Homework 4: Games Theory

MICROECONOMICS - ECON 401A

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1 Differentiated Bertrand

Two firms $i \in \{1, 2\}$ sell imperfectly substitutable goods. They compete in Bertrand fashion by posting prices $p_i \in [0, 1]$. When they set prices (p_1, p_2) the demand for firm 1's good is $q_1(p_1, p_2) = 1 - p_1 + p_2$ and the demand for firm 2's good is $q_2(p_2, p_1) = 1 - p_2 + p_1$. There are no costs of production, so the profit functions of the firms are

$$\pi_1(p_1, p_2) = p_1(1 - p_1 + p_2)$$
$$\pi_2(p_2, p_1) = p_2(1 - p_2 + p_1)$$

(a) Calculate the best-response functions $BR_1(p_2)$ and $BR_2(p_1)$ and draw them in (p_1, p_2)-space.

For $BR_1(p_2)$:

$$\begin{aligned}\pi_1(p_1, p_2) &= p_1(1 - p_1 + p_2) \\ \frac{d\pi_1}{dp_1} &= 1 - 2p_1 + p_2 = 0\end{aligned}$$

From the F.O.C. we get:

$$\begin{aligned}p_2^* &= 2p_1 - 1 \\ p_1^* &= \frac{1 + p_2}{2}\end{aligned}$$

In the same way, for $BR_2(p_1)$:

$$\begin{aligned}\pi_2(p_2, p_1) &= p_2(1 - p_2 + p_1)\\ \frac{d\pi_2}{dp_2} &= 1 - 2p_2 + p_1 = 0\end{aligned}$$

Therefore, from the F.O.C.:

$$\begin{aligned}p_1^* &= 2p_2 - 1\\ p_2^* &= \frac{1 + p_1}{2}\end{aligned}$$

So the best response functions are:

$$\begin{aligned}p_1^* &= \frac{1 + p_2}{2}\\ p_2^* &= \frac{1 + p_1}{2}\end{aligned}$$

(b) Calculate the Nash equilibrium prices (p_1^*, p_2^*).

The Nash equilibrium is the intersection of the best response functions, so:

$$\begin{aligned}p_1^* &= \frac{1 + p_2}{2}\\ p_2^* &= \frac{1 + p_1}{2}\end{aligned}$$

Substituting p_1^* into p_2^*:

$$\begin{aligned}p_2^* &= 2(2p_2 - 1) - 1\\ p_2^* &= 4p_2 - 3\\ p_2^* &= \frac{3}{3} = 1\end{aligned}$$

and substituting p_2^* into p_1^*:

$$\begin{aligned}p_1^* &= 2(1) - 1\\ p_1^* &= 1\end{aligned}$$

So the Nash equilibrium is $p_1^* = 1$ and $p_2^* = 1$.

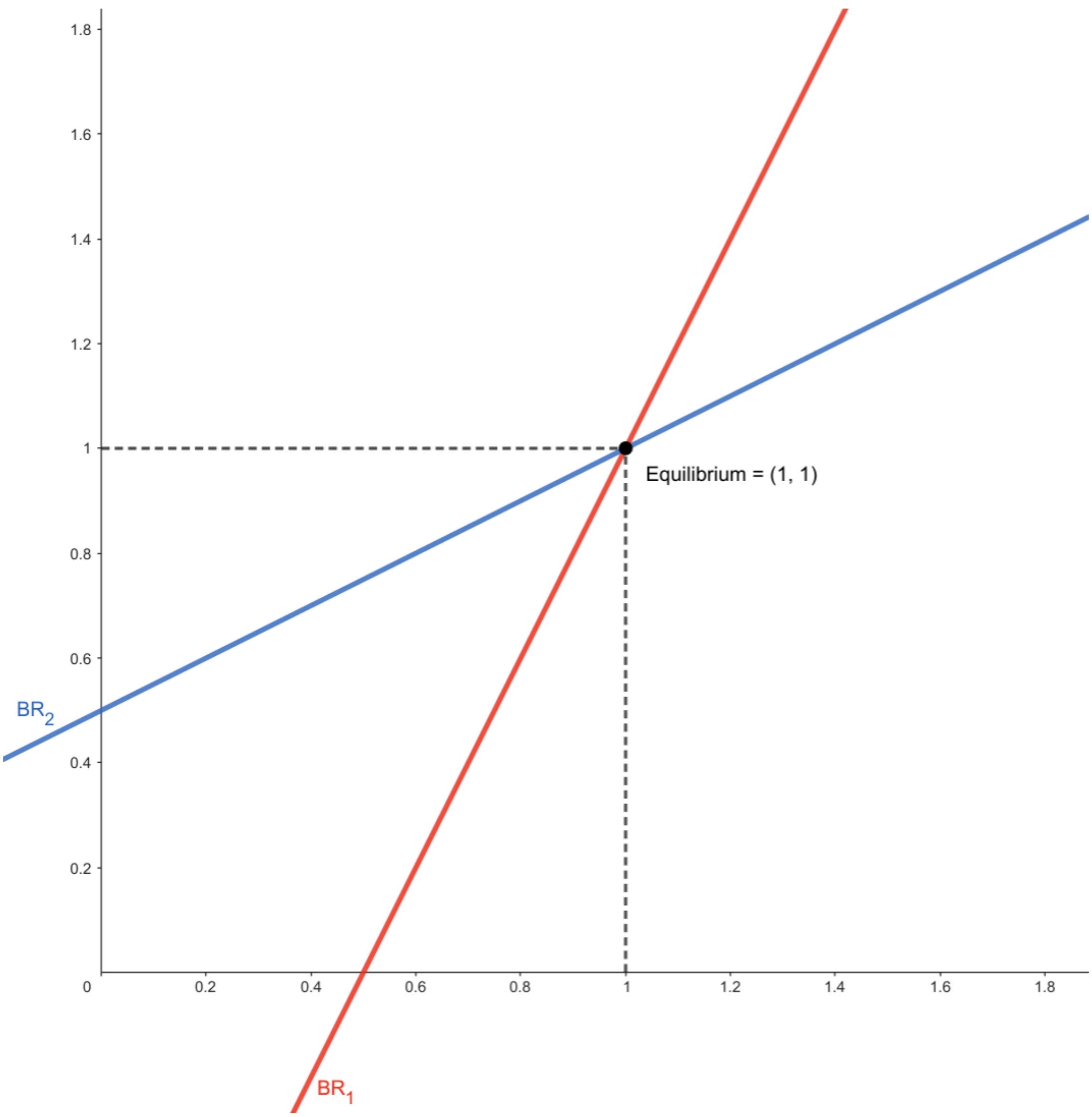

Figure 1: Best response functions

2 Sales

There are two firms $i \in \{1, 2\}$ and three customers $\{A, B, C\}$. The firms choose prices $\{p_1, p_2\}$ simultaneously. Customer A only wants to buy from firm 1 and has a value of v. Customer B only wants to buy from firm 2 and has a value of v. Customer C values both products at v and buys from the cheapest firm (and flips a coin if the prices are the same). There are no costs so, assuming $p_i \leq v$, firm i's profits are

$$\pi_i = \begin{cases} p_i & \text{if } p_i > p_j \\ \frac{3}{2}p_i & \text{if } p_i = p_j \\ 2p_i & \text{if } p_i < p_j \end{cases}$$

(a) Argue there is no pure strategy Nash equilibrium.

Let's assume that there exists a Nash equilibrium in pure strategies. In this case, the problem for each firm is to maximize its profit given the optimal strategy of the opposing firm:

$$\max_{p_i} \pi_i(p_i, p_j^*)$$

Where p_j^* is the optimal strategy of the opposing firm. In the case of firm 1, it maximizes its profit when:

$$p_1^* = p_2^* - \epsilon_1, \quad \epsilon_1 > 0$$

We need to add the ϵ_1 term to ensure that $p_1^* < p_2^*$, since otherwise the profit of firm 1 would be $3/2p_1^*$, and, therefore, firm 1 wouldn't be maximizing its profit.

Similarly, firm 2 maximizes its profit when:

$$p_2^* = p_1^* - \epsilon_2, \quad \epsilon_2 > 0$$

Substituting p_2^* into the equation for p_1^* and vice versa, it is obtained that:

$$p_1^* = p_1^* - \epsilon_2 - \epsilon_1 \implies \epsilon_1 + \epsilon_2 = 0$$

Which is a contradiction, since $\epsilon_1, \epsilon_2 > 0$ in order to maximize their profits. Therefore, there does not exist a Nash equilibrium in pure strategies.
Intuitively, both firms must choose a price lower than the other in order to maximize their profits. This iterative decision-making process will lead to a simulated price war where both firms will end up choosing a price of zero. However, in this scenario, both firms have incentives to raise their prices to $\bar{p}$ to maximize their profits, even though they are not earning the maximum of $2\bar{p}$. At this point, the iterative process will restart, leading to the same conclusions.

(b) We now derive the symmetric mixed strategy equilibrium. Suppose both firms choose random price with cdf $F(p)$ and support $[p, \bar{p}]$. Argue that $\bar{p} \leq v$. Write down firm 1's profit from price $p \in [p, \bar{p}]$.

The profit function for firm 1, given the optimal strategy of firm 2, is:

$$\pi(p) = p(1 - F(p)) + pF(p)$$

Because $F(p)$ represents the probability to choose a price lower that p, and $1-F(p)$ the probability to choose a price higher than p.

$$\begin{aligned}
\pi(p) &= p(1-F(p)) + 2pF(p) \\
\pi(p) &= p - pF(p) + 2pF(p) \\
\pi(p) &= p(1+F(p))
\end{aligned}$$

So the profit function for firm 1 (and 2) in terms of the cdf $F(p)$ is:

$$\pi(p) = p\,(1+F(p))$$

Consumers only buy if the price is lower than their valuation, so the maximum price a company can charge is v. If it charged more, consumers would not buy and the company would not make any profit. Therefore, $\bar{p} \leq v$.

(c) Argue that $\bar{p} = v$ in equilibrium.
(d) Derive the distribution of prices $F(p)$ in equilibrium. What is the support of prices $[\underline{p}, \bar{p}]$?

In equilibrium, both firms should be indifferent between any combination of prices, including the case where both firms choose the same price. Therefore, the expected profit is:

$$\begin{aligned}
\pi(p) = p\,(1+F(p)) &= \frac{3}{2}p \\
1 + F(p) &= \frac{3}{2} \\
F(p) &= \frac{1}{2}
\end{aligned}$$

This result states that the equilibrium price accumulates a mass of $\frac{1}{2}$ at p, a single point. This kind of distribution functions are from a family called Dirac delta functions, which are defined as:

$$\delta(x) = \begin{cases} \infty & \text{if } x = p \\ 0 & \text{otherwise} \end{cases}$$

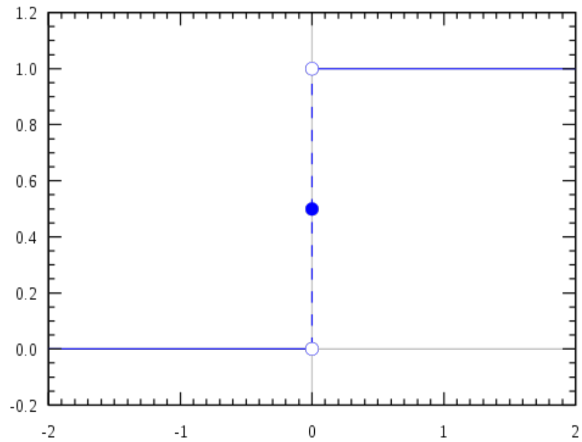

Figure 2: Dirac Delta CDF Centered in Zero

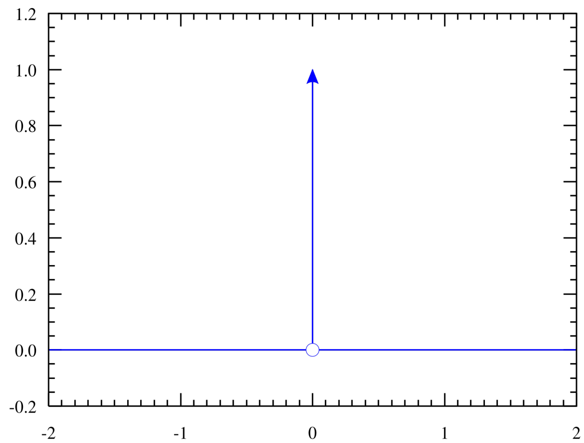

Figure 3: Dirac Delta PDF Centered in Zero

Answer (c), (d): As we argued, the pdf of the equilibrium price follows a Dirac delta function centered in the equilibrium price. the function is defined as:

$$f(x) = \begin{cases} \infty & \text{if } x = p \\ 0 & \text{otherwise} \end{cases}$$

As we explored in the previous questions, the equilibrium price should be lower or equal to the customer's valuation v in order to ensure at least one customer buys the product. Also, if the price is lower than v, firms have incentives to start a price war with no equilibrium. Therefore, the equilibrium price is v, and the support of the Dirac delta function is $[v, v]$, even though the function is defined for all real numbers.

3 Double Marginalization

A manufacturer M sells his product to consumers through a retailer R. First, the manufacturer sets a wholesale price $w \geq 0$. The retailer sees this and chooses the final price p $\geq$ w. The demand function is $Q(p) = 1 - p$. For simplicity, assume the manufacturer and retailer have zero cost. The profit functions are thus given by $\pi M = w(1-p)$ for the manufacturer and $\pi R = (p-w)(1-p)$ for the retailer.

(a) Given w what is the optimal price p of the retailer?

$$\begin{aligned}
\pi_R &= (p-w)(1-p) \\
\pi_R &= p - p^2 - w + wp \\
\frac{d\pi_R}{dp} &= 1 - 2p + w = 0 \\
\frac{1+w}{2} &= p
\end{aligned}$$

The optimal price of the retailer is $\frac{1+w}{2}$.

(b) Using backwards induction, what is the optimal wholesale price w of the manufacturer?

$$\begin{aligned}
\pi_M &= w(1-p) \\
\frac{1+w}{2} &= p \\
\pi_M &= w(1 - \frac{1+w}{2}) \\
\pi_M &= (w - \frac{w+w^2}{2}) \\
\pi_M &= w - \frac{w}{2} - \frac{w^2}{2} \\
\frac{d\pi_M}{dw} &= 1 - \frac{1}{2} - \frac{2w}{2} = 0 \\
w &= \frac{1}{2} = 0.5 \\
p &= 0.75
\end{aligned}$$

The optimal wholesale price of the manufacturer is 0.75.

(c) What are the firms' profits in equilibrium?

$$
\begin{aligned}
\pi_R &= (p-w)(1-p) \\
\pi_R &= (0.75-0.5)(1-0.75) \\
\pi_R &= (0.25)(0.25) \\
\pi_R &= 0.0625
\end{aligned}
$$

$$
\begin{aligned}
\pi_M &= w(1-p) \\
\pi_M &= 0.5(1-0.75) \\
\pi_M &= 0.5(0.25) \\
\pi_M &= 0.125
\end{aligned}
$$

The firms' profits in equilibrium are $\pi_R = 0.0625$ and $\pi_M = 0.125$.

(d) Now assume that manufacturer and retailer integrate vertically and charge a price p to maximize joint profits $\pi(p) = p(1-p)$. What is the optimal price p?

$$
\begin{aligned}
\pi(p) &= p(1-p) \\
\pi(p) &= p - p^2 \\
\frac{d\pi(p)}{dp} &= 1-2p = 0 \\
p &= \frac{1}{2} \\
\pi(p) &= 0.25
\end{aligned}
$$

The optimal price is 0.5 and the joint profit is 0.25.

(e) How do industry profits in the vertically integrated firm compare to the equilibrium in (c)? Explain the difference.

$$
\begin{aligned}
\pi_R + \pi_M = 0.0625 + 0.125 = 0.1875 \\
\pi(p) = 0.25
\end{aligned}
$$

The increase in total profit due to vertical integration (from 0.1875 to 0.25) exemplify the economic theory that suggests vertical integration can yield advantages by mitigating the inefficiencies linked to double marginalization, thereby improving the integrated entity's profitability.

4 Tax Competition

Each US state independently chooses its own taxes, seeking to attract firms and workers from other states. For example, Kansas famously slashed its income tax rates in 2012; the Governor claimed this would ”bring businesses from across the nation to the Midwest” and that he would ”keep pruning state government any place that we can” in order to balance the budget. Is such tax competition good for the US population as a whole (in the same way competition across firms is good for welfare) or do states impose negative externalities on one another?

Tax competition allows state governments to compete by offering incentives to investors or controlling internal markets. Certain businesses and individuals would face obstacles as a result of this competition, but others would be encouraged to enter the market. These kinds of regulations allow governments to control their own markets to suit their own requirements.

The following is a comparison of the advantages and disadvantages of tax competition and its possible effects for the US population:

Advantages:

- As a result of the tax structures offered, consumers will have the freedom to choose between various states, thus increasing their utilities according to their initial preferences.
- The effectiveness and innovation of policies aimed at increasing incentives for investment in different nations would be increased by establishing a competitive market.
- States would have to maintain high fiscal discipline, evident in the management of finances before society, due to concern about losing business.

Disadvantages:

- The uneven distribution of resources and wealth among the states as a result of each state's failure to provide investors with the same incentives.
- States will likely keep competing with one another to provide bigger incentives, which can result in lower taxes and less money for public services.Reduced financing for basic public services may have long-term consequences for growth, impacting infrastructure, health, safety, and education standards, as well as perhaps driving migration to other states.

In conclusion, permitting tax rivalry across states may spur innovation and growth, but such competition needs to be carefully managed to prevent unfavorable effects. It is imperative that policymakers compete to satisfy and safeguard the interests and well-being of societies while also being cognizant of the demands of individual states.

5 Ransoms

Under Italian law, families are barred from paying ransom to kidnappers. Why does this law exist? Is it a good law?

The Italian law referred to is known as the ”Anti-Kidnapping Law.” It was implemented in response to a rising wave of kidnappings, particularly noticeable during the 1970s and 1980s, when Italy faced serious kidnapping problems perpetrated by both common criminals and terrorist groups like the Red Brigades. The main aim of this law was to minimize incentives for committing kidnappings by eliminating the possibility of families paying ransoms. Additionally, since the law also demands cooperation with authorities, it presumably increases the risk of capture for kidnappers.

In a context without this law, families faced a situation where the cost of not cooperating with the kidnappers was the life of their relative, while the cost of cooperating was paying a ransom. Kidnappers were aware of this dilemma, so to coerce the families' decision, they made credible threats, thereby creating a balance skewed in favor of cooperating with the kidnappers.

With the Italian law, the government aims to increase the cost of cooperating with kidnappers to a point where the threat's credibility might be insufficient for families to decide to pay the ransom. In this scenario, the outcome depends on the family's fear of the law and the credibility and brutality of the kidnappers.

In cases where kidnappers are not credible, the law might be effective as families would incur a high cost if they collaborate rather than reporting to authorities. Such cases could involve kidnappings of middle or upper-class families, where the kidnapping is random, and its effectiveness depends on the speed of ransom payment.

In scenarios where kidnappers are credible, the law might not be effective, as families are aware of the consequences of non-cooperation. This could be the case for families with some kind of relationship with the kidnappers, where the purpose of the kidnapping is not solely to extract money but also to obtain information or enact vengeance.

Another condition necessary for the effectiveness of this law is the authorities' ability to recover kidnapped relatives unharmed. If their capability is low, the cost of not cooperating with kidnappers remains high, rendering the law ineffective.