M B L – E x e r c i s e s

```
EXERCISE @ex:myLabel

    % All of the following "attributes" are optional.
    % Attributes MUST be listed before code and text

    % The curent exercise can only be solved, if dependent
    % exercises have been passed before
    REQUIREMENT=ex:myEx1,ex:myEx2

    % forbid shuffling of single/multiple choice answers
    ORDER=static                    % default is shuffle

    % display (short!) single/multiple choice answers horizontally
    CHOICE_ALIGNMENT=horizontal     % default is vertical

    % forbid student to reanswer exercise
    DISABLE_RETRY=true              % default is false

    % Limited time to solve the question [seconds].
    TIME=4                          % default is -1 := not timed

    % total score weighting for this exercise. Default score per
    % answer is 1. The sum of scores of all answers is weighted
    % to the attribute given here.
    SCORES=4                        % default is 1

    % The number of randomized variable instances for the
    % exercise; default value is 5.
    INSTANCES=3

    % generates random variables and calculates the solution.
    CODE
        let a/b = randZ(-3,3)      % a,b in {-3,…,3}\{0}, a≠b
        let c = a + b              % (assume: a=1, b=3) c=4
        let d = a > b              % d=false
        let u = true
        let f(x) = x^2 + c         % f(x) = x^2 + 4
        let g(x) = x^2 + term(c)   % g(x) = x^2 + 1 + 3
        let h(x) = 2x + 5x         % h(x) = 2x + 5x
        let y(x) = opt(h)          % y(x) = 7x   (opt := optimize)
        let w(x) = sin(2x) + 3x
        let wd(x) = diff(w,x)      % wd(x) = 2cos(2x)+3
        let s = sqrt(-1)^2         % s = -1
        let v = [1,2,3,4]          % vector with 4 elements
        let I = eye<3>()           % A = [[1,0,0],[0,1,0],[0,0,1]]

    Multiple Choice:
    [x]  correct answer
    [ ]  incorrect answer
    % Dynamic answers. Only the name of a variable can be given
```

```
% here. Calculations must be done in part CODE.
[:d] correct iff $a > b$ is true
[:u] correct answer, given per variable

Single Choice:
(x) correct answer
( ) incorrect answer

% Asks "1 + 3 = [  ]" and provides a keyboard with
% integer numbers.
$a+b=$ #c

% Asks "∫ (x^2+4) dx = [  ] +C (C∈R)" and provides a
% keyboard for terms.
% The student answer is first differentiated w.r.t.
% variable x, and then it is evaluated, i.e. compared to
% variable f.
$\int (f) \, dx=$ #f,DIFF=x $+C (C\in\RR)$

% Asks "Garfield is a _ _ _ .".
% The keyboard only shows letters {a,c,t} in shuffled order.
Garfield is a #"cat".

% Per default, gap answers give a hint on the number of
% characters needed. Also the keyboard is restricted to the
% letters that occur in the solution.
% Use the following attributes to change that behavior.
Garfield is a #"cat",HIDE_LENGTH,SHOW_ALL_LETTERS.

% Asks "Order the numbers ascendingly: [3] [2] [4] [1]"
% A correct answer is rewarded with 3 scores.
%  → multiple attributes can be combined with "," as separator
Order the numbers ascendingly: #v,ARRANGE,SCORE=3


% Asks "The 3x3 identity matrix is [   ]".
% Per default, the student only needs to give the elements
% of the solution matrix. If attributes ROWS and/or
% COLS are set to dynamic, then the student also
% needs to find the matrix dimensions.
The 3x3 identity matrix is #I,ROWS=dynamic,COLS=dynamic
```

```
% Asks "(√-1)² = [  ]". Since variable s is integral (-1), the
% default keyboard would only contain numeral keys.
% To "confuse" students, a keyboard with key "i" would be
% nice. Attribute KEYBOARD sets an existing¹ or custom
% keyboard.
$term(s)=$ #s,KEYBOARD=myKeyboardName

% Asks "1 + 3 = [A][B][C]..." and provides one correct
% and (n-1) incorrect answers (e.g. {2,5,6,4,1} for CHOICES=5)
$a+b=$ #c,CHOICES=5

% Asks "1 + 3 = [A][B][C]..." and provides one correct
% and (n-1) incorrect answers (e.g. {10,5,6,4,20} for
% CHOICES=2 and manually added answers 10 and 20)
$a+b=$ #c,CHOICES=2+"10"+"20"

% Asks "3+4 = [7][1+6][2+3][4+3]". All answers are provided
% manually. Two answers are correct. Generally, at least one
% correct answer must be provided. If wished, also all
% provided answers may be correct and the student can click on
% any answer. If answers include variables, they are replaced,
% but the resulting term is not optimized (if wished, this has
% to be done in the part CODE)
% assume a=3, cc = 7
$3+4=$ #cc,CHOICES=0+"7"+"1+6"+"2+3"+"4+a"


% Asks "f(x)=sin(2x)+3x, f'(x) = [  ]" and shows (e.g.)
% tokens    [+]  [*]  [sin(2x)]  [3]  [2]  [cos(2x)]  [4]  [1]
% that must be selected to build the solution term,
% here: "f'(x)=2*sin(2*x)+3".
% Tokens are automatically derived from the solution term.
% The constant 1.5 is the factor of automatically generated
% tokens (based on the number of essential tokens). If this
% number is equal to 1, then each generated token is useful
% for the solution.
% Additional tokens (for student confusion) can be provided by
% adding +"MY_TOKEN_1"+"MY_TOKEN_2"…. Tokens are in SMPL
% math format.
$"f"(x)=w, f'(x)=$ #wd,TOKENS=1.5+"pi"
```

1 https://github.com/mathebuddy/mathebuddy/blob/main/app/lib/keyboard_layouts.dart

Keyboard definition

```
% keyboards must currently be inserted manually into the app code.
KEYBOARD myKeyboardName
    % Keys are separated by one or more spaces.
    %
    % Special key "!B" is the backspace key.
    % Special key "!E" is the enter key.
    %
    % A "submatrix" where each of the elements is the same
    % is rendered as large key (e.g. "sqrt(" in the example spans
    % two columns, keys "!B" and "!E" each span two rows).
    7 8 9  +     -     !B
    4 5 6  *     /     !B
    1 2 3  ^(    )     !E
    0 i pi sqrt( sqrt( !E
```

G A M E C o d e n a m e "E v e n t"

(refer to slides in Sciebo)

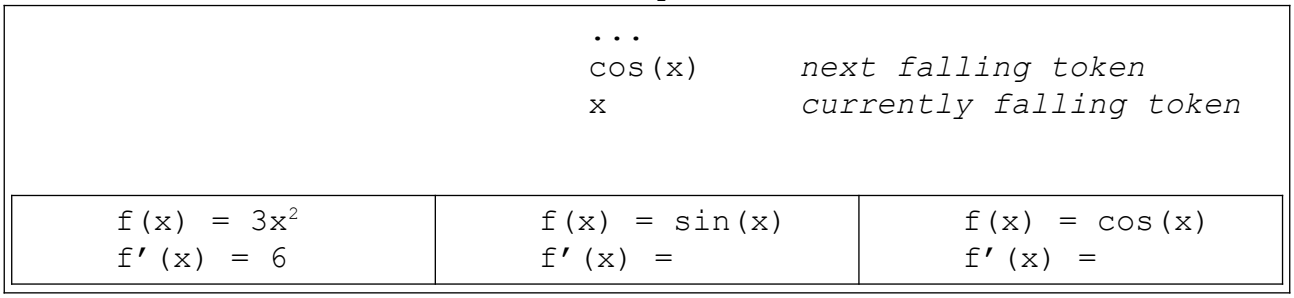
```
EVENT        % level name
#####

EXERCISE     % level contains n≥1 exercises
    TIME=5
    …
EXERCISE
    …
```

G A M E C o d e n a m e : "T e t r i s" (… it's not Tetris)

Answers (or term-tokens) are falling down from top to bottom
→ the student must move them left or right (or keep middle) in limited time
→ speed accelerates…

Example

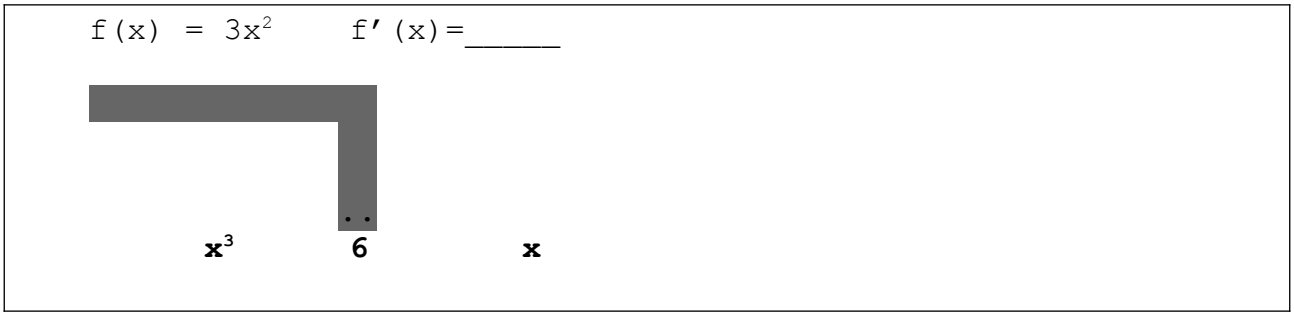



```
TETRIS       % level name
######

EXERCISE     % level contains exactly 1 exercise
    % shown columns (maybe two columns are the limit on
    %   smartphones with small displays…??)
    COLUMNS=3
    $f(x)=ff$
    $f'(x)=$ #ffd,TOKENS
    % code instances; each instance must provide all referenced
    % variables
    CODE
        let a/b = rand(2,7)
        let ff(x) = a x^b          % "*" can be omitted :-)
        let ffd(x) = diff(ff)
    CODE
        let ff(x) = sin(x)
        let ffd(x) = diff(ff)
    CODE
        let ff(x) = cos(x)
        let ffd(x) = diff(ff)
    …
```

G A M E C o d e n a m e : "S n a k e"

The answer (or tokens) are randomly placed in a 2D-grid. A snake, that keeps moving into a direction until changed, must be controlled towards the next correct solution. If the solution is "eaten", the snake gets larger. Speed accelerates...



S M P L – L o o p s & C o n d i t i o n s

```
let x = 5
% execute block {..}, while x > 0 is true
while x > 0 {
   x = x - 1
}

% execute block once and repeat loop as long as x < 5 is true
do {
    x = x + 1
} while x < 5

let f = 1
% set (new) variable k to 1, 2, 3, 4, 5 and run the block each
% iteration
for k from 1 to 5 {    % bounds can also be terms
    f = f * k
}

x = 4
let s = 0          % s := sign(x)
if x > 0 {
    s = 1
} elif s < 0 {     % elif := else if
    s = -1
} else {
    s = 0          % redundant here, since s is already 0
}
```

M A T H - R U N T I M E :

OPERAND-TYPE	EXAMPLE(s)	
boolean	**true, false**	
int	**-3, 4**	
rational	**4/7**	
real	**3.14, -1.337**	
irrational	**pi, e**	
complex	**4+5i**	
vector	**[1,2,3+4i]**	
matrix	**[[1,2],[3,4],[5,6]]**	(3x2 matrix)
set	**{ 3, 4, 4/3 }**	
identifier	**blub**	
string	**"hello, world!"**	

List of nullary functions (ups; dimensions are some kind of non-null-arity):

eye<3>()	identity matrix with 3 rows and 3 columns
zeros<3>()	zero vector with 3 elements
zeros<3,4>()	zero matrix with 3 rows and 4 columns
ones<3>()	one's vector with 3 elements
ones<3,4>()	one's matrix with 3 rows and 4 columns

List of unary functions: **TODO: acos, asin, ...**

abs(x)	absolute value of x (int, vector, cmplx, …)
arg(x)	argument of complex x
ceil(x)	smallest integer that is ≥ x
cols(x)	number of columns of matrix x
conj(x)	conjugate complex number of x
cos(x)	cosine of x
det(x)	determinant of matrix x
exp(x)	exponential function of x
fac(x)	factorial of x
floor(x)	smallest integer xthat is ≤ x
imag(x)	imaginary part of complex x
is_invertible(x)	true, if mat x is invertible, otherwise false
is_symmetric(x)	true, if mat x is symmetrix, otherwise false
is_zero(x)	true, if (value,vec,mat,...) x is approx zero
len(x)	number of elements of set or vector
ln(x)	natural logarithm
max(x)	maximum element of vector, matrix, set, …
min(x)	minimum element of vector, matrix, setm, …
norm(x)	euclidean norm of vector x
opt(x)	optimize term x (e.g. 2x+3x → 5x)
real(x)	real part of complex x
round(x)	round to nearest integer
rows(x)	number of rows of a matrix
shuffle(x)	shuffles the elements of a vector
sin(x)	sine
sqrt(x)	square root
tan(x)	tan
term(x)	term of variable x (refer to exercise example)
transpose(x)	transpose matrix x
triu(x)	upper triangular part of matrix x

List of binary functions:

binomial(n,k)	binomial coefficient
complex(x,y)	returns a complex number x+y*i
col(a,k)	gets column k (first is 0) from matrix a
cross(x,y)	cross product of vetors x and y
diff(f,x)	automatic differentiation of function f w.r.t variable x
dot(x,y)	dot (scalar) product of vectors x and y
rand(x,y)	draws a random number in range [x,y]

```
  rand<n>(x,y)        draws a random vector with n elements …
  rand<m,n>(x,y)      draws a random matrix with m rows and n cols …
  randZ(x,y)          same as rand(x,y) without zero
  randZ<n>(x,y)       same as rand<n>(x,y) without zeros
  randZ<m,n>(x,y)     same as rand<m,n>(x,y) without zeros
  row(a,k)            gets row k (first is 0) from matrix a

List of quaternary functions:

  int(f,x,a,b)        Numerically approximates integral of function
                      f w.r.t. variable x in bounds from a to b
```

SYNTHESIS OF TERM TOKENS:

```
example:
  input term t:
    2*sin(2*x)+3  as tree:  +(*(2,sin(*(2,x))),3)
  number of output tokens N:
    6
  output:
    {+,*,2,sin(2*x),3,cos(2*x)}  (underlined := incorrect)

depth=0: {+,2*sin(2*x),3}     n=3 < N
depth=1: {+,*,2,sin(2*x),3}   n=5 < N
depth=2: {+,*,2,sin(,),x,3}   n=7 > N
→ choose smallest depth with n<N and fill remaining tokens with
  shuffling:
     change constants:
         new := old + randZ(-1,1) * rand(1,floor(value/2))
         keep positive;
     change sin ↔ cos ↔ exp
→ if N is not given, then choose depth=1, and all tokens are
needed for the solution TODO: handling of tokens that are used
twice or more in the solution
```

```
Algorithm:
Input:
t := the input term
N := the number of output tokens
Algorithm:
T := { t }
for each Ti in T:
Ti := set of operator(s) and operand(s) for Ti
Ti := shuffle(Ti)
T := T uu Ti
```

```
n := |T|
if n < N then depth++ and goto 2.
END
Output:
T

Algorithm for shuffle(U):
Input:
     V := term
Algorithm:

Output:
     U
```