The Distillation of Human Knowledge

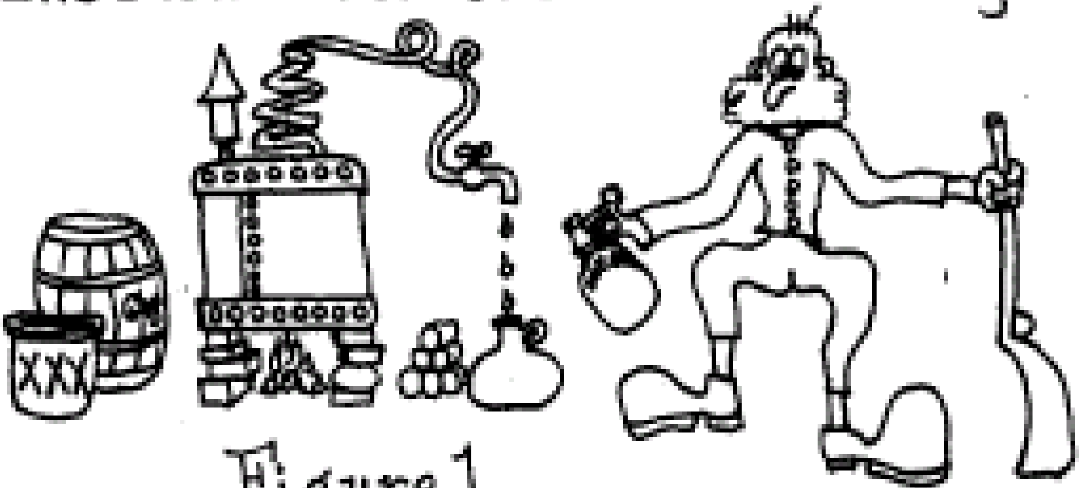


Figure 1

An experimental catalytic converter being developed in the hills of Kentucky in a top-secret research project converts sour mash into a lead-free fuel which can put new life into your tired old carburetor. This work is being conducted under a joint research grant from the Mafia and the Central Intelligence Agency.

Exact details of the device are, of course, classified. However, a key component of the system is a long square pipe with a second square duct running down the middle. (See Figure 2) The space between the two pipes is packed with fermented rye mash. A critical question is

"What is the exact temperature distribution in the material?"

By Neal Wagner, Univ. of Texas San Antonio
(original source: http://www.cs.utsa.edu/~wagner/CS2073/distill/dsgif.html) PROBLEM 17-1 [illegible]

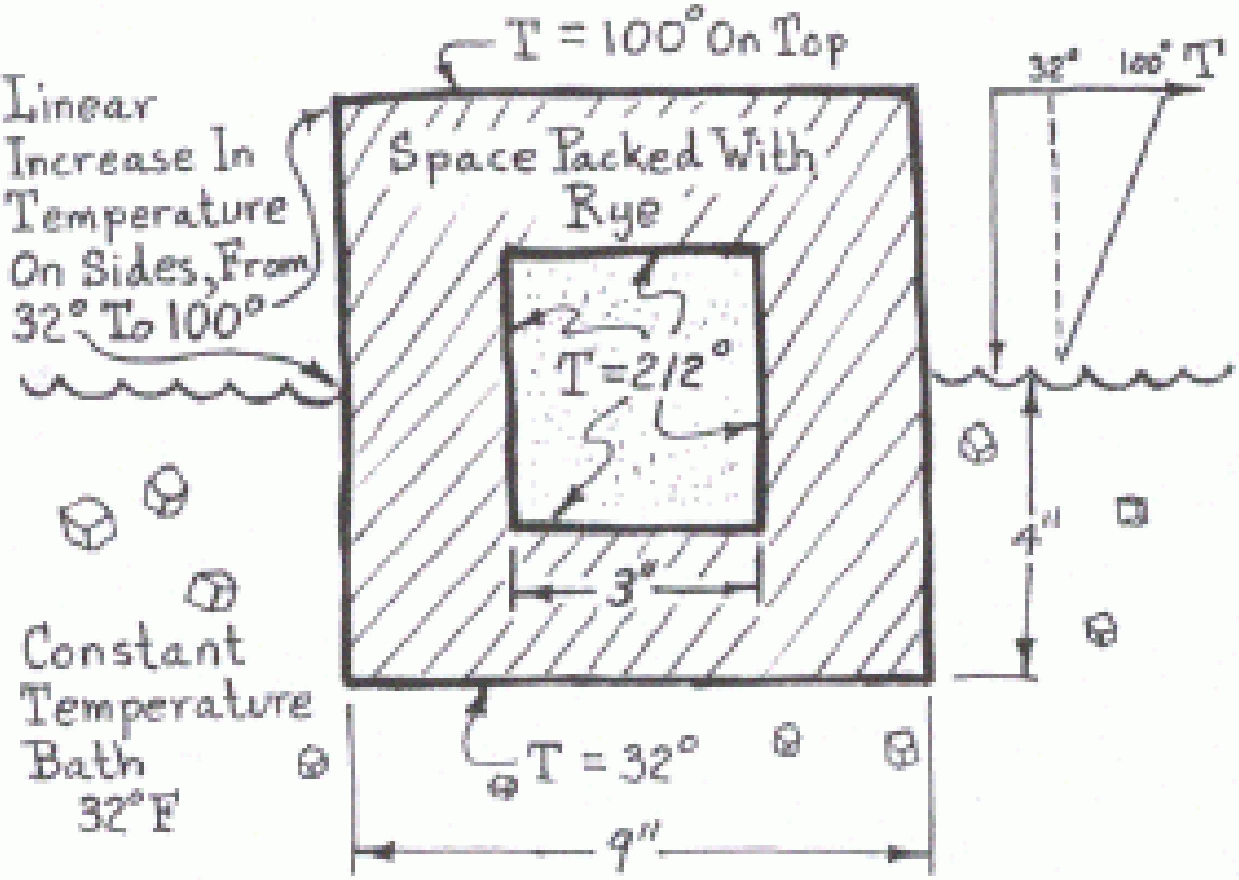


Figure 2

Drawn by: Moonbean McSwine, Inventor
Signed: X ← His mark

We need to use the computer to calculate the temperature distribution in the rye packed between the two square ducts. The outer pipe is 9 inches square and the inner pipe is 3 inches square and runs down the middle.

As part of the Likker Condensing System, the lower part of the outer pipe is submerged in a Constant Temperature

Bath, consisting of ice cubes floating in Kickapoo Joy Juice. This keeps the whole outer wall of the lower part at exactly 32° F.

As part of the Likker Eggstrakter System, the inner pipe is filled with steam which keeps its walls at a constant 212 °F.

The top surface is part of the Alky Regenerator & Revenoo Offiser Roaster. That surface is kept at a constant 100° by the hickory fire.

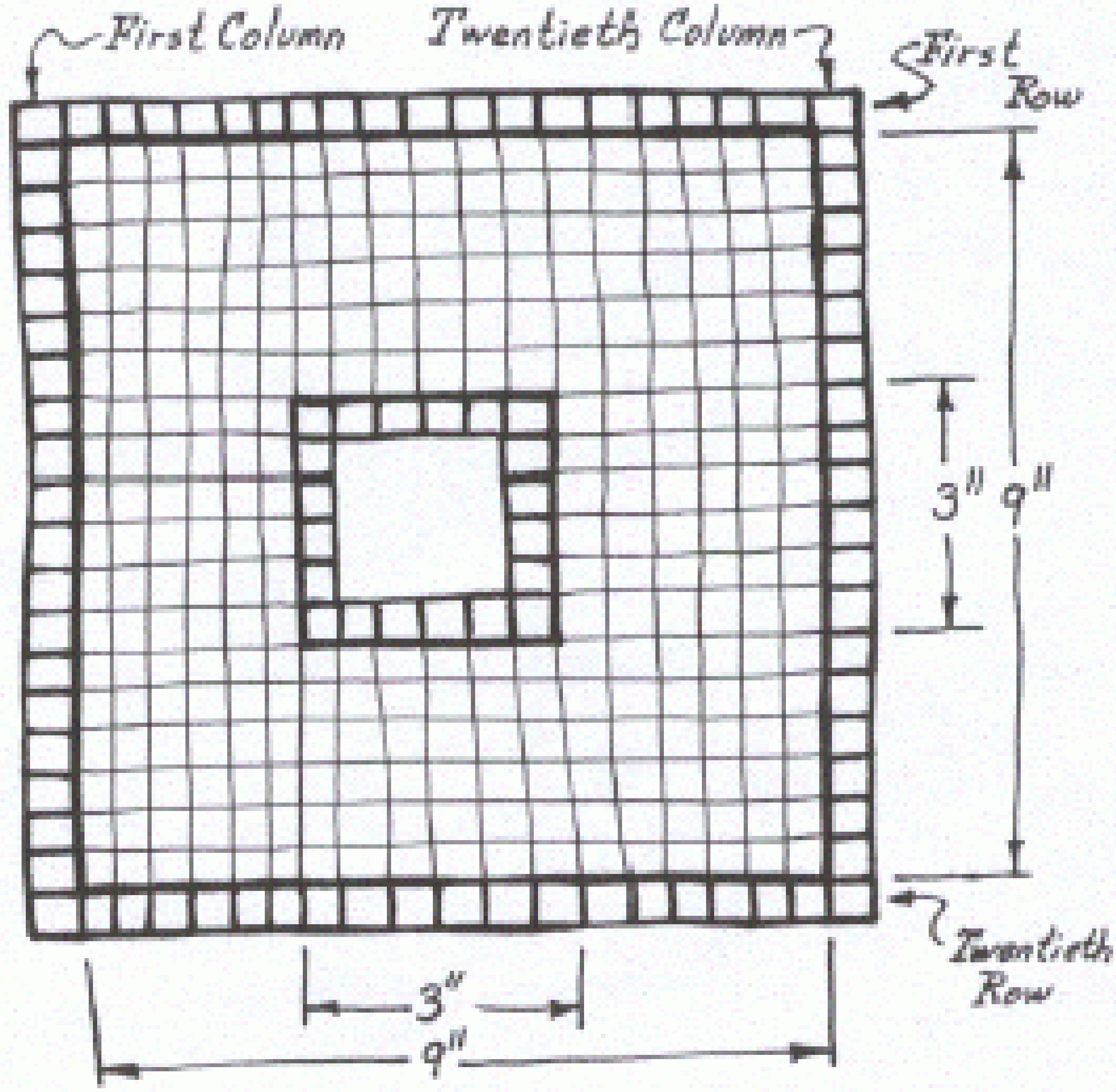
The sides of the outer pipe vary in temperature, from 100° at the top down to 32° at the height of the ice bath. The bath comes 4" up the side of the pipe, as shown. Thus, the bottom 4" are at 32° and then the pipe wall increases linearly in temperature up to 100°.

Thus, we know the Boundary Temperature all around the walls in contact with the rye mash. The Catcher in the Rye is

How can we figure the temperature in the material itself, knowing the boundary temperatures?

I'll tell you how! It's not very hard at all. Just keep an eye out for the Revenoo Offiser while I tell you.

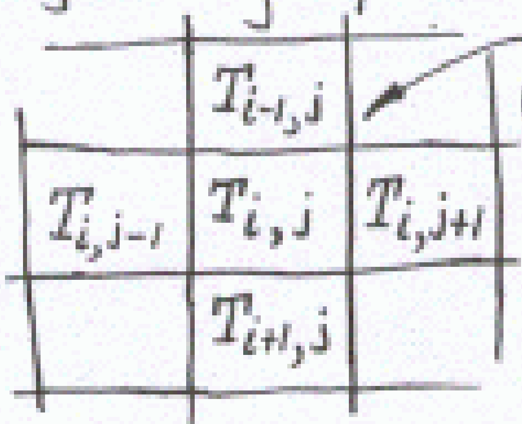
Suppose we were to subdivide the space between the pipes into a lot of small square regions as shown below:



Notice I added an extra row of squares inside the inner pipe and outside the outer pipe. I'll tell you about those Phantom Squares later.

Now, if the squares are small enough, then it is reasonable to assume that the temperature is uniform within each square. In other words, we could say the square in the 7th row and 12th column is at 53½° and we wouldn't need to say how the temperature varied across the square. With small squares, there wouldn't be much variation.

Also, common sense tells us that the temperature in each square would be the Average of the temperatures at the four neighboring squares:



Typical Square in the i^{th} Row and j^{th} Column

$$T_{i,j} = \frac{1}{4}\left(T_{i-1,j} + T_{i+1,j} + T_{i,j-1} + T_{i,j+1}\right)$$

MAGIC FORMULA

Looking at the figure on page 258, we see we could set up a big 20 by 20 matrix in the computer and use it to store the temperatures of all the little squares.

But we don't Know the temperatures of all those squares!

True, but we Do know the temperatures of all the Phantom Squares around the boundaries. We can fill those values into our matrix, since they aren't going to change.

What about all the squares between the two ducts?

They're the problem, aren't they? Just to get started, let's arbitrarily set them all to some reasonable value between 32° and 212°, say 90° or so.

You call that Science? It sounds pretty random to me!

It _is_ pretty random because it _Doesn't Really Matter_. This is just a starting guess. Now we'll iteratively apply our Magic Formula and use it over and over to improve our estimates of each temperature. When we're through, we'll know very closely just what the actual temperature of each square really is.

Start with the first inside square and iteratively step from square to square, applying the Magic Formula to each inside square. Store the new temperature for the square in place of the old value.

What do we do when we reach the last square?

Go right back to the first square and do it all over again! After a number of passes, the temperatures of the squares won't change much from iteration to iteration, so you'll know you have pretty much found the final temperature distribution you were looking for!

Say that again?

O.K. Here's the process. You cycle from square to square applying the Magic Formula as you go. Each time, *before* recording the new temperature for a square, see how much of a change that is from what it was before. Don't bother recording this change unless it is a *bigger* change than any other square made during that pass. Thus, during the pass, you keep track of the biggest (absolute value) change in temperature of any square.

That's easy enough!

When you finish with the last interior square, check what the biggest change was. If, during that pass, some square changed temperature by more than 1/2°, go back and do it all over again. After a while, the values will settle down and no square will change by more than 1/2° during a pass.

At that point, quit and write out the final temperature matrix.

Hint #1: Save time, $$, and space in the computer by making use of the symmetry about the centerline of the ducts.

Hint #2: Make sure you don't try applying the Magic Formula to one of the boundary Phantom Squares! Their temperatures never change!

Incidentally, you have just solved Laplace's equation, $\nabla^2 u = 0$. If you wanted a more accurate answer, you could use smaller squares and a tighter tolerance.

Solved it? I don't even <u>Know</u> Mr. Laplace!

Hint #3: Watch out for the Revenoo Offisers!

Cracker Jim sez he was out fishin' on the lake 'tother evenin when two Revenoors rowed by him with a still they had just confiscated. An ole man and a boy passed them rowin' in tother direkshun. They rowed on quite a ways without sayin' a word, but finally the boy spoke. Cracker Jim sez they wuz purty far away, but voices carry over a lake in the still of the evenin. He could make out the boy say

Paw. Is thet our'n?