Practica 8 - Derivaciones

Ejercicio 1

- $f.\,xs = \langle \forall_i : 0 < i < \#xs : xs.\,i = xs.0 \rangle$

Casos base

$f.\,[]$
$= \{\text{Especificación}\}$
$\langle \forall_i : 0 < i < \#[] : [].\,i = \{].0 \rangle$
$= \{\text{Definición de \#}\}$
$\langle \forall_i : 0 < i < 0 : [].\,i = \{].0 \rangle$
$= \{\text{Rango vacío}\}$
$True$

$f.\,[x]$
$= \{\text{Especificación}\}$
$\langle \forall_i : 0 < i < \#[x] : [x].\,i = \{x].0 \rangle$
$= \{\text{Definición de \#}\}$
$\langle \forall_i : 0 < i < 1 : [x].\,i = \{x].0 \rangle$
$= \{\text{Rango vacío}\}$
$True$

Caso inductivo

$f.\,(x \triangleright xs)$
$= \{\text{Especificación}\}$
$\langle \forall_i : 0 < i < \#(x \triangleright xs) : (x \triangleright xs).\,i = (x \triangleright xs).0 \rangle$
$= \{\text{Definición de \#}]$
$\langle \forall_i : 0 < i < 1 + \#xs : (x \triangleright xs).\,i = (x \triangleright xs).0 \rangle$
$= \{\text{Propiedad de .}]$
$\langle \forall_i : 0 < i < 1 + \#xs : (x \triangleright xs).\,i = x \rangle$
$= \{\text{Separación de un término}]$
$(x \triangleright xs).1 = x \wedge \langle \forall_i : 0 < i < \#xs : (x \triangleright xs).\,(i+1) = x \rangle$
$= \{\text{Propiedad de .}\}$
$xs.0 = x \wedge \langle \forall_i : 0 < i < \#xs : xs.\,i = x \rangle$
$= \{\text{Hipótesis inductiva}\}$
$xs.0 = x \wedge f.\,xs$

- $f.\,xs.\,x = \langle \exists_i : 0 \leq i < \#xs : xs.\,i = x \rangle$

Caso base

$f.\,[].\,x$

$= \{\text{Especificación}\}$

$\langle \exists_i : 0 \leq i < \#[] : [].\,i = x \rangle$

$= \{\text{Definición de } \#\}$

$\langle \exists_i : 0 \leq i < 0 : [].\,i = x \rangle$

$= \{\text{Rango vacío}\}$

$False$

Caso inductivo

$f.\,(y \triangleright xs).\,x$

$= \{\text{Especificación}\}$

$\langle \exists_i : 0 \leq i < \#(y \triangleright xs) : (y \triangleright xs).\,i = x \rangle$

$= \{\text{Definición de } \#\}$

$\langle \exists_i : 0 \leq i < 1 + \#xs : (y \triangleright xs).\,i = x \rangle$

$= \{\text{Separación de un término}\}$

$(y \triangleright xs).0 = x \vee \langle \exists_i : 0 \leq i < \#xs : (y \triangleright xs).\,(i + 1) = x \rangle$

$= \{\text{Propiedad de .}\}$

$y = x \vee \langle \exists_i : 0 \leq i < \#xs : xs.\,i = x \rangle$

$= \{\text{Hipótesis inductiva}\}$

$y = x \vee f.\,xs.\,x$

- $f.\,xs.\,x = \langle \forall_i : 0 \leq i < \#xs : xs.\,i = x \rangle$

Caso base

$f.\,[].\,x$

$= \{\text{Especificación}\}$

$\langle \forall_i : 0 \leq i < \#[] : [].\,i = x \rangle$

$= \{\text{Definición de } \#\}$

$\langle \forall_i : 0 \leq i < 0 : [].\,i = x \rangle$

$= \{\text{Rango vacío}\}$

$True$

Caso inductivo

$f.\,(y \triangleright xs).\,x$

$= \{\text{Especificación}\}$

$\langle \forall_i : 0 \leq i < \#(y \triangleright xs) : (y \triangleright xs).\,i = x \rangle$

$= \{\text{Definición de } \#\}$

$\langle \forall_i : 0 \leq i < 1 + \#xs : (y \triangleright xs).\,i = x \rangle$

$= \{\text{Separación de un término}\}$

$(y \triangleright xs).0 = x \wedge \langle \forall_i : 0 \leq i < \#xs : (y \triangleright xs).\,(i + 1) = x \rangle$

$= \{\text{Propiedad de .}\}$

$y = x \wedge \langle \forall_i : 0 \leq i < \#xs : xs.\,i = x \rangle$

$= \{\text{Hipótesis inductiva}\}$
$y = x \wedge f.\,xs.\,x$

- $f.\,xs.\,ys = \langle \forall_i : 0 \le i < \#xs \vee 0 \le i < \#ys : \#xs = \#ys \wedge xs.\,i = ys.\,i \rangle$

Casos base

$f.\,[].\,[]$
$= \{\text{Especificación}\}$
$\langle \forall_i : 0 \le i < \#[] \vee 0 \le i < \#[] : \#[] = \#[] \wedge [].\,i = [].\,i \rangle$
$= \{\text{Definición de } \#\}$
$\langle \forall_i : 0 \le i < 0 \vee 0 \le i < 0 : 0 = 0 \wedge [].\,i = [].\,i \rangle$
$= \{\text{Rango vacío}\}$
$True$

$f.\,(x \triangleright xs).\,[]$
$= \{\text{Especificación}\}$
$\langle \forall_i : 0 \le i < \#(x \triangleright xs) \vee 0 \le i < \#[] : \#(x \triangleright xs) = \#[] \wedge (x \triangleright xs).\,i = [].\,i \rangle$
$= \{\text{Definición de } \#\}$
$\langle \forall_i : 0 \le i < 1 + \#xs \vee 0 \le i < 0 : 1 + \#xs = 0 \wedge (x \triangleright xs).\,i = [].\,i \rangle$
$= \{\text{Lógica y álgebra}\}$
$\langle \forall_i : 0 \le i < 1 + \#xs : False \wedge (x \triangleright xs).\,i = [].\,i \rangle$
$= \{\text{Lógica}\}$
$\langle \forall_i : 0 \le i < 1 + \#xs : False \rangle$
$= \{\text{Término constante}\}$
$False$

Análogamente se resuelve $f.\,[].\,(y \triangleright ys)$

$f.\,(x \triangleright xs).\,(y \triangleright ys)$
$= \{\text{Especificación}\}$
$\langle \forall_i : 0 \le i < \#(x \triangleright xs) \vee 0 \le i < \#(y \triangleright ys) : \#(x \triangleright xs) = \#(y \triangleright ys) \wedge (x \triangleright xs).\,i = (y \triangleright ys).\,i \rangle$
$= \{\text{Definición de } \#\}$
$\langle \forall_i : 0 \le i < 1 + \#xs \vee 0 \le i < 1 + \#ys : 1 + \#xs = 1 + \#ys \wedge (x \triangleright xs).\,i = (y \triangleright ys).\,i \rangle$
$= \{\text{Aritmética}\}$
$\langle \forall_i : 0 \le i < 1 + \#xs \vee 0 \le i < 1 + \#ys : \#xs = \#ys \wedge (x \triangleright xs).\,i = (y \triangleright ys).\,i \rangle$
$= \{\text{Separación de un término}\}$
$\#xs = \#ys \wedge (x \triangleright xs).0 = (y \triangleright ys).0 \wedge \langle \forall_i : 0 \le i < \#xs \vee 0 \le i < \#ys : \#xs = \#ys \wedge (x \triangleright xs).\,(i+1) = (y \triangleright ys).\,(i+1) \rangle$
$= \{\text{Propiedad de .}\}$
$\#xs = \#ys \wedge x = y \wedge \langle \forall_i : 0 \le i < \#xs \vee 0 \le i < \#ys : \#xs = \#ys \wedge xs.\,i = ys.\,i \rangle$
$= \{\text{Hipótesis inductiva}\}$
$\#xs = \#ys \wedge x = y \wedge f.\,xs.\,ys$

Ejercicio 2