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Design of an array of folded patches

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Abstract

Abstract

Tchebyshev array factor design

A Tchebyshev array factor will be designed in this part, so that the specific structure of the single element of the array is disregarded for the moment. The array is represented by a linear distribution of five elements $(n_{el} = 5)$, which are supposed to be uniformly spaced and with a non-uniform amplitude for the feed arrangement, that anyway is going to be symmetrical (starting from the element in the middle, located in the origin of the geometrical axes). Also the "mean lobe to side lobet ratio" (R) will be one of the input design variables. Thus, the variables of this design part are the element inter-spacing (d_{ont} , which needs to be optimal such that the beamwidth is minimized), the feed amplitude of the single antenna (each one represented by a C_i , with $i \in \{-2, -1, 0, 1, 2\}$, the i^{th} current coefficient - these unknowns are actually just three and not five, due to the symmetrical amplitude distribution hyphotesis), the operating resonant frequency required for the single antenna (f) and some other quantities related to them, such as the tapering efficiency (η_T) and the beamwidth (BW_{fn}) . Moreover, the difference between the beamwidth of the Tchebyshev array (Uniform Spacing Non Uniform Amplitude case, shorten NUA) and that of a uniform array having the same number of elements and inter-spacing value but a uniform amplitude feed distribution (Uniform Spacing Uniform Amplitude, shorten UA) will be discussed. The input design variables related to the Tchebyshev array factor (i.e. n_{el} , R and f) are listed in **table 1**.

Starting from the optimal inter-element spacing, it requires some secondary variables in orderer to be evaluated. Firstly, a coefficient (γ , depending on R and indirectly on the number of array elements) needs to be calculated, among with the wavelength in the free space (λ , as the ratio between the light speed in the free space - c - and the operating frequency f). Having those quantities, d_{opt} minimizing the BW_{fn} can be computed by using the **eq. (1)** group.

Array factor input design variabiles		
Parameter	Value	
# elements	$n_{el} = 2N + 1 = 5$	
Mean lobe/side lobe ratio	$R = 120 \cong 41.58 dB$	
Frequency	f = 2.1 GHz	

Table 1: Table trial

$$d_{opt} \rightsquigarrow \min\{BW_{fn}\}$$

$$d_{opt} = \lambda \left[1 - \frac{\arccos\left(\frac{1}{\gamma}\right)}{\pi} \right] \tag{1}$$

$$\gamma = \cosh\left[\frac{1}{2N}\ln\left(R + \sqrt{R^2 - 1}\right)\right]$$

The array factor evaluation through the Tchebyshev polynomial comes as the next step. The Tchebyshev polynomial approximation to the second order $(T_2(x))$ is sufficient so the current coefficients can be obtained, remembering that a symmetrical amplitude distribution hypothesis has been made. In this case the current coefficients C_i , $i \in \{-2, -1, 0, 1, 2\}$ follow the rule $C_i = C_{-i}$, thus their symbolic representation can be simplified as follows: C_n , $n \in \{0, 1, 2\}$ (or just $n = \overline{0, 2}$). Since the *Riblet variation* to the *Dolph-Tchebyshev synthesis model* has been used, the variable of $T_2(x)$ becomes $x = a + b\cos(u)$ and the formulation for $d_{opt} \in (\lambda/2, \lambda]$ differs from the standard model. The coefficients a and b are related to the maximum value selected in the sub-domain of $T_2(x)$ (the window of visible radiation lobes). This maximum, called for example x_1 , corresponds to the main lobe in which it can be convertend by evaluating the array factor $|T_2(x_1)|$ (some additional references to the specific formulas that need to be used in order to compute can be found in [1] and [2]). That said, the current coefficients can be extracted from $T_2(x)$ (so by using eq. (2)). Resulting uniquely out of a C_n dependence, the tapering efficiency is obtained by using eq. (3).

Next, both non uniform amplitude (Tchebyshev array, NUA) and uniform amplitude (UA) cases are compared. The comparison shows how the BW_{fn} in the UA case (i.e. $BW_{fn}^{[UA]}$) is narrower than that of NUA case (i.e. $BW_{fn}^{[NUA]}$). This result (see **eq. (4)** and **table 2**) is generally effective and the comparison has been made to show that this particular design case confirms the general condition.

All the numerical quantities related to the Tchebyshev array design are gathered together in **table 2**. After these quantities are calculated, some important design considerations can be made about the array efficiency. A Non-Uniform Amplitude Array Factor has been designed by using the Riblet variation of the Dolph-Tchebyshev array synthesis model. Considering the *maximum to minimum feed ratio*:

$$r_{\rm max\,/\,min} = \frac{C_{\rm max}}{C_{\rm min}}$$

the less $r_{\rm max/min}$ is, the more efficient distribution of current is reached. In this particular design, the requirement was to to find the d_{opt} which minimizes the beamwidth, starting from the input design variables. Thus, the $r_{\rm max/min}$ is a straight consequence of the current coefficients and its optimal value has not been the seek of this project. Anyway, for this design, $r_{\rm max/min} \cong 4.39$ meaning that if a damage of the element with the $C_{\rm max}$ (i.e. C_0) level of feed occurs, most part of the efficiency will be lost. In any case, the tapering efficiency shows how it will not be possible to take advantage of 21 % of the array in an ideal situation, remembering that this design model can be discerned by the real circumstance in terms of the Tchebyshev error (see [1]).

$$T_2[x = a + b\cos u] = C_0 + 2C_1\cos u + C_2\cos 2u = (2a^2 + b^2 - 1) + 4ab\cos u + b^2\cos 2u$$
 (2)

$$\eta_T = \frac{1}{2N+1} \frac{||C_0 + 2C_1 + 2_2||^2}{C_0^2 + 2C_1^2 + 2C_2^2}$$
 (3)

$$BW_{fn}^{[UA]}\!<\!BW_{fn}^{[NUA]}$$

$$BW_{fn}^{[NUA]} = 2\frac{180}{\pi} \left[\frac{\pi}{2} - \arccos\left(\frac{\cos\left(\frac{\cos\left(\frac{\pi}{2N} - a\right)}{b}\right)}{k_0 d}\right) \right]$$
(4)

$$BW_{fn}^{[UA]} = \frac{2\lambda}{N d} \frac{180}{\pi}$$

Now, discussing the results is mandatory:

Parameter	Value
Feed coefficients $[A]$	$egin{array}{c c c c c c c c c c c c c c c c c c c $
Normalized feed coefficients to C_{max}	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
Tapering efficiency	$\eta_T = 79\%$
Beamwidth	Tchebyshev Uniform 50.6° 34.8°

Table 2: Tchebyshev array design results

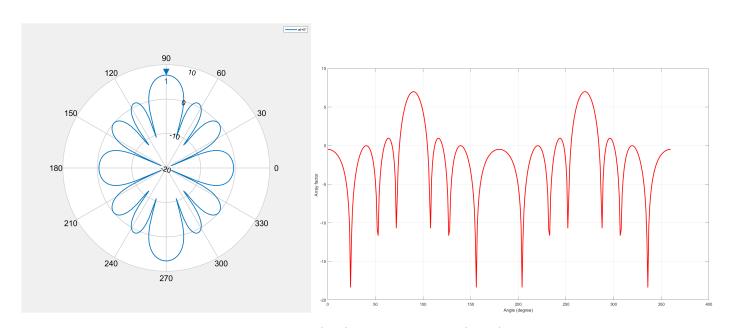


Figure 1: Array factor polar (left) and rectangular (right) diagrams

Rectangular folded patch design

The main components of a rectangular folded patch are: the patch, the substrate (generally accessory, but used in this project), the ground, the rectangular shorting pin between the patch and the ground, and the feed. More details about them will be presented in a short while. Before that, some other remarks are necessary: this antenna will be the element of the array, which will be designed starting actually from a PIFA (*Planar Inverted F Antenna*), given the limitations of the Antenna Toolbox, which will be discussed and overcome later on. A general PIFA realized with a dielectric substrate is shown in fig. 2.

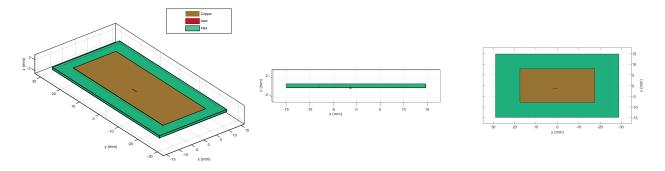


Figure 2: PIFA realized with dielectric substrate

By imposing the particular condition by which the width of the rectangular shorting (w_{sc}) equals the patch width size (W_{patch}) , the PIFA and the folded patch antenna will be two equivalent structures:

$$W_{patch} = W_{sc} \tag{5}$$

This remark on the PIFA is necessary because generally its structure is not equivalent to that of the folded patch antenna because of the possible variability of the shorting width (w_{sc}) , which doesn't always satisfy the above imposed condition. That said, the design requirements are listed below: A preliminary evaluation of the patch parameters have been realized by leaning on a theoretical set of formulas [Balanis]. That's the reason why the characteristics of the patch shown into the table are called "pre-optimized features" (same thing applies to the ground component). Thus, an optimization process of all those parameters will be performed in some following steps. Just before that, the formulas of the theoretical model will be pointed out:

$$L_{patch} + W_{patch} - w_{sc} = \frac{\lambda_{FR4}}{4} + h_{FR4}$$

$$W_{patch} = \frac{\lambda}{2} \sqrt{\frac{2}{\varepsilon_{FR4} + 1}}$$

$$\varepsilon_{eff} = \frac{\varepsilon_{FR4} + 1}{2} + \frac{\varepsilon_{FR4} - 1}{2} \left(1 + 12 \frac{h_{FR4}}{W_{patch}}\right)^{-\frac{1}{2}}$$

$$L_{eff} = \frac{\lambda_{FR4}}{4}$$

$$\Delta L = 0.412 h \left[\frac{(\varepsilon_{eff} + 0.3) \left(\frac{W_{patch}}{h_{FR4}} + 0.268\right)}{(\varepsilon_{eff} - 0.258) \left(\frac{W_{patch}}{h_{FR4}} + 0.8\right)} \right]$$

$$L = L_{eff} - 2\Delta L$$
(6)

Folded patch design parameters		
Parameter/Component	Value/Type/Material	
Frequency	2.1 GHz	
Matched input resistance	$R_{in} = 50 \Omega$	
Substrate	FR4	
Relative permittivity	$\varepsilon_{FR4} = 4.8$	
Relative permeability	$\mu_{FR4} \cong 1$	
Loss tangent	$\left\{ \tan(\delta) \right\}_{FR4} = 0.0260$	
Thickness	$h_{FR4} = 0.8 mm$	
Patch	Copper	
(pre-optimized features)		
Conductivity	$\kappa_{copper} = 5.96 \cdot 10^7 S/m$	
Thickness	$h_{patch} = 3.556 \cdot 10^{-5} m$	
Length	$L_{patch} \cong \frac{\lambda_{FR4}}{\Lambda} = 0.0171 m$	
Width	$W_{patch} \cong 0.419 m$	
Ground	Copper	
(pre-optimized features)	(same conductivity listed	
(pre-optimized reatures)	above)	
Thickness	$h_{GND} = h_{patch}$	
Length	$L_{GND} = 0.04 m$	
Width	$W_{GND} = 0.06 m$	
Feed	Coaxial cable	

$$R_{r} = \frac{120 \,\lambda}{W_{patch}} \left[1 - \frac{1}{24} \left(2\pi \, \frac{h_{FR4}}{\lambda} \right)^{2} \right]^{-1} \tag{8}$$

$$\Theta_E = 2 \arccos \sqrt{\frac{7.03 \,\lambda^2}{4 \,(3 \,L_e^2 + h_{FR4}^2) \,\pi^2}}$$

$$\Theta_H = 2 \arccos \sqrt{\frac{1}{2 + 2\pi \,\frac{W_{patch}}{\lambda}}}$$
(9)

$$\ell_{feed} = \frac{L_{patch}}{\pi} \arccos \sqrt{\frac{R_{in}}{R_r}}$$
 (10)

Where Θ_i ($i=\{E,H\}$) are the half-power beamwidth values given by the E-cut and the H-cut. $\lambda=c/f$ is the free-space wavelength ($c=299792458\,m/s$ is the light-speed in the free space). R_{in} is the input impedance (a resistance), while R_r is the radiation resistance

Refinement with MatLab MoM

Substrate thickness selection

Three thickness levels were available for the FR4 substrate required this project $(h_{FR4}^{(i)}=\{h_{FR4}^{(1)},h_{FR4}^{(2)},h_{FR4}^{(3)}\}=\{0.8,1.0,1.6\}$ mm)

FR4 substrate	project thickness	levels available
$h_{FR4}^{(1)} = 0.8 mm$	$h_{FR4}^{(2)} = 1.0 mm$	$h_{FR4}^{(3)} = 1.6 mm$

The Antenna Toolbox gives specific information about the mesh density level that should be adopted for the design of the patch antenna components. The only issue is that these details are given only for particular ranges of the ratio indicator called relative thickness or electrical thickness h_{λ} . The electrical thickness depends on the ratio between the substrate thickness (h_{FR4}) and the wavelength related to the substrate medium (λ_{FR4}) . When a mesh is configured in the Antenna Toolbox environment, a specific parameter needs to be adjusted: the maximum edge length of the generic triangle covering the geometry of the antenna (e_{\max}) . In the case of a relative length h_{λ} comparable to 1/10, it's recommended to select a $e_{\max} \cong \lambda/10$. A substrate thickness respecting this relationship is called a thick substrate. None of the available substrates verify this condition. Among them, only the thinnest substrate and the second to last one (thus $h_{FR4} = 0.8 \, mm$ and $h_{FR4} = 1.0 \, mm$) are part of a range which the Antenna Toolbox provides instructions of. It's the thin substrate range: the automatic mesh mode should be adopted for a thin substrate, namely having a relative thickness less or equal than one fifth $(h_{\lambda} \leq 1/50)$. In the specific project case:

$$h_{FR4} = 0.8 \, mm \implies h_{\lambda} = \frac{1}{81} \qquad h_{FR4} = 1.0 \, mm \implies h_{\lambda} = \frac{1}{62}$$
 (11)

Thinner substrate choice rationale. The quality factor depending on $tan\delta$ is generally low in the FR4 substrate case. This means the FR4 is a big power dispersor. Since increasing h_{FR4} will provoke just more losses in terms of a radiation efficiency drop and since the only thickness values of $0.8 \ mm$ and $1.0 \ mm$ would give reliable/accurate results in the Antenna Toolbox simulations, the $0.8 \ mm$ will be adopted.

Mesh density refinement

Although a mesh density choice has already been made by selecting the best maximum edge length $e_{\rm max}$, the accuracy achievable by using the mesh automatic mode in the case of substrates belonging to the *thin substrate range* will be proved hereafter. An initial study of the mesh density level influence on the reflection coefficient (Γ in dB) evaluated at the resonant frequency ($f=2.1\,GHz$) has been realized, thus a $\Gamma_{2.1\,GHz}=F(e_{\rm max})$ function has been plotted with an initial step of $\Delta e_{\rm max}=2.5\cdot 1^{-4}\,m$ between every two mesh densities related to their specific $e_{\rm max}$. This first simulation considered a broader range of $e_{\rm max}$ variation: $[2.5\cdot 10^{-4}\,m$, $6.0\cdot 10^{-4}\,m$].

Since the resulting plot (fig. 3) has shown big uncertainty of the reflection coefficient value at the resonant frequency ($\Gamma_{2.1GHz}$) at almost every mesh $e_{\rm max}$ level (primarily due to the big step selected between one density level and another), some more detailed tests have been run, by considering a slightly narrower range ($[2.5\cdot10^{-4}\,m,\,5.0^{-4}\,m]$) and a tchicker evaluation of the maximum edge values (so that the mesh variation step has been remarkably reduced to $1.0^{-4}\,m$). Specifically, the step between two mesh densities level in terms of the maximum edge length of each one has been reduced from a $\Delta e_m = 2.5\cdot10^{-4}\,4$ to $\Delta e_m = 1.0\cdot10^{-4}$. In all these simulation, an important fact needs to be noted. Even very small variation on the maximum edge length value involved considerable inconsistencies in almost every part of the mesh range in terms of big variations of the frequency at wich Γ reaches its minimum. Thus, considering the frequency f^* at which $\min(\Gamma)$ is actually obtained, instead of evaluationg it at the theoretical resonating frequency value at every mesh level, not only the standard test comparing $e_{\rm max}$ and Γ has been run, but also some mesh refinement plots representing the relationship between f^* and $e_{\rm max}$, Δf^* and $e_{\rm max}$ and also $\Gamma_{dB}(f^*)$ and $e_{\rm max}$ have been taken into account (where Δf^* is the difference between f^* and the resonant frequency

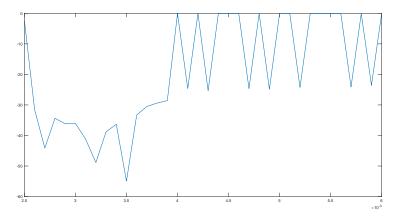


Figure 3: Minimum of the reflection coefficient $\Gamma[dB]$ in the frequency range $2.0 \div 2.2\,GHz$ depending on the varying mesh density level

 $f=2.1\,GHz)$. More parameter relationships have been collected and this led to the setting of the mesh density choice in terms of $e_{\rm max}$ that has been selected inside the most stable region (i.e. showing the smallest deviation of the reflection coefficient minimum from the resonant frequency). In the end it's been specifically taken the 'automatic' $e_{\rm max}$ (= $3.5^{-4}\,m$) suggested by the Antenna Toolbox, since this value belongs to the stable region and seems to give the most accurate results. The $e_{\rm max}$ values belonging to the stable region ([$3.1\cdot10^{-4}\,m,3.7\cdot10^{-4}\,m$]) exibit slight deviations from the resonant frequency ($\Delta f^* \cong [0.01\,GHz,0.03\,GHz]$) and the minima of the reflection coefficient vary as follows: [$-24\,dB$, $-33\,dB$].

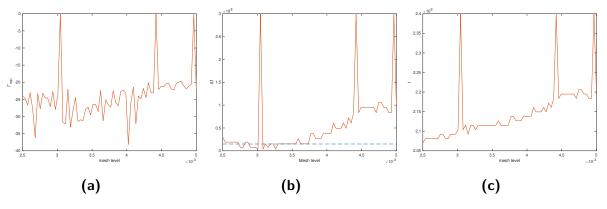


Figure 4: (a) (s_m, Γ_{dB}) plot, (b) $(s_m, \Delta f)$ plot, and (c) (s_m, f) plot

Patch parameters refinement

After the selection of the maximum edge length of the mesh (consequently, of its density level), a more refined computation of the reflection computation will be made, depending on the patch (i.e. on its length and width), but also on the feed location. Firstly, only a parametrical variation of the feed position across the patch length direction has been considered, depending on variations of L_{patch} and W_{patch} . This means that the first refinement of the feed position has been evaluated starting from its theoretical equation (depending indirectly by W_{patch}). The change of the feed location has been taken into account in the computation of every step of the simulation, thus in every evaluation of the reflection coefficient. The patch size variations provoked wide modifications of the reflection coefficient, which values depending on that have been represented by an initial contour plot (with variations of the patch size in a broader range and with a larger step between one value and another).

After that, a simulation in a narrower range variation of the patch size has been run in order to choose from there a set of values of Γ (i.e. a set of couples (L_{patch}, W_{patch})) that should put the patch antenna in the best resonant condition. A set of 20 values has been selected from the second simulation range for the next and more specific simulation. In this third case, the reflection coefficient has been plotted in a range around the resonant frequency $([2.0\,GHz,2.2\,GHz])$ in order to find which is the best combination for the patch size that makes actually resonate the antenna at the project frequency. Another determining and discriminating factor was the input impedance $(Z_{in}=R_{in}+jY_{in},W_{in})$ where the real part of Z_{in} is the imput resistance, while the imaginary one is the input reactance), because a impedance matching (at $50\,\Omega$) needed to be achieved for the project. In the ideal case, of course, a reflection coefficient $\Gamma^{(id)}=0.00\to -\infty\,dB$ would be required in order to reach the perfect impedance matching (perfect matching with input resistance at $50.00\,\Omega$ and null reactance). As a real result, before an accurate matching, the Γ value related to all the couple candidates $((L_{patch}, W_{patch}))$ spaced ranged from $-24\,dB$ to $-30\,dB$. Another design choice contributed to the final patch size choice: the feed location varying across the patch width direction. Thus, a parametrical impedance matching depending on that has been run on the best couple candidates. The resulting values are:

$$\Gamma_{final} = -54.94 \, dB$$
 $R_{in} = 49.86 \, \Omega$ $Y_{in} = 0.11 \, S$

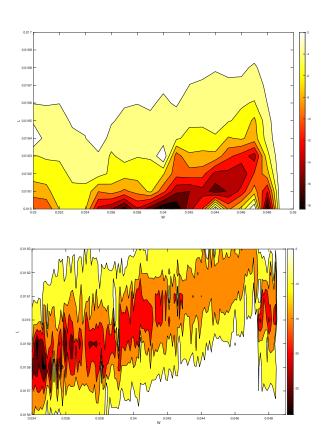


Figure 5: First and second (refined) contour plots depending on the patch size $(L_{patch} and W_{patch} variations)$

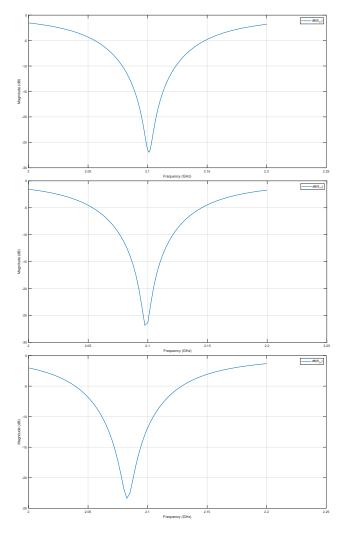


Figure 6: Some of the best Γ plots depending on specific patch size candidates $(L_{\it patch}, W_{\it patch})$

Patch parameters

$$L + W - w_{SC} = \frac{\lambda}{4} + h_{sub}$$

$$W = \frac{\lambda_0}{2} \sqrt{\frac{2}{\varepsilon_r + 1}}$$
(12)

$$BW_E = 2 \arccos \sqrt{\frac{7.03 \lambda_0^2}{4 (3 L_e^2 + h^2) \pi^2}}$$

$$BW_H = 2 \arccos \sqrt{\frac{1}{2 + k_0 W}}$$
(13)

$$\ell_{feed} = \frac{L}{\pi} \arccos \sqrt{\frac{R_{in}}{R_r}}$$
 (14)

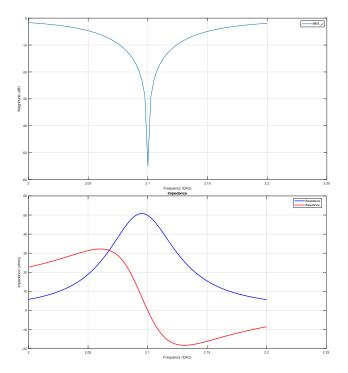


Figure 7: Final Γ and impedance matching plots after further refinement including w_{feed} change

From the PIFA pifa to a PCB stack Since the Antenna Array Designer is not able to generate an array of PIFAs, the Sensor Array Designer among with the PCB Antenna Designer have been used so that all the parameters required for the project could be analyzed. Just before that, a comparison between the single antenna realized with the PIFA structure and the antenna created by using a PCB stack was performed. A single limitation in the use of the PCB Antenna Designer has been encountered, but it has been overcome with an approximation strategy. The limitation consisted in the absence of a specific option or combination of commands that would allow to realize the rectangular shorting pin between the patch and the ground. The only thing the designer can do is to replace this kind of shorting pin with a series of small diameter (e.g. $0.4 \, mm$) cylindrical shorting pins close to one edge of the patch, across the patch width direction. With this design choice, a very similar behaviour to that of the PIFA structure was possible to simulate. To support this results, a comparison of the 2D patterns (elevation and azymuth cut) of the PIFA to those of the PCB stack antenna has been realized in terms of the mean square error values, shorten MSE, related to the two specific two-dimensional patterns considered (MSE_{el} in the elevation cut directivity pattern and MSE_{az} in the azimuth cut directivity pattern). Furthermore, since the Sensor Array Designer allows the plotting of some patterns such as the directivity, a comparison between the 2D patterns of the antenna array of PIFAs and the array of PCBs has been presented. All this discussion was necessary, because moving to the last part of the project required a deeper study of the array of antennas, that will be displayed by using the PCB antenna as the element of the array.

Single PIFA and PCB stack comparison.

$$MSE_{az} \cong 0.16 \, dB \qquad MSE_{el} = 0.055 \, dB$$
 (15)

- Overall PCB and PIFA arrays comparison. In this case the comparison has been made between
 - The main lobe (ML) levels in the two arrays (as a difference between the two directivity (i.e. D_0)values corresponding to that angle):

$$\Delta D_{0,az}^{(ML)} = 2.37 dB \qquad \Delta D_{0,az}^{(SL)} = 0.02 dB$$

$$\Delta D_{0,el}^{(ML)} = 1.93 dB \qquad \Delta D_{0,el}^{(SL)} = 4.01 dB$$
(16)

- The first side lobe levels in the two arrays

both in the broadside case.

Overall patch antenna array performance

In this last part of the project, the array factor and the the element antenna (PIFA or PCB stack) designs will be combined so that the total effect will be examined. As it's been already mentioned in the previous paragraph, some of the information describing the overall array performance can be asked from both the Sensor Array Analyzer (where the array of PIFAs was designed) and the PCB/Antenna Array Designer (where the single patch antenna made starting from the PCB stack and also the whole array can be constructed). The performance of the overall array will be evaluated in two cases: in the broadside case (90°) but also at 45° off the boresight direction. First of all, it is very simple to identify the phase shift coefficients in the broadside case because they all equal 0° (there's no actual phase shift between antennas). In the second case, some manual calculation can be made in order to insert the phase shifts between the single antennas (this is the case of the PCB stack array), and it can be also automatically computed by using the array of PIFAs. The generic procedure is shown in Eqn. x, where $n = \{1,5\}$

$$u_0 = \alpha d_{opt} = 2\pi \frac{d_{opt}}{\lambda} \cos(\theta_0)$$

$$\alpha_n = n \alpha d_{opt} = nu_0$$
(17)

Total array gain

In order to get and describe the last plots of the project, it is not possible to recur to the Sensor Array Analyzer because some of the commands, such as and EHfields and patternMultiply, are not available in this tool. Thus, in this last part, only the array of PCBs will be used, because it's compatible with these commands. The total array gain will be computed and displayed in two different cases: by mean of a fullwave model (with no approximation) and by using the pattern multiplication principle. It will be indirectly plotted because of tool limitation, specifically related to the patternMultiply, which doesn't get any gain plot, but it can get the directivity plot. Since these two are proportional, so that the gain is obtainable by multiplying the directivity by the radiation efficiency (η_R) .

References

References

- [1] Constantine A. Balanis. *Antenna Theory: Analysis and Design*. John Wiley & Sons Inc. 2016.
- [2] Sophocles J. Orfandis. *Electromagnetic Waves and Antennas*. The MathWorks, Inc., 1999-2016.

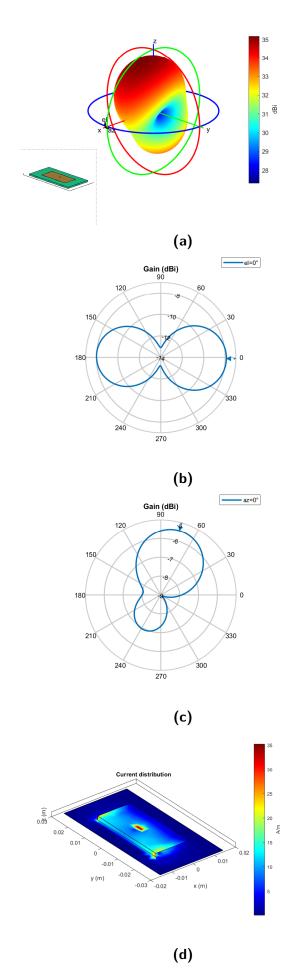


Figure 8: Gain patterns (a) in 3D, (b) in the nulle elevation plane, (c) in the null azimuth plane and (d) 3D current plot on the patch antenna

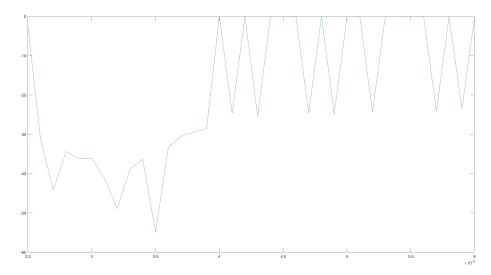


Figure 9: Minimum of the reflection coefficient $\Gamma[dB]$ in the frequency range $2.0 \div 2.2\,GHz$ depending on the varying mesh density level

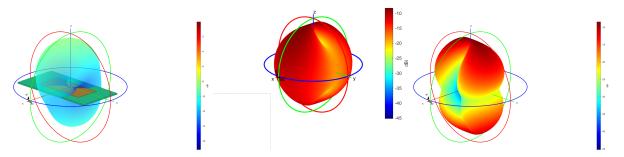


Figure 10: Gain pattern (left), gain pattern with vertical polarization (center) and with the horizontal one (right)

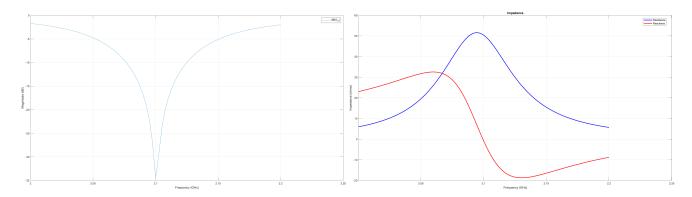


Figure 11: Reflection coefficient (left) and impedances (right) plots depending on $f \in 2.0 \div 2.1 \, GHz$

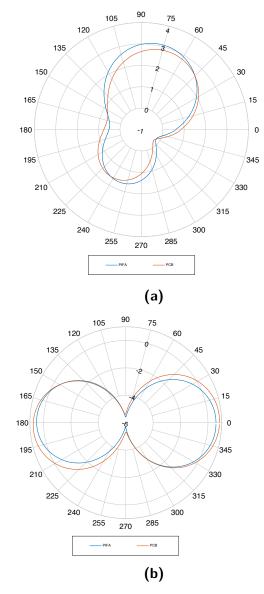


Figure 12: PIFA and PCB single antenna directivity patterns (dB) in the Azimuth cut ($\theta_{el}=0^{\circ}$, (a)) and in the Elevation cut ($\phi_{az}=0^{\circ}$, (b))

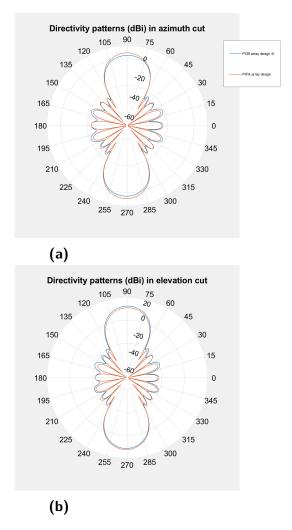


Figure 13: PIFA and PCB arrays directivity patterns (dB) in the Azimuth cut $(\theta_{el}=0^\circ,$ (a)) and in the Elevation cut $(\phi_{az}=0^\circ,$ (b))

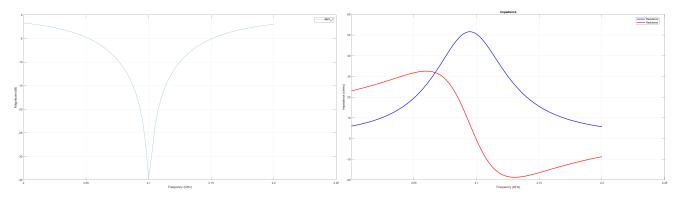


Figure 14: Reflection coefficient (left) and impedances (right) plots depending on $f \in 2.0 \div 2.1\,GHz$. This are the resulting plots before W_{feed} optimization.