# Mathematical Foundation of Data Science

**Teacher: 赵明雄**

## Problem: Crossing the Desert


| Student Name: | Student ID: |
|---|---|
| MD MAHFUZUR RAHMAN （毒液） | 20183290375 |
| SAMEER ZULKAR SYED(陈也军) | 20183290716 |
| MD APEL MAHMUD(马瑞) | 20183290541 |
| MD HASAN ALI SHEIKH(哈桑) | 20183290502 |
| MD ABDUL WADUD PRINCE(吴磊) | 20183290404 |
| AL HASIB(阿布) | 20183290253 |

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- The best strategy under known conditions of the whole weather conditions
- The weather is unknown in general the best strategy
- Multi-player game under the conditions of the strategy
- Model evaluation and promotion
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# Overview

- ***In Question One,*** the weather conditions are known throughout the journey. The desert map is abstracted and simplified, and the key path is calculated by the shortest circuit algorithm to obtain a directional map. On this basis, a dynamic planning model is constructed, and the optimal strategy in a given initial state is solved by using the memory search implementation algorithm. Through the multi-search algorithm, all the initial states are searched to obtain the global optimal strategy of the specified level. In the optimal strategy of the first and second levels, the final remaining funds are 10470 yuan and 12730 yuan.
- ***In question two***, because of the uncertainty of the environment, the cumulative effect of uncertainty will affect the final decision, so we have established a simplified model based on key nodes, and thus established the user's decision path. At the same time, in order to ensure the player's earnings and best deal with uncertain environment, players should adopt some specific strategies, such as because the next day can still be hot, so in hot weather should also walk as usual. Finally, the planning model can be established with the remaining funds as the target function, time and resources as the constraints, and the connection of parameters can be established, so as to analyze the basic analysis strategies that the user should adopt, such as going to the end point when only a certain amount of resources or time remains, analyzing the relationship between revenue and cost to decide whether ore should be mined, etc.
- ***In question three***, there is a multi-player game. In view of making all the decisions at the starting point, a complete information static game model is established. After simplifying the map, the decision set is calculated using the dynamic planning model, the winning matrix is obtained, and 2 Nash equilibriums are obtained. Then we establish a linear planning model and find out the hybrid strategy. For each step to make a decision and the weather is unknown.

# Overview-Question background

- In the Desert Crossing game, players use a map to buy a certain amount of water and food with initial funds and walk through the desert from the starting point. Different weather conditions can be encountered along the way, and additional funds or resources can be restock in mines and villages, with the goal of reaching the end within a specified time frame and retaining as much money as possible.

- Based on the key nodes that the player may make decisions, we will discusses
  - the decision-making process of dynamic planning,
  - the stage decision path in uncertain state,
  - the game under multi-player conditions,
  - and gradually solves the problems in this background after simplifying the map through the network model.

# Overview-Target task

- ***Question 1:*** There must be a global optimization strategy for a given map when the weather conditions are known throughout the course and only one player is available.

- ***Question 2:*** How to make the optimal decision in the uncertain state of the environment, so as to maximize profits.

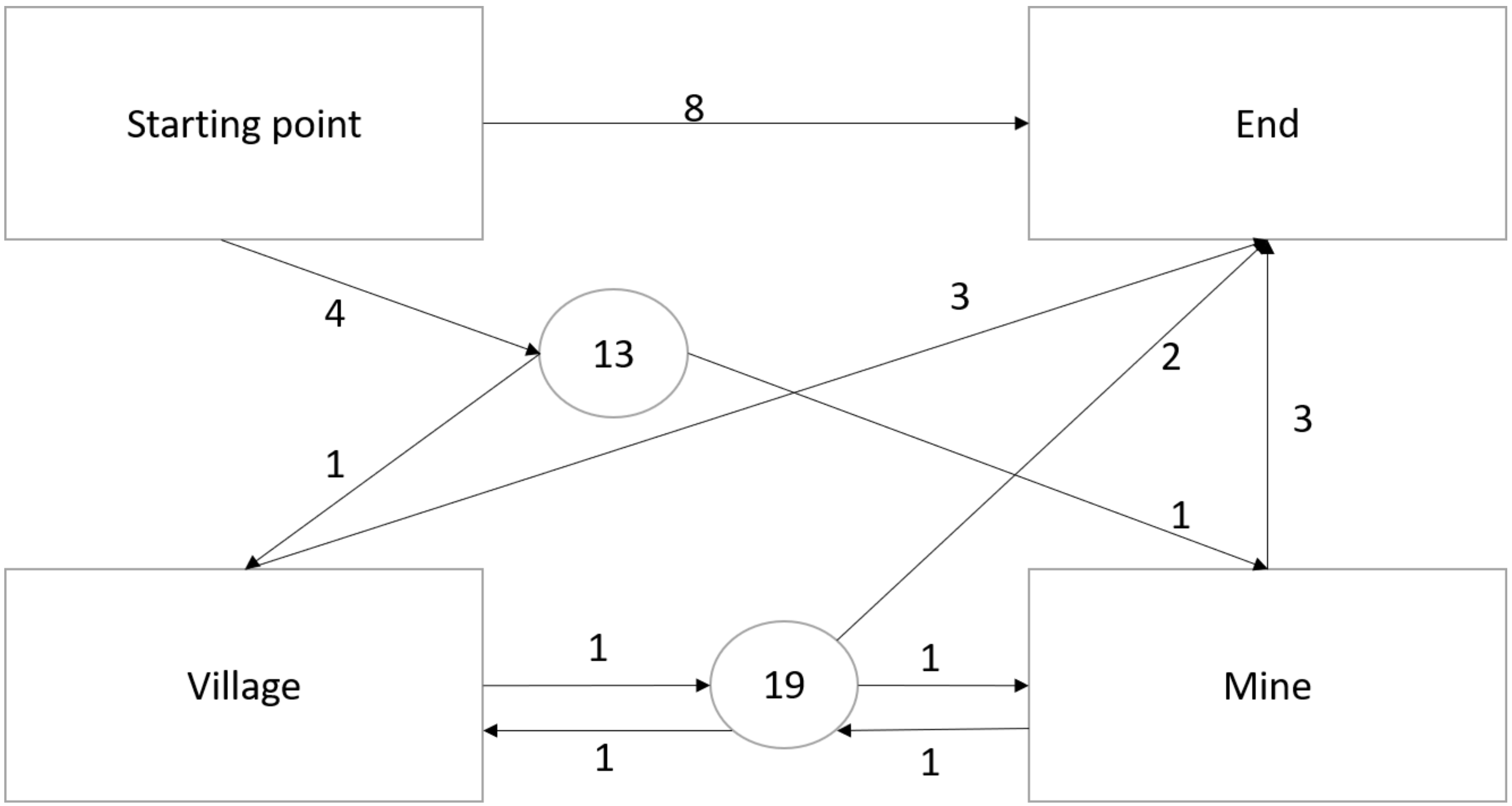
- ***Question 3:*** How to make a strategy in advance or make the best strategy based on the environment when multiple people are involved and decisions are influenced by each other.

# Symbol Description and model foundation

*Table 1 symbols and meaning*

| Symbol | Meaning | Symbol | Meaning | Symbol | Meaning |
|---|---|---|---|---|---|
| $cost$ | spend | $m_{init}$ | Initial capital | $w_{cost}$ | Benchmark price of water |
| $Cap_{basic}$ | Basic income | $c_{total}$ | Total cost | $f_{cost}$ | Base price of food |
| $w_{sun}$ | Basic water | $w_{hot}$ | High temperature basic water consumption | $w_{dust}$ | Basic water consumption of sandstorm |
| $f_{sun}$ | Consumption on sunny days | $f_{hot}$ | High temperature basic food consumption | $f_{dust}$ | Basic food consumption in sandstorms |
| $m_{left}$ | Clear basic food consumption | $f_{left}$ | Surplus food | $w_{left}$ | Remaining water |
| $f_{buy}$ | Remaining money | $w_{buy}$ | Purchased water | $d_{total}$ | Total days |
| $d_{cur}$ | Food purchased | $Point$ | Player position | $K$ | Number of stages |
| $S$ | Current day State | $X$ | Decision making | $D$ | Decision set |
| $d_{min}$ | Detour the mine to the end | $cap_{total}$ | All benefits | $cost_{total}$ | All spent |
| $c_{sun}$ | Clear basic consumption | $c_{hot}$ | Hot basic consumption | $c_{dust}$ | Basic consumption of sand and dust |
| | | | | | |

# The network model is simplified



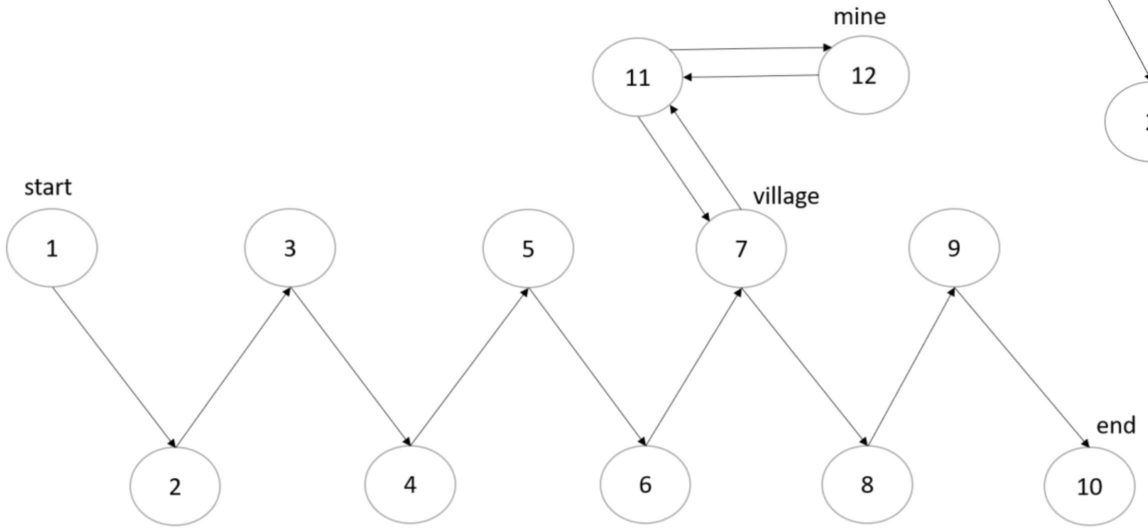

The network model simplifies the indication

# The best strategy under known conditions of the whole weather conditions

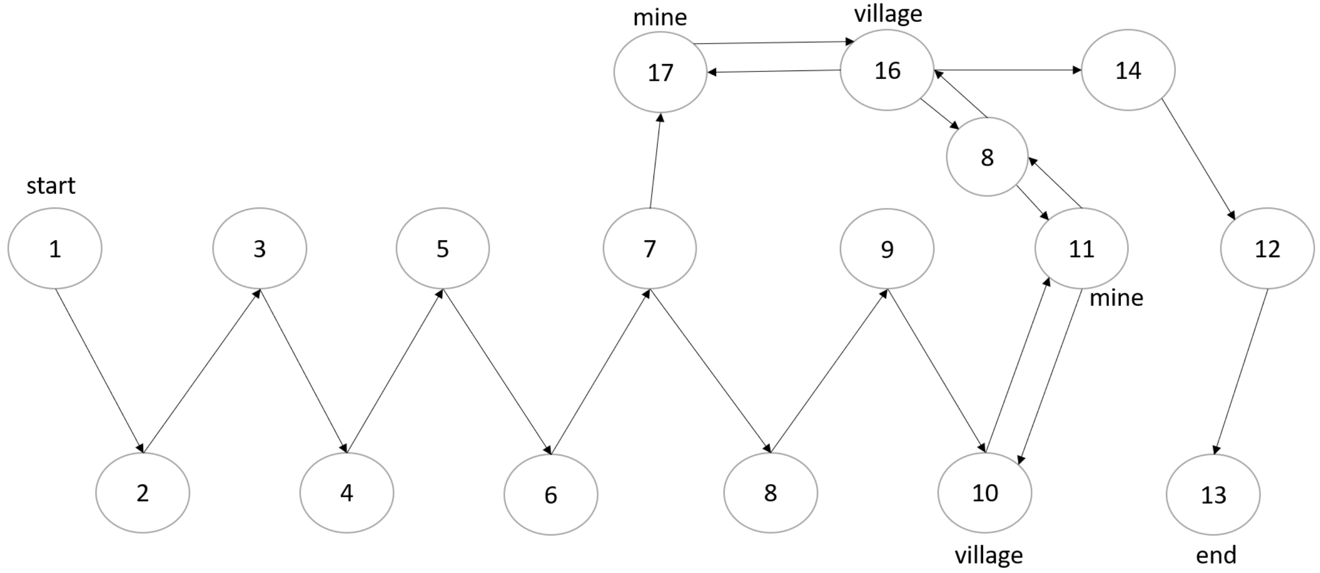
- Analysis
  - In this question, the weather conditions are known in advance for each day throughout the game window, and the game has a limited time limit, i.e. the game ends in a limited step.

# Model Building

**Vector Model**




First Level




Second Level

# Dynamic planning model

- Stage variables

k = dcur ……………………………………(1)

(2) The state variable

sk = g(point,wleft,fleft) …………………………………(2)

(3) Behavioral decision-making

*Table 2 A set of decisions made when a player is in a mine in the first pass*

| Decision number | Weather | Path | Weather to mine | Decision number | Weather | Path | Weather to mine |
|---|---|---|---|---|---|---|---|
| $A_1$ | Sunny | Move to point 11 | No | E | High temperature | Stay | Yes |
| B | Sunny | Stay | Yes | F | High temperature | Stay | No |
| C | Sunny | Stay | No | G | Sandstorm | Stay | Yes |
| $D_1$ | High temperature | Move to point 11 | No | H | Sandstorm | Stay | No |

# State transfer function

$$w_{left} = \begin{cases} w_{left} - n * w_{sun} & s_k \in \text{sunny} \\ w_{left} - n * w_{hot} & s_k \in \text{High temperature} \\ w_{left} - w_{dust} & s_k \in \text{sandstorm} \end{cases} \quad \text{.............(3)}$$

$$f_{left} = \begin{cases} f_{left} - n * f_{sun} & s_k \in \text{sunny} \\ f_{left} - n * f_{hot} & s_k \in \text{High temperature} \\ f_{left} - f_{dust} & s_k \in \text{sandstorm} \end{cases} \quad \text{.............(4)}$$

$$s_{k+1} = g(point_{k+1}, w_{left}, f_{left}) \quad \text{.............(5)}$$

In (3)-(5),

$$n = \begin{cases} 3 & s_k \in \text{mining} \\ 2 & s_k \in \text{move} \\ 1 & s_k \in \text{stay} \cap \text{No mining} \end{cases} \quad \text{...............(6)}$$

# Stage profit and loss function

$$cost(s_k,x_k) = -p * w_{cost} * w_{buy} - p * f_{cost} * f_{buy} + q * cap_{basic} \quad \text{................(7)}$$

$$p = \begin{cases} 2 & s_k \in \text{Supply \& k not equal 0} \\ 1 & k = 0 \\ 0 & else \end{cases} \quad \text{.............(8)}$$

Based on the above instructions, the dynamic planning model can be summarized as:

$$\begin{cases} cost_{total}(s_0) = m_{init} - m_{left}(s_0) \\ cost_{total}(s_{k+1}) = cost(s_k, x_k) + cost_{total}(s_k) \quad k = 0, 1, 2, ..., d_{total} - 1 \end{cases} \quad \text{....(9)(10)}$$

# Model Solution

- Algorithm

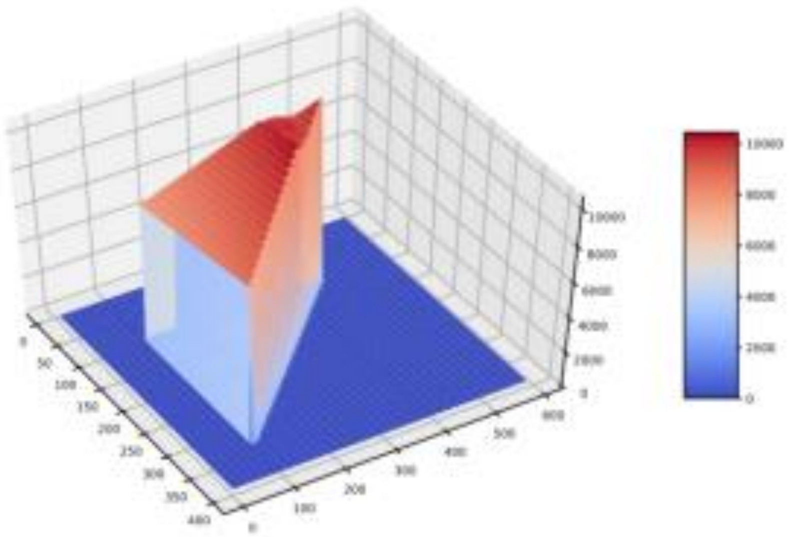


*Figure 3 Level 1: The final remaining funds for optimal strategies under different initial replenishment conditions*

# The Weather is Unknown

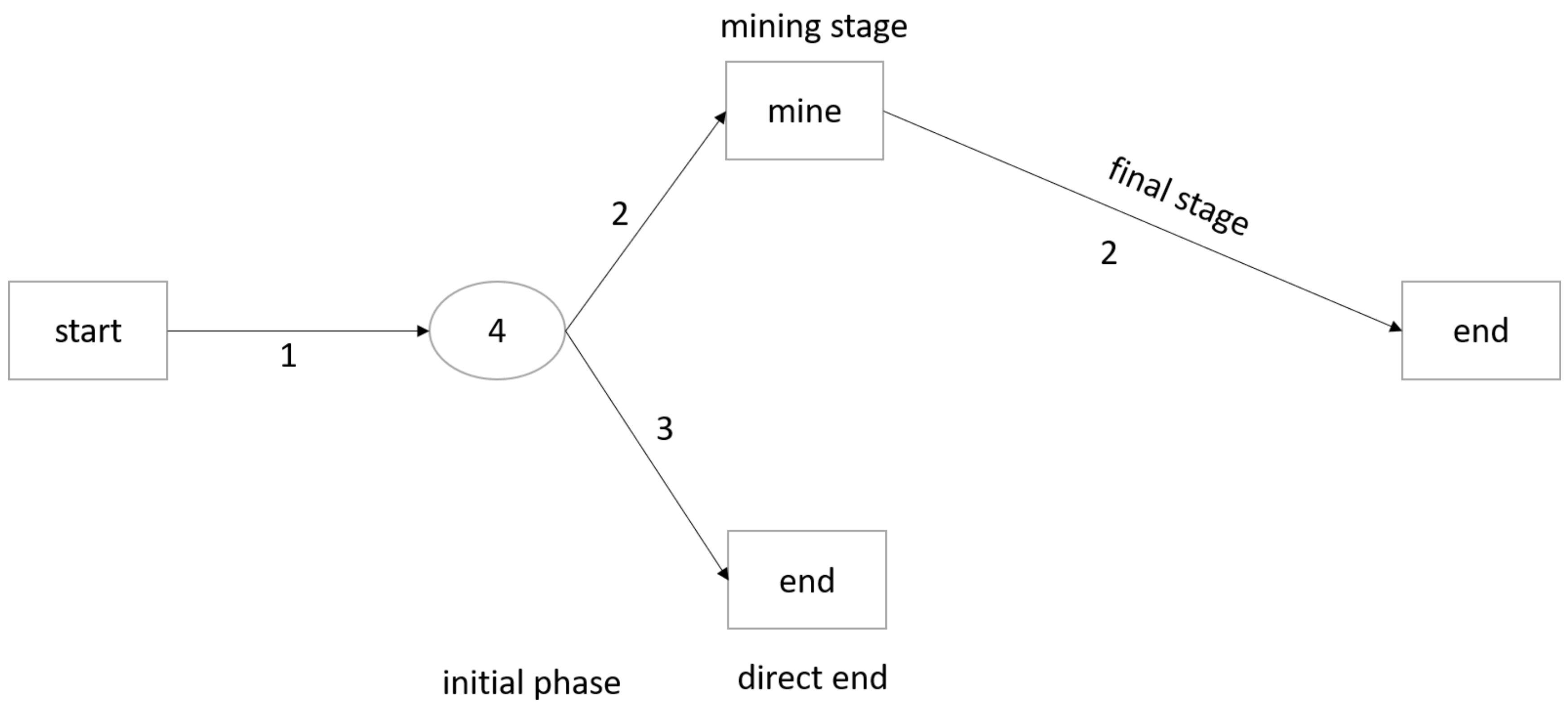
- Problem Analysis
- Decision Model
- Influencing Factors
- The Decision Path Analysis
- Solution Model (Level 3)
- Solution Model (Level 4)

# Problem Analysis

- Must reach the End Point within a specified time
- Must not run out of Food or Water
- The probability of weather is important decision making factor.

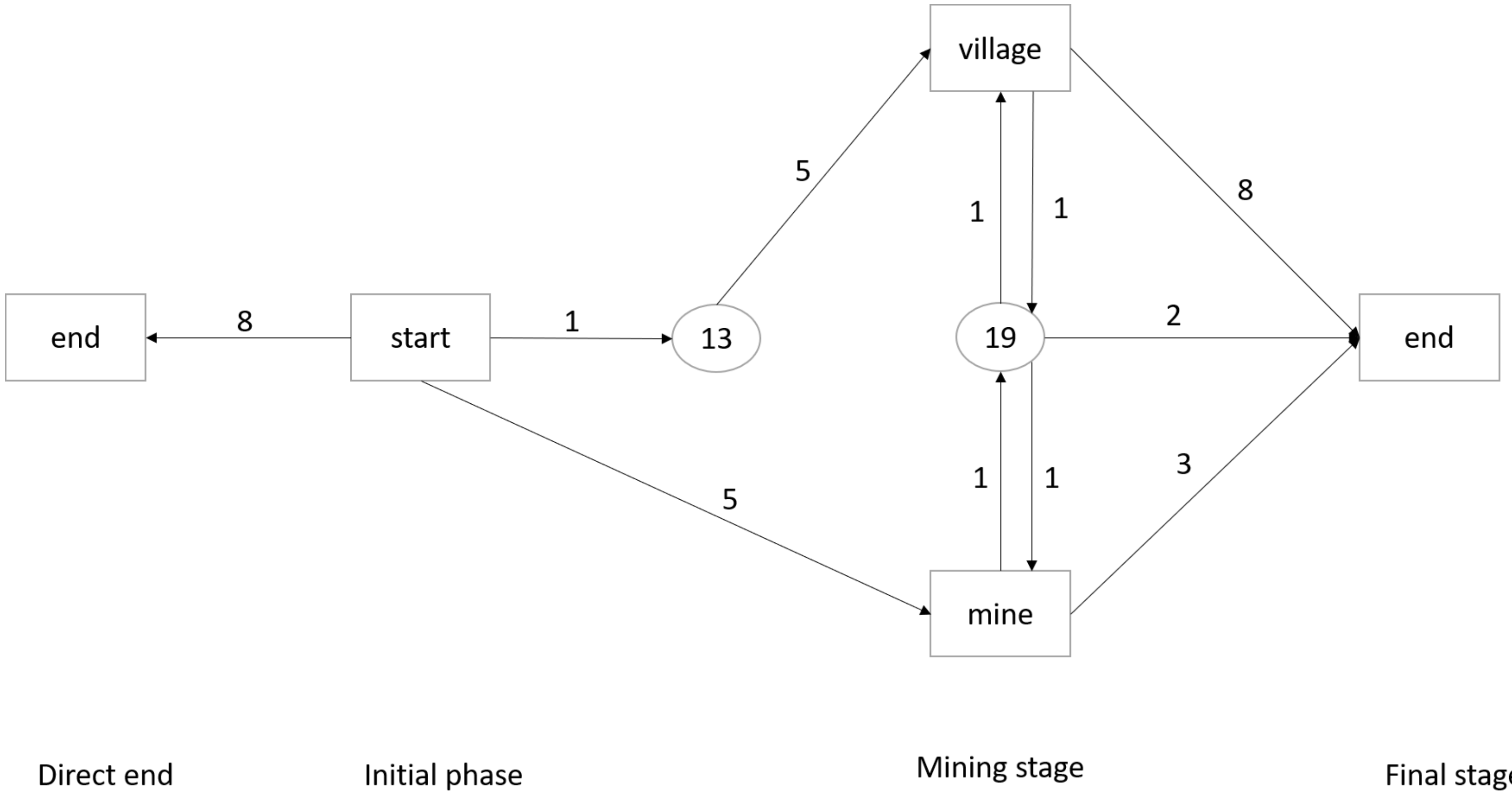
# Decision Modeling

Simplified Decision Path (Level 3):



# Decision Path Analysis

Decision Path (Level 4):



# Influencing Factors.

- Village Presence
- High Temperature
- Sandstorm

$$\max\ m_{left} = m_{init} - cost_{total} + cap_{total},$$

$$\text{s.t. } cost_{total} = (w_{buy} + w_{village} * 2) * w_{cost} + (f_{buy} + f_{village} * 2) * f_{cost},$$

$$cap_{total} = d_{mine} * cap_{basic},$$

$$d_{total} \geq dis + detour + d_{dust} + d_{mine}.$$

$$w_{buy} + w_{village} \geq w_{hot} * (d_{mine} * 3 + dis_{cur} * 2 + detour * 2) + w_{dust} * d_{dust},$$

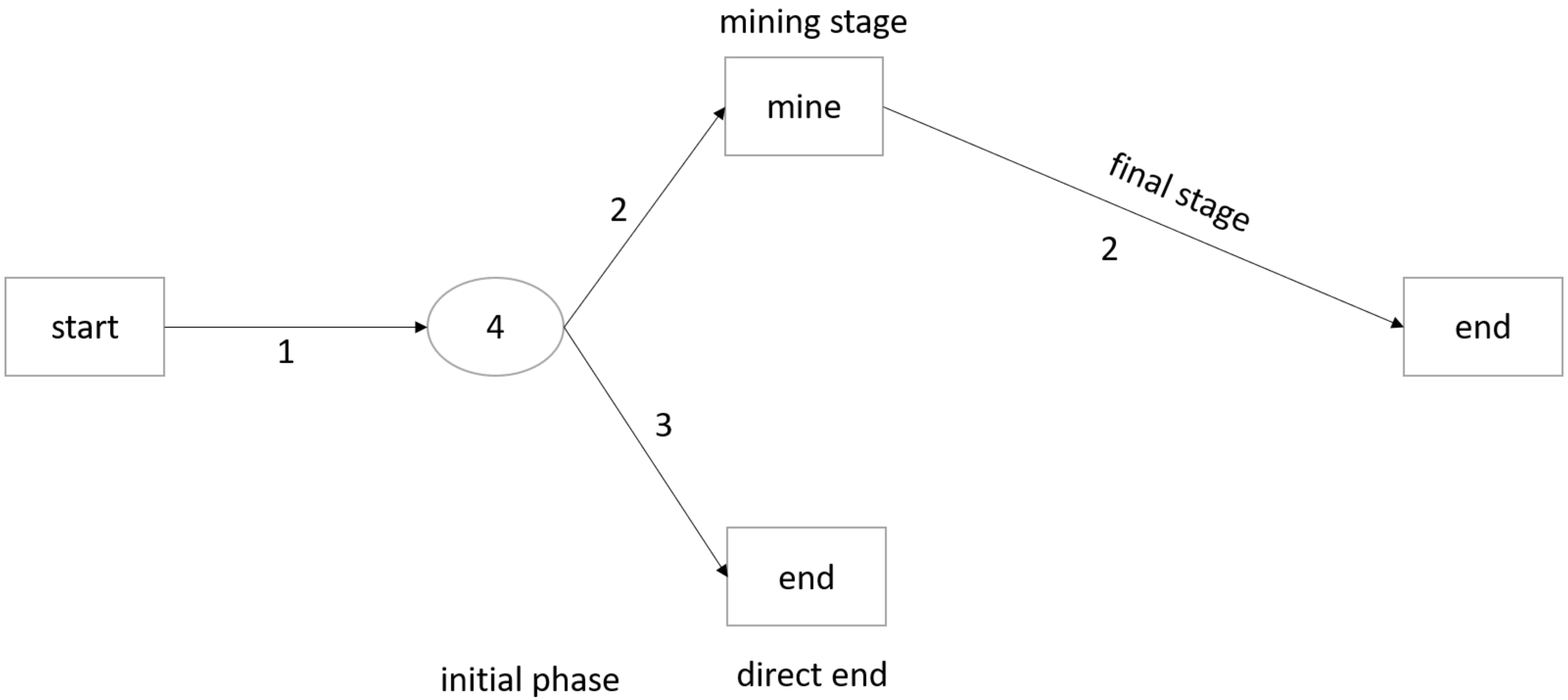
$$f_{buy} + f_{village} \geq f_{hot} * (d_{mine} * 3 + dis_{cur} * 2 + detour * 2) + f_{dust} * d_{dust}$$

# Path Selection

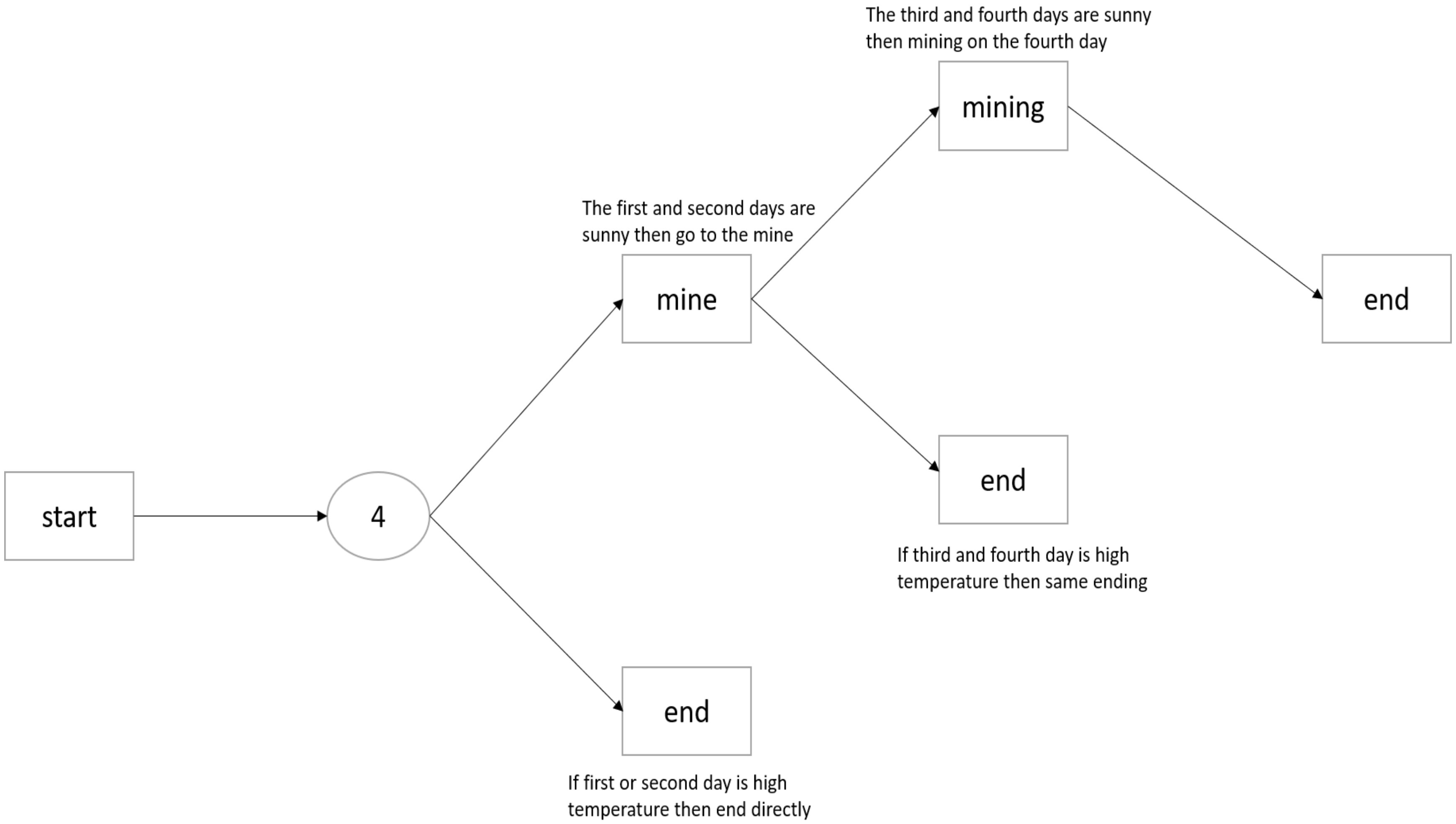
- Does the income from the mine cover the cost of going to the mine?
- Is the presence of the village, is it cost effective to mine given both the cost of making the detour and also the cost of buying necessary resources at the village?
- If resource being carried is  just enough to reach the end point even with adverse weather conditions and there is no chance of buying more, immediately head to exit.
- In the event of sandstorm, assume the deadline has also reduced as you can’t move in sandstorm.

# Solution Model (Level 3)

Simplified Path Model for level 3



# Path Selection (decision) model for level 3:



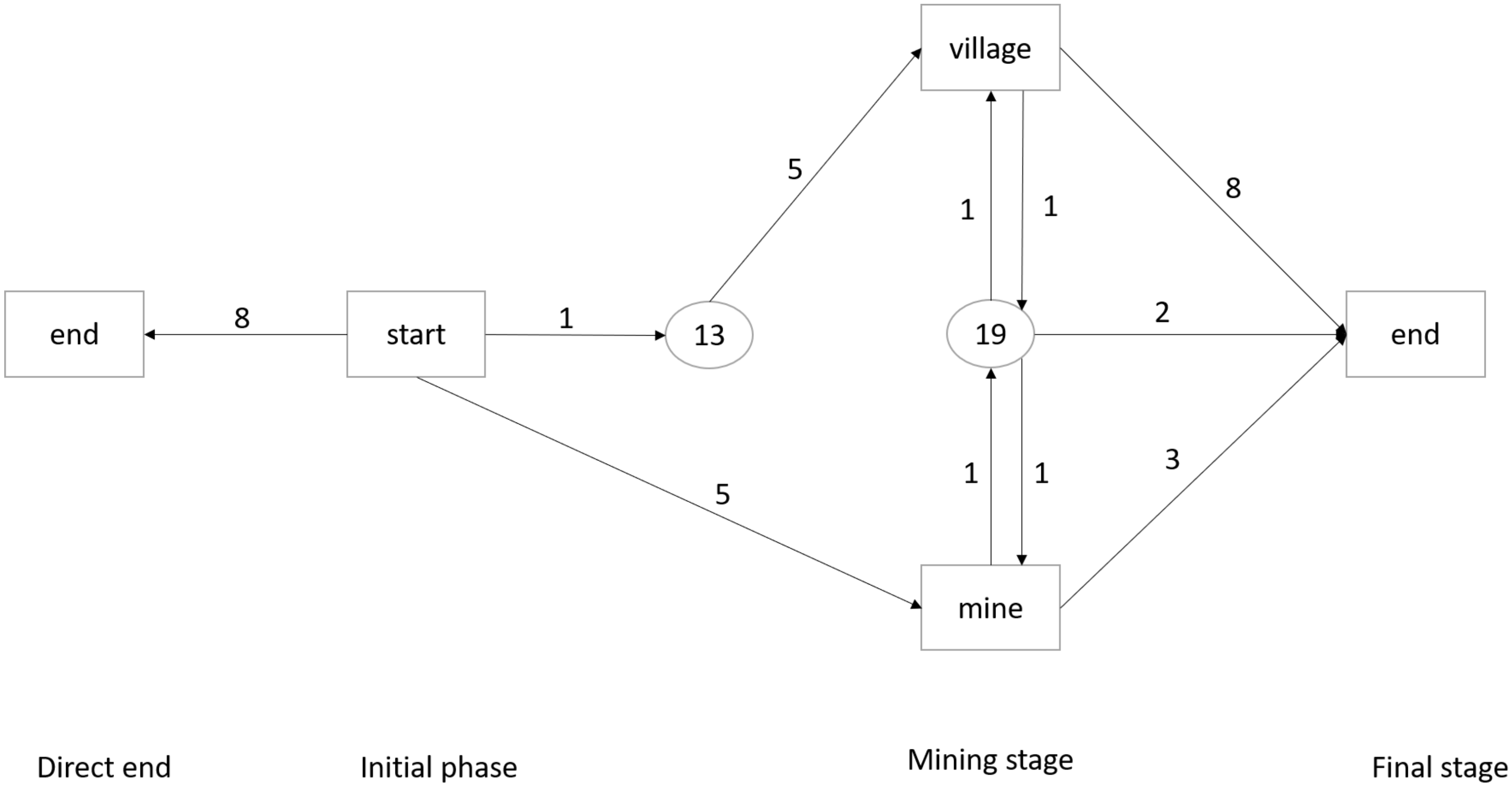
# Summary of decision making policy (Level 3)

*Table 3 Best strategy for the third level*

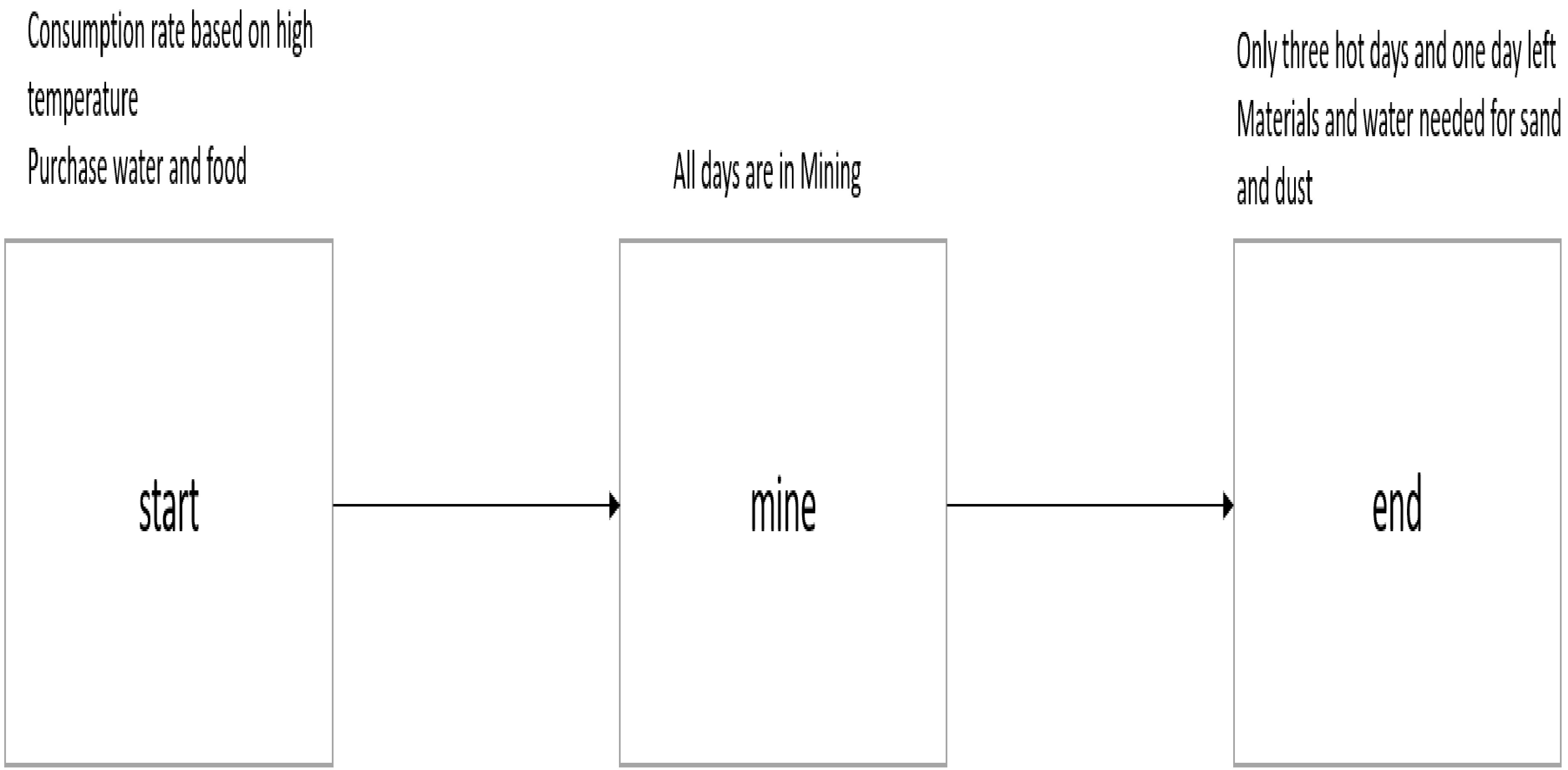
| | |
|---|---|
| Initial purchase | $4 * w_{hot} = 216kg$ water and $4 * f_{hot} = 144kg$ food，spend 2520 yuan |
| Basic strategy | Go to position 4 on Day 1, follow 4-6-13 to the end if the next day is hot, and follow 4-3-9-11-13 to the end if the next day is not hot, where mining is mining early on the fourth day if the 3/4th day is all sunny. |
| The remaining funds | if there is no mining, remains $m_{init} - 216kg * w_{cost} - 144kg * f_{cost} = 7480$ yuan；If you can mine，remains 7480+200=7680 yuan. |

# Solution Model (Level 4)

Simplified Path Model (Level 4):



# Path Selection (decision) Model for Level 4:



# Summary of Decision making policy (Level 4)

*4 The best strategy for the fourth level*

| | |
|---|---|
| Initial purchase | 720kg of water and 480kg of food, or in proportion to consumption in clear weather |
| Basic strategy | Go directly to the mine, keep moving forward in addition to sandstorms, and choose to mine no matter what the weather is like when you arrive at the mine |
| End condition | If the remaining water during mining will be less than 192kg or less than 128kg, choose to follow the shortest path to the end. |

# Multi-player game under the condition of the strategy

- Problem Analysis

  In this problem, there are many decision makers in the game, each decision maker maintains its own decision variables and objective functions, and the behavior of decision makers affect each other, so we use the game model to study.

  At level 5, the number of players is 2, each player has full information on day 0, and each player's course of action needs to be determined on day 0 and cannot be changed thereafter, resulting in a completely static game of information. Under the analysis framework of static model, we need to determine the game's people, decision sets, utility functions, and winning matrix, so as to calculate the game's Nash equilibrium and formulate the optimal game strategy.

  In the sixth level, the number of players is 3, and each player knows the action plan and the number of resources left for the rest of the player's day after the action ends, and then determines their action plan for the next day. Similar to level five, each player wants to take the optimal path from the starting point to the end.

# The fifth pass model is established and solved

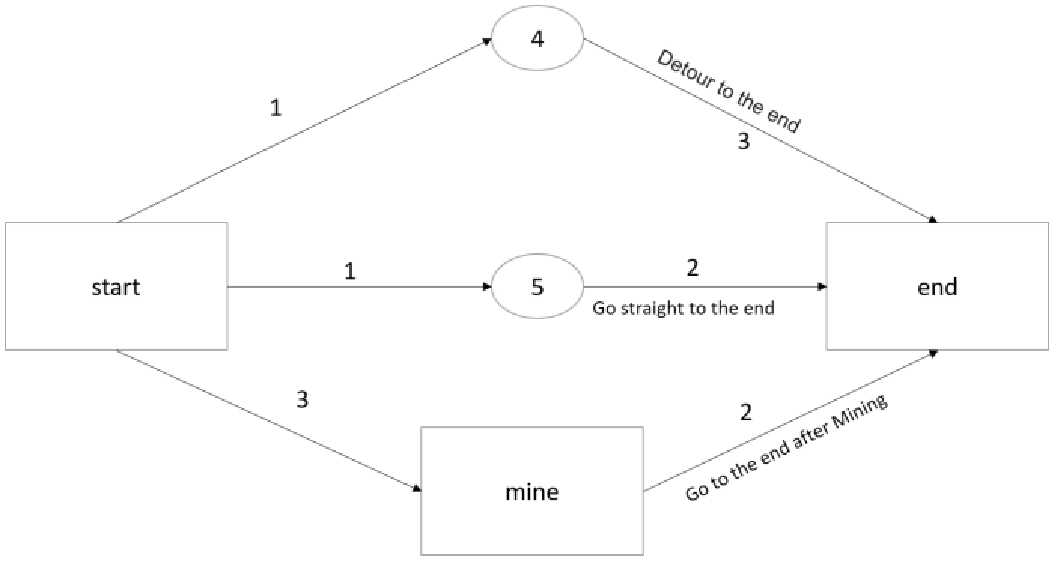
**Modeling**

First of all, we set up a complete information static game model for the fifth level. (1) People in the bureau. In a response, the participants who have the right to decide on their own course of action are referred to as the in-office persons. In this model, the person is the player. We use I to represent a collection of people in the bureau. 2) Decision collection In a game of countermeasures, a practical and complete course of action available to the people in the bureau is called a strategy. Each person in each game participating in the countermeasure, i , i ϵ I, has its own decision set Di (3) Win the matrix Each of the two sides meets the x1 ∈ D1,x2 ∈ D2 decision ( x1, x2 ), using u( x1, x2 ) to represent the effectiveness of the game in the game. Use the winning matrix M = | u(i, j) | to represent the benefits of all game possibilities. (4) Nash equalizer Both sides of the game strive to maximize their effectiveness in the game through decision-making, with the player represented separately 1 and player 2 will select the strategy

$$\begin{cases} u_1(x_1{}^*, x_2{}^*) \geq u_1(x_1, x_2{}^*) \\ u_2(x_1{}^*, x_2{}^*) \geq u_2(x_1{}^*, x_2) \\ x_1 \epsilon D_1 \\ x_2 \epsilon D_2 \end{cases}$$

# The fifth pass model is established and solved

Solution




**Decision path selection**

# The fifth pass model is established and solved

## 1. One player's strategy:

*Table 5 One player's strategy*

| Policy Number | Strategy description | Final amount |
|---|---|---|
| Strategy 1 | Mining first and then heading to the end | ￥9325 |
| Strategy 2 | Go straight to the end without mining | ￥9535 |
| Strategy 3 | Go straight to the end without mining | ￥9425 |

## 2. Two player's strategy:

| Strategy number | Strategy description | The final amount for each player |
|---|---|---|
| Strategy 4 | Both players mine first and then head to the end | ￥8515 |
| Strategy 5 | Neither player mine directly to the end point | ￥9070 |
| Strategy 6 | Neither player digs a mine to make the detour to the end | ￥8850 |

## Game decision and its issues:

- **The participants in the game (people in the game) are 2 players;**
- **There are three strategies for the game: go directly to the end, make a detour to the end, dig and then go to the end;**
- **The decision set of the 2 players in the game is the same;**
- **The purpose of both sides of the game is to successfully reach the end and make the remaining amount the largest;**

**Utility function of the player 1 matrix:**

$$M_1 = | u_1(i, j) |_{3\times 3} = \begin{pmatrix} 9070 & 9425 & 9325 \\ 9535 & 8850 & 9325 \\ 9535 & 9425 & 8515 \end{pmatrix}$$

**Utility function of the player 2 matrix:**

$$M_2 = | u_2(i, j) |_{3\times 3} = \begin{pmatrix} 9070 & 9535 & 9535 \\ 9425 & 8850 & 9425 \\ 9325 & 9325 & 8515 \end{pmatrix}$$

# The fifth pass model is established and solved

**Final amount under the two player’s game:**

| Player 2 / Player 1 | Go directly to the end | Take a detour to the end | After mining, go to the end |
|---|---|---|---|
| Go directly to the end | 9070, 9070 | 9425, 9535 | 9325, 9535 |
| Take a detour to the end | 9535, 9425 | 8850, 8850 | 9325, 9425 |
| After mining, go to the end point | 9535, 9325 | 9425, 9325 | 8515, 8515 |

# The sixth pass model is established and solved

**Modeling:**

Players are rational players; the goal is to maximize the remaining amount to win the game;

•Players do not communicate before making decisions, cannot produce collusion, cooperation;

• All players make decisions at the same time;

• Regardless of malicious competition between players, they deliberately follow the same path as other players to consume each other's resource’s purpose.

**We will mark Markov's decision-making process as follows: M =< S,X,Ps,x,R >**

# The sixth pass model is established and solved

(1) **State collection**
(2) **Action collection**
(3) **The probability of state transfer**
(4) **Return**

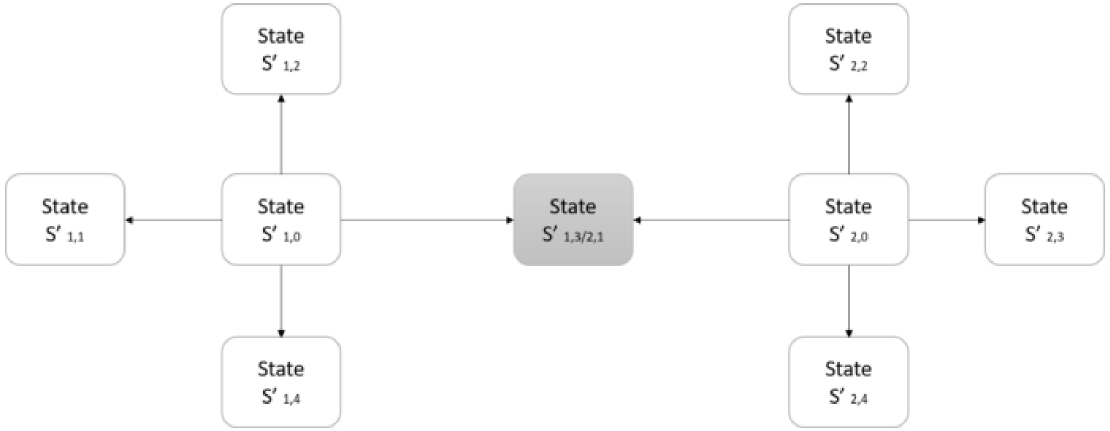



*Figure 12 Level 6: Action Set and Game Diagram*

# The sixth pass model is established and solved

## Status information contained in the status collection

| Serial Number | Serial Information | Serial Number | Serial Information |
|---|---|---|---|
| A | Personal money | G | Player 3 money |
| B | My remaining resources | H | Player 3 remaining resources |
| C | My backpack capacity | I | Player 3 strategy of the day |
| D | Player 2 money | J | Weather |
| E | Player 2 remaining resources | K | Basic game parameters |
| F | Player 2 strategy of the day | L | Basic map of the game |
| | | | |

## Risk Factor:

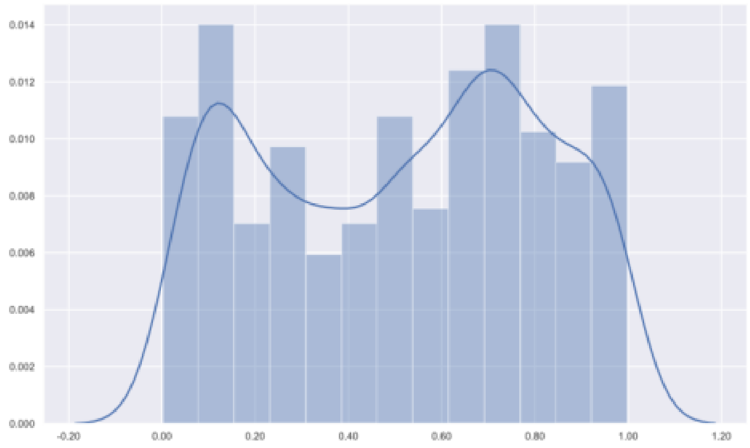



*Figure 14 An estimate of the degree of risk acceptance for all failed gamers*

# Model evaluation and promotion

- ***Question one***
  - For question one, after simplifying the original map into a directional graph model, a complete dynamic planning model, including state variables, decision variables, state transfer functions, profit and loss functions, etc., is established by analyzing the decision-making scheme and constraints in the rules of the game. In the implementation of the algorithm, combined with multiple searches, memory search, pruning and other optimization means, significantly improve the computational speed.
- ***Question two***
  - In the context of environmental uncertainty, the model establishes the player's decision path by analyzing the accumulation of uncertainties, by adjusting the Dijkstra algorithm, creating a simplified network with key locations as the core. At the same time, the relationship between the player's earnings and individual behaviors is established, thus establishing several basic decision-making bases, in the specific environment, only need to carry out cost analysis and environmental analysis, you can get the player's general best strategy.

# Model evaluation and promotion

- ***Question three***
  - For question three, based on the idea of static game and dynamic game, the basic model of player's behavior strategy is established based on the conclusions of the first two questions, under the circumstances of multi-player participation and the decision-making results will affect each other.
  - For the static full information game at level five, it is assumed that there is no exchange of information between players. The model abstracts out three decision-making choices, and calculating the probability of the strategy from the perspective of hybrid strategy can guarantee the maximum amount mean from the game of large number of disks, but the guiding significance of the decision-making of a game is limited.

# Summary