Mathematical Foundation of Data Science

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Problem: Crossing the Desert

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Overview

- ***In Question One,*** the weather conditions are known throughout the journey. The desert map is abstracted and simplified, and the key path is calculated by the shortest circuit algorithm to obtain a directional map. On this basis, a dynamic planning model is constructed, and the optimal strategy in a given initial state is solved by using the memory search implementation algorithm. Through the multi-search algorithm, all the initial states are searched to obtain the global optimal strategy of the specified level. In the optimal strategy of the first and second levels, the final remaining funds are 10470 yuan and 12730 yuan.
- ***In question two***, because of the uncertainty of the environment, the cumulative effect of uncertainty will affect the final decision, so we have established a simplified model based on key nodes, and thus established the user's decision path. At the same time, in order to ensure the player's earnings and best deal with uncertain environment, players should adopt some specific strategies, such as because the next day can still be hot, so in hot weather should also walk as usual. Finally, the planning model can be established with the remaining funds as the target function, time and resources as the constraints, and the connection of parameters can be established, so as to analyze the basic analysis strategies that the user should adopt, such as going to the end point when only a certain amount of resources or time remains, analyzing the relationship between revenue and cost to decide whether ore should be mined, etc.
- ***In question three***, there is a multi-player game. In view of making all the decisions at the starting point, a complete information static game model is established. After simplifying the map, the decision set is calculated using the dynamic planning model, the winning matrix is obtained, and 2 Nash equilibriums are obtained. Then we establish a linear planning model and find out the hybrid strategy. For each step to make a decision and the weather is unknown.

Resource allocation problem

- Problem Introduction
  - The first and second levels are similar. Both players make decisions when the weather is certain. There is an optimal strategy. After simplifying the model, we can establish a state transition equation and solve the optimal strategy through dynamic programming.

Solution ideas

(4) State transfer function

At each stage, players may change resources and positions, resulting in state changes. The state transfer function is shown in formula (3)-(5).

$$w_{left} = \begin{cases} w_{left} - n * w_{sun} & s_k \in \text{sunny} \\ w_{left} - n * w_{hot} & s_k \in \text{High temperature} \\ w_{left} - w_{dust} & s_k \in \text{sandstorm} \end{cases} \qquad \text{.............(3)}$$

$$f_{left} = \begin{cases} f_{left} - n * f_{sun} & s_k \in \text{sunny} \\ f_{left} - n * f_{hot} & s_k \in \text{High temperature} \\ f_{left} - f_{dust} & s_k \in \text{sandstorm} \end{cases} \qquad \text{.............(4)}$$

$$s_{k+1} = g(point_{k+1}, w_{left}, f_{left}) \qquad \text{.............(5)}$$

In (3)-(5),

$$n = \begin{cases} 3 & s_k \in \text{mining} \\ 2 & s_k \in \text{move} \\ 1 & s_k \in \text{stay} \cap \text{No mining} \end{cases} \qquad \text{...............(6)}$$

Solution ideas

(5) Stage profit and loss function

The goal of the game is to retain as much money as possible when the end point is reached, and to use the change of funds as a stage profit and loss function.

$$cost(s_k,x_k) = -p * w_{cost} * w_{buy} - p * f_{cost} * f_{buy} + q * cap_{basic} \quad \text{..............(7)}$$

$$p = \begin{cases} 2 & s_k \in \text{Supply \& k not equal 0} \\ 1 & k = 0 \\ 0 & else \end{cases} \quad \text{..........(8)}$$

Based on the above instructions, the dynamic planning model can be summarized as:

$$\begin{cases} cost_{total}(s_0) = m_{init} - m_{left}(s_0) \\ cost_{total}(s_{k+1}) = cost(s_k, x_k) + cost_{total}(s_k) \quad k = 0, 1, 2, ..., d_{total} - 1 \end{cases} \quad \text{....(9)(10)}$$

Where $x_k \in D_k(s_k)$, $m_{left}(s_0)$ is the remaining funds after day 0 replenishment.

Traversing all the states at the end of the last phase, with the least total cost (the most money remaining), is the optimal strategy in that initial state. In the model solution, all initial states of a specific level are considered to obtain the global optimal strategy for that level.

Optimal Decision problem

- Problem Introduction
  - The third and fourth hurdles are all uncertain decision-making problems. Because all the weather is uncertain, the decision-making problems are more complicated. We need to simplify the decision-making path, analyze the benefits of choosing different strategies in different weather, and finally make a decision.

Solving Process

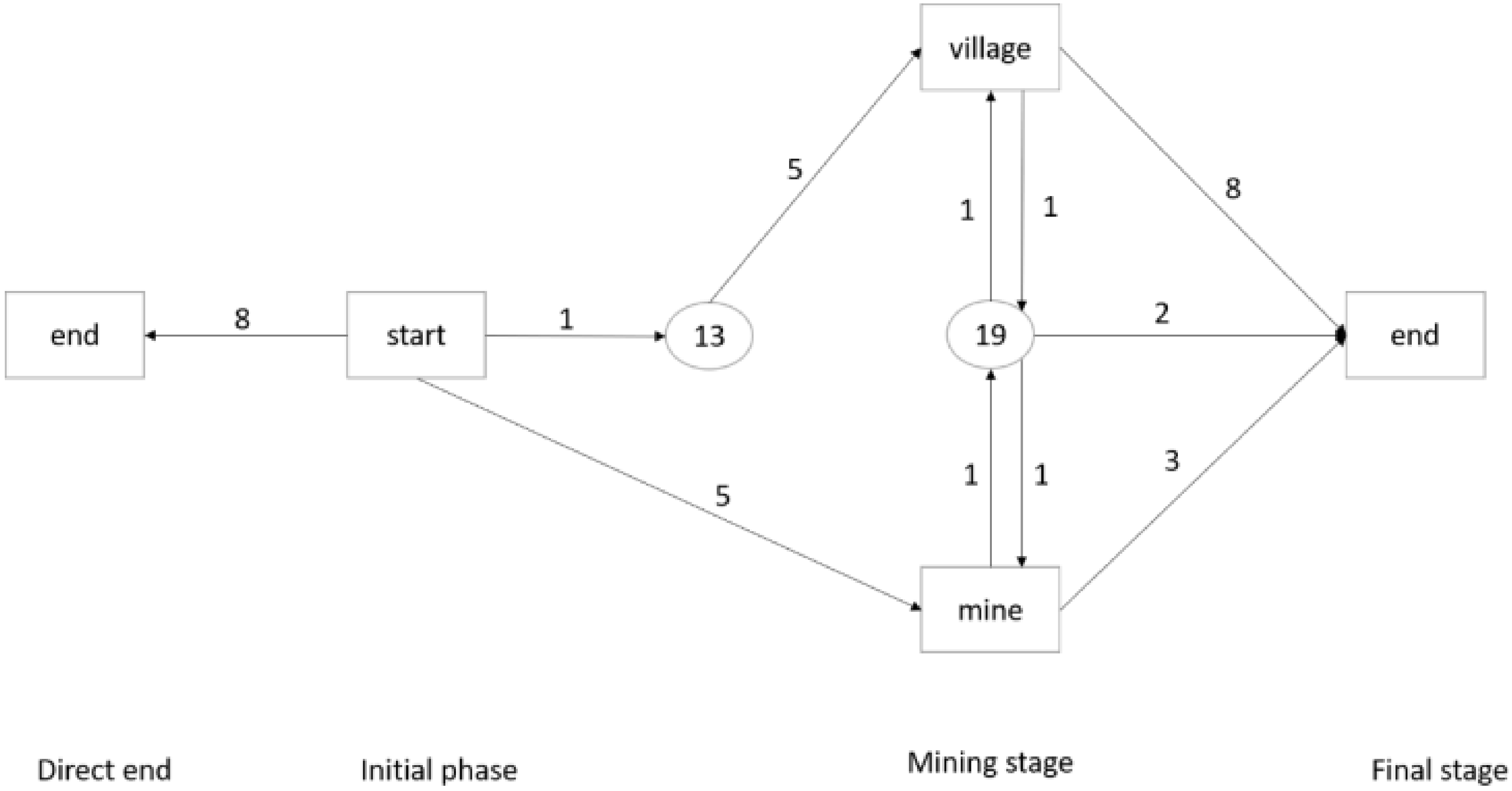

Figure 6 Level 4 - Decision path

Solving Process

$$\max m_{left} = m_{init} - cost_{total} + cap_{total}, \quad (11)$$

$$\text{s.t. } cost_{total} = (w_{buy} + w_{village} * 2) * w_{cost} + (f_{buy} + f_{village} * 2) * f_{cost}, \quad (12)$$

$$cap_{total} = d_{mine} * cap_{basic}, \quad (13)$$

$$d_{total} \geq dis + detour + d_{dust} + d_{mine}, \quad (14)$$

$$w_{buy} + w_{village} \geq w_{hot} * (d_{mine} * 3 + dis_{cur} * 2 + detour * 2) + w_{dust} * d_{dust}, \quad (15)$$

$$f_{buy} + f_{village} \geq f_{hot} * (d_{mine} * 3 + dis_{cur} * 2 + detour * 2) + f_{dust} * d_{dust} \quad (16)$$

Solving Process

4 The best strategy for the fourth level

Initial purchase	720kg of water and 480kg of food, or in proportion to consumption in clear weather
Basic strategy	Go directly to the mine, keep moving forward in addition to sandstorms, and choose to mine no matter what the weather is like when you arrive at the mine
End condition	If the remaining water during mining will be less than 192kg or less than 128kg, choose to follow the shortest path to the end.

Dynamic game Problem

- Problem Introduction
  - The fifth and sixth levels are all about multi-player participation. The weather in the fifth level is determined, and the weather in the sixth level is uncertain. The complexity of the problem lies in the fact that there is no communication between players and there is a game relationship. If the decisions are the same, the comprehensive income will be low. Considering that in the actual game process, the players are all risky, so we can treat the players The risk propensity of the system is defined, and then the Markov state transition process and Monte Carlo simulation are used to simulate, so as to obtain the final analysis.

Solving Process

(3) The probability of state transfer

In order to depict the player's process of considering the current state and making a decision, i.e. the probability of making an action in the state of sk, the probability of risk acceptance is introduced as the probability of state transfer

$$T(S, a, S') \sim P_r(s' \mid s, a) \quad \text{..........................(21)}$$

When multiple players' optimal paths conflict, if a player's optimal path is also the only viable path, he will have no choice. In addition, the probability of state transition varies for players with different risk appetites, as shown in Table 10.

Table 10 Differences in the probability of state transfer for different risk appetites

Risk appetite	Impact on the probability of state transition
Risk appetite	The player moves to the conflict point with a probability of 66.6% – 100%
Risk neutral	Player moves to the point of conflict with a probability of 33.3% to 66.6%.
Risk aversion	players move to conflict points with a 0% probability of 33.3%.

Solving Process

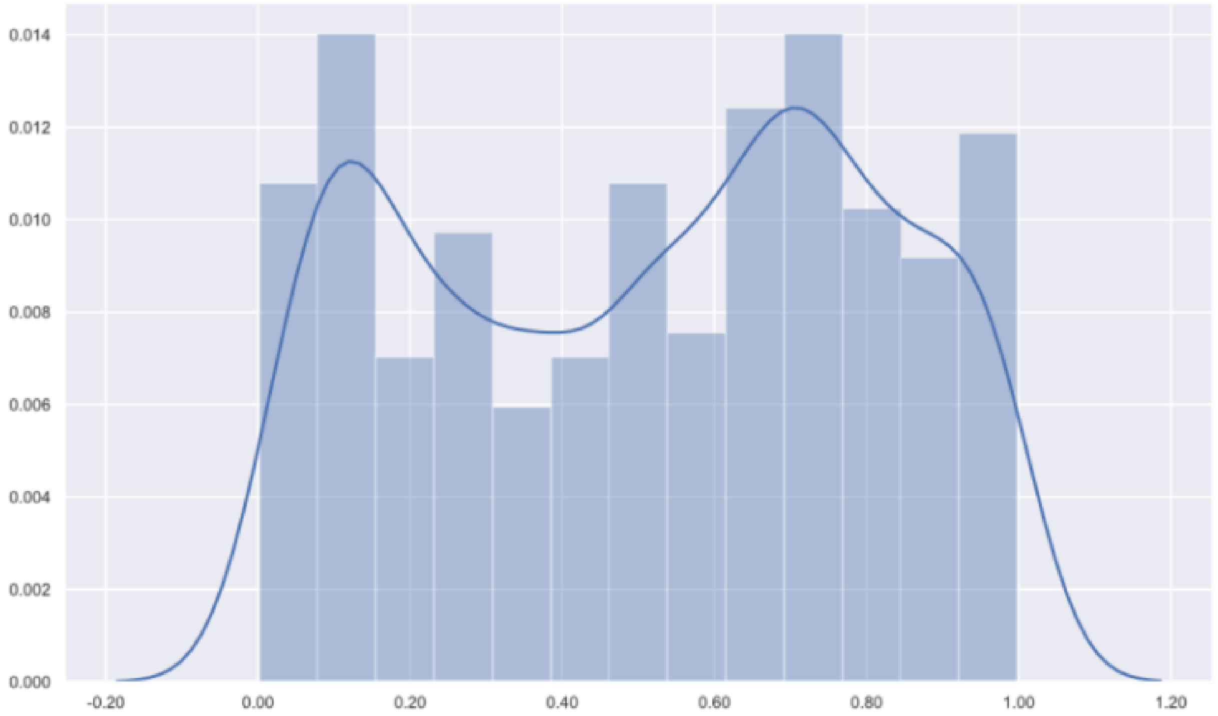

Figure 14 An estimate of the degree of risk acceptance for all failed gamers