Chapter 2, Section 2. Exercises 1, 4-6

MTH 594, Prof. Mikael Vejdemo-Johansson
Differential Geometry Independent Study

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Exercise 2.2.1

Show that, if γ is a unit-speed plane curve,

$$\dot{n}_s = -k_s \mathbf{t}$$

Knowing the unit normal and its derivative:
$$n_s = (-sin\phi, \ cos\phi) \ \ , \ \ \dot{n}_s = \dot{\phi}(-cos\phi, \ -sin\phi)$$

And unit tangent, which can be signed:
$$\mathbf{t} = (cos\phi, \ sin\phi) \ \Rightarrow \ -\mathbf{t} = (-cos\phi, \ -sin\phi)$$

And knowing that signed curvature $k_s = \dot{\phi}$

$\dot{n}_s = -k_s \mathbf{t}$ can be expanded as such:

$$
\begin{aligned}
\dot{n}_s &= -\mathbf{t}k_s && \text{Commuting } \mathbf{t} \text{ to front of expression} \\
\dot{n}_s &= -(cos\phi, \ sin\phi)k_s && \text{Expanding } \mathbf{t} \\
\dot{n}_s &= (-cos\phi, \ -sin\phi)k_s && \text{Signing } \mathbf{t} \\
\dot{n}_s &= (-cos\phi, \ -sin\phi)\dot{\phi} && \text{Expanding } k_s
\end{aligned}
$$