Uniwersytet Wrocławski

Faculty
of Mathematics
and Computer Science

Mathematical
Institute

Chaos theory

Sylwia Majchrowska

Life finds a way

Tiny variations … never repeat, and vastly affect the outcome.

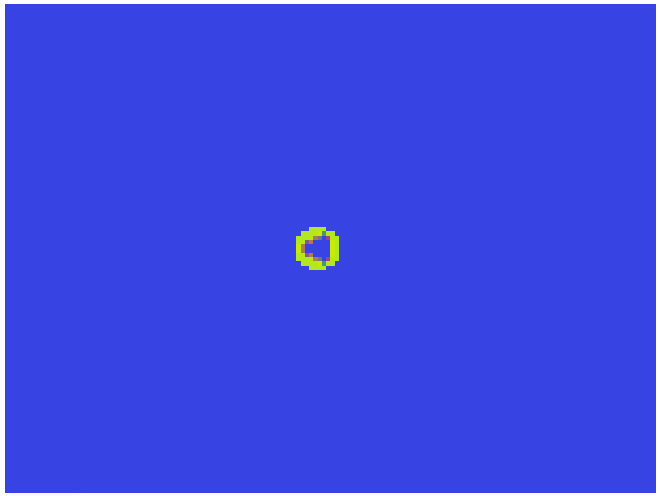
Jeff Goldblum (“Ian Malcolm”), Jurassic Park

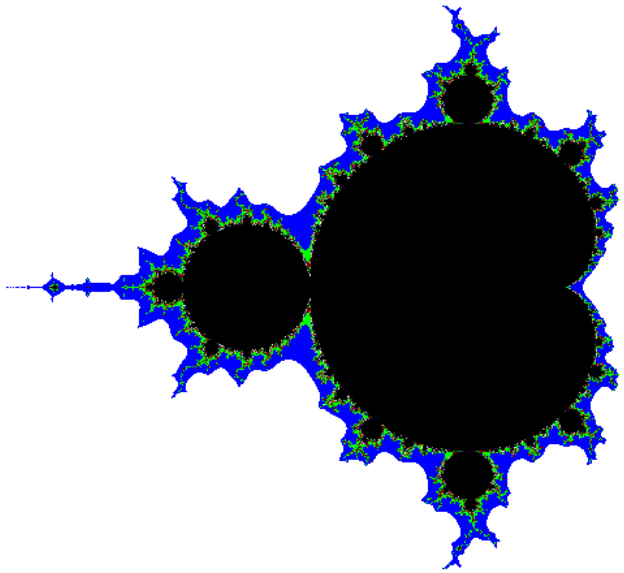
What is Chaos Theory?

"As far as the laws of mathematics refer to reality, they are not certain, and as far as they are certain, they do not refer to reality."

Albert Einstein

Nature of chaos





Laplace's demon

(23 March 1749 – 5 March 1827)

"We may regard the present state of the universe as the effect of its past and the cause of its future. An intellect which at any given moment knew all of the forces that animate nature and the mutual positions of the beings that compose it, if this intellect were vast enough to submit the data to analysis, could condense into a single formula the movement of the greatest bodies of the universe and that of the lightest atom; for such an intellect nothing could be uncertain and the future just like the past would be present before its eyes.„

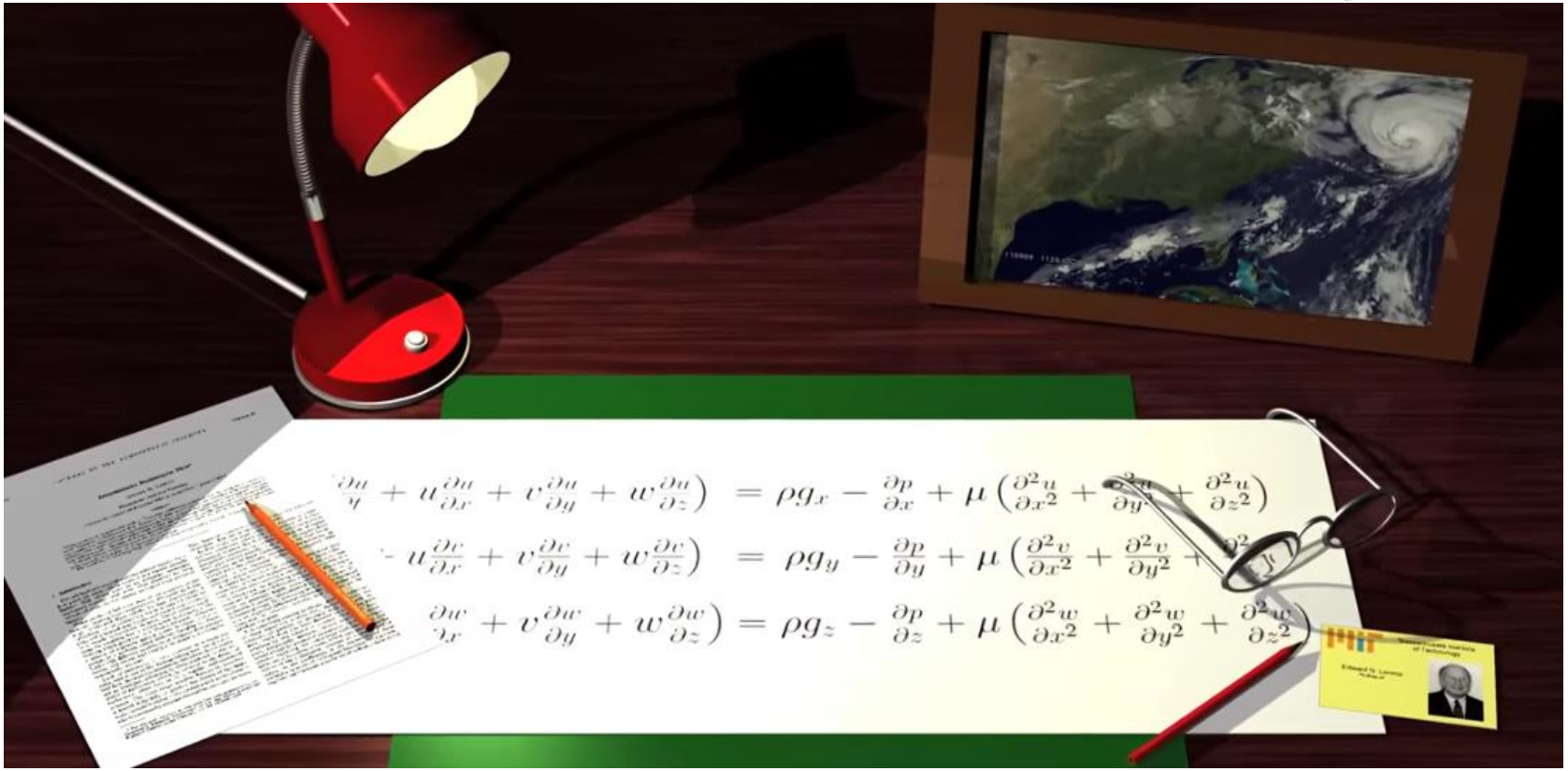
Marquis Pierre Simon de Laplace

"*Prediction is difficult, especially the future.„*

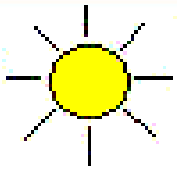
Niels Bohr

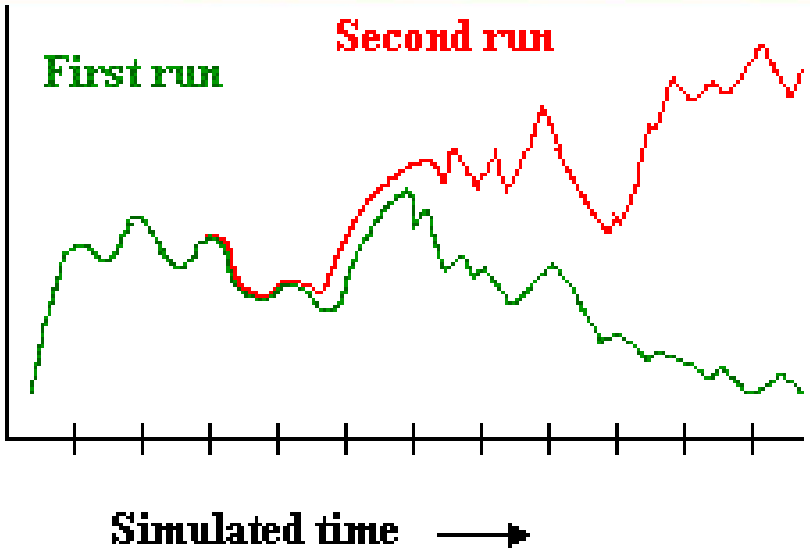
How Chaos Theory was born and why?

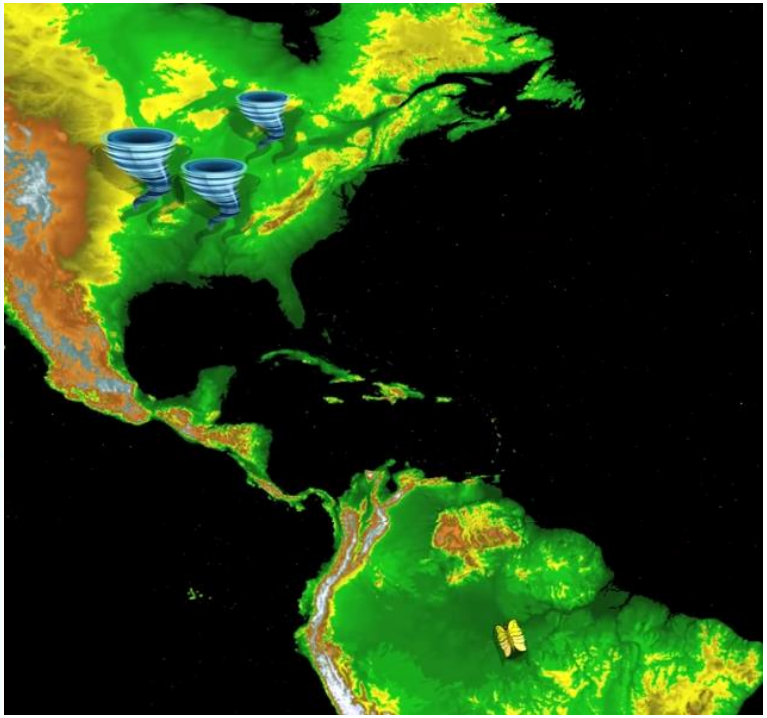
In 1963, Edward Lorenz (1917-2008), studied convection in the Earth's atmosphere.

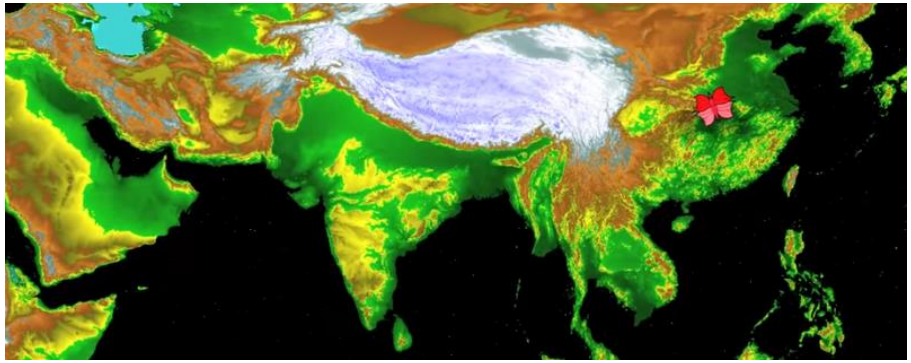


Butterflies make all the difference









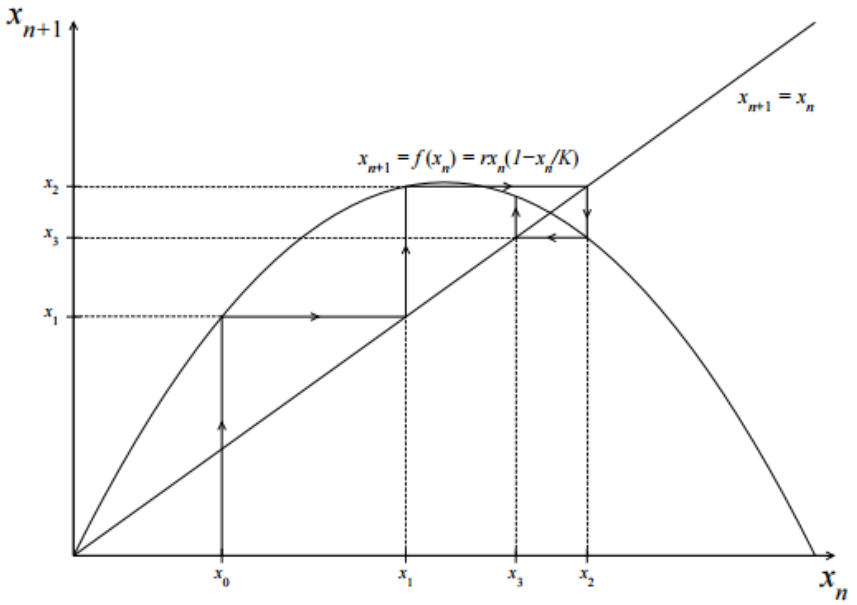
Sensitivity to initial conditions

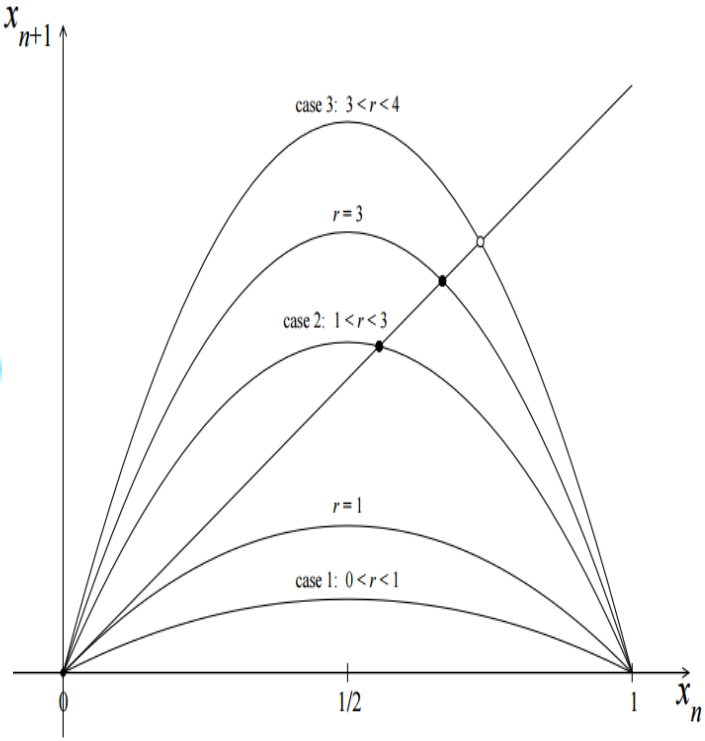
Discrete Logistic Equation

$$x_{n+1} = r\left(1 - \frac{x_n}{K}\right) x_n$$

$$\bar{x}_n = \frac{x_n}{K}$$

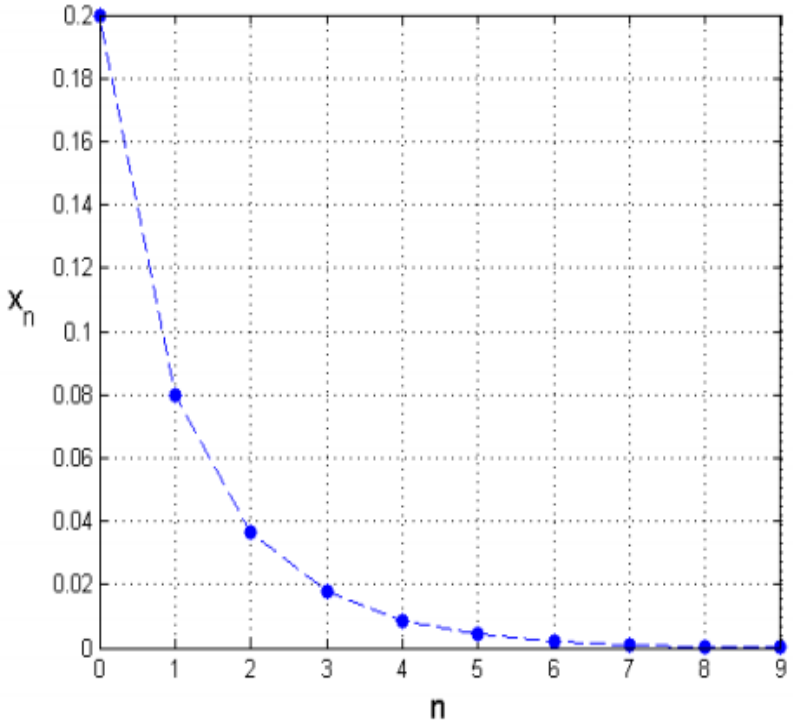
$$x_{n+1} = f(x_n) = r x_n (1 - x_n)$$



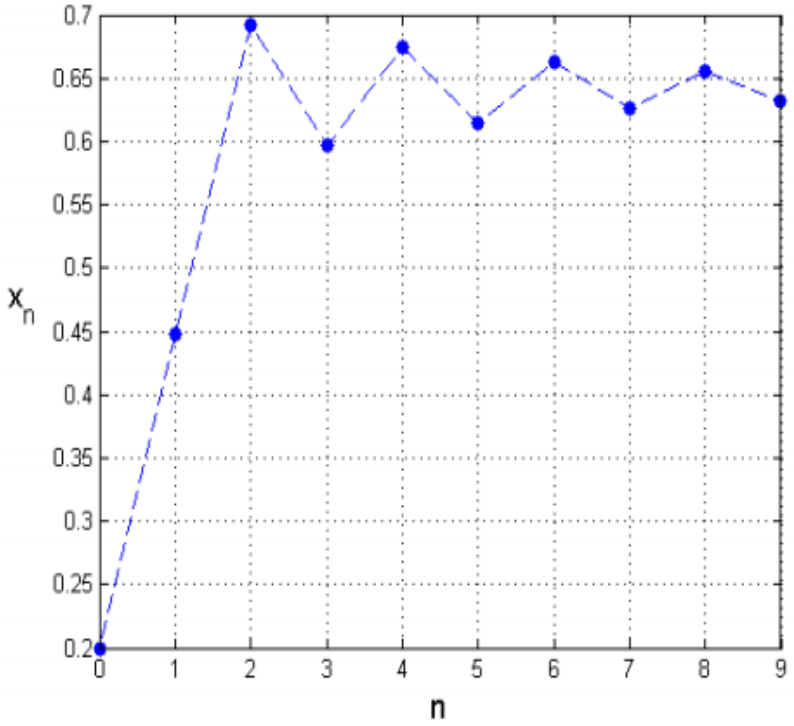


Analysis of the Discrete Logistic Equation

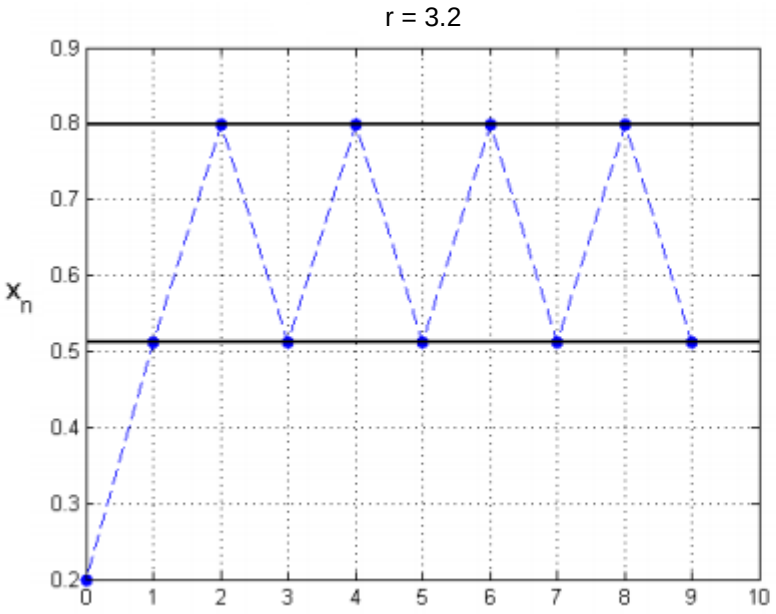
r = 0.5

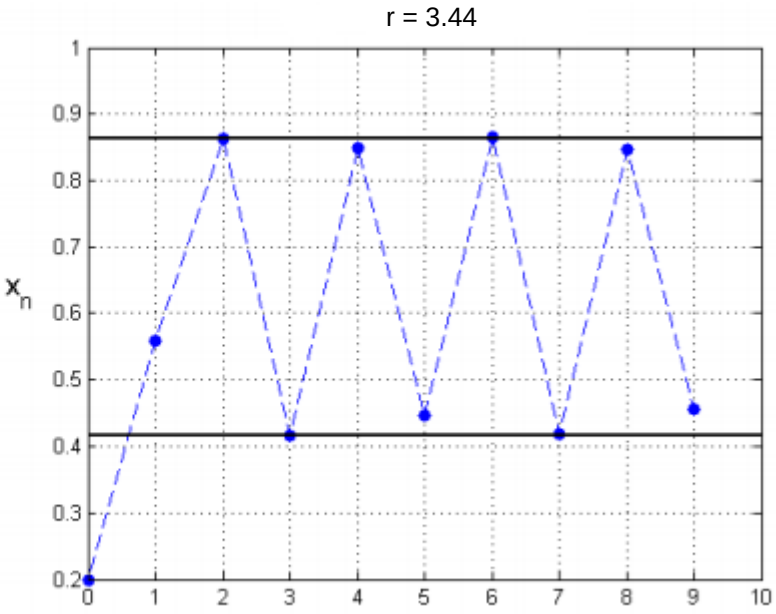


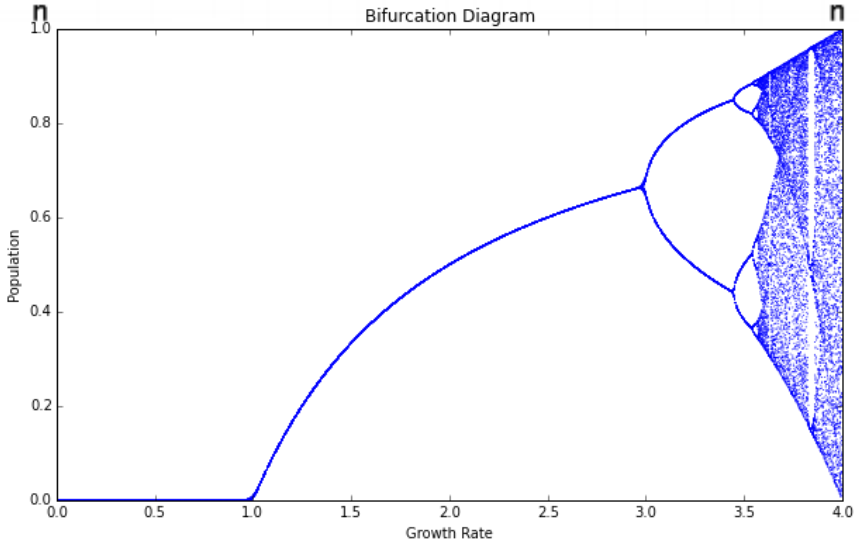
r = 2.8



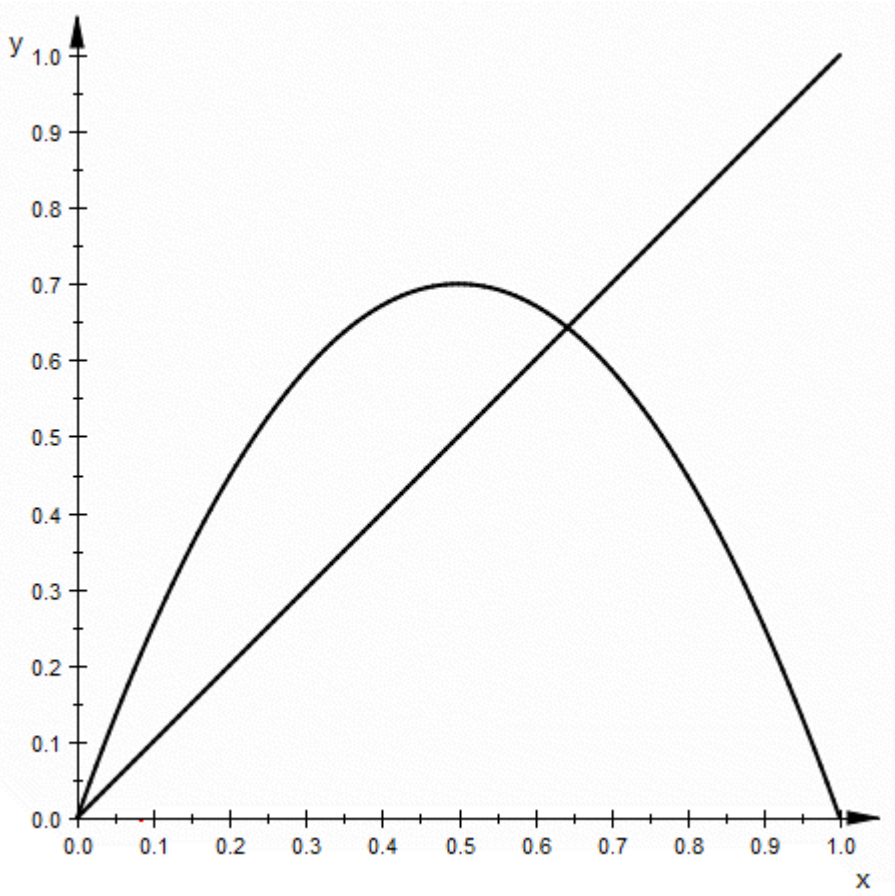
Analysis of the Discrete Logistic Equation

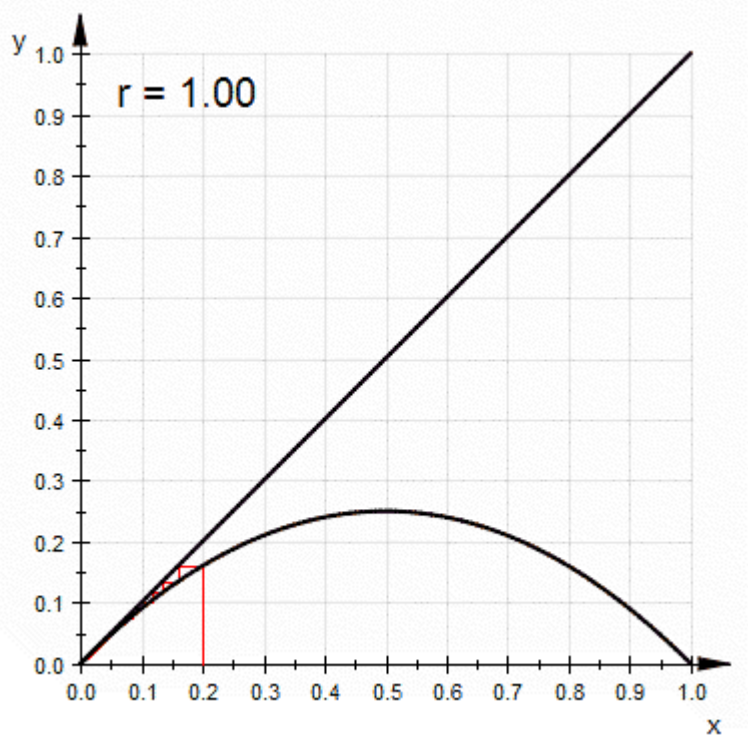




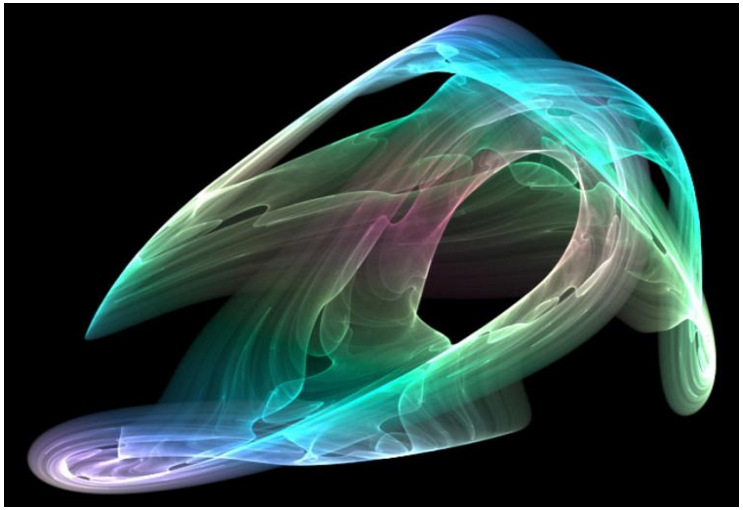


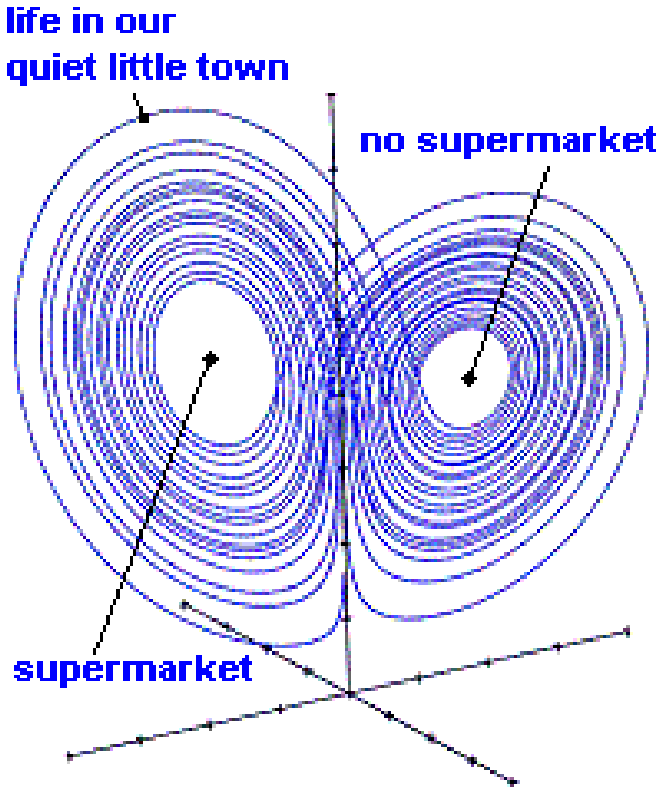
Verhulst diagrams





Attractive, strange behaviour

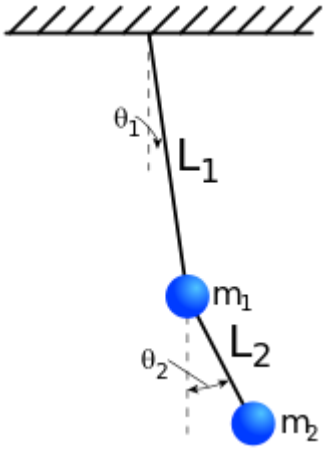




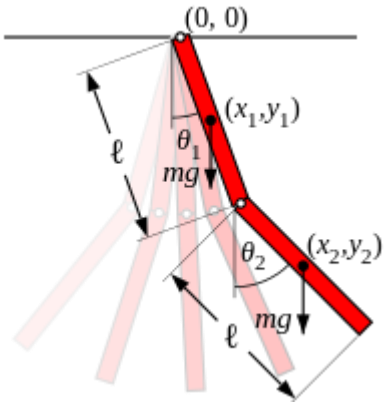
First, a highly regular motion towards the attractor … then a much more irregular motion on it.

Ian Stewart, The Magical Maze

Double pendulum



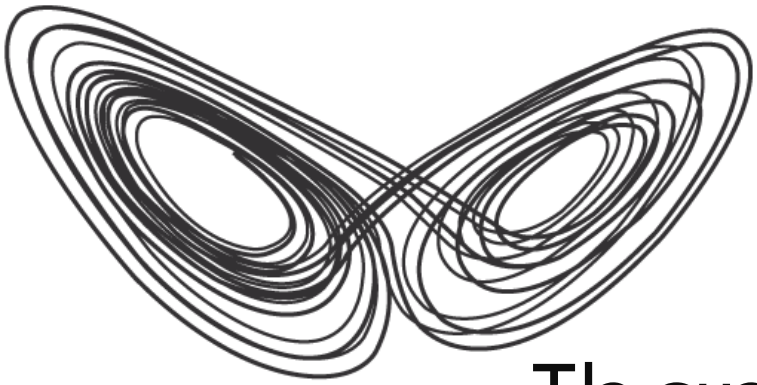
Trajectories of a double pendulum

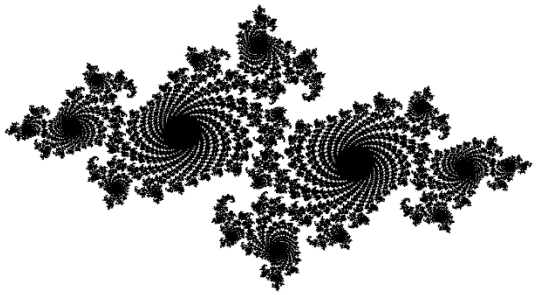


In short

Those scientists are damned clever *

* either that, or they are out of their minds





Thank you for your attention ☺