(!t[p].flag) {
            t[p] = line;
            return;
        }
        double del1 = calc(line, l) - calc(t[p], l);
        double del2 = calc(line, r) - calc(t[p], r);
        if(del1 > eps &&del2  > eps) {
            t[p] = line;
            return;
        }
        if(del1 < eps && del2 < eps) return;

        int mid = l + r >> 1;
        if(calc(line, mid) - calc(t[p], mid) > eps) {
            swap(t[p], line);
        }
        double cr = cross(t[p], line);
        if ((double) mid - cr > eps) {
            insert(p << 1, l, mid, ql, qr, line);
        }
        else {
            insert(p << 1 | 1, mid + 1, r, ql, qr, line);
```

```
        }
        return;
    }
    int mid = l + r >> 1;
    if(ql <= mid) insert(p << 1, l, mid, ql, qr, line);
    if(qr > mid) insert(p << 1 | 1, mid + 1, r, ql, qr, line);
    return;

}

double query(int p, int l, int r, int qx) {
    if(l == r) {
        return calc(t[p], qx);
    }
    int mid = l + r >> 1;
    double ret = calc(t[p], qx);
    if(qx <= mid) ret = max(ret, query(p << 1, l , mid, qx));
    else ret = max(ret, query(p << 1 | 1, mid + 1, r, qx));
    return ret;
}

void solve() {
    cin>>n;
    string op;
    double s, p;
    int t;
    for(int i = 1; i <= n; ++i) {
        cin>>op;
        //cout<<i<<" "<<op<<endl;
        if(op[0] == 'Q') {
            cin>>t;
            cout<<(int) (query(1, 1, sc, t) / 100)<<"\n";
        }
        else {
            cin>>s>>p;
            s -= p;
            node newline;
            newline.b = s;
            newline.k = p;
            newline.flag = 1;
            insert(1, 1, sc, 1, sc, newline);
        }
    }
}

signed main() {
    ios::sync_with_stdio(false);
    cin.tie(nullptr);

```

```
    int t = 1;
    //cin>>t;
    while(t--) {
        solve();
    }

    return 0;
}
```

min25筛

定义积性函数$f(x)$，且$f(p^k)=p^k(p^k-1)$（p是一个质数），求

$$\sum_{i=1}^{n} f(i)$$

$$g(n,j)=\sum_{i=1}^{n} F(i)\ [i\in \mathrm{p}\ ||\ i\text{的最小质因子大于第j个素数}]$$

$$g(n,j)=\begin{cases} g(n,j-1) & {p_j}^2>n \\ g(n,j-1)-F(p_j)\cdot\left(g\left(\left\lfloor\frac{n}{p_j}\right\rfloor,j-1\right)-\sum_{i=1}^{j-1}F(p_j)\right) & {p_j}^2\le n \end{cases}$$

$$S(n,j)=\sum_{i=1}^{n} f(i)\ [i\text{的最小质因子大于第}j\text{个质数}]$$

$$S(n,j)=g(n,|P|)-\sum_{i=1}^{j-1} f(p_i)+\sum_{k=j+1}^{{p_k}^2\le n}\sum_{e=1}^{{p_k}^e\le n} f({p_k}^e)\left(S\left(\left\lfloor\frac{n}{{p_k}^e}\right\rfloor,k\right)+[e>1]\right)$$

```
#include<bits/stdc++.h>
#define int long long
using namespace std;
typedef long long ll;
const int N = 1e6 + 1010, mod = 1e9 + 7;
int n;

int ksm(int a, int b) {
    int ret = 1;
    while(b) {
        if(b & 1) ret = ret * a % mod;
        a = a * a % mod;
        b >>= 1;
    }
    return ret;
```

```
}


namespace min25 {
    int sq;
    int g1[N], g2[N], w[N], id1[N], id2[N], tot;
    int p[N], pr[N], cnt;
    int sp1[N], sp2[N];
    int inv2, inv6;
    void init() {
        sq = sqrt(n);
        cnt = 0;
        inv2 = ksm(2, mod - 2), inv6 = ksm(6, mod - 2);
        for(int i = 2; i <= sq; ++i) {
            if(!p[i]) {
                p[i] = pr[++cnt] = i;
                sp1[cnt] = (sp1[cnt - 1] + i) % mod;
                sp2[cnt] = (sp2[cnt - 1] + i*i%mod) % mod;
            }
            for(int j = 1; j <= cnt && i * pr[j] <= sq; ++j) {
                p[i * pr[j]] = pr[j];
                if(p[i] == pr[j]) break;
            }
        }
    }
    void getG() {
        for(ll l = 1, r; l <= n; l = r + 1) {
            r = n / (n / l);
            int x = n / l ;
            w[++tot] = x;
            x %= mod;
            g1[tot] = (x * (x + 1)%mod * inv2 % mod - 1+ mod)% mod;
            g2[tot] = (x * (x + 1)%mod * (x * 2 + 1)% mod * inv6%mod - 1 + mod) % mod;

            w[tot] <= sq ? id1[w[tot]] = tot : id2[n / w[tot]] = tot;
        }
        for(int j = 1; j <= cnt; ++j) {
            for(int i = 1; i <= tot && pr[j] * pr[j] <= w[i]; ++i) {
                int tmp = w[i] / pr[j];
                int p = tmp <= sq? id1[tmp] : id2[n / tmp];
                g1[i] = (g1[i] - pr[j] * (g1[p] - sp1[j - 1] + mod) % mod + mod) % mod;
                g2[i] = (g2[i] - pr[j] * pr[j] %mod * (g2[p] - sp2[j - 1] + mod) % mod + mod) % mod;
            }
        }
    }
    int getS(int i, int j) {
```

```
        if(pr[j] >= i) return 0;
        int p = i <= sq? id1[i] : id2[n / i];
        int ans = ((g2[p] - g1[p] + mod) % mod - (sp2[j] - sp1[j] + mod) % mod + mod) % mod;
        for(int k = j + 1; pr[k] * pr[k] <= i && k <= cnt; ++k) {
            int pe = pr[k];
            for(int e = 1; pe <= i; ++e, pe = pe * pr[k]) {
                int x = pe % mod;
                ans = (ans + x * (x - 1) % mod * (getS(i / pe, k) + (e > 1)) % mod) % mod;
            }
        }
        return ans;
    }
    int getans(ll n) {
        init();
        getG();

        return getS(n, 0) + 1;
    }
}

signed main() {
    scanf("%lld", &n);
    printf("%lld", min25::getans(n));
    return 0;
}
```

杜教筛

$$ans_1 = \sum_{i=1}^{n} \varphi(i)$$

$$ans_2 = \sum_{i=1}^{n} \mu(i)$$

$$g(1)S(n) = \sum_{i=1}^{n}(f * g)(i) - \sum_{i=2}^{n} g(i)S(\lfloor \frac{n}{i} \rfloor)$$

```
#include<bits/stdc++.h>
#define int long long
using namespace std;
typedef long long ll;
const int N = 5e6 + 1010;
ll T;
ll sphi[N], smu[N];
ll n;
int p[N], pr[N], cnt;
int mu[N], phi[N];
map<ll, ll> mpphi;
map<ll, ll> mpmu;
void init() {
    for(int i = 2; i < N; ++i) {
        if(!p[i]) {
            p[i] = i;
            pr[++cnt] = i;
            mu[i] = -1;
            phi[i] = i - 1;
        }
        for(int j = 1; j <= cnt && i * pr[j] < N; ++j) {
            p[i * pr[j]] = pr[j];
            if(p[i] == pr[j]) {
                mu[i * pr[j]] = 0;
                phi[i * pr[j]] = phi[i] * pr[j];
                break;
            }
            else {
                mu[i * pr[j]] = -mu[i];
                phi[i * pr[j]] = phi[i] * (pr[j] - 1);
```

```
                }
            }
        }
    mu[1] = 1;
    phi[1] = 1;
    for(int i = 1; i < N; ++i) {
        smu[i] = smu[i - 1] + mu[i];
        sphi[i] = sphi[i - 1] + phi[i];
    }
}
ll getphi(int n) {
    if(n < N) return sphi[n];
    if(mpphi.count(n)) return mpphi[n];
    ll sum = (ll)(1 + n) * n / 2;
    for(int l = 2; l <= n; ++l) {
        int r = n / (n / l);
        sum -= (ll)(r - l + 1) * getphi(n / l);
        l = r;
    }
    return mpphi[n] = sum;
}
ll getmu(int n) {
    if(n < N) return smu[n];
    if(mpmu.count(n)) return mpmu[n];
    ll sum = 1;
    for(int l = 2; l <= n; ++l) {
        int r = n / (n / l);
        sum -= (ll)(r - l + 1) * getmu(n / l);
        l = r;
    }
    return mpmu[n] = sum;
}
signed main() {
    init();
    scanf("%lld", &T);
    while(T--) {
        scanf("%lld", &n);
        printf("%lld %lld\n", getphi(n), getmu(n));
    }
    return 0;
}
```

卢卡斯定理

```cpp
ll lucas(int n,int m,int p){
    if(m==0) return 1;
    return lucas(n/p,m/p,p)*C(n%p,m%p)%p;
}
```

扩展卢卡斯定理

```cpp
#include<bits/stdc++.h>
using namespace std;
#define ll long long
const int N = 2e6+100;
ll a[N], p[N], pe[N];
ll fac[N];
int cnt = 0;
ll T, n, m;
ll mod;
int now;
void exgcd(ll a, ll b, ll &x, ll &y){
    if(!b){
        x = 1;
        y = 0;
        return;
    }
    exgcd(b, a%b, y, x);
    y -= (a / b) * x;
}
ll qpow(ll a,ll b,ll p){
    ll c=1;
    while(b){
        if(b&1)c=(c*a)%p;
        a=(a*a)%p;
        b>>=1;
    }
    return c;
}
ll inv(ll a,ll p){
    ll x, y;
    exgcd(a, p, x, y);
    x %= p;
    if(x < 0) x += p;
    return x;
}

void init() {
    int pp = mod;
    for(int i = 2; i * i <= pp; ++i){
```

```
        if(pp%i) continue;
        ll tmp=1;
        p[++cnt] = i;
        pe[cnt] = 1;
        while(pp % i == 0){
            pp/=i;
            pe[cnt] *= i;
        }
    }
    if(pp > 1) {
        p[++cnt] = pp;
        pe[cnt] = pp;
    }

}


ll facdiv(ll n, ll p, ll pk){
    if(!n) return 1;
    ll ans=1;
    for(ll i = 1; i < pk; ++i){
        if(i % p) ans = (ans * i) % pk;
    }
    ans = qpow(ans, n / pk, pk);
    for(ll i = 1; i <= n % pk; ++i){
        if(i % p) ans = (ans * i) % pk;
    }
    return ans * facdiv(n / p, p, pk) % pk;
}
ll C(ll n, ll m, ll p, ll pk){
    if(n < m) return 0;
    ll f1 = facdiv(n, p, pk), f2 = facdiv(m, p, pk), f3 = facdiv(n - m, p, pk), cnt=0;
    ll t1 = n, t2 = m, t3 = n - m;
    for(; t1; t1/=p) cnt += t1/p;
    for(; t2; t2/=p) cnt -= t2/p;
    for(; t3; t3/=p) cnt -= t3/p;
    return ((f1*inv(f2,pk) % pk)*inv(f3,pk)%pk)*qpow(p,cnt,pk)%pk;
}


ll exlucas(ll n,ll m,int pp){
    ll x;
    ll ret = 0;
    for(int i = 1; i <= cnt; ++i) {
        x = C(n, m, p[i], pe[i]);
        ret = (ret + ((mod / pe[i] * x) % mod * inv(mod / pe[i], pe[i]) % mod))%mod;
    }
```

```
    return ret;
}
int main(){
    //scanf("%lld%lld", &mod, &T);
    //T = 1;
    scanf("%lld%lld%lld",&n,&m, &mod);
    init();
    printf("%lld\n",exlucas(n,m,mod));

    // while(T--) {
    //  scanf("%lld%lld",&n,&m);
    //  printf("%lld\n",exlucas(n,m,mod));
    // }
    return 0;
}
```

卡特兰数

卡特兰数，一个特殊的数列。通项公式为：

$$Cat_n = \frac{C_{2n}^{n}}{n+1}$$

从0开始的前几项为：$1, 1, 2, 5, 14, 42, 132, \cdots$，所以有的题可以直接打个表看看（比如这个）

然后是它是怎么推出来的，最主要的就是从$(0,0)$到(n,n)不穿过直线$y=x$的路径计数（不想上图了，可以手画一个）。首先我们随便走的走法就是$2n$步里面选n步向上剩下n步向右，就是C_{2n}^{n}。

然后减去不合法的方案数。我们发现，如果穿过直线$y=x$，那必然接触直线$y=x+1$。然后我们把第一个接触点之后向右和向上的走法反转，那么它就会走到$(n-1,n+1)$，走法数显然是C_{2n}^{n+1}。于是一个公式就是

$$Cat_n = C_{2n}^{n} - C_{2n}^{n+1}$$

还有一些其他的公式：

$$Cat_n = \frac{Cat_{n-1}(4n-2)}{n+1}$$

$$Cat_n = \sum_{i=1}^{n} Cat_{i-1} Cat_{n-i} (n \geq 2)$$

最后是一些常用的卡特兰数模型：

1. 一个01串，n个0n个1。使任意前缀中0的个数不小于1的个数的方案数为Cat_n。
2. n个点的有标号二叉树的个数为Cat_n。
3. 一个栈的进栈序列为$1, 2, \cdots n$,则不同的出栈序列个数为Cat_n。
4. 圆上$2n$个点，用n条线段成对连接，不相交的方案数为Cat_n。
5. 将一个凸多边形剖分成n个三角形的方案数为Cat_n。

prufer序列

一个长度为n-2的Prufer序列，唯一对应一棵n个点固定形态的无根树。

性质：

1. prufer序列中，点u出现的次数，等于点u在树中的度数-1
2. n个点的无根树，唯一对应长度为n-2的prufer序列，序列每个数都在1到n的范围内。
3. Cayley定理：n个点的无向完全图的生成树的计数: n^{n-2}，即n个点的有标号无根树的计数
4. n个节点的度依次为$d1, d2, \ldots, dn$的无根树共有$\frac{(n-2)!}{\prod_{i=1}^{n}(d_i-1)!}$个，因为此时Prufer编码中的数字i恰好出现di-1次，(n-2)!是总排列数
5. n个点的有标号有根树的计数：$n^{n-2} * n = n^{n-1}$

矩阵树定理

给出一个无向无权图,设 A 为邻接矩阵, D 为度数矩阵($D[i][i]$ =节点 i 的度数,其他的无值)。

则基尔霍夫(Kirchhoff)矩阵即为 : $K = D - A$

然后令 K' 为 K 去掉**第k行与第k列**(k任意)的结果($n - 1$阶主子式),

$\det(K')$ 即为该图的生成树个数。

- ## 有向扩展

前面都是无向图,神奇的是有向图的情况也是可以做的。

(邻接矩阵 A 的意义同有向图邻接矩阵)

那么现在的矩阵 D 就要变一下了。

若 $D[i][i] = \sum_{j=1}^{n} A[j][i]$,即**到该点的边权总和(入)**。

此时求的就是**外向树** (从根向外)

若 $D[i][i] = \sum_{j=1}^{n} A[i][j]$,即从**从该点出发的边权总和(出)**。

此时求的就是**内向树** (从外向根)

(如果考场上不小心忘掉了,可以手玩小样例)

(同样可以加权!)

此外,既然是有向的,那么就需要**指定根**。

前面提过要任意去掉第 k 行与第 k 列,是因为无向图所以不用在意谁为根。

在有向树的时候需要理解为指定根,结论是 : 去掉哪一行就是那一个元素为根。

二项式反演（3个形式）

$$f(n)=\sum_{i=0}^{n}(-1)^i\binom{n}{i}g(i)$$
$$g(n)=\sum_{i=0}^{n}(-1)^i\binom{n}{i}f(i)$$

$$f(n)=\sum_{i=0}^{n}\binom{n}{i}h(i)\Leftrightarrow\frac{h(n)}{(-1)^n}=\sum_{i=0}^{n}(-1)^i\binom{n}{i}f(i)$$

$$f(n)=\sum_{i=n}^{m}\binom{i}{n}g(i)\Leftrightarrow g(n)=\sum_{i=n}^{m}(-1)^{i-n}\binom{i}{n}f(i)$$

第一类斯特林数

n个不同元素构成m个圆的排列方案数

$$s_u(n,m)=s_u(n-1,m-1)+s_u(n-1,m)*(n-1)$$

第二类斯特林数

n个不同元素构成m个集合的排列方案数

$$S(n,m)=S(n-1,m-1)+m\times S(n-1,m)$$

$$s_u(n,m)=s_u(n-1,m-1)+s_u(n-1,m)*(n-1)$$

$$S(n,m)=\frac{1}{m!}\sum_{k=0}^{m}(-1)^k\frac{m!}{k!(m-k)!}(m-k)^n$$

$$S(n,m)=\frac{1}{m!}\sum_{k=0}^{m}m!\frac{(-1)^k}{k!}\frac{(m-k)^n}{(m-k)!}$$

$$S(n,m)=\sum_{k=0}^{m}\frac{(-1)^k}{k!}\frac{(m-k)^n}{(m-k)!}$$

下降幂

下降幂:

$$x^{\underline{m}}=x(x-1)\cdots(x-m+1)=m!\binom{x}{m}=\frac{x!}{(x-m)!}$$

下降幂的差分:

$$(x+1)^{\underline{m}}-x^{\underline{m}}=mx^{\underline{m-1}}$$

下降幂的定和式:

$$\sum_{a\le x<b}x^{\underline{m}}=\frac{b^{\underline{m+1}}-a^{\underline{m+1}}}{m+1}$$

贝尔数

贝尔数B_n是基数（元素个数）为n的集合的划分方法的数目。集合S的一个划分是定义为S的两两不相交的非空子集的族，它们的并是S。

正文

首先根据贝尔数的定义，有

$$B_n=\sum_{m=0}^{n} S(n,m)$$

其中$S(n,m)$是第二类斯特林数。
那么再由第二类斯特林数的展开式可得

$$\begin{aligned}\text{原式}&=\sum_{m=0}^{n}\frac{1}{m!}\sum_{k=0}^{m}(-1)^k C(m,k)(m-k)^n\\&=\sum_{m=0}^{n}\sum_{k=0}^{m}\frac{(-1)^k}{k!}\frac{(m-k)^n}{(m-k)!}\end{aligned}$$

这样子，设$A_i=\frac{(-1)^i}{i!}$，$B_i=\frac{i^n}{i!}$，这样子就是

$$\text{原式}=\sum_{m=0}^{n}\sum_{k=0}^{m}A_kB_{m-k}$$

可以NTT，但是太麻烦，我们注意到对于A_i这一项，它只会与B_0，B_1，$B_2 ... B_{n-i}$相乘，就是一个前缀和的形式，所以A_i这一项的贡献就算了出来，这样子的话，预处理A,B以及其前缀和，然后for一遍，把每一项A_i的贡献算出来加上去就可以了，这样子是$O(nlogn)$的（要算快速幂）。

$$B_{n+1}=\sum_{i=0}^{n}\binom{n}{i}B_i$$

$$B_{p+n}\equiv B_n+B_{n+1} \pmod p$$

$$B_{p^m+n}\equiv mB_n+B_{n+1} \pmod p$$

```
void init()
{
    vis.clear();
    b.clear();
    a[1][1]=1;
    for(int i=2;i<=p;++i) for(int j=1;j<=i;++j) {
        if(j==1) a[i][j]=a[i-1][i-1];
        else {
            a[i][j]=(a[i][j-1]+a[i-1][j-1])%p;
        }
    }
}

int get_ans(int x)
{
    if(x<=p) {
```

```
        return a[x][0];
    }
    if(vis[x]) return b[x];
    vis[x]=1;
    return b[x]=(get_ans(x-p)+get_ans(x-p+1))%p;

```