Array of folded patches

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Tchebyshev array factor design

The design parameters for the array are:

Parameter	Value
# elements	$2N+1=5$
Mean lobe/side lobe ratio	$R=120 \cong 41.58\, dB$
Frequency	$f = 2.1\, GHz$

It's been specifically required to find the optimal inter-element spacing so that the minimum of the beamwidth will be reached:

$$d_{opt} \rightsquigarrow \min\{BW_{fn}\}$$

$$d_{opt} = \lambda \left[1 - \frac{\arccos\left(\frac{1}{\gamma}\right)}{\pi}\right] \tag{1}$$

$$\gamma = \cosh\left[\frac{1}{2N} \ln\left(R + \sqrt{R^2-1}\right)\right]$$

where $\lambda = \frac{c}{f}$ is the frequency in the free-space . In this case, $d_{opt} \in \left(\frac{\lambda}{2}, \lambda\right]$, which means that The coefficients a and b related to the Tchebyshev polynomial approximation for the array will be chosen by following the $d_{opt} \in \left(\frac{\lambda}{2}, \lambda\right]$ condition:

$$T_2\left[x = a + b\cos u\right] = C_0 + 2C_1 \cos u + C_2 \cos 2u = (2a^2+b^2-1) + 4ab\cos u + b^2\cos 2u \tag{2}$$

Once the amplitude current feed coefficients are computed (C_n, $n=\overline{0,2}$), the tapering efficiency can be calculated:

$$\eta_T = \frac{1}{2N+1} \frac{||C_0 + 2C_1 + 2_2||^2}{C_0^2 + 2C_1^2 + 2C_2^2} \tag{3}$$

Let's consider two cases of uniform spacing array and:

$$\text{Uniform Amplitude (UA)} \quad || \quad \text{Non-uniform Amplitude (NUA, Tchebyshev)} \tag{4}$$

The comparison will show how

$$BW_{fn}^{[UA]} < BW_{fn}^{[NUA]}$$

$$BW_{fn}^{[NUA]} = 2\frac{180}{\pi}\left[\frac{\pi}{2} - \arccos\left(\frac{\arccos\left(\frac{\cos\left(\frac{\pi}{2N}-a\right)}{b}\right)}{k_0 d}\right)\right] \tag{5}$$

$$BW_{fn}^{[UA]} = \frac{2\lambda}{N\,d}\frac{180}{\pi}$$

Parameter	Value
Feed coefficients $[A]$	C_0 || $C_1 = C_{-1}$ || $C_2 = C_{-2}$ 41.2 || 29.8 || 9.6
Normalized feed coefficients to $C_{\max}$	C_0^* || $C_1^* = C_{-1}^*$ || $C_2^* = C_{-2}^*$ 1.000 || 0.7215 || 0.2336
Tapering efficiency	$\eta_T = 79\,\%$
Beamwidth	Tchebyshev || Uniform 50.6° || 34.8°

Now, discussing the results is mandatory:

Max/min feed ratio Even if this is the design of a Non-Uniform Amplitude Array, the less the ratio $r_{\max\,/\,\min} = \frac{C_{\max}}{C_{\min}}$ is, the more efficient distribution of current is reached. In this particular design:

$$r_{\max\,/\,\min} \cong 4.39 \tag{6}$$

meaning that if a damage of the element with the $C_{\max}$ level of feed occurs, most part of the efficiency will be lost. In any case, the tapering efficiency shows how it will not be possible to take advantage of $21\,\%$ of the array in an ideal situation, remembering that this design model can be discerned by the real circumstance in terms of the Tchebyshev error [Balanis].

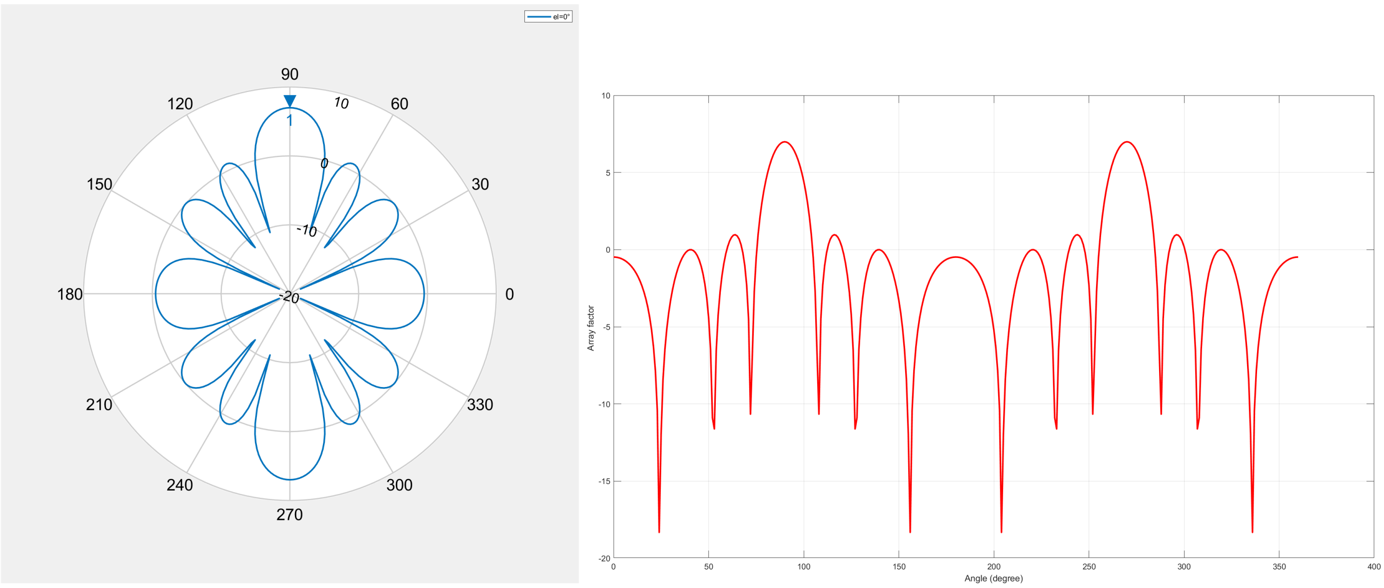
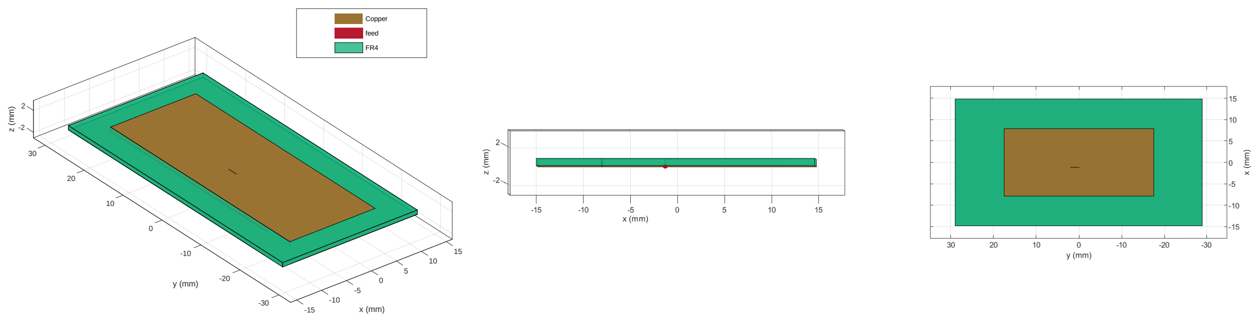


Figure 1: Array factor polar (left) and rectangular (right) diagrams

Rectangular folded patch design

The main components of a rectangular folded patch are: the patch, the substrate (generally accessory, but used in this project), the ground, the rectangular shorting pin between the patch and the ground, and the feed. More details about them will be presented in a short while. Before that, some other remarks are necessary: this antenna will be the element of the array, which will be designed starting actually from a PIFA (*Planar Inverted F Antenna*), given the limitations of the `Antenna Toolbox`, which will be discussed and overcome later on. A general PIFA realized with a dielectric substrate looks like:



By imposing the particular condition by which the width of the rectangular shorting (w_{sc}) equals the patch width size (W_{patch}), the PIFA and the folded patch antenna will be two equivalent structures:

$$W_{patch} = w_{sc} \tag{7}$$

This remark on the PIFA is necessary because generally its structure is not equivalent to that of the folded patch antenna because of the possible variability of the shorting width (w_{sc}), which doesn't always satisfy the above imposed condition. That said, the design requirements are listed below: A preliminary evaluation of the patch parameters have been realized by leaning on a theoretical

Folded patch design parameters	
Parameter/Component	**Value/Type/Material**
Frequency	$2.1\,GHz$
Matched input resistance	$R_{in} = 50\,\Omega$
Substrate	FR4
Relative permittivity Relative permeability Loss tangent Thickness	$\varepsilon_{FR4} = 4.8$ $\mu_{FR4} \cong 1$ $\{\tan(\delta)\}_{FR4} = 0.0260$ $h_{FR4} = 0.8\,mm$
Patch (pre-optimized features)	Copper
Conductivity Thickness Length Width	$\kappa_{copper} = 5.96{\cdot}10^{7}\,S/m$ $h_{patch} = 3.556 \cdot 10^{-5}\,m$ $L_{patch} \cong \frac{\lambda_{FR4}}{4} = 0.0171\,m$ $W_{patch} \cong 0.419\,m$
Ground (pre-optimized features)	Copper (same conductivity listed above)
Thickness Length Width	$h_{GND} = h_{patch}$ $L_{GND} = 0.04\,m$ $W_{GND} = 0.06\,m$
Feed	Coaxial cable

set of formulas [Balanis]. That's the reason why the characteristics of the patch shown into the table are called "*pre-optimized features*" (same thing applies to the ground component). Thus, an optimization process of all those parameters will be performed in some following steps. Just before that, the formulas of the theoretical model will be pointed out:

$$\begin{aligned} L_{patch} + W_{patch} - w_{sc} &= \frac{\lambda_{FR4}}{4} + h_{FR4} \\ W_{patch} &= \frac{\lambda}{2}\sqrt{\frac{2}{\varepsilon_{FR4}+1}} \end{aligned} \tag{8}$$

$$\begin{aligned} \varepsilon_{eff} &= \frac{\varepsilon_{FR4}+1}{2} + \frac{\varepsilon_{FR4}-1}{2}\left(1+12\frac{h_{FR4}}{W_{patch}}\right)^{-\frac{1}{2}} \\ L_{eff} &= \frac{\lambda_{FR4}}{4} \\ \Delta L &= 0.412\, h \left[\frac{(\varepsilon_{eff}+0.3)\left(\frac{W_{patch}}{h_{FR4}}+0.268\right)}{(\varepsilon_{eff}-0.258)\left(\frac{W_{patch}}{h_{FR4}}+0.8\right)}\right] \\ L &= L_{eff} - 2\Delta L \end{aligned} \tag{9}$$

$$R_r = \frac{120\,\lambda}{W_{patch}}\left[1-\frac{1}{24}\left(2\pi\frac{h_{FR4}}{\lambda}\right)^2\right]^{-1} \tag{10}$$

$$\begin{aligned} \Theta_E &= 2\arccos\sqrt{\frac{7.03\,\lambda^2}{4\,(3\,L_e^2+h_{FR4}^2)\,\pi^2}} \\ \Theta_H &= 2\arccos\sqrt{\frac{1}{2+2\pi\frac{W_{patch}}{\lambda}}} \end{aligned} \tag{11}$$

$$\ell_{feed} = \frac{L_{patch}}{\pi}\arccos\sqrt{\frac{R_{in}}{R_r}} \tag{12}$$

Where Θ_i $(i = \{E, H\})$ are the half-power beamwidth values given by the E-cut and the H-cut. $\lambda = c/f$ is the free-space wavelength ($c = 299792458\,m/s$ is the light-speed in the free space). R_{in} is the input impedance (a resistance), while R_r is the radiation resistance

Refinement with `MatLab MoM`

Substrate thickness selection

Three thickness levels were available for the $FR4$ substrate required this project ($h_{FR4}^{(i)} = \{h_{FR4}^{(1)}, h_{FR4}^{(2)}, h_{FR4}^{(3)}\} = \{0.8, 1.0, 1.6\}\,mm$)

FR4 substrate project thickness levels available		
$h_{FR4}^{(1)} = 0.8\,mm$	$h_{FR4}^{(2)} = 1.0\,mm$	$h_{FR4}^{(3)} = 1.6\,mm$

The `Antenna Toolbox` gives specific information about the mesh density level that should be adopted for the design of the patch antenna components. The only issue is that these details are given only for particular ranges of the ratio indicator called *relative thickness* or *electrical thickness* h_λ. The electrical thickness depends on the ratio between the substrate thickness (h_{FR4}) and the wavelength related to the substrate medium (λ_{FR4}). When a mesh is configured in the `Antenna Toolbox` environment, a specific parameter needs to be adjusted: the maximum edge length of the generic triangle covering the geometry of the antenna ($e_{\max}$). In the case of a relative length h_λ comparable to $1/10$, it's recommended to select a $e_{\max} \cong \lambda/10$. A substrate thickness respecting this relationship is called a *thick substrate*. None of the available substrates verify this condition. Among them, only the thinnest substrate and the second to last one (thus $h_{FR4} = 0.8\,mm$ and $h_{FR4} = 1.0\,mm$) are part of a range which the `Antenna Toolbox` provides instructions of. It's the *thin substrate range*: the automatic mesh mode should be adopted for a thin substrate, namely having a relative thickness less or equal than one fifth ($h_\lambda \leq 1/50$). In the specific project case:

$$h_{FR4} = 0.8\,mm \rightsquigarrow h_\lambda = \frac{1}{81} \qquad h_{FR4} = 1.0\,mm \rightsquigarrow h_\lambda = \frac{1}{62} \tag{13}$$

> ***Thinner substrate choice* rationale.** The quality factor depending on $tan\delta$ is generally low in the FR4 substrate case. This means the FR4 is a big power dispersor. Since increasing h_{FR4} will provoke just more losses in terms of a radiation efficiency drop and since the only thickness values of $0.8\,mm$ and $1.0\,mm$ would give reliable/accurate results in the `Antenna Toolbox` simulations, the $0.8\,mm$ will be adopted.

Mesh density refinement Although a mesh density choice has already been made, the accuracy achievable by using the mesh automatic mode in the case of substrates belonging to the *thin substrate range* will be proved hereafter. An initial study of the mesh density level influence on the reflection coefficient (Γ in dB) evaluated at the resonant frequency ($f = 2.1\,GHz$) has been realized. Since

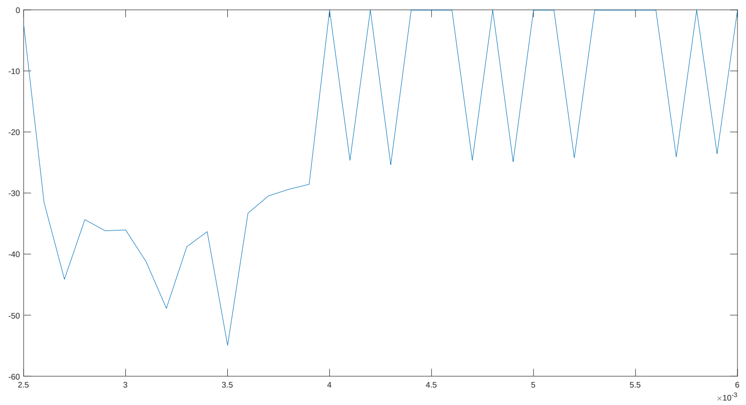
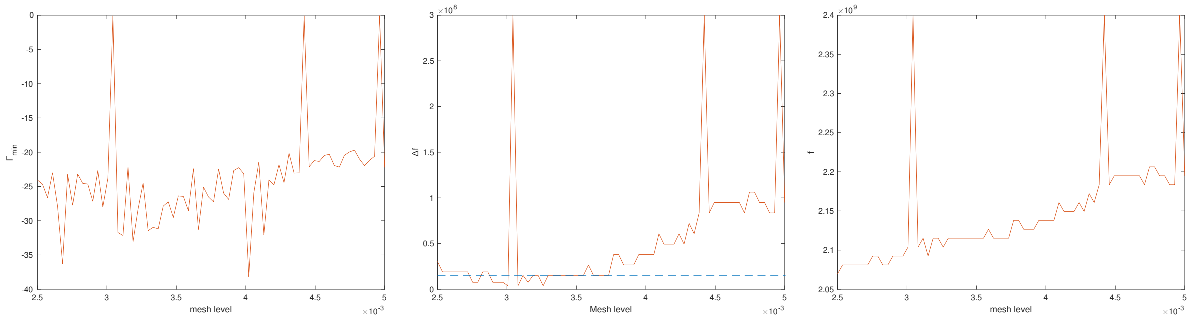


Figure 2: Minimum of the reflection coefficient $\Gamma\,[dB]$ in the frequency range $2.0 \div 2.2\,GHz$ depending on the varying mesh density level

the resulting plot has shown big uncertainty of the Γ_{dB} value at almost every frequency (primarily due to the big step selected between one density level and another), some more detailed test have been made. Specifically, the step has been reduced from a $\Delta s_m = 2.5 \cdot 10{-}4$ variation on the mesh density level to $\Delta s_m = 1.0 \cdot 10^{-4}$. Moreover, considering the frequency f^* at which $\min(\Gamma_{dB})$ is obtained, depending ofn variations of the mesh density level, not only the standard test comparing s_m and Γ_{dB} has been run, but also some mesh refinement plotting representing the relationship between f^* vs s_m, Δf^* vs s_m and also $\Gamma_{dB}(f^*)$ vs s_m have been taken into account. This led to more parameter relationships and to the setting of the mesh density choice s_m that has been selected inside a more accurate range.

Patch parameters

$$\begin{aligned} L + W - w_{SC} &= \frac{\lambda}{4} + h_{sub} \\ W &= \frac{\lambda_0}{2}\sqrt{\frac{2}{\varepsilon_r + 1}} \end{aligned} \tag{14}$$



(a) (b) (c)

Figure 4: (a) (s_m, Γ_{dB}) plot, (b) $(s_m, \Delta f)$ plot, and (c) (s_m, f) plot

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\begin{aligned}
BW_E &= 2 \arccos \sqrt{\frac{7.03\,\lambda_0^2}{4\,(3\,L_e^2 + h^2)\,\pi^2}} \\
BW_H &= 2 \arccos \sqrt{\frac{1}{2 + k_0\,W}}
\end{aligned} \tag{15}
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$$
\ell_{feed} = \frac{L}{\pi} \arccos \sqrt{\frac{R_{in}}{R_r}} \tag{16}
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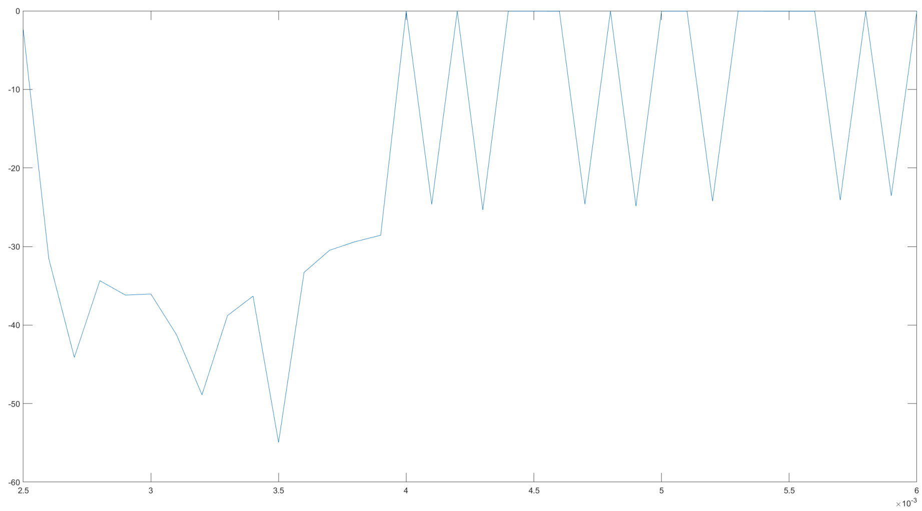

Figure 5: Minimum of the reflection coefficient $\Gamma\,[dB]$ in the frequency range $2.0 \div 2.2\,GHz$ depending on the varying mesh density level

Overall array performance evaluation

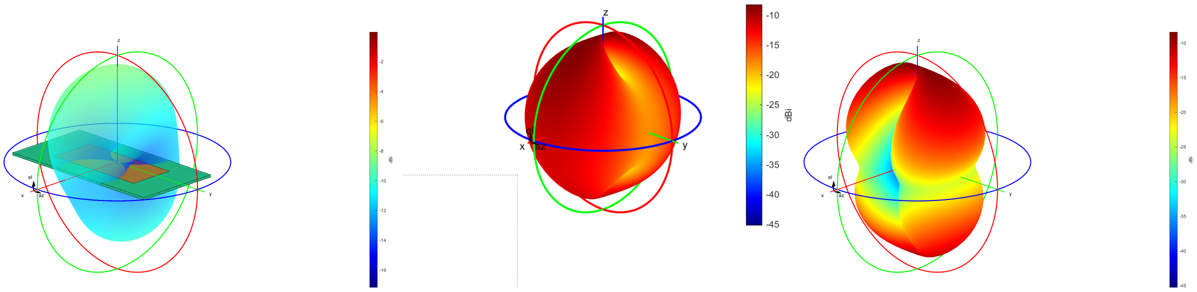

Figure 6: Gain pattern (left), gain pattern with vertical polarization (center) and with the horizontal one (right)

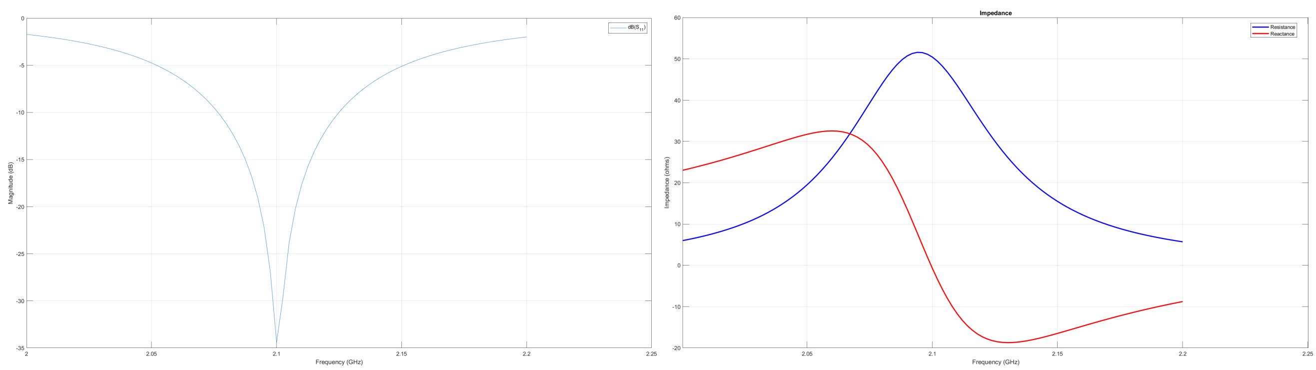

Figure 7: Reflection coefficient (left) and impedances (right) plots depending on $f \in 2.0 \div 2.1\, GHz$