

1 Predicting the integration of social information in pragmatic word learning across
2 development

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¹³

Abstract

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¹⁵ *Keywords:* language acquisition, social cognition, pragmatics, Bayesian modeling

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17 Predicting the integration of social information in pragmatic word learning across
18 development

19 **Introduction**

20 . . .

21 **Adults**

22 **Assessment of general and speaker specific expectations (Experiment 1 and 2)**

23 As a first step, we assessed whether adults make general and common ground
24 inferences when tested separately. In experiment 1, participants could infer which object a
25 novel word referred to by assuming that the speaker communicated in an informative way.
26 The speaker was located between two tables, one with two novel objects, A and B, and the
27 other with only object A (Fig. 1A). When the speaker turned to the table with the two
28 objects (A and B) and used a novel word to request one of them, participants could infer
29 that the word referred to object B. This follows from the counter-factual inferences that, if
30 the (informative) speaker would have wanted to refer to object A, they would have turned to
31 the table with the single object, because this would have been the least ambiguous way to
32 refer to this object. As a consequence, turning to the table with two objects most likely
33 reflects an intention to refer to object B. In the control condition, both tables contained both
34 objects and no inference could be made based on the speaker's behavior.

35 Participants made the inference consistent with the assumption that the speaker
36 communicated informatively: They selected object B above chance (0.5) in the test condition
37 ($t(39) = 5.51, p < .001$, see Fig. 2) and more often compared to the control condition ($\beta =$
38 $1.49, se = 0.50, p = .003$).

39 In experiment 2, we tested if participants use common ground information that is
40 specific to a speaker to infer the referent of a novel word. In the preference manipulation
41 (Fig. 1B . left), the speaker expressed preference for one of two objects. When the speaker

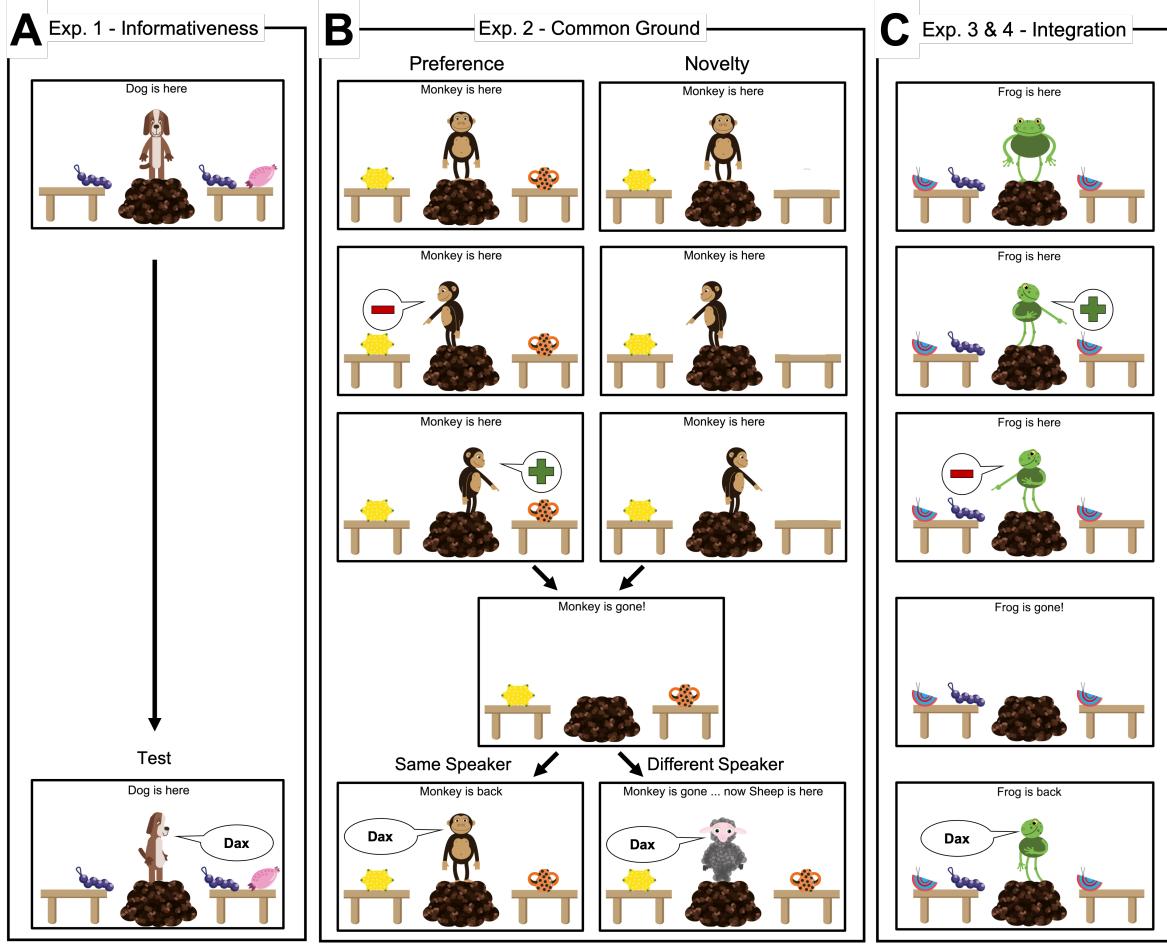


Figure 1. Schematic experimental procedure with screenshots from the adult experiments. In all conditions, at test (bottom), the speaker ambiguously requested an object using a non-word (e.g. “dax”). Participants clicked on the object they thought the speaker referred to. Informativeness (Experiment 1, left) translated to making one object less frequent in context. Common ground (Experiment 2, middle) was manipulated by making one object preferred by or new to the speaker. Green plus signs represent utterances that expressed preference and red minus of dispreference (see main text for details). Experiment 3 (right) combined manipulations. When expressing e.g. preference for an object on a table with two objects (panel 3 from top), the respective object was temporarily enlarged. Condition for Experiment 3 shown here: preference - same speaker - incongruent.

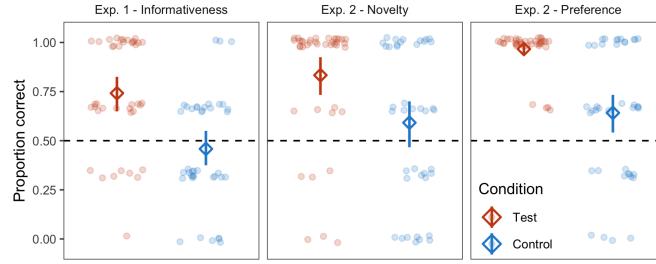


Figure 2. Results from experiment 1 and 2 for adults. For preference and novelty, control refers to a different speaker (see Fig. 1B). Transparent dots show data from individual participants, diamonds represent condition means, error bars are 95% CIs. Dashed line indicates performance expected by chance.

42 later used a novel word to requested an object, participants could infer the referent by
 43 assuming that the speaker was talking about the object they liked. In a control condition, a
 44 different speaker, whose preferences were unknown, made the request. Adults selected the
 45 preferred object above chance when the same speaker made the request ($t(39) = 29.14, p <$
 46 .001, see Fig. 2) and more so compared to when a different speaker asked for an object ($\beta =$
 47 2.97, $se = 0.70, p < .001$).

48 In the novelty manipulation (Fig 1B - right), the speaker encountered one object on
 49 one of the tables. Later, a second object appeared. The second object was therefore new to
 50 the speaker. When the same speaker later used a novel word to requested an object (in a
 51 slightly excited way), participants could infer that it was the new object they are referring to.
 52 In contrast, when a different speaker, tho whom both objects were equally new, made a
 53 request, no inference could be made based on the speaker's state of knowledge. In line with
 54 this reasoning, participants selected the new object above chance with the same speaker
 55 ($t(39) = 6.77, p < .001$, see Fig. 2), and more often compared to the different speaker ($\beta =$
 56 1.70, $se = 0.39, p < .001$). Taken together, experiment 1 and 2 showed that adults use
 57 general and speaker specific expectations to identify the referent of a novel word. In the
 58 following, we studied how these expectations are integrated when combined experimentally.

59 **Model predictions for information integration evaluated against new data**
 60 **(Experiment 3).**

61 We modeled the integration of general and common ground expectations as a process
 62 of probabilistic inference. We used the results of experiment 1 and 2 to inform key
 63 parameters in the model’s architecture. Within the RSA framework, general expectations
 64 that speakers (or listeners) communicate in a cooperative and informative way are reflected
 65 in the model’s recursive structure. When interpreting an utterance, the hypothetical listener
 66 imagines that it was generated by a pragmatic speaker whose goal is to produce the most
 67 informative utterance. The pragmatic speaker is modeled as reasoning about what the most
 68 informative utterance would be for a naive listener, who does not know the novel words.
 69 Importantly, the utterance in this case includes not just the novel word but also the turning
 70 to one of the tables. The conditional probability that the listener interprets an utterance as
 71 referring to a particular referent is defined as:

$$P_L(r_s|u) \propto P_S(u|r_s)P_S(r_s)$$

72 Here, the term $P_S(u|r_s)$ denotes the likelihood that the pragmatic speaker will produce
 73 a particular utterance u to refer to a referent r . It is defined in terms of a utility function
 74 $U_S(u; s)$ consisting of the surprisal of u for a naive listener, who interprets u according its
 75 literal semantics:

$$P_S(u|r_s) \propto \exp(\alpha U_S(u; s))$$

76 This utility is maximized in order to decide which utterance to use to communicate
 77 about a particular referent, reflecting the expectation that the speaker communicates in an
 78 informative way. The absolute strength of $P_S(u|r_s)$ depends on a scalar value, α , which can
 79 be interpreted as an indicator of how rational the speaker is in choosing utterances (i.e. as α
 80 increases, the speaker is more likely to choose the most informative utterance). We used the

81 data from experiment 1 to specify how rational participants thought speakers were in the
82 current setup. That is, we inferred which value of α would generate model predictions
83 (assuming equal prior probability for each object) that corresponded to the average
84 proportion of correct responses measured in experiment 1.

85 We treated common ground expectations as changes in the prior probability that a
86 particular referent is being referred to ($P_S(r_s)$ in the first equation). For example, if the
87 speaker expresses preference for object A, and the same speaker later requests an object,
88 then A has a higher probability of being the referent of a subsequent utterance, regardless of
89 any additional pragmatic considerations. We used the proportions measured in experiment 2
90 to specify the probability distribution across referents for the different common ground
91 manipulations (novelty or preference, same or different speaker).

92 With α and $P_S(r_s)$ fixed based on experiment 1 and 2, we generated model predictions
93 for situations in which general and common ground expectations had to be integrated (Fig.
94 1C). Expectations could either be congruent or incongruent. If, for example, object A was
95 the more informative object (unique to that table) and also the one the speaker expressed
96 preference for, expectations were congruent. Next, the speaker who made the request could
97 either be the same as the one in the previous interaction or a different one. Finally, common
98 ground information could be manipulated in the form of preference or novelty. This resulted
99 in a total of 2 (novelty or preference) \times 2 (same or different speaker) \times 2 (congruent or
100 incongruent) = 8 conditions for which we generated model predictions. In order to capture
101 that behavioral data is to some extend noisy, all models included a noise parameter. Noise
102 parameters ranged between 0 and 1 and reflect the proportion of responses that are
103 estimated (based on the data) to be random instead of in line with model predictions.
104 Including the noise parameter greatly improved the model fit to the data (see SI Appendix
105 for details). We did not preregister the inclusion of a noise parameter for experiment 3 but
106 did so for all subsequent experiments.

107 In experiment 3, we collected new behavioral data for each of the eight conditions.

108 Experimentally, we combined the procedures of experiment 1 and 2. The test setup was
109 identical to experiment 1, however, before making a request, the speaker interacted with the
110 objects so that some of them were preferred by or new to them (Fig. 1C). We discuss and
111 visualize the results as the proportion with which participants chose the more informative
112 object (i.e., the object that would be the more informative referent when only general
113 expectations are considered). Before comparing responses to model predictions we fit a
114 generalized linear mixed model (GLMM) to the data to test if participant's responses varied
115 across conditions. Participants distinguished between congruent and incongruent trials when
116 the speaker remained the same (model term: `alignment x speaker`; $\beta = -2.90$, $se = 0.56$, p
117 $< .001$).

118 Model predictions were highly correlated with the average response in each condition
119 (Fig. 3A and B). We compared the fit of the pragmatics model described above to two
120 alternative models: the *flat prior* model ignored the speaker specific expectation and the
121 *prior only* model ignored the informativeness inference. This analysis tests whether the
122 trade-off between expectations in the pragmatics model captured the structure in the data
123 better compared to models that assume that participants only consider one type of
124 expectation at a time. Model fit was much better for the pragmatic compared to the flat
125 prior (Bayes Factor (BF) = 4.2e+53) or the prior only model (BF = 2.5e+34), suggesting
126 that participants considered and integrated both types of expectations. The estimated
127 proportion of random responses according to the pragmatics model was 0.30 (95% Highest
128 Density Interval (HDI): 0.23 - 0.36). This value was considerably lower for the pragmatics
129 model, compared to the alternative models (see SI Appendix), lending additional support to
130 the conclusion that the pragmatics model better captured the behavioral data.

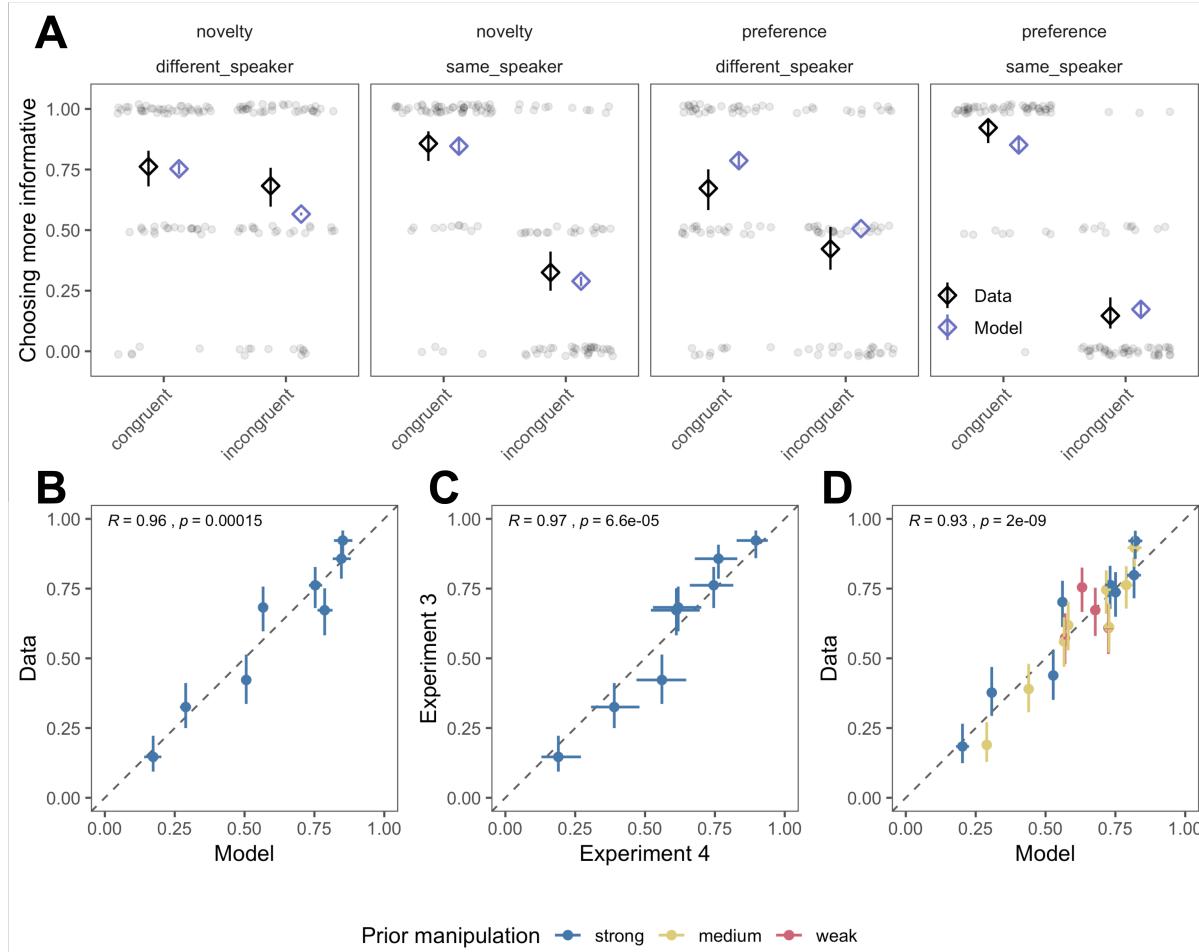


Figure 3. Results from experiment 3 and 4 for adults. Data and model predictions by condition for experiment 3 (A). Transparent dots show data from individual participants, diamonds represent condition means. Correlation between model predictions and data in experiment 3 (B), between data in experiment 3 and data for the strong prior manipulation in experiment 4 (direct replication - C) and between model predictions and data in experiment 4 (D). Coefficients and p-values are based on Pearson correlation statistics. Error bars represent 95% HDIs.

¹³¹ **Replication and extension to different levels of speaker specific expectation**

¹³² **(Experiment 4).**

¹³³ The goal of experiment 4 was twofold: First, to replicate the results of experiment 3
¹³⁴ and second, to test the generalizability of the pragmatics model to a larger range of

135 conditions. Compared to experiment 3, we manipulated the strength of the common ground
 136 expectations. For both, preference and novelty, we planned to have a strong, a medium and
 137 a weak manipulation. Expectation strength was manipulated by changing the way the
 138 speaker interacted with the objects prior to the request and measured as the proportion with
 139 which participants chose the preferred/novel object in the same speaker condition (see SI
 140 Appendix for results). For novelty, we succeeded in finding three qualitatively different
 141 manipulations. For preference, we piloted a number of manipulations but did not find a way
 142 to indicate only weak preference. Experiment 4 therefore included strong and medium
 143 manipulations for novelty and preference and a weak manipulation for novelty. The general
 144 expectation was the same as in experiment 3. For each level we crossed general and common
 145 ground expectations in the same way as in experiment 3, resulting in a total of 8 (strong) +
 146 8 (medium) + 4 (weak) = 20 conditions. The strong manipulation was a direct replication of
 147 experiment 3. Model predictions were generated in the same way as in experiment 3.

148 Again, before evaluating model predictions, we fit GLMMs to the data for each level of
 149 common ground manipulation. Participants distinguished between congruent and
 150 incongruent trials when the speaker remained the same for the strong ($\beta = -1.60$, $se = 0.52$,
 151 $p = .002$)¹ and medium ($\beta = -2$, $se = 0.51$, $p < .001$), but not the weak manipulation ($\beta =$
 152 -0.68 , $se = 0.55$, $p = .219$, model term: `alignment x speaker` in all cases). This pattern
 153 validates our manipulations. If the speaker specific manipulation is weak, changing the
 154 speaker has no differential impact on the informativeness inference. Results from the strong
 155 manipulation in experiment 4 further replicated findings from experiment 3 (Fig. 3C).

156 Model predictions from the pragmatics model were again highly correlated with the
 157 average response in each condition (Fig. 3D). We evaluated model fit for the same models as
 158 in experiment 3 and found again that the pragmatics model fit the data much better
 159 compared to the flat prior ($BF = 4.7e+71$) or the prior only model ($BF = 8.9e+82$). The

¹Here, this effect differed between novelty and preference: model term manipulation x alignment x speaker;
 $\beta = -2.96$, $se = 0.73$, $p < .001$

160 inferred level of noise based on the data for the pragmatics model was 0.36 (95% HDI: 0.31 -
161 0.41), which was again lower compared to the alternative models (see SI Appendix).

162 Summarizing experiment 3 and 4, we saw that adults flexibly integrated different
163 expectations about a speaker in a word learning scenario. This process was best described by
164 a model that treated information integration as a form of probabilistic inference. Next, we
165 took a developmental perspective and studied if the same model could be used to describe
166 children's word learning.

167 **Children**

168 **Assessment of general and speaker specific expectations (Experiment 1 and 2)**

169 We approached children's word learning in the same way as adults'. First, we measured
170 the strength of general (informativeness) and common ground (preference) expectations and
171 used these measurements to set model parameters. Then we generated model predictions for
172 how expectations should be integrated. Finally, we compared model predictions to new
173 behavioral data. In this, we took a truly developmental perspective. We sampled children
174 continuously across the entire age range and generated model predictions for the average
175 developmental trajectory in each condition. In all the following experiments, we tested
176 children between 3.0 and 4.9 years of age. For experiment 1 and 2, we also tested
177 two-year-olds but did not find sufficient evidence that they make general and/or common
178 ground inferences in the tasks we used.

179 Children were only tested in the test condition in experiment 1. To compare children's
180 performance to chance level, we binned age by year. Four-year-olds selected the more
181 informative object (i.e. the object that was unique to the location the speaker turned to)
182 above chance ($t(29) = 2.80, p = .009$). Three-year-olds, on the other hand, did not ($t(31) =$
183 $-1.31, p = .198$). Consequently, when we fit a GLMM to the data with age as a continuous
184 predictor, performance increased with age ($\beta = 0.38, se = 0.11, p < .001$, see Fig. 4). Thus,
185 children's expectation that speakers communicate in an informative way increased with age.

186 In experiment 2, we assessed whether children use common ground information to
 187 identify the referent of a novel word. We tested children with the novelty as well as the
 188 preference manipulation but found little evidence that children distinguished between
 189 requests made by the same speaker or a different speaker in the case of novelty. We therefore
 190 dropped this manipulation and focused on preference for the remainder of the study.

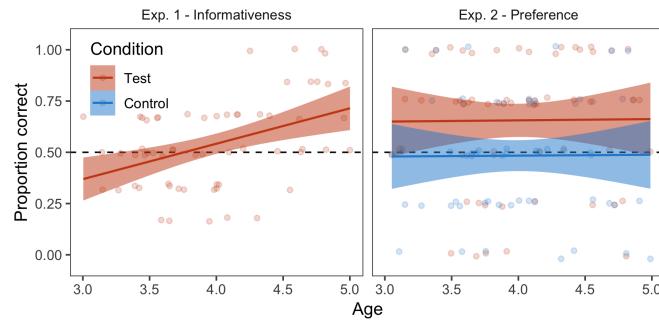


Figure 4. Results from experiment 1 and 2 for children. For preference, control refers to the different speaker condition (see Fig. 1B). Transparent dots show data from individual participants, regression lines show fitted linear models with 95% CIs. Dashed line indicates performance expected by chance.

191 For preference, four-year-olds selected the preferred object above chance when the same
 192 speaker made the request ($t(30) = 4.14, p < .001$), whereas three-year-olds did not ($t(29) =$
 193 $1.62, p = .117$). However, when we fit a GLMM to the data with age as a continuous
 194 predictor, we found an effect of speaker identity ($\beta = 0.92, se = 0.30, p = .002$) but no effect
 195 of age ($\beta = 0.04, se = 0.18, p = .83$) or interaction between speaker identity and age ($\beta =$
 196 $-0.04, se = 0.24, p = .886$, Fig. 4). Thus, children across the age range used common ground
 197 information to infer the referent of a novel word.

198 **Model predictions for information integration evaluated against new data**
 199 **(Experiment 3)**

200 We used the measurements from experiment 1 and 2 to specify α and $P_S(r_s)$ in the
 201 pragmatics model. However, instead of setting these parameters to a single value, we used

202 the data to infer the intercept and slope that best described the developmental trajectory for
 203 each parameter. For $P_S(r_s)$, this was done separately for the same speaker and the different
 204 speaker condition (see SI Appendix for details). These parameter settings were then used to
 205 generate age sensitive model predictions in 2 (same or different speaker) x 2 (congruent or
 206 incongruent) = 4 conditions. As for adults, all models included a noise parameter, which was
 207 estimated based on the data.

208 In experiment 3, we combined the procedures of experiment 1 and 2 and collected new
 209 data from children between 3.0 and 4.9 years of age in each of the four conditions (Fig. 1C).
 210 Before comparing the data to model predictions, we fit a GLMM to the data to test if
 211 children distinguished between conditions. Children's propensity to differentiate between
 212 congruent and incongruent trials for the same or a different speaker increased with age
 213 (model term: `age x alignment x speaker`; $\beta = -1.18$, $se = 0.50$, $p = .018$).

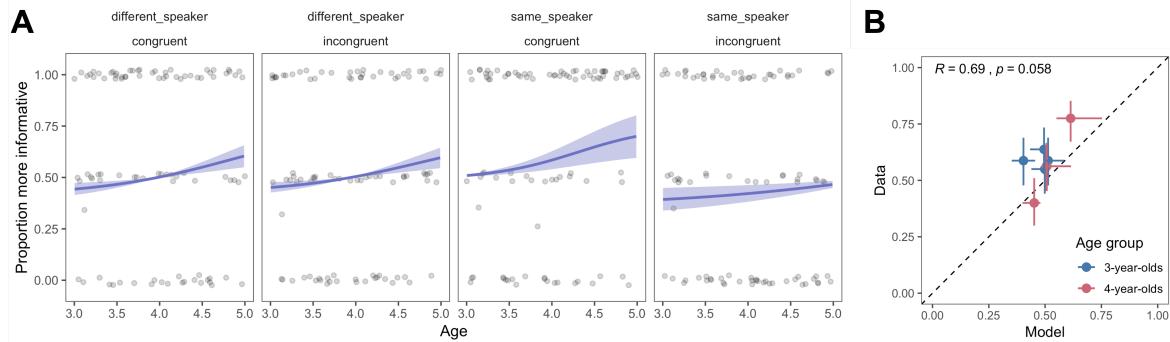


Figure 5. Results from experiment 3 for children. Purple lines show model predictions with 95% HDIs, transparent dots show data from individual participants (A). Correlation between model predictions and condition means binned by year (B). Coefficient and p-value are based on Pearson correlation statistic. Error bars represent 95% HDIs.

214 Next, we evaluated the model predictions (Fig 5A). For the correlational analysis, we
 215 binned model predictions and data by year. There was a substantial correlation between the
 216 predicted and measured average response in each age bin (Fig. 5B). For the model
 217 comparison, we treated age continuously. As for adults, we found a much better model fit for

the pragmatics model compared to the flat prior ($BF = 577$) or the prior only model ($BF = 10560$). The inferred level of noise based on the data for the pragmatics model was 0.51 (95% HDI: 0.26 - 0.77), which was lower compared to the alternative models considered (see SI Appendix). The high level of inferred noise moved the model predictions in all conditions close to chance level (Fig. 5A). We therefore compared two additional sets of models with different parameterizations that emphasized differences between conditions in the model predictions more. This analysis was not preregistered. Parameter free models did not include a noise parameter and developmental noise models allowed the noise parameter to change with age. In each case, the pragmatics model provided a better fit compared to the alternative models (flat prior: parameter free $BF = 334$, developmental noise $BF = 16361$; prior only: parameter free $BF = 20$, developmental noise $BF = 1e+06$). Taken together, these results suggest that children's word learning is best described as a flexible integration of general and common ground expectations.

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Discussion

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Methods

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All experimental procedures, sample sizes and statistical analysis were pre-registered

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(<https://osf.io/u7kxe/>). Experimental stimuli, data files and analysis scripts are freely

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available in an online repository (<https://github.com/manuelbohn/mcc>).

237

Participants

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Adult participants were recruited via Amazon Mechanical Turk (MTurk) and received

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payment equivalent to an hourly wage of ~ \$9. Experiment 1 and each manipulation of

240

experiment 2 had $N = 40$ participants. Sample size in experiment 3 was $N = 121$. $N = 167$

241

participated in the experiments to measure the strong, medium and weak preference and

242

novelty manipulations. Finally, experiment 4 had $N = 286$ participants.

243 Children were recruited from the floor of the Children's Discovery Museum in San Jose,
244 California, USA. Parents gave informed consent and provided demographic information. We
245 collected data from a total of 243 children between 3.0 and 4.9 years of age. We excluded 15
246 due to less than 75% of reported exposure to English, five because they responded incorrect
247 on 2/2 training trials, three because of equipment malfunction and two because they quit
248 before half of the test trials were completed. The final sample size in each experiment was as
249 follows: $N = 62$ (41 girls, mean age = 4) in experiment 1, $N = 61$ (28 girls, mean age =
250 3.99) in experiment 2 and $N = 96$ (54 girls, mean age = 3.96) in experiment 3.

251 **Experimental procedures**

252 All experiments were framed as games in which participants would learn words from
253 animals. They were implemented in HTML/JavaScript as a website. Adults were directed to
254 the website via MTurk and responded by clicking objects. Children were guided through the
255 game by an experimenter and responded by touching objects on the screen. For each animal
256 character, we recorded a set of utterances (one native English speaker per animal) that were
257 used to provide information and make requests. All studies started with an introduction to
258 the animals and two training trials in which familiar objects were requested (car and ball).
259 Subsequent test trials in each condition were presented in a random order.

260 The setup of experiment 1 for adults is shown in Fig. 1A. In the beginning of each
261 trial, the animal introduced themselves (e.g. "Hi, I'm Dog") and then turned towards the
262 table with the two objects. The same utterance was used to make a request in all adult
263 studies ("Oh cool, there is a [non-word] on the table, how neat, can you give me the
264 [non-word]?"). In the test condition, there was one object on the other table whereas in the
265 control condition there were two. Participants received six trials, three per condition.

266 The setup for experiment 2 is shown in Fig. 1B. In the preference manipulation, the
267 animal introduced themselves, then turned to one of the tables and expressed either that
268 they liked ("Oh wow, I really like that one") or disliked ("Oh bleh, I really don't like that

269 one") the object before turning to the other side and expressing the respective other attitude.
270 Next the animal disappeared and, after a short pause, either the same or a different animal
271 returned and requested an object while facing straight ahead. This procedure was the strong
272 preference manipulation. In the medium version, the animal only expressed preference and
273 did so in a more subtle way (simply saying: "Oh, wow").

274 In the novelty manipulation one of the tables was initially empty. The animal turned to
275 one of the sides and commented either on the presence ("Aha, look at that") or the absence
276 ("Hm..., nothing there") of an object before turning to the other side and commenting in a
277 complementary way. After shortly disappearing, the same animal repeated the sequence
278 above. When the animal left a second time, a new object appeared on the empty table. Next,
279 either the same or a different animal returned and requested an object. This corresponded to
280 the strong manipulation. For the medium manipulation, the animal turned to each table
281 only once before the new object appeared. In the weak manipulation, the animal only
282 commented on the present object once and never turned to the empty table. Participants
283 always received six trials, three with the same and three with the different speaker.

284 For experiment 3 and 4 we inserted the common ground manipulation before the
285 request in the setup of experiment 1 (Fig. 1C). For example, the animal turned to the table
286 with one object and express that they liked object A, then turned to the other table and
287 express that they did not like object B. Next, after quickly disappearing, the animal
288 reappeared, turned to the table with two objects and make a request. To make it clear,
289 which of the objects the speaker commented on while being turned to the table with the two
290 objects during the common ground manipulation, the object was temporarily enlarged.
291 Participants completed eight trials for one of the common ground manipulations with two
292 trials per condition (same/different speaker x congruent/incongruent).

293 Experiment 1 for children was modeled after Frank and Goodman (2014). Instead of
294 on tables, objects were presented as hanging in trees. After introducing themselves, the
295 animal turned to the tree with two objects and said: "This is a tree with a [non-word], how

296 neat, a tree with a [non-word]”). Next, the trees and the objects in them disappeared and
297 new trees replaced them. The two objects from the tree the animal turned to previously were
298 now spread across the two trees (one object per tree, position counterbalanced). While
299 facing straight, the animal first said “Here are some more trees” and then asked the child to
300 pick the tree with the object that corresponded to the novel word (“Which of these trees has
301 a [non-word]”). Children received six trials in a single test condition.

302 Experiment 2 for children was identical to the strong preference manipulation for
303 adults. Children received eight trials, four with the same and four with a different animal
304 returning.

305 In experiment 3 for children, we again inserted the preference manipulation into the
306 setup of experiment 1. After greeting the child, the animal turned to one of the trees,
307 pointed to an object (object was temporarily enlarged and moved closer to the animal) and
308 expressed liking or disliking. Then the animal turned to the other tree and expressed the
309 other attitude for the other kind of object. Next, the animal disappeared and either the
310 same or a different animal returned. The rest of the trial was identical to the label and
311 request phase of experiment 1. Children received eight trials, two per condition
312 (same/different speaker x congruent/incongruent).

313 Analysis

314 All analysis were run in R (R Core Team, 2018). GLMMs were fit via the function
315 `glmer` from the package `lme4` (Bates, Mächler, Bolker, & Walker, 2015). Probabilistic
316 models and model comparisons were implemented in WebPPL (Goodman & Stuhlmüller,
317 2014) using the r package `rwebppl` (Braginsky, Tessler, & Hawkins, 2019). The flat prior
318 model had the same structure as the pragmatics model but used a uniform prior over objects.
319 The prior only model predicted choice based on the prior probability for each object alone.
320 Note that the proportions that informed the prior distribution in the pragmatics and prior
321 only model were measured when participants chose between two objects in experiment 2. In

322 experiment 3, however, three objects were involved. For each object we used the proportion
323 measured in experiment 2 as the prior probability. This approach spread out the absolute
324 probability mass but conserved the relative relation between objects. Bayes Factors for
325 model comparisons are based on marginal likelihoods of each model given the data. The
326 corresponding model code can be found in the associated online repository.

327

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