

计算方法 B

Programming Assignment #3

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非线性方程求根

1 问题描述

分别编写用 Newton 法和弦截法求根的通用程序。再利用你的通用程序分别求下面方程的根

$$f(x) = 2x^4 + 24x^3 + 61x - 16x + 1 = 0$$

其中, Newton 迭代法分别取初值 $x_0 = 0$ 和 $x_0 = 3$; 弦截法的初值分别取为 $x_0 = 0$, $x_1 = 0.5$ 以及 $x_0 = 0.1$, $x_1 = 1.5$;

取误差限 ε 为 $1.0\text{E}-9$, 即当 $|f(x_k)| < \varepsilon$ 时, 停止迭代。将计算结果列成表格, 要求给出初值、每步的迭代结果, 以及最终的迭代结果 (包括迭代步数); 比较或分析两种计算方法的优劣。

2 计算结果

3 结果分析

4 算法分析

5 实验结论

迭代步数 k	x_k	$f(x_k)$
$k = 0$ (初值)	0.0000000000E+000	1.0000000000E+000
$k = 1$	6.2500000000E-002	2.4417114258E-001
$k = 2$	9.2675144823E-002	6.0357821710E-002
$k = 3$	1.0750916023E-001	1.4994760152E-002
$k = 4$	1.1485323376E-001	3.7248898748E-003
$k = 5$	1.1848368152E-001	9.1626064336E-004
$k = 6$	1.2024260677E-001	2.1577268802E-004
$k = 7$	1.2102581790E-001	4.2847681852E-005
$k = 8$	1.2128383271E-001	4.6530959359E-006
$k = 9$	1.2131962667E-001	8.9569062833E-008
$k = 10$	1.2132034327E-001	3.5900726836E-011
$ f(x_{10}) < \varepsilon$, 停止迭代		

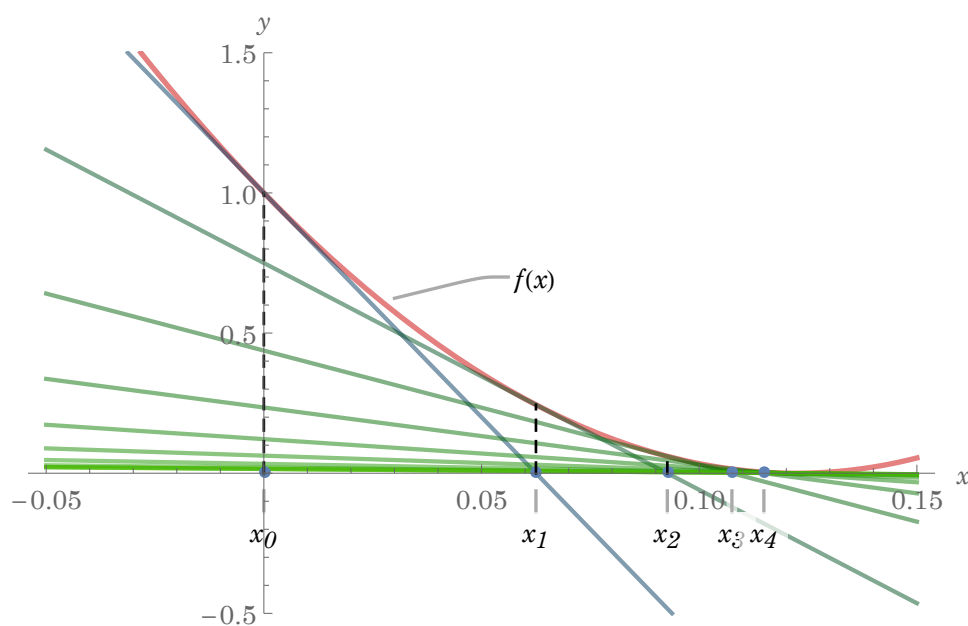


图 1: Newton 法迭代结果 1

迭代步数 k	x_k	$f(x_k)$
$k = 0$ (初值)	3.0000000000E+000	1.3120000000E+003
$k = 1$	1.9192751236E+000	3.9180729077E+002
$k = 2$	1.1936133228E+000	1.1368262566E+002
$k = 3$	7.3112145458E-001	3.1859876182E+001
$k = 4$	4.5362080609E-001	8.6190504416E+000
$k = 5$	2.9663693142E-001	2.2633471161E+000
$k = 6$	2.1197535112E-001	5.8197426653E-001
$k = 7$	1.6779403531E-001	1.4770709460E-001
$k = 8$	1.4519438879E-001	3.7206334228E-002
$k = 9$	1.3376760731E-001	9.3253679860E-003
$k = 10$	1.2803649704E-001	2.3222638207E-003
$k = 11$	1.2519603327E-001	5.6755417123E-004
$k = 12$	1.2383872242E-001	1.2927049695E-004
$k = 13$	1.2327102895E-001	2.2587102744E-005
$k = 14$	1.2311856261E-001	1.6284754711E-006
$k = 15$	1.2310571808E-001	1.1556329893E-008
$k = 16$	1.2310562562E-001	5.9885429948E-013
$ f(x_{16}) < \varepsilon$, 停止迭代		

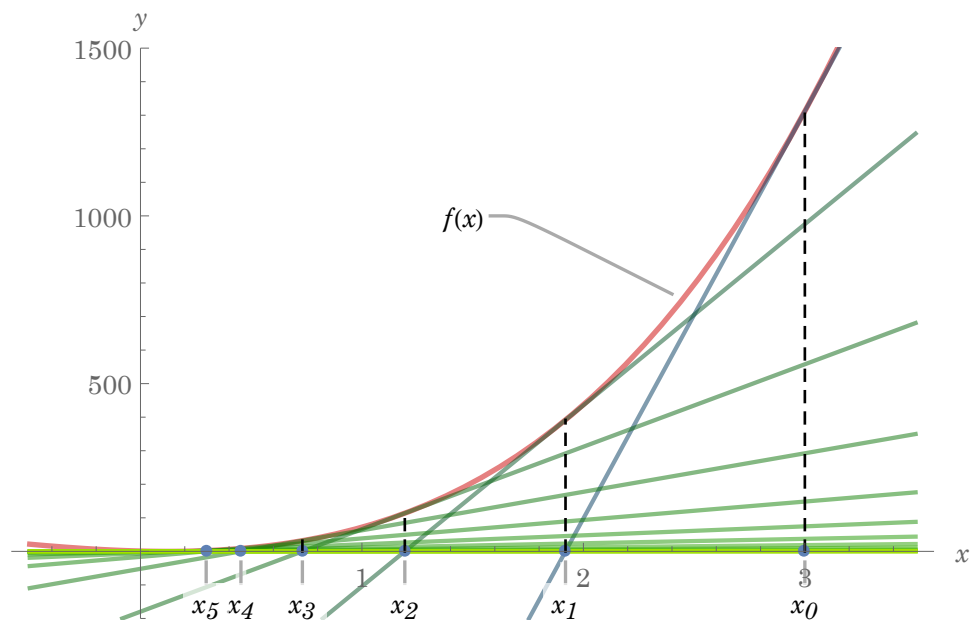


图 2: Newton 法迭代结果 2

迭代步数 k	x_k	$f(x_k)$
$k = 0$ (初值)	+0.0000000000E+000	1.0000000000E+000
$k = 1$ (初值)	+5.0000000000E-001	1.1375000000E+001
$k = 2$	-4.8192771084E-002	1.9100839450E+000
$k = 3$	-1.5882177241E-001	4.9849583298E+000
$k = 4$	+2.0528956326E-002	6.9745241532E-001
$k = 5$	+4.9704099508E-002	3.5839401507E-001
$k = 6$	+8.0543024888E-002	1.1965360731E-001
$k = 7$	+9.5999095782E-002	4.7582804260E-002
$k = 8$	+1.0620354980E-001	1.7777847602E-002
$k = 9$	+1.1229022963E-001	6.8102183582E-003
$k = 10$	+1.1606968076E-001	2.5795784215E-003
$k = 11$	+1.1837415259E-001	9.7415265691E-004
$k = 12$	+1.1977247782E-001	3.6072356011E-004
$k = 13$	+1.2059475518E-001	1.2741741436E-004
$k = 14$	+1.2104383231E-001	3.9878767168E-005
$k = 15$	+1.2124841215E-001	9.3453570276E-006
$k = 16$	+1.2131102788E-001	1.1695181286E-006
$k = 17$	+1.2131998479E-001	4.4816937383E-008
$k = 18$	+1.2132034170E-001	2.3240176450E-010
$ f(x_{18}) < \varepsilon$, 停止迭代		

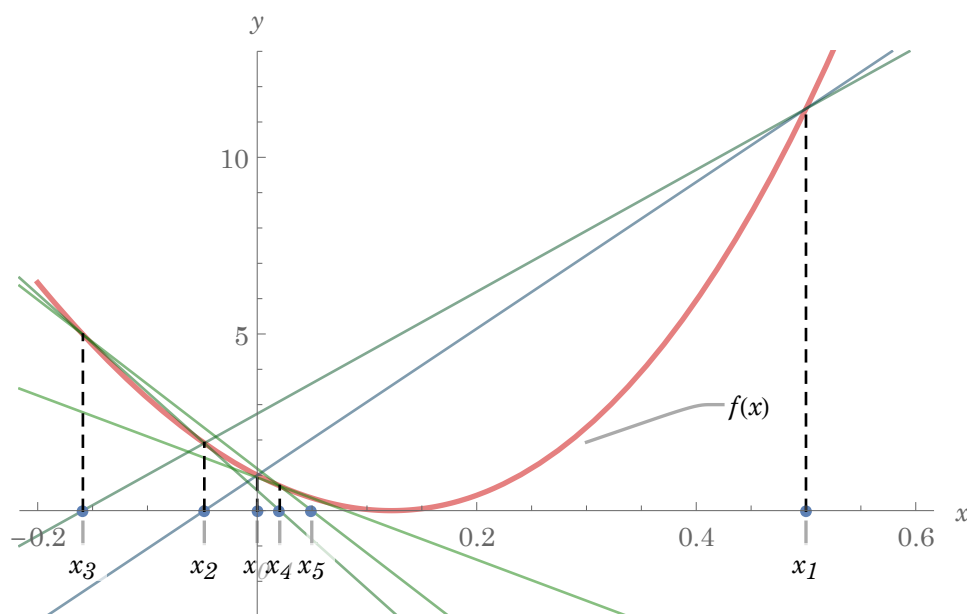
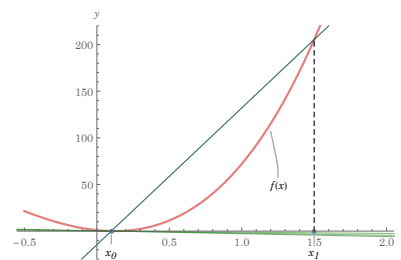
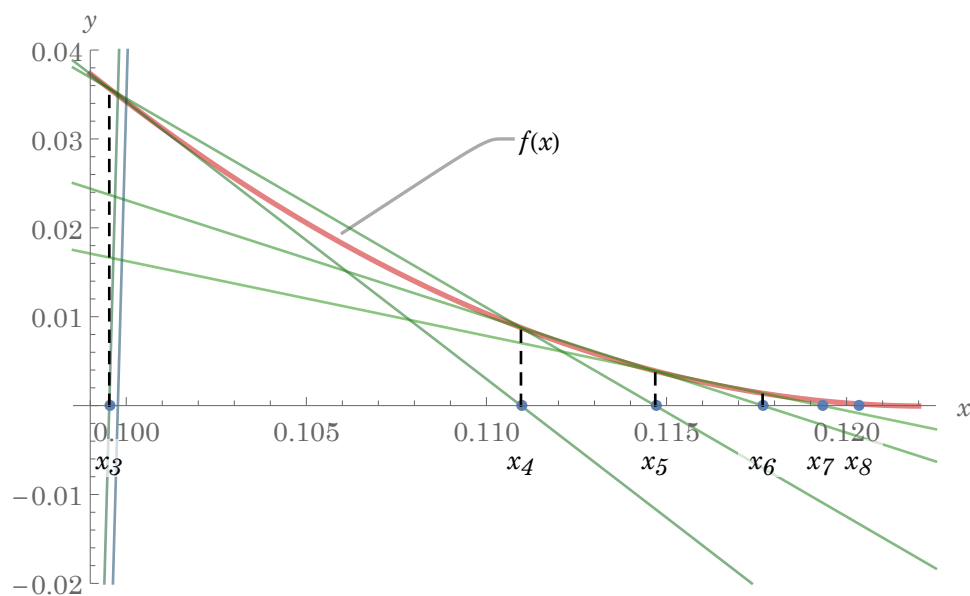


图 3: 弦截法迭代结果 1

迭代步数 k	x_k	$f(x_k)$
$k = 0$ (初值)	1.0000000000E-001	3.4200000000E-002
$k = 1$ (初值)	1.5000000000E+000	2.0537500000E+002
$k = 2$	9.9766826661E-002	3.4920022729E-002
$k = 3$	9.9528703766E-002	3.5662994682E-002
$k = 4$	1.1095871197E-001	8.7722830757E-003
$k = 5$	1.1468740718E-001	3.8969392942E-003
$k = 6$	1.1766781216E-001	1.3876451955E-003
$k = 7$	1.1931598272E-001	5.3099453171E-004
$k = 8$	1.2033760050E-001	1.9023231922E-004
$k = 9$	1.2090792407E-001	6.3397280115E-005
$k = 10$	1.2119299484E-001	1.7038550552E-005
$k = 11$	1.2129776891E-001	2.8550135170E-006
$k = 12$	1.2131885895E-001	1.8556909953E-007
$k = 13$	1.2132032505E-001	2.3119210990E-009
$k = 14$	1.2132034354E-001	1.9196866319E-012
$ f(x_{14}) < \varepsilon$, 停止迭代		



(a) 整体示意图



(b) 局部放大图

图 4: 弦截法迭代结果 2