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Professor: Marrocco Gaetano
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Università di Roma
Tor Vergata

# Design of an array of folded patches

Blasi Luca, Mastrofini Alessandro, Mucenica Stefan Leonard

**Abstract**

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## Tchebyshev array factor design

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The design parameters for the array are:

| Parameter | Value |
|---|---|
| # elements | $2N+1=5$ |
| Mean lobe/side lobe ratio | $R=120 \cong 41.58\,dB$ |
| Frequency | $f\ =2.1\,GHz$ |

It's been specifically required to find the optimal inter-element spacing so that the minimum of the beamwidth will be reached:

$$d_{opt} \rightsquigarrow \min\{BW_{fn}\}$$

$$d_{opt} = \lambda \left[ 1 - \frac{\arccos\left(\frac{1}{\gamma}\right)}{\pi} \right] \tag{1}$$

$$\gamma = \cosh\left[\frac{1}{2N}\ \ln\left(R + \sqrt{R^2-1}\right)\right]$$

where $\lambda = \frac{c}{f}$ is the frequency in the free-space . In this case, $d_{opt} \in \left(\frac{\lambda}{2}, \lambda\right]$, which means that The coefficients $a$ and $b$ related to the Tchebyshev polynomial approximation for the array will be chosen by following the $d_{opt} \in \left(\frac{\lambda}{2}, \lambda\right]$ condition:

$$T_2\left[x = a + b\cos\,u\right] = C_0\ + 2\,C_1\ \cos\,u\ + C_2\cos\,2u\ = \left(2a^2+b^2-1\right)\ +\ 4ab\,\cos\,u + b^2\,\cos\,2u \tag{2}$$

Once the amplitude current feed coefficients are computed $(C_n\,,\ n=\overline{0,2})$, the tapering efficiency can be calculated:

$$\eta_T\ = \frac{1}{2N+1}\frac{||C_0+2C_1+2_2||^2}{C_0^2+2C_1^2+2C_2^2} \tag{3}$$

Let's consider two cases of uniform spacing array and:

$$\text{Uniform Amplitude (UA)} \quad || \quad \text{Non-uniform Amplitude (NUA, Tchebyshev)} \tag{4}$$

The comparison will show how

$$BW_{fn}^{[UA]} < BW_{fn}^{[NUA]}$$

$$BW_{fn}^{[NUA]} = 2\frac{180}{\pi}\left[\frac{\pi}{2} - \arccos\left(\frac{\arccos\left(\frac{\cos\left(\frac{\pi}{2N}-a\right)}{b}\right)}{k_0 d}\right)\right] \tag{5}$$

$$BW_{fn}^{[UA]} = \frac{2\lambda}{N\,d}\,\frac{180}{\pi}$$

| Parameter | Value |
|---|---|
| Feed coefficients $[A]$ | $C_0$ \|\| $C_1 = C_{-1}$ \|\| $C_2 = C_{-2}$<br>41.2 \|\| 29.8 \|\| 9.6 |
| Normalized feed coefficients to $C_{\max}$ | $C_0^*$ \|\| $C_1^* = C_{-1}^*$ \|\| $C_2^* = C_{-2}^*$<br>1.000 \|\| 0.7215 \|\| 0.2336 |
| Tapering efficiency | $\eta_T = 79\,\%$ |
| **Beamwidth** | Tchebyshev \|\| Uniform<br>50.6° \|\| 34.8° |

Now, discussing the results is mandatory:

## Max/min feed ratio

Even if this is the design of a Non-Uniform Amplitude Array, the less the ratio $r_{\max/\min} = \frac{C_{\max}}{C_{\min}}$ is, the more efficient distribution of current is reached. In this particular design:

$$r_{\max/\min} \cong 4.39 \tag{6}$$

meaning that if a damage of the element with the $C_{\max}$ level of feed occurs, most part of the efficiency will be lost. In any case, the tapering efficiency shows how it will not be possible to take advantage of 21 % of the array in an ideal situation, remembering that this design model can be discerned by the real circumstance in terms of the Tchebyshev error [1].

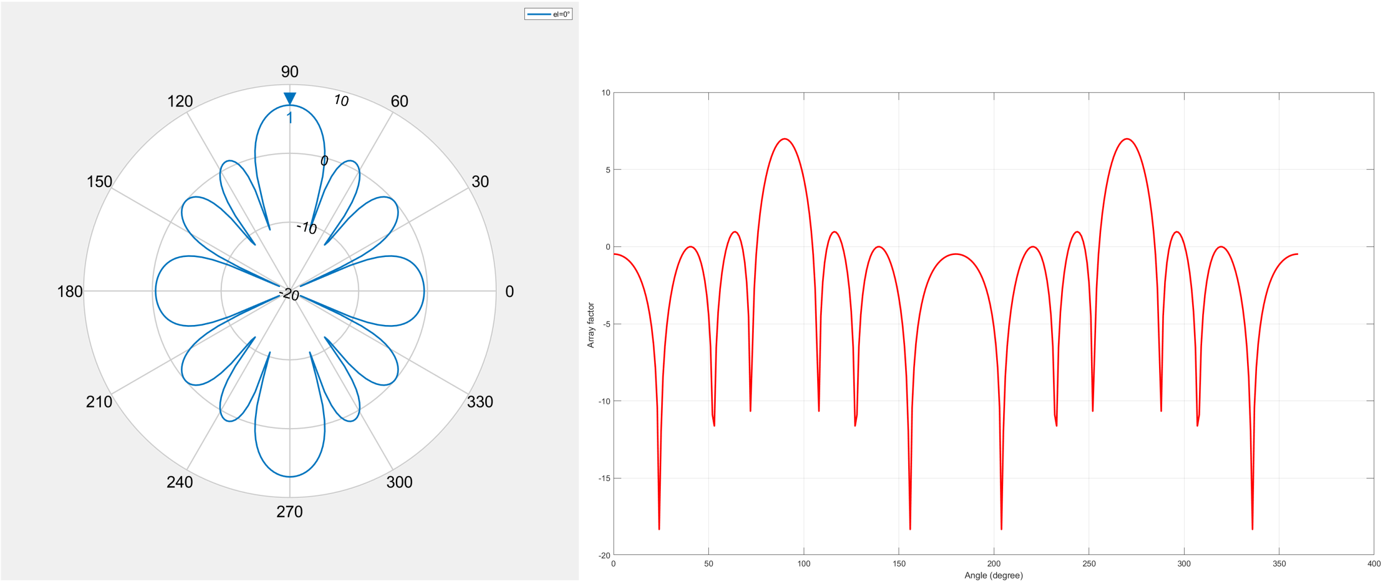



**Figure 1:** Array factor polar (left) and rectangular (right) diagrams

# Rectangular folded patch design

The main components of a rectangular folded patch are: the patch, the substrate (generally accessory, but used in this project), the ground, the rectangular shorting pin between the patch and the ground, and the feed. More details about them will be presented in a short while. Before that, some other remarks are necessary: this antenna will be the element of the array, which will be designed starting actually from a PIFA (*Planar Inverted F Antenna*), given the limitations of the `Antenna Toolbox`, which will be discussed and overcome later on. A general PIFA realized with a dielectric substrate is shown in fig. 2 .

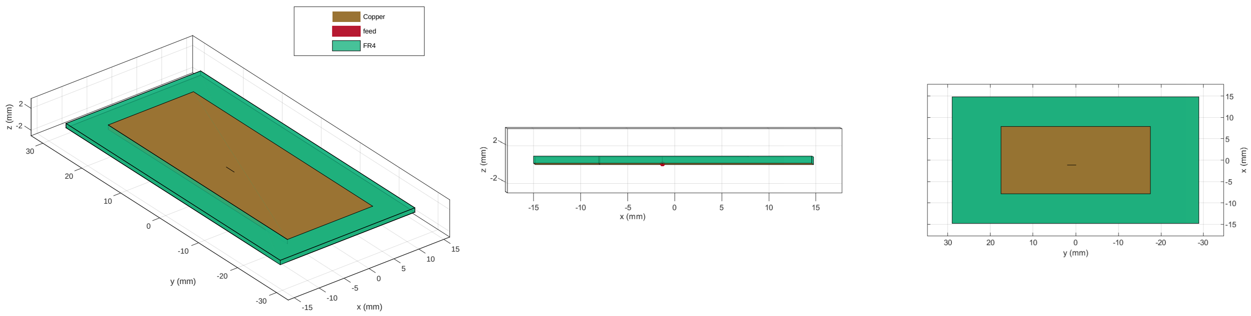



**Figure 2:** PIFA realized with dielectric substrate

By imposing the particular condition by which the width of the rectangular shorting ($w_{sc}$) equals the patch width size ($W_{patch}$), the PIFA and the folded patch antenna will be two equivalent structures:

$$W_{patch} = w_{sc} \tag{7}$$

This remark on the PIFA is necessary because generally its structure is not equivalent to that of the folded patch antenna because of the possible variability of the shorting width ($w_{sc}$), which doesn't always satisfy the above imposed condition. That said, the design requirements are listed below: A preliminary evaluation of the patch parameters have been realized by leaning on a theoretical set of formulas [Balanis]. That's the reason why the characteristics of the patch shown into the table are called "*pre-optimized features*" (same thing applies to the ground component). Thus, an

| Folded patch design parameters | |
|---|---|
| **Parameter/Component** | **Value/Type/Material** |
| Frequency | $2.1\,GHz$ |
| Matched input resistance | $R_{in} = 50\,\Omega$ |
| Substrate | FR4 |
| Relative permittivity<br>Relative permeability<br>Loss tangent<br>Thickness | $\varepsilon_{FR4} = 4.8$<br>$\mu_{FR4} \cong 1$<br>$\{\tan(\delta)\}_{FR4} = 0.0260$<br>$h_{FR4} = 0.8\,mm$ |
| Patch<br>(pre-optimized features) | Copper |
| Conductivity<br>Thickness<br>Length<br>Width | $\kappa_{copper} = 5.96 \cdot 10^{7}\,S/m$<br>$h_{patch} = 3.556 \cdot 10^{-5}\,m$<br>$L_{patch} \cong \frac{\lambda_{FR4}}{4} = 0.0171\,m$<br>$W_{patch} \cong 0.419\,m$ |
| Ground<br>(pre-optimized features) | Copper<br>(same conductivity listed above) |
| Thickness<br>Length<br>Width | $h_{GND} = h_{patch}$<br>$L_{GND} = 0.04\,m$<br>$W_{GND} = 0.06\,m$ |
| Feed | Coaxial cable |

optimization process of all those parameters will be performed in some following steps. Just before that, the formulas of the theoretical model will be pointed out:

$$L_{patch} + W_{patch} - w_{sc} = \frac{\lambda_{FR4}}{4} + h_{FR4}$$
$$W_{patch} = \frac{\lambda}{2}\sqrt{\frac{2}{\varepsilon_{FR4} + 1}} \tag{8}$$

$$\varepsilon_{eff} = \frac{\varepsilon_{FR4} + 1}{2} + \frac{\varepsilon_{FR4} - 1}{2}\left(1 + 12\frac{h_{FR4}}{W_{patch}}\right)^{-\frac{1}{2}}$$
$$L_{eff} = \frac{\lambda_{FR4}}{4}$$
$$\Delta L = 0.412\,h\left[\frac{(\varepsilon_{eff} + 0.3)\left(\frac{W_{patch}}{h_{FR4}} + 0.268\right)}{(\varepsilon_{eff} - 0.258)\left(\frac{W_{patch}}{h_{FR4}} + 0.8\right)}\right] \tag{9}$$
$$L = L_{eff} - 2\Delta L$$

$$R_r = \frac{120\,\lambda}{W_{patch}}\left[1 - \frac{1}{24}\left(2\pi\frac{h_{FR4}}{\lambda}\right)^2\right]^{-1} \tag{10}$$

$$\Theta_E = 2\arccos\sqrt{\frac{7.03\,\lambda^2}{4\,(3\,L_e^2 + h_{FR4}^2)\,\pi^2}}$$
$$\Theta_H = 2\arccos\sqrt{\frac{1}{2 + 2\pi\frac{W_{patch}}{\lambda}}} \tag{11}$$

$$\ell_{feed} = \frac{L_{patch}}{\pi} \arccos \sqrt{\frac{R_{in}}{R_r}} \tag{12}$$

Where $\Theta_i$ ($i = \{E, H\}$) are the half-power beamwidth values given by the E-cut and the H-cut. $\lambda = c/f$ is the free-space wavelength ($c = 299792458\, m/s$ is the light-speed in the free space). $R_{in}$ is the input impedance (a resistance), while $R_r$ is the radiation resistance

## Refinement with MatLab MoM

## Substrate thickness selection

Three thickness levels were available for the $FR4$ substrate required this project ($h^{(i)}_{FR4} = \{h^{(1)}_{FR4}, h^{(2)}_{FR4}, h^{(3)}_{FR4}\} = \{0.8, 1.0, 1.6\}\, mm$)

| FR4 substrate project thickness levels available | | |
|---|---|---|
| $h^{(1)}_{FR4} = 0.8\, mm$ | $h^{(2)}_{FR4} = 1.0\, mm$ | $h^{(3)}_{FR4} = 1.6\, mm$ |

The Antenna Toolbox gives specific information about the mesh density level that should be adopted for the design of the patch antenna components. The only issue is that these details are given only for particular ranges of the ratio indicator called *relative thickness* or *electrical thickness* $h_\lambda$. The electrical thickness depends on the ratio between the substrate thickness ($h_{FR4}$) and the wavelength related to the substrate medium ($\lambda_{FR4}$). When a mesh is configured in the Antenna Toolbox environment, a specific parameter needs to be adjusted: the maximum edge length of the generic triangle covering the geometry of the antenna ($e_{\max}$). In the case of a relative length $h_\lambda$ comparable to $1/10$, it's recommended to select a $e_{\max} \cong \lambda/10$. A substrate thickness respecting this relationship is called a *thick substrate*. None of the available substrates verify this condition. Among them, only the thinnest substrate and the second to last one (thus $h_{FR4} = 0.8\, mm$ and $h_{FR4} = 1.0\, mm$) are part of a range which the Antenna Toolbox provides instructions of. It's the *thin substrate range*: the automatic mesh mode should be adopted for a thin substrate, namely having a relative thickness less or equal than one fifth ($h_\lambda \leq 1/50$). In the specific project case:

$$h_{FR4} = 0.8\, mm \rightsquigarrow h_\lambda = \frac{1}{81} \qquad h_{FR4} = 1.0\, mm \rightsquigarrow h_\lambda = \frac{1}{62} \tag{13}$$

> ***Thinner substrate choice* rationale.** The quality factor depending on $tan\delta$ is generally low in the FR4 substrate case. This means the FR4 is a big power dispersor. Since increasing $h_{FR4}$ will provoke just more losses in terms of a radiation efficiency drop and since the only thickness values of $0.8\, mm$ and $1.0\, mm$ would give reliable/accurate results in the Antenna Toolbox simulations, the $0.8\, mm$ will be adopted.

## Mesh density refinement

Although a mesh density choice has already been made by selecting the best maximum edge length $e_{\max}$, the accuracy achievable by using the mesh automatic mode in the case of substrates belonging to the *thin substrate range* will be proved hereafter. An initial study of the mesh density level influence on the reflection coefficient ($\Gamma$ in $dB$) evaluated at the resonant frequency ($f = 2.1\, GHz$) has been realized, thus a $\Gamma_{2.1\, GHz} = F(e_{\max})$ function has been plotted with an initial step of $\Delta e_{\max} = 2.5 \cdot 1^{-4}\, m$ between every two mesh densities related to their specific $e_{\max}$. This first simulation considered a broader range of $e_{\max}$ variation: $[2.5 \cdot 10^{-4}\, m\, ,\, 6.0 \cdot 10^{-4}\, m]$.

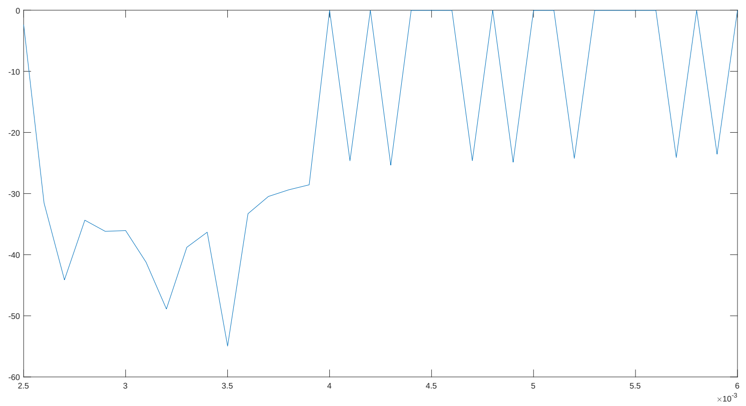
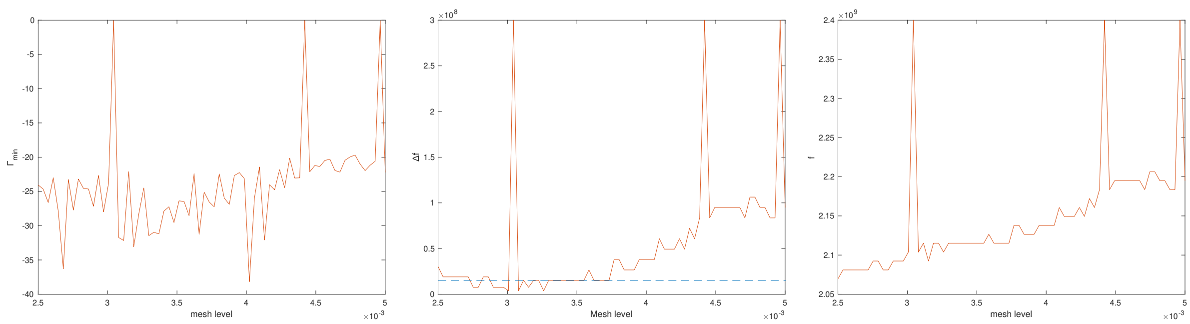


**Figure 3:** Minimum of the reflection coefficient $\Gamma\,[dB]$ in the frequency range $2.0 \div 2.2\,GHz$ depending on the varying mesh density level

Since the resulting plot (fig. 3) has shown big uncertainty of the reflection coefficient value at the resonant frequency ($\Gamma_{2.1\,GHz}$) at almost every mesh $e_{\max}$ level (primarily due to the big step selected between one density level and another), some more detailed tests have been run, by considering a slightly narrower range ($[2.5{\cdot}10^{-4}\,m\,,\,5.0^{-4}\,m]$ ) and a tchicker evaluation of the maximum edge values (so that the mesh variation step has been remarkably reduced to $1.0^{-4}\,m$). Specifically, the step between two mesh densities level in terms of the maximum edge length of each one has been reduced from a $\Delta e_m = 2.5{\cdot}10{-}4$ to $\Delta e_m = 1.0{\cdot}10^{-4}$. In all these simulation, an important fact needs to be noted. Even very small variation on the maximum edge length value involved considerable inconsistencies in almost every part of the mesh range in terms of big variations of the frequency at wich $\Gamma$ reaches its minimum. Thus, considering the frequency $f^*$ at which $\min(\Gamma)$ is actually obtained, instead of evaluationg it at the theoretical resonating frequency value at every mesh level, not only the standard test comparing $e_{\max}$ and $\Gamma$ has been run, but also some mesh refinement plots representing the relationship between $f^*$ and $e_{\max}$, $\Delta f^*$ and $e_{\max}$ and also $\Gamma_{dB}(f^*)$ and $e_{\max}$ have been taken into account (where $\Delta f^*$ is the difference between $f^*$ and the resonant frequency $f = 2.1\,GHz$). More parameter relationships have been collected and this led to the setting of the mesh density choice in terms of $e_{\max}$ that has been selected inside the most stable region (i.e. showing the smallest deviation of the reflection coefficient minimum from the resonant frequency). In the end it's been specifically taken the 'automatic' $e_{\max}$ ($= 3.5^{-4}\,m$) suggested by the `Antenna Toolbox`, since this value belongs to the stable region and seems to give the most accurate results. The $e_{\max}$ values belonging to the stable region ($[3.1 \cdot 10^{-4}\,m, 3.7{\cdot}10^{-4}\,m]$) exibit slight deviations from the resonant frequency ($\Delta f^* \cong [0.01\,GHz, 0.03\,GHz]$) and the minima of the reflection coefficient vary as follows: $[-24\,dB\,,\,-33\,dB]$.




**(a)** **(b)** **(c)**

**Figure 4:** (a) $(s_m, \Gamma_{dB})$ plot, (b) $(s_m, \Delta f)$ plot, and (c) $(s_m, f)$ plot

## Patch parameters refinement

After the selection of the maximum edge length of the mesh (consequently, of its density level), a more refined computation of the reflection computation will be made, depending on the patch (i.e. on its length and width), but also on the feed location. Firstly, only a parametrical variation of the feed position across the patch length direction has been considered, depending on variations of $L_{patch}$ and $W_{patch}$. This means that the first refinement of the feed position has been evaluated starting from its theoretical equation (depending indirectly by $W_{patch}$). The change of the feed location has been taken into account in the computation of every step of the simulation, thus in every evaluation of the reflection coefficient. The patch size variations provoked wide modifications of the reflection coefficient, which values depending on that have been represented by an initial contour plot (with variations of the patch size in a broader range and with a larger step between one value and another). After that, a simulation in a narrower range variation of the patch size has been run in order to choose from there a set of values of $\Gamma$ (i.e. a set of couples $(L_{patch}, W_{patch})$) that should put the patch antenna in the best resonant condition. A set of 20 values has been selected from the second simulation range for the next and more specific simulation. In this third case, the reflection coefficient has been plotted in a range around the resonant frequency ($[2.0\,GHz, 2.2\,GHz]$) in order to find which is the best combination for the patch size that makes actually resonate the antenna at the project frequency. Another determining and discriminating factor was the input impedance ($Z_{in} = R_{in} + \jmath Y_{in}$, where the real part of $Z_{in}$ is the imput resistance, while the imaginary one is the input reactance), because a impedance matching (at $50\,\Omega$) needed to be achieved for the project. In the ideal case, of course, a reflection coefficient $\Gamma^{(id)} = 0.00 \;\rightarrow\; -\infty\,dB$ would be required in order to reach the perfect impedance matching (perfect matching with input resistance at $50.00\,\Omega$ and null reactance). As a real result, before an accurate matching, the $\Gamma$ value related to all the couple candidates ($(L_{patch}, W_{patch})$) spaced ranged from $-24\,dB$ to $-30\,dB$. Another design choice contributed to the final patch size choice: the feed location varying across the patch width direction. Thus, a parametrical impedance matching depending on that has been run on the best couple candidates. The resulting values are:

$$\Gamma_{final} = -54.94\,dB \qquad R_{in} = 49.86\,\Omega \qquad Y_{in} = 0.11\,S$$

## Patch parameters

$$\begin{aligned} L + W - w_{SC} &= \frac{\lambda}{4} + h_{sub} \\ W &= \frac{\lambda_0}{2}\sqrt{\frac{2}{\varepsilon_r + 1}} \end{aligned} \tag{14}$$

$$\begin{aligned} BW_E &= 2\ \arccos\sqrt{\frac{7.03\,\lambda_0^2}{4\,(3\,L_e^2 + h^2)\,\pi^2}} \\ BW_H &= 2\ \arccos\sqrt{\frac{1}{2 + k_0\,W}} \end{aligned} \tag{15}$$

$$\ell_{feed} = \frac{L}{\pi}\ \arccos\sqrt{\frac{R_{in}}{R_r}} \tag{16}$$

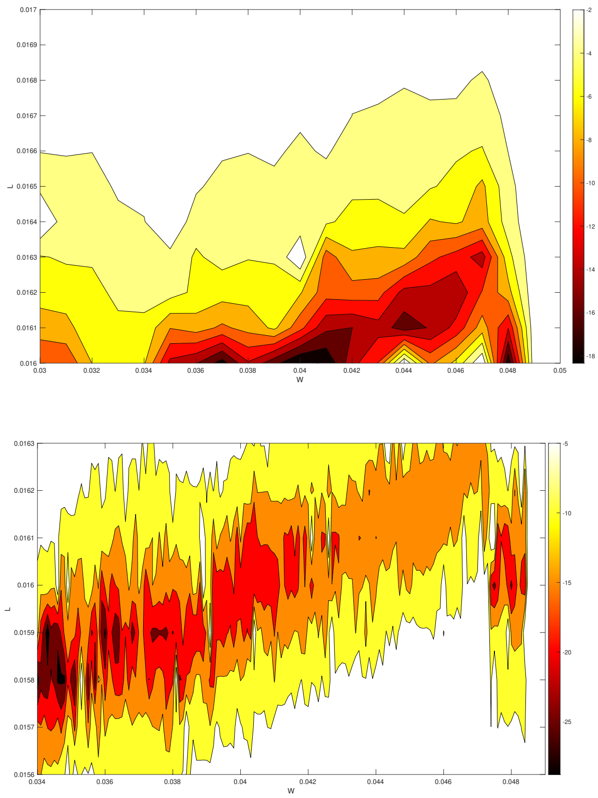


**Figure 5:** First and second (refined) contour plots depending on the patch size ($L_{patch} and W_{patch}$ variations)

## Overall array performance evaluation

## References


## References



[1] Constantine A. Balanis. *Antenna Theory: Analysis and Design*. John Wiley & Sons Inc, 2016.

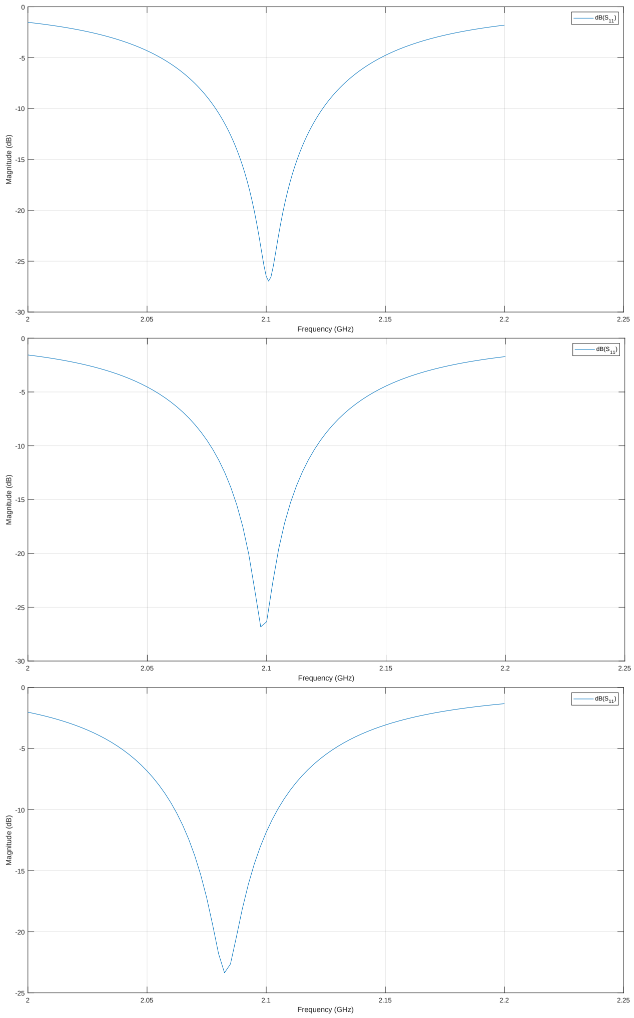



**Figure 6:** Some of the best $\Gamma$ plots depending on specific patch size candidates $(L_{patch}, W_{patch})$

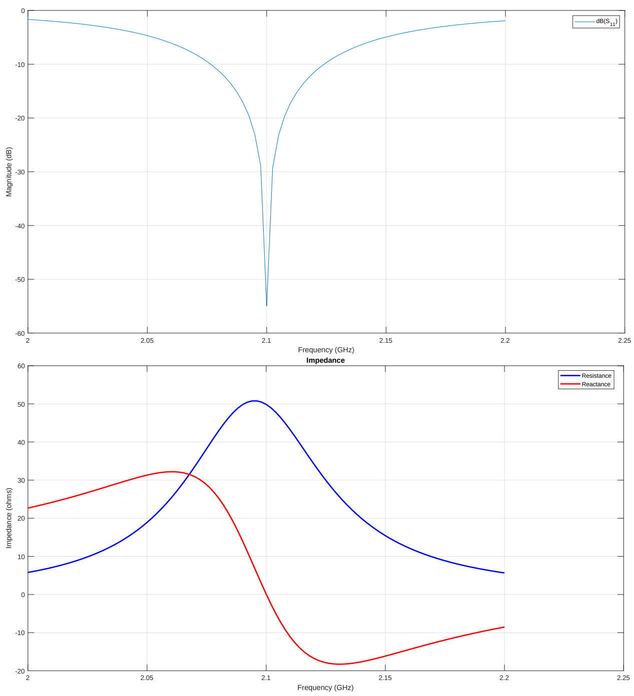
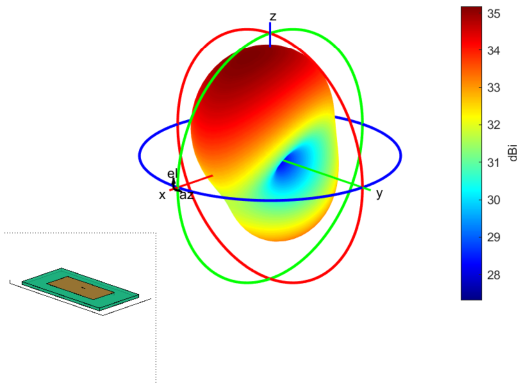



**Figure 7:** Final $\Gamma$ and impedance matching plots after further refinement including $w_{feed}$ change

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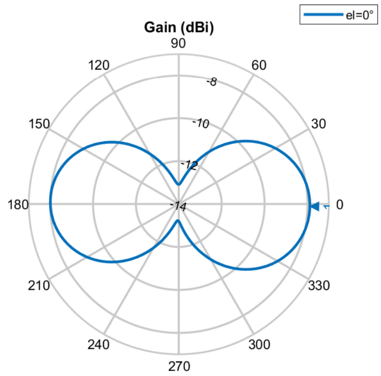


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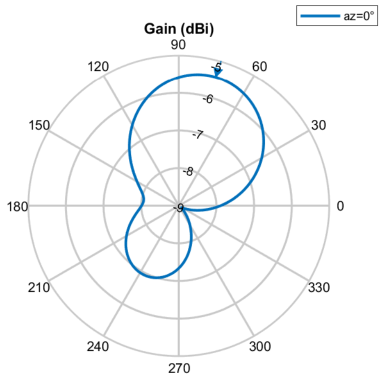


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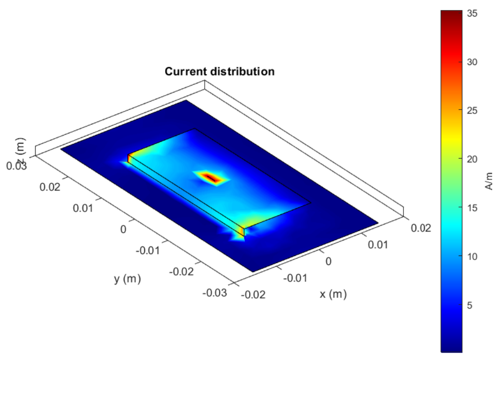


**(d)**

**Figure 8:** Gain patterns (a) in 3D, (b) in the nulle elevation plane, (c) in the null azimuth plane and (d) 3D current plot on the patch antenna

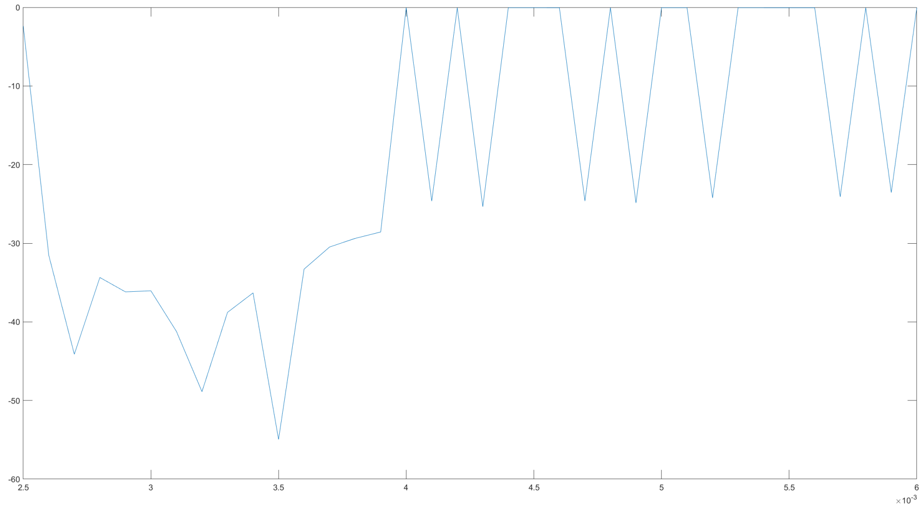


**Figure 9:** Minimum of the reflection coefficient $\Gamma\,[dB]$ in the frequency range $2.0 \div 2.2\,GHz$ depending on the varying mesh density level

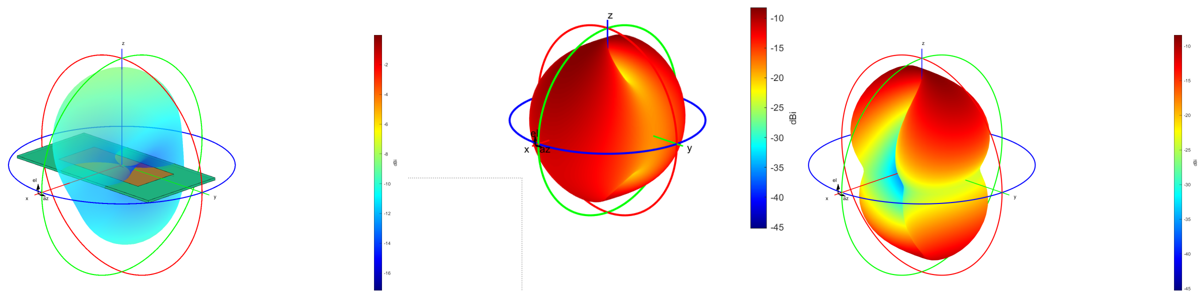


**Figure 10:** Gain pattern (left), gain pattern with vertical polarization (center) and with the horizontal one (right)

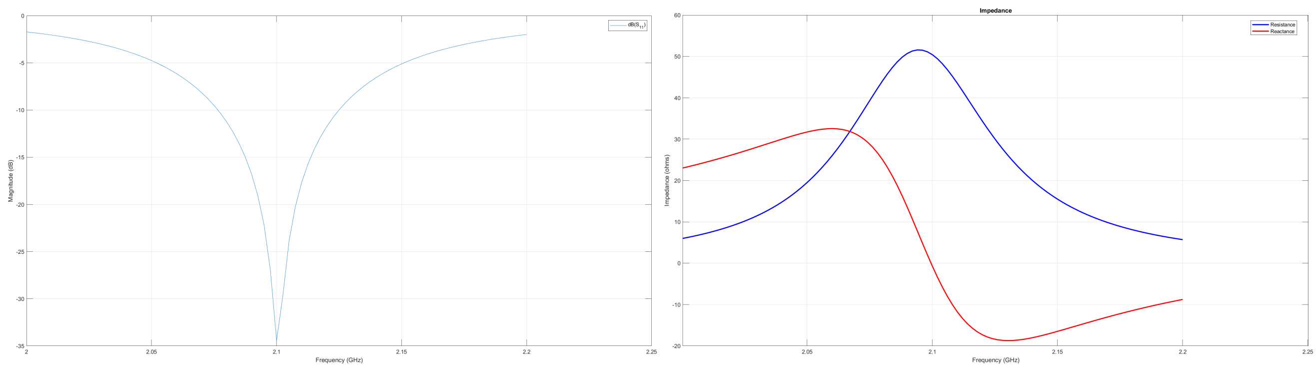

**Figure 11:** Reflection coefficient (left) and impedances (right) plots depending on $f\ \in\ 2.0{\div}2.1\,GHz$