

Chapter 4, Section 2. Exercises 1, 2, and 4 through 9

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Differential Geometry Independent Study

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Exercise 4.2.8

Show that translations and invertible linear transformations of \mathbb{R}^3 take smooth surfaces to smooth surfaces.

Translations and invertible linear transformations induce diffeomorphisms on surfaces; a diffeomorphism being an isomorphism (an invertible morphism or mapping that preserves length) between two smooth manifolds.

Let S be a surface, and \tilde{S} be S subject to a translation or invertible linear transformation.

Because S is a smooth bijective mapping of U to $S \cap W$, \tilde{S} will be smooth as well.

Because these two surfaces are smooth and a diffeomorphism is between two smooth manifolds (and we are trusting that S is locally Euclidean), we can then see that a translation/invertible linear transformation of a surface will preserve that surface's smoothness.