Chapter 3, Section 3. Exercises 1, 2 and 3

MTH 594, Prof. Mikael Vejdemo-Johansson
Differential Geometry Independent Study

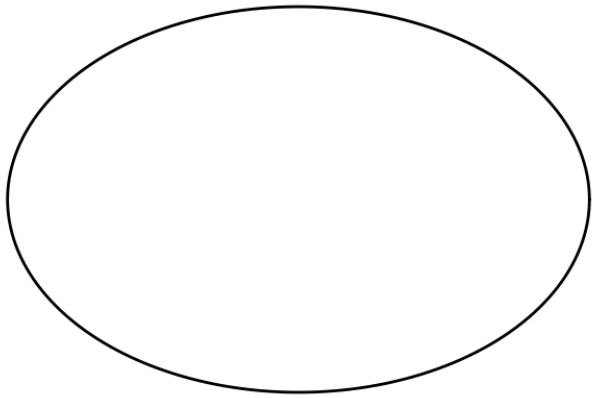
Matthew Connelly

October 2, 2018

Exercise 3.3.1

Show that the ellipse in Example 3.1.2 is convex.

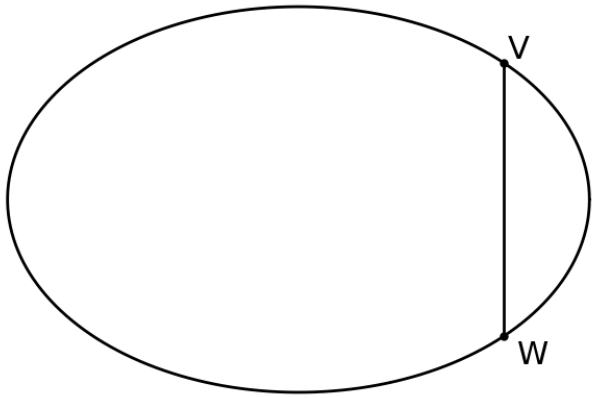
$\gamma(t) = (p\ cost, q\ sint)$ is our ellipse.



Our ellipse is convex if a straight line segment joining any two points of $Int(\gamma)$ is contained entirely within $Int(\gamma)$.

Because our ellipse is symmetrical about the x-axis, if it is convex above the x-axis, then it should also be convex below; meaning, there should be no surprises after $t = \pi$ (half of γ's period).

We can then check $Int(\gamma)$ using a chord inscribed in the ellipse, between two diametrically opposed points:



This chord can be written as:

$$\overline{VW} = \gamma(-t) - \gamma(t)$$

$$\overline{VW} = y(-t) - y(t)$$