Array of folded patches

Wireless Electromagnetic Technologies - University of Rome Tor Vergata

Professor: Marrocco Gaetano, Students: Blasi Luca, Mastrofini Alessandro, Mucenica Stefan Leonard

Tchebyshev array factor design

The design parameters for the array are:

Parameter	Value
# elements	$2N+1=5$
Mean lobe/side lobe ratio	$R=120 \cong 41.58\,dB$
Frequency	$f\ = 2.1\,GHz$

It's been specifically required to find the optimal inter-element spacing so that the minimum of the beamwidth will be reached:

$$d_{opt} \rightsquigarrow \min\{BW_{fn}\}$$

$$d_{opt} = \lambda \left[1 - \frac{\arccos\left(\frac{1}{\gamma}\right)}{\pi}\right] \tag{1}$$

$$\gamma = \cosh\left[\frac{1}{2N}\ \ln\left(R + \sqrt{R^2-1}\right)\right]$$

where $\lambda = \frac{c}{f}$ is the frequency in the free-space . In this case, $d_{opt} \in \left(\frac{\lambda}{2}, \lambda\right]$, which means that The coefficients a and b related to the Tchebyshev polynomial approximation for the array will be chosen by following the $d_{opt} \in \left(\frac{\lambda}{2}, \lambda\right]$ condition:

$$T_2\,[x = a + b\cos u] = C_0\ +\ 2\,C_1\ \cos u\ +\ C_2 \cos 2u\ =\ (2a^2 + b^2 - 1)\ +\ 4ab\ \cos u + b^2\ \cos 2u \tag{2}$$

Once the amplitude current feed coefficients are computed (C_n , $n = \overline{0,2}$), the tapering efficiency can be calculated:

$$\eta_T\ =\ \frac{1}{2N+1}\ \frac{||C_0 + 2C_1 + 2_2||^2}{C_0^2 + 2C_1^2 + 2C_2^2} \tag{3}$$

Let's consider two cases of uniform spacing array and:

$$\text{Uniform Amplitude (UA)} \quad || \quad \text{Non-uniform Amplitude (NUA, Tchebyshev)} \tag{4}$$

The comparison will show how

$$BW_{fn}^{[UA]} < BW_{fn}^{[NUA]}$$

$$BW_{fn}^{[NUA]} = 2\frac{180}{\pi}\left[\frac{\pi}{2} - \arccos\left(\frac{\arccos\left(\frac{\cos\left(\frac{\pi}{2N}-a\right)}{b}\right)}{k_0 d}\right)\right] \tag{5}$$

$$BW_{fn}^{[UA]} = \frac{2\lambda}{N\,d}\ \frac{180}{\pi}$$

Parameter	Value
Feed coefficients $[A]$	C_0 || $C_1 = C_{-1}$ || $C_2 = C_{-2}$ 41.2 || 29.8 || 9.6
Normalized feed coefficients to $C_{\max}$	C_0^* || $C_1^* = C_{-1}^*$ || $C_2^* = C_{-2}^*$ 1.000 || 0.7215 || 0.2336
Tapering efficiency	$\eta_T = 79\,\%$
Beamwidth	Tchebyshev || Uniform 50.6° || 34.8°

Now, discussing the results is mandatory:

Max/min feed ratio Even if this is the design of a Non-Uniform Amplitude Array, the less the ratio $r_{\max\,/\,\min} = \frac{C_{\max}}{C_{\min}}$ is, the more efficient distribution of current is reached. In this particular design:

$$r_{\max\,/\,\min} \cong 4.39 \tag{6}$$

meaning that if a damage of the element with the $C_{\max}$ level of feed occurs, most part of the efficiency will be lost. In any case, the tapering efficiency shows how it will not be possible to take advantage of $21\,\%$ of the array in an ideal situation, remembering that this design model can be discerned by the real circumstance in terms of the Tchebyshev error [Balanis].

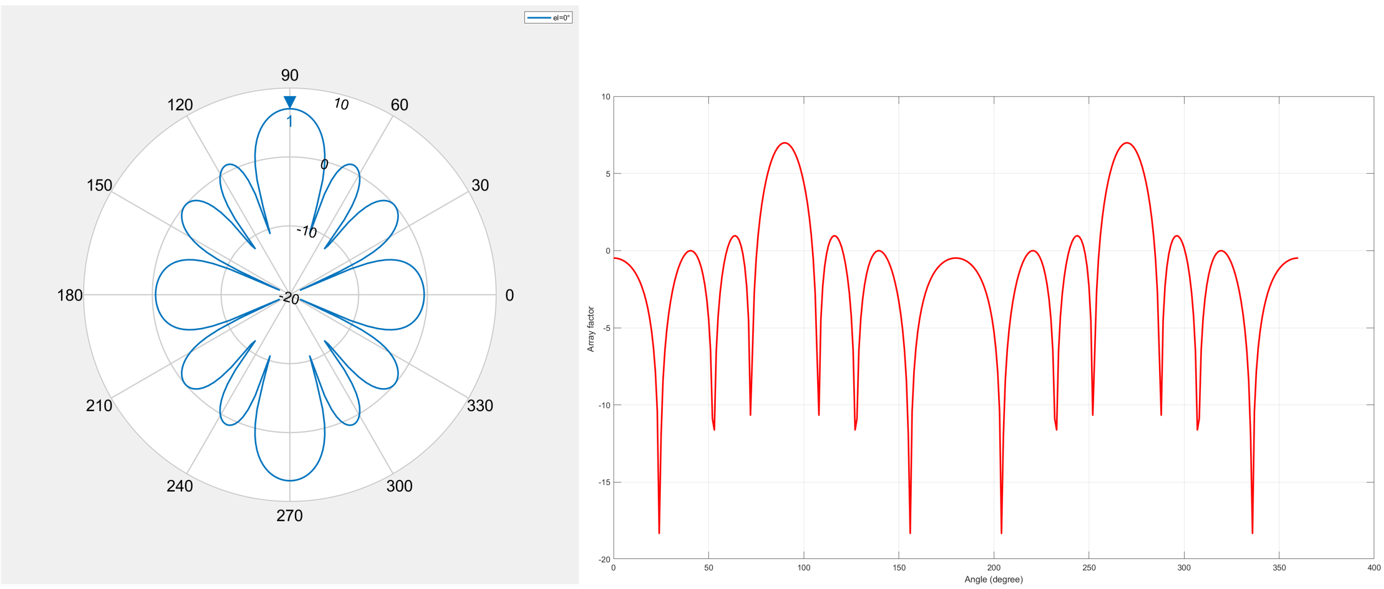
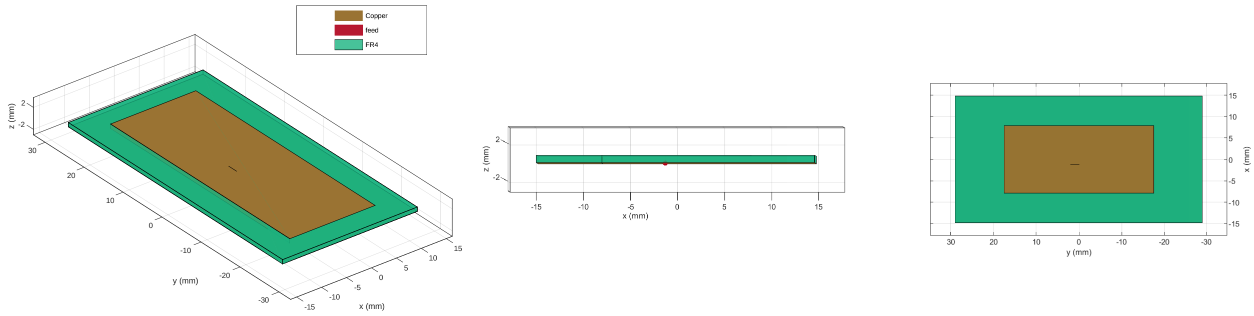


Figure 1: Array factor polar (left) and rectangular (right) diagrams

Rectangular folded patch design

The main components of a rectangular folded patch are: the patch, the substrate (generally accessory, but used in this project), the ground, the rectangular shorting pin between the patch and the ground, and the feed. More details about them will be presented in a short while. Before that, some other remarks are necessary: this antenna will be the element of the array, which will be designed starting actually from a PIFA (*Planar Inverted F Antenna*), given the limitations of the `Antenna Toolbox`, which will be discussed and overcome later on. A general PIFA realized with a dielectric substrate looks like:



By imposing the particular condition by which the width of the rectangular shorting (w_{sc}) equals the patch width size (W_{patch}), the PIFA and the folded patch antenna will be two equivalent structures:

$$W_{patch} = w_{sc} \tag{7}$$

This remark on the PIFA is necessary because generally its structure is not equivalent to that of the folded patch antenna because of the possible variability of the shorting width (w_{sc}), which doesn't always satisfy the above imposed condition. That said, the design requirements are listed below: A preliminary evaluation of the patch parameters have been realized by leaning on a theoretical

Folded patch design parameters	
Parameter/Component	**Value/Type/Material**
Frequency	$2.1\,GHz$
Matched input resistance	$R_{in} = 50\,\Omega$
Substrate	FR4
Relative permittivity Relative permeability Loss tangent Thickness	$\varepsilon_{FR4} = 4.8$ $\mu_{FR4} \cong 1$ $\{\tan(\delta)\}_{FR4} = 0.0260$ $h_{FR4} = 0.8\,mm$
Patch (pre-optimized features)	Copper
Conductivity Thickness Length Width	$\kappa_{copper} = 5.96{\cdot}10^7\,S/m$ $h_{patch} = 3.556 \cdot 10^{-5}\,m$ $L_{patch} \cong \frac{\lambda_{FR4}}{4} = 0.0171\,m$ $W_{patch} \cong 0.419\,m$
Ground (pre-optimized features)	Copper (same conductivity listed above)
Thickness Length Width	$h_{GND} = h_{patch}$ $L_{GND} = 0.04\,m$ $W_{GND} = 0.06\,m$
Feed	Coaxial cable

set of formulas [Balanis]. That's the reason why the characteristics of the patch shown into the table are called "*pre-optimized features*" (same thing applies to the ground component). Thus, an optimization process of all those parameters will be performed in some following steps. Just before that, the formulas of the theoretical model will be pointed out:

$$
\begin{aligned}
L_{patch} + W_{patch} - w_{sc} &= \frac{\lambda_{FR4}}{4} + h_{FR4} \\
W_{patch} &= \frac{\lambda}{2}\sqrt{\frac{2}{\varepsilon_{FR4} + 1}}
\end{aligned} \tag{8}
$$

$$
\begin{aligned}
\varepsilon_{eff} &= \frac{\varepsilon_{FR4}+1}{2} + \frac{\varepsilon_{FR4}-1}{2}\left(1 + 12\frac{h_{FR4}}{W_{patch}}\right)^{-\frac{1}{2}} \\
L_{eff} &= \frac{\lambda_{FR4}}{4} \\
\Delta L &= 0.412\, h \left[\frac{(\varepsilon_{eff}+0.3)\left(\frac{W_{patch}}{h_{FR4}}+0.268\right)}{(\varepsilon_{eff}-0.258)\left(\frac{W_{patch}}{h_{FR4}}+0.8\right)}\right] \\
L &= L_{eff} - 2\Delta L
\end{aligned} \tag{9}
$$

$$
R_r = \frac{120\,\lambda}{W_{patch}}\left[1 - \frac{1}{24}\left(2\pi\frac{h_{FR4}}{\lambda}\right)^2\right]^{-1} \tag{10}
$$

$$
\begin{aligned}
\Theta_E &= 2 \arccos\sqrt{\frac{7.03\,\lambda^2}{4\,(3\,L_e^2 + h_{FR4}^2)\,\pi^2}} \\
\Theta_H &= 2 \arccos\sqrt{\frac{1}{2 + 2\pi\frac{W_{patch}}{\lambda}}}
\end{aligned} \tag{11}
$$

$$
\ell_{feed} = \frac{L_{patch}}{\pi} \arccos\sqrt{\frac{R_{in}}{R_r}} \tag{12}
$$

Where Θ_i $(i = \{E, H\})$ are the half-power beamwidth values given by the E-cut and the H-cut. $\lambda = c/f$ is the free-space wavelength ($c = 299792458\, m/s$ is the light-speed in the free space). R_{in} is the input impedance (a resistance), while R_r is the radiation resistance

Substrate thickness selection Three thickness levels were available for the $FR4$ substrate required this project $(h^{(i)}_{FR4} = \{h^{(1)}_{FR4}, h^{(2)}_{FR4}, h^{(3)}_{FR4}\} = \{0.8, 1.0, 1.6\}\, mm)$

FR4 substrate project thickness levels available		
$h^{(1)}_{FR4} = 0.8\, mm$	$h^{(2)}_{FR4} = 1.0\, mm$	$h^{(3)}_{FR4} = 1.6\, mm$

The `Antenna Toolbox` gives specific information about the mesh density level that should be adopted for the design of the patch antenna components. The only issue is that these details are given only for particular ranges of the ratio indicator called *relative thickness* or *electrical thickness* h_λ. The electrical thickness depends on the ratio between the substrate thickness (h_{FR4}) and the wavelength related to the substrate medium (λ_{FR4}). When a mesh is configured in the `Antenna Toolbox` environment, a specific parameter needs to be adjusted: the maximum edge length of the generic triangle covering the geometry of the antenna $(e_{\max})$. In the case of a relative length h_λ comparable to $1/10$, it's recommended to select a $e_{\max} \cong \lambda/10$. A substrate thickness respecting this relationship is called a *thick substrate*. None of the available substrates verify this condition.

Among them, only the thinnest substrate and the second to last one (thus $h_{FR4} = 0.8\,mm$ and $h_{FR4} = 1.0\,mm$) are part of a range which the Antenna Toolbox provides instructions of. It's the *thin substrate range*: the automatic mesh mode should be adopted for a thin substrate, namely having a relative thickness less or equal than one fifth ($h_\lambda \leq 1/50$). In the specific project case:

$$h_{FR4} = 0.8\,mm \rightsquigarrow h_\lambda = \frac{1}{81} \qquad h_{FR4} = 1.0\,mm \rightsquigarrow h_\lambda = \frac{1}{62} \tag{13}$$

***Thinner substrate choice* rationale**. The quality factor depending on $tan\delta$ is generally low in the FR4 substrate case. This means the FR4 is a big power dispersor. Since increasing h_{FR4} will provoke just more losses in terms of a radiation efficiency drop and since the only thickness values of $0.8\,mm$ and $1.0\,mm$ would give reliable/accurate results in the Antenna Toolbox simulations, the $0.8\,mm$ will be adopted.

Mesh density refinement Although a mesh density choice has already been made by selecting the best maximum edge length $e_{\max}$, the accuracy achievable by using the mesh automatic mode in the case of substrates belonging to the *thin substrate range* will be proved hereafter. An initial study of the mesh density level influence on the reflection coefficient (Γ in dB) evaluated at the resonant frequency ($f = 2.1\,GHz$) has been realized, thus a $\Gamma_{2.1\,GHz} = F(e_{\max})$ function has been plotted with an initial step of $\Delta e_{\max} = 2.5\cdot 1^{-4}\,m$ between every two mesh densities related to their specific $e_{\max}$. This first simulation considered a broader range of $e_{\max}$ variation: $[2.5\cdot 10^{-4}\,m\,,\,6.0\cdot 10^{-4}\,m]$. Since the resulting plot has shown big uncertainty of the reflection coefficient value at the resonant

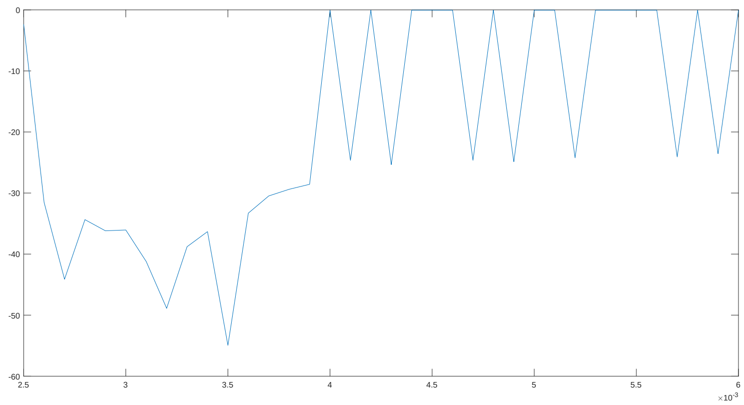
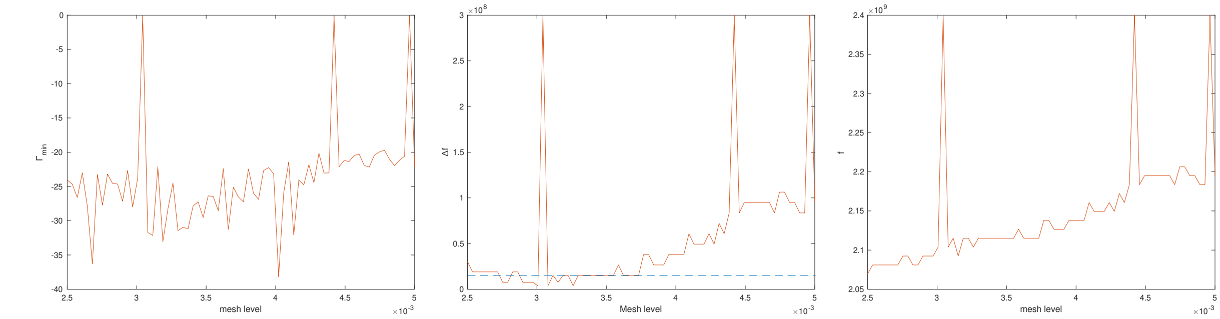


Figure 2: Minimum of the reflection coefficient $\Gamma\,[dB]$ in the frequency range $2.0 \div 2.2\,GHz$ depending on the varying mesh density level

frequency ($\Gamma_{2.1\,GHz}$) at almost every mesh $e_{\max}$ level (primarily due to the big step selected between one density level and another), some more detailed tests have been run, by considering a slightly narrower range ($[2.5\cdot 10^{-4}\,m\,,\,5.0^{-4}\,m]$) and a tchicker evaluation of the maximum edge values (so that the mesh variation step has been remarkably reduced to $1.0^{-4}\,m$). Specifically, the step between two mesh densities level in terms of the maximum edge length of each one has been reduced from a $\Delta e_m = 2.5\cdot 10{-}4$ to $\Delta e_m = 1.0\cdot 10^{-4}$. In all these simulation, an important fact needs to be noted. Even very small variation on the maximum edge length value involved considerable inconsistencies in almost every part of the mesh range in terms of big variations of the frequency at wich Γ reaches its minimum. Thus, considering the frequency f^* at which $\min(\Gamma)$ is actually obtained, instead of evaluatong it at the theoretical resonating frequency value at every mesh level, not only the standard test comparing $e_{\max}$ and Γ has been run, but also some mesh refinement plots representing the relationship between f^* and $e_{\max}$, Δf^* and $e_{\max}$ and also $\Gamma_{dB}(f^*)$ and $e_{\max}$ have been taken into account (where Δf^* is the difference between f^* and the resonant frequency $f = 2.1\,GHz$). More parameter relationships have been collected and this led to the setting of the mesh density choice in terms of $e_{\max}$ that has been selected inside the most stable region (i.e. showing the smallest deviation of the reflection coefficient minimum from the resonant frequency). In the end it's been specifically taken the 'automatic' $e_{\max}$ $(= 3.5^{-4}\,m)$ suggested by the Antenna Toolbox, since this value belongs to the stable region and seems to give the most accurate results. The $e_{\max}$ values belonging to the stable region $([3.1 \cdot 10^{-4}\,m, 3.7 \cdot 10^{-4}\,m])$ exibit slight deviations from the resonant frequency $(\Delta f^* \cong [0.01\,GHz, 0.03\,GHz])$ and the minima of the reflection coefficient vary as follows: $[-24\,dB\,,\,-33\,dB]$.



(a) (b) (c)

Figure 4: (a) (s_m, Γ_{dB}) plot, (b) $(s_m, \Delta f)$ plot, and (c) (s_m, f) plot

Patch parameters refinement After the selection of the maximum edge length of the mesh (consequently, of its density level), a more refined computation of the reflection computation will be made, depending on the patch (i.e. on its length and width), but also on the feed location. Firstly, only a parametrical variation of the feed position across the patch length direction has been considered, depending on variations of L_{patch} and W_{patch}. This means that the first refinement of the feed position has been evaluated starting from its theoretical equation (depending indirectly by W_{patch}). The change of the feed location has been taken into account in the computation of every step of the simulation, thus in every evaluation of the reflection coefficient. The patch size variations provoked wide modifications of the reflection coefficient, which values depending on that have been represented by an initial contour plot (with variations of the patch size in a broader range and with a larger step between one value and another). After that, a simulation in a narrower range variation of the patch size has been run in order to choose from there a set of values of Γ (i.e. a set of couples (L_{patch}, W_{patch})) that should put the patch antenna in the best resonant condition. A set of 20 values has been selected from the second simulation range for the next and more specific simulation. In this third case, the reflection coefficient has been plotted in a range around the resonant frequency $([2.0\,GHz, 2.2\,GHz])$ in order to find which is the best combination for the patch size that makes actually resonate the antenna at the project frequency. Another determining and discriminating factor was the input impedance ($Z_{in} = R_{in} + jY_{in}$, where the real part of Z_{in} is the imput resistance, while the imaginary one is the input reactance), because a impedance matching (at $50\,\Omega$) needed to be achieved for the project. In the ideal case, of course, a reflection coefficient $\Gamma^{(id)} = 0.00 \rightarrow -\infty\,dB$ would be required in order to reach the perfect impedance matching (perfect matching with input resistance at $50.00\,\Omega$ and null reactance). As a real result, before an accurate matching, the Γ value related to all the couple candidates $((L_{patch}, W_{patch}))$ spaced ranged from $-24\,dB$ to $-30\,dB$. Another design choice contributed to the final patch size choice: the feed location varying across the patch width direction. Thus, a parametrical impedance matching depending on that has been run on the best couple candidates. The resulting values are:

$$\Gamma_{final} = -54.94\,dB \qquad R_{in} = 49.86\,\Omega \qquad Y_{in} = 0.11\,S$$

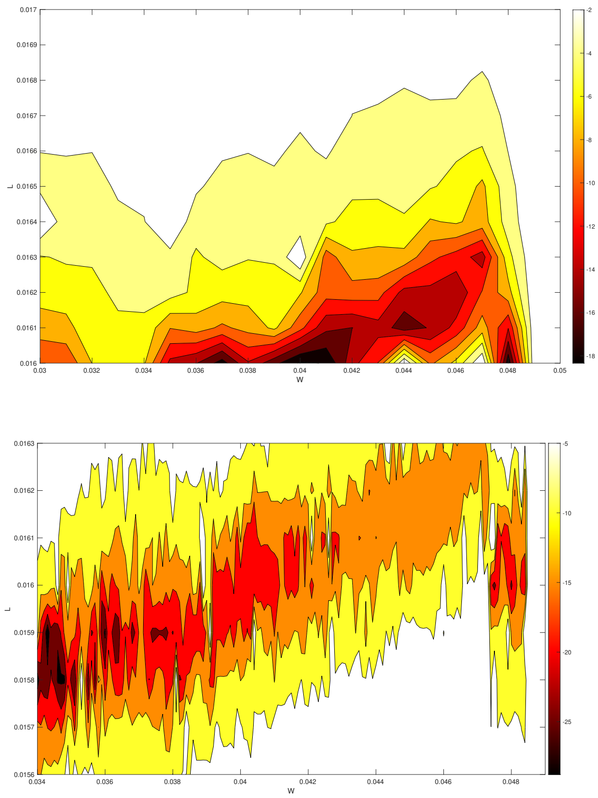


Figure 5: First and second (refined) contour plots depending on the patch size ($L_{patch} and W_{patch}$ variations)

From the PIFA pifa to a PCB stack Since the `Antenna Array Designer` is not able to generate an array of PIFAs , the `Sensor Array Designer` among with the `PCB Antenna Designer` have been used so that all the parameters required for the project could be analyzed. Just before that, a comparison between the single antenna realized with the PIFA structure and the antenna created by using a PCB stack was performed. A single limitation in the use of the `PCB Antenna Designer` has been encountered, but it has been overcome with an approximation strategy. The limitation consisted in the absence of a specific option or combination of commands that would allow to realize the rectangular shorting pin between the patch and the ground. The only thing the designer can do is to replace this kind of shorting pin with a series of small diameter (e.g. $0.4\,mm$) cylindrical shorting pins close to one edge of the patch, across the patch width direction. With this design choice, a very similar behaviour to that of the PIFA structure was possible to simulate. To support this results, a comparison of the 2D patterns (elevation and azymuth cut) of the PIFA to those of the PCB stack antenna has been realized in terms of the mean square error values, shorten MSE, related to the two specific two-dimensional patterns considered (MSE_{el} in the elevation cut directivity pattern and MSE_{az} in the azimuth cut directivity pattern). Furthermore, since the `Sensor Array Designer` allows the plotting of some patterns such as the directivity, a comparison between the 2D patterns of the antenna array of PIFAs and the array of PCBs has been presented. All this discussion was necessary, because moving to the last part of the project required a deeper study of the array of antennas, that will be displayed by using the PCB antenna as the element of the array.

- **Single PIFA and PCB stack comparison**.

$$MSE_{az} \cong 0.16\,dB \qquad MSE_{el} = 0.055\,dB \tag{14}$$

- **Overall PCB and PIFA arrays comparison**. In this case the comparison has been made between

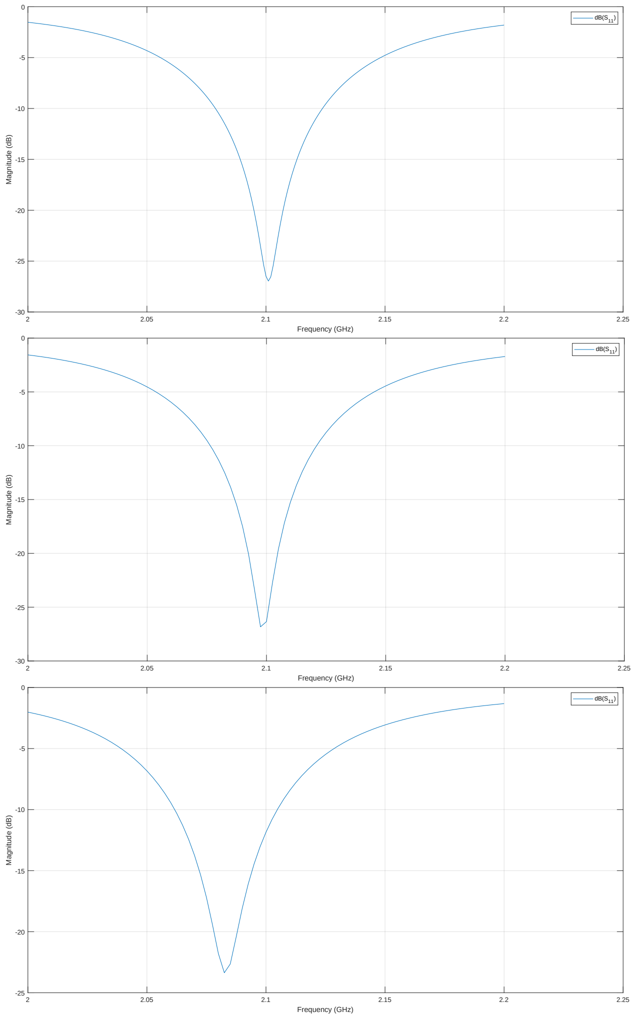


Figure 6: Some of the best Γ plots depending on specific patch size candidates (L_{patch}, W_{patch})

- The main lobe (ML) levels in the two arrays (as a difference between the two directivity (i.e. D_0)values corresponding to that angle):
-

$$\begin{aligned} \Delta D_{0,az}^{(ML)} &= 2.37\,dB \qquad & \Delta D_{0,az}^{(SL)} &= 0.02\,dB \\ \Delta D_{0,el}^{(ML)} &= 1.93\,dB \qquad & \Delta D_{0,el}^{(SL)} &= 4.01\,dB \end{aligned} \tag{15}$$

- The first side lobe levels in the two arrays

both in the broadside case.

Overall patch antenna array performance

In this last part of the project, the array factor and the the element antenna (PIFA or PCB stack) designs will be combined so that the total effect will be examined. As it's been already mentioned in the previous paragraph, some of the information describing the overall array performance can be asked from both the `Sensor Array Analyzer` (where the array of PIFAs was designed) and the `PCB/Antenna Array Designer` (where the single patch antenna made starting from the PCB stack and also the whole array can be constructed). The performance of the overall array will be evaluated in two cases: in the broadside case (90°) but also at 45° off the boresight direction. First of all, it

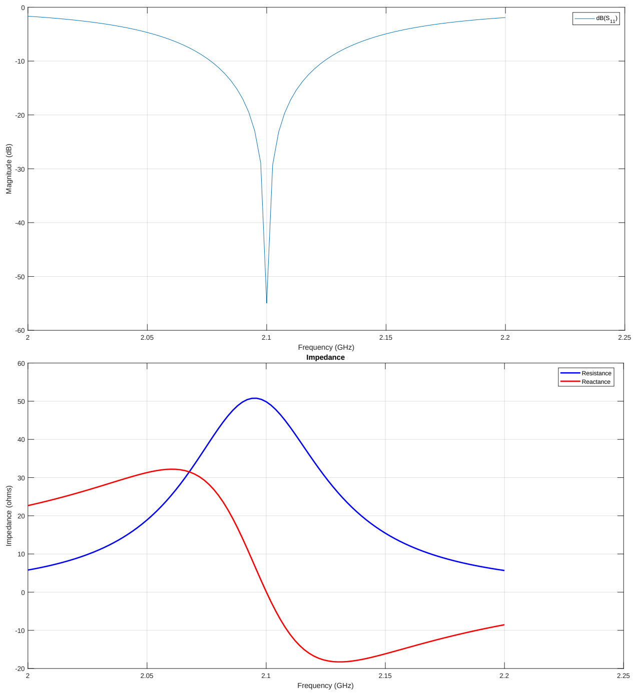


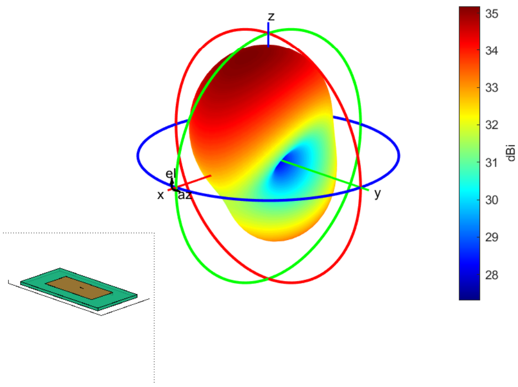
Figure 7: Final Γ and impedance matching plots after further refinement including w_{feed} change

is very simple to identify the phase shift coefficients in the broadside case because they all equal 0° (there's no actual phase shift between antennas). In the second case, some manual calculation can be made in order to insert the phase shifts between the single antennas (this is the case of the PCB stack array), and it can be also automatically computed by using the array of PIFAs. The generic procedure is shown in Eqn. x, where $n = \{1, 5\}$

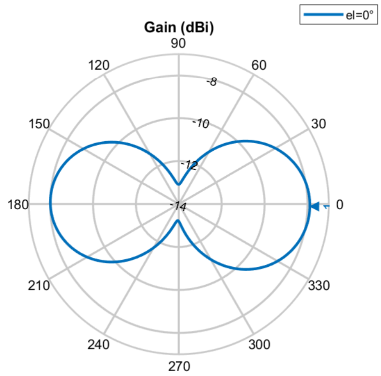
$$\begin{aligned} u_0 = \alpha d_{opt} &= 2\pi \frac{d_{opt}}{\lambda} \cos(\theta_0) \\ \alpha_n = n\,\alpha\,d_{opt} &= nu_0 \end{aligned} \tag{16}$$

Total array gain

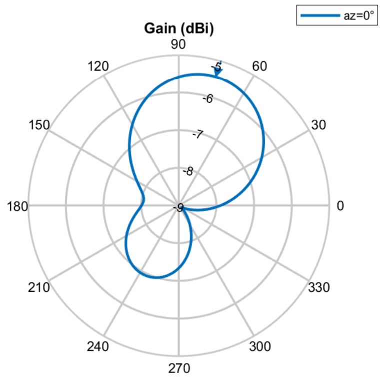
In order to get and describe the last plots of the project, it is not possible to recur to the `Sensor Array Analyzer` because some of the commands, such as and `EHfields` and `patternMultiply`, are not available in this tool. Thus, in this last part, only the array of PCBs will be used, because it's compatible with these commands. The total array gain will be computed and displayed in two different cases: by mean of a fullwave model (with no approximation) and by using the pattern multiplication principle. It will be indirectly plotted because of tool limitation, specifically related to the `patternMultiply`, which doesn't get any gain plot, but it can get the directivity plot. Since these two are proportional, so that the gain is obtainable by multiplying the directivity by the radiation efficiency (η_R).



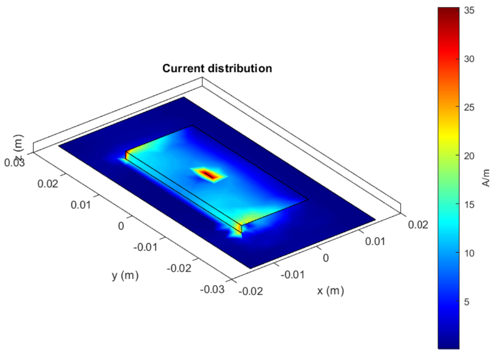
(a)



(b)

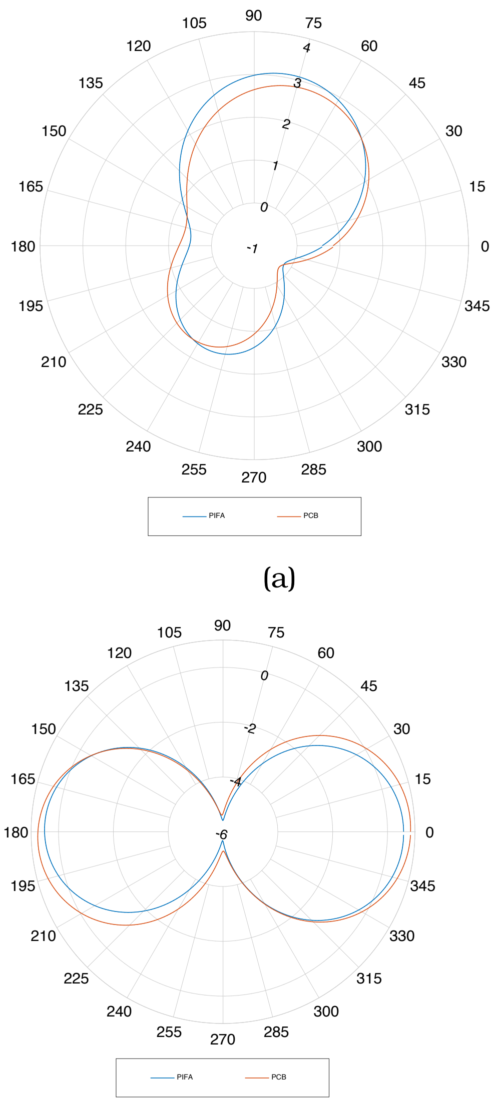


(c)



(d)

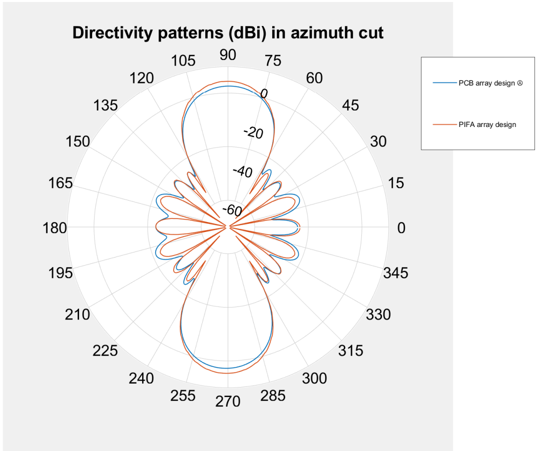
Figure 8: Gain patterns (a) in 3D, (b) in the nulle elevation plane, (c) in the null azimuth plane and (d) 3D current plot on the patch antenna



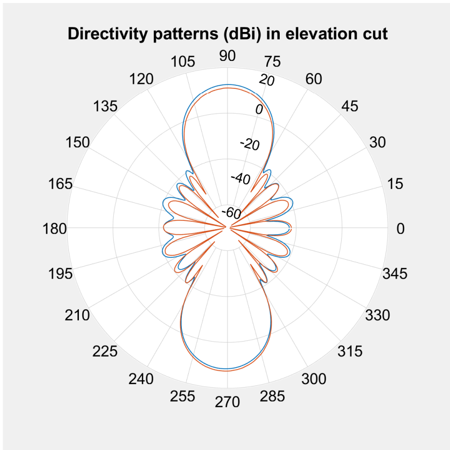
(a)

(b)

Figure 9: PIFA and PCB single antenna directivity patterns (dB) in the Azimuth cut ($\theta_{el} = 0°$, (a)) and in the Elevation cut ($\phi_{az} = 0°$, (b))



(a)



(b)

Figure 10: PIFA and PCB arrays directivity patterns (dB) in the Azimuth cut ($\theta_{el} = 0°$, (a)) and in the Elevation cut ($\phi_{az} = 0°$, (b))

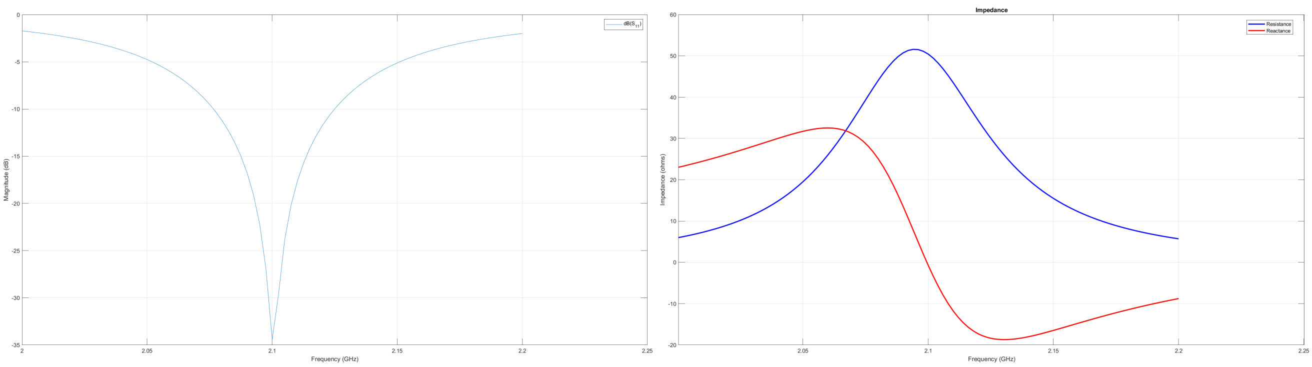


Figure 11: Reflection coefficient (left) and impedances (right) plots depending on $f \in 2.0 \div 2.1\,GHz$. This are the resulting plots before W_{feed} optimization.