Chapter 4, Section 1. Exercises 1, 2, 3 and 5

MTH 594, Prof. Mikael Vejdemo-Johansson
Differential Geometry Independent Study

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Exercise 4.1.1

Show that any open disc in the xy-plane is a surface.

Let an open disc in the xy-plane be defined as

$$D = \{(x, y) \in \mathbb{R}^2 \mid \|(x, y) - a\| < r\}$$

where a is the disc's center and r is its radius > 0.

Then, let there be an open ball in $\mathbb{R}^3$ defined similarly as

$$W = \{(x, y, z) \in \mathbb{R}^3 \mid \|(x, y, z) - a\| < r\}$$

with a and r representing its own center and radius.

By definition, $D \subset W$, which implies $D \subset \mathbb{R}^3$; this will eliminate the need to define a separate open disc in $\mathbb{R}^2$ for the following statement on homeomorphism.

We can then see that

$$D \cap W = D$$

which means that $D \cap W$ is homeomorphic to D.

Or, in more formal notation:

$$\sigma : D \to D \cap W$$

and

$$\sigma^{-1} : D \cap W \to D$$

This second statement inverting σ is especially important; mapping one set to its intersection with another set is trivial, but the inversion of that mapping may not yield a continuous result with preservation of range. Here, however, we are successful in mapping a surface to an open disc and vice versa.

It should also be noted that the atlas of D is a single surface patch, reinforcing the notion that the intersection of D and some $\mathbb{R}^3$ set will be homeomorphic to itself (D) for all points in $\mathbb{R}^2$ that are also in D.