Chapter 4, Section 3. Exercise 1 only.

MTH 594, Prof. Mikael Vejdemo-Johansson
Differential Geometry Independent Study

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Exercise 4.3.1

If S is a smooth surface, define the notion of a *smooth function* $S \to \mathbb{R}$. Show that, if S is a smooth surface, each component of the inclusion map $S \to \mathbb{R}^3$ is a smooth function $S \to \mathbb{R}$.

<u>Smooth Function $S \to \mathbb{R}$</u>

Surface S can be parametrized as follows:

$$\sigma : U \to \mathbb{R}^3$$

Then, to define a smooth function f that will map S to $\mathbb{R}$ we must be able to map U to R smoothly, which can be done with the following composition:

$$f \circ \sigma : U \to \mathbb{R}$$

or more explicitly:

$$f(\sigma(U)) = \mathbb{R}$$
$$f(\mathbb{R}^3) = \mathbb{R}.$$

<u>Inclusion Map $S \to \mathbb{R}^3$</u>

Let some function g be defined as:

$$g : S \to \mathbb{R}^3$$

If g is an inclusion map, then $\forall s \in S$, $g(s) = s$. That is, $g(s) \subset \mathbb{R}^3$, as is to be expected of a $\mathbb{R}^3$ surface.

If $g(s)$ were surjective (meaning, if $g(s) \subseteq \mathbb{R}^3$), then it would be an identity function $I(s \in \mathbb{R}^3) = s$.

$g(s)$ can then be written as a vector-valued function: $g(s) =< x, y, z >$,
where $s_n =< x_n, y_n, z_n >$ (as implied above with the mention of $g(s) = s$). $\therefore$, each component of s, being scalar and belonging to $\mathbb{R}$, will then be mapped to $\mathbb{R} \in S$.