计算方法 B

Homework #2

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Question 1

$f(x) = \sqrt{x}$ 在离散点有 $f(81) = 9,\ f(100) = 10,\ f(121) = 11$，用插值方法计算 $\sqrt{108}$ 的近似值，根据误差公式给出误差界。

二次插值函数为

$$
\begin{aligned}
L_2(x) &= \frac{(x-100)(x-121)}{(81-100)(81-121)} f(81) + \frac{(x-81)(x-121)}{(100-81)(100-121)} f(100) \\
&\quad + \frac{(x-81)(x-100)}{(121-81)(121-100)} f(121) \\
&= \frac{1}{7980}(-x^2 + 601x + 29700)
\end{aligned}
$$

计算函数在 $x = 108$ 处的近似值为

$$
\begin{aligned}
\sqrt{108} = f(108) &\approx L_2(108) \\
&= \frac{1}{7980}(-108^2 + 601 \times 108 + 29700) \\
&= \frac{6912}{665} \\
&\approx 10.394
\end{aligned}
$$

二次插值函数的误差为

$$
\begin{aligned}
R_2(108) &= \frac{f^{(3)}(\xi)}{3!}(108-81)(108-100)(108-121), && \xi \in [81, 121] \\
&= -\frac{351}{2}\xi^{-2.5}, && \xi \in [81, 121]
\end{aligned}
$$

$R_2(x)$ 在区间上是单调递增的，有

$$
-0.002972 \approx -\frac{13}{4374} \leqslant R_2(108) \leqslant -\frac{351}{322102} \approx -0.001090
$$

Question 2

利用下面的函数值表，做出差商表，写出相应的牛顿插值多项式，并计算 $f(1.5)$ 的近似值

x	1.0	2.0	3.0	4.0
$f(x)$	2.0	4.0	8.0	5.0

Question 2 continued on next page. . .

计算给定数据的一至三阶差商：

$$
\begin{aligned}
f[1,2] &= \frac{4.0-2.0}{2-1} = 2 \\
f[2,3] &= \frac{8.0-4.0}{3-2} = 4 \\
f[3,4] &= \frac{5.0-8.0}{4-3} = -3 \\
f[1,2,3] &= \frac{f[2,3]-f[1,2]}{3-1} = \frac{4-2}{3-1} = 1 \\
f[2,3,4] &= \frac{f[3,4]-f[2,3]}{4-2} = \frac{(-3)-4}{2} = -3.5 \\
f[1,2,3,4] &= \frac{f[2,3,4]-f[1,2,3]}{4-1} = \frac{(-3.5)-1}{3} = -1.5
\end{aligned}
$$

差商表为

i	x_i	$f(x_i)$	$f[x_{i-1}, x_i]$	$f[x_{i-2}, x_{i-1}, x_i]$	$f[x_{i-3}, x_{i-2}, x_{i-1}, x_i]$
0	1	2.0			
1	2	4.0	2		
2	3	8.0	4	1	
3	4	5.0	-3	-3.5	-1.5

相应的 Newton 插值多项式为

$$
\begin{aligned}
N_3(x) &= f(1) + (x-1)f[1,2] + (x-1)(x-2)f[1,2,3] + (x-1)(x-2)(x-3)f[1,2,3,4] \\
&= 2.0 + 2(x-1) + (x-1)(x-2) - 1.5(x-1)(x-2)(x-3) \\
&= -1.5x^3 + 10x^2 - 17.5x + 11
\end{aligned}
$$

计算近似值有

$$
\begin{aligned}
f(1.5) &\approx N_3(1.5) \\
&= -1.5 \times 1.5^3 + 10 \times 1.5^2 - 17.5 \times 1.5 + 11 \\
&= \frac{35}{16} \\
&= 2.188
\end{aligned}
$$

Question 3

利用数据 $f(0) = 2.0$, $f(1) = 0.5$, $f(3) = 0.25$, $f'(3) = 0.6$，构造出三次插值多项式，写出其插值余项，并计算 $f(2)$ 的近似值。

Question 3 continued on next page…

定义序列 $\{z_0 = 0,\, z_1 = 1,\, z_2 = 3, z_3 = 3\}$，用 Newton 插值构造 Hermite 插值多项式，差商表为

i	z_i	$f(z_i)$	$f[z_{i-1}, z_i]$	$f[z_{i-2}, z_{i-1}, z_i]$	$f[z_{i-3}, z_{i-2}, z_{i-1}, z_i]$
0	0	2.0			
1	1	0.5	-1.5		
2	3	0.25	-0.125	$\frac{11}{24}$	
3	3	0.25	0.6	$\frac{29}{80}$	$-\frac{23}{720}$

其中用 $f'(3) = 0.6$ 代替了 $f[z_2, z_3]$ 即 $f[3, 3]$。
得到三次插值多项式

$$\begin{aligned} H_3(x) &= f(0) + (x-0)f[0,1] + (x-0)(x-1)f[0,1,3] + (x-0)(x-1)(x-3)f[0,1,3,3] \\ &= 2.0 - 1.5x + \frac{11}{24}x(x-1) - \frac{23}{720}x(x-1)(x-3) \\ &= \frac{1}{720}(-23x^3 + 422x^2 - 1479x + 1440) \end{aligned}$$

计算近似值有

$$\begin{aligned} f(2) &\approx H_3(2) \\ &= \frac{1}{720}(-23 \times 2^3 + 422 \times 2^2 - 1479 \times 2 + 1440) \\ &= -\frac{7}{360} \\ &\approx -0.01944 \end{aligned}$$