Chapter 4, Section 1. Exercises 1, 2, 3 and 5

MTH 594, Prof. Mikael Vejdemo-Johansson
Differential Geometry Independent Study

Matthew Connelly

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Exercise 4.1.5

Show that every open subset of a surface is a surface.

Let $S \in \mathbb{R}^3$ be a surface.
Let $U \in \mathbb{R}^2$ and $W \in \mathbb{R}^3$ be open sets in their respective spaces.

Then, $S \cap W$ will be a surface patch (and an open subset of surface S) if U can be mapped to $S \cap W$ smoothly and bijectively.
σ will be this mapping function:

$$\sigma : U \rightarrow S \cap W$$

Then, let there be a group of subsets, defined as

$$S_2 \subseteq S \cap W$$

$$W_2 \subseteq W$$

$$U_2 \subseteq U$$

and, a smooth, bijective mapping function analogous to σ:

$$\sigma_2 : U_2 \rightarrow S_2$$

The following comparison can now be made:

$$S_2 \cap W_2 = (S \cap W) \cap W_2$$

Which means that the open subset of surface S is a surface because it can be mapped to a surface patch homeomorphically. This general definition can then be applied to all surface subsets.