Chapter 4, Section 2. Exercises 1, 2, and 4 through 9

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Differential Geometry Independent Study

Matthew Connelly

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Exercise 4.2.8

Show that translations and invertible linear transformations of $\mathbb{R}^3$ take smooth surfaces to smooth surfaces.

Translations and invertible linear transformations induce diffeomorphisms on surfaces; a diffeomorphism being an isomorphism (an invertible morphism or mapping that preserves length) between two smooth manifolds.

Let S be a surface, and $\widetilde{S}$ be S subject to a translation or invertible linear transformation.
Because S is a smooth bijective mapping of U to $S \cap W$, $\widetilde{S}$ will be smooth as well.
Because these two surfaces are smooth and a diffeomorphism is between two smooth manifolds (and we are trusting that S is locally Euclidean), we can then see that a translation/invertible linear transformation of a surface will preserve that surface's smoothness.