Chapter 3, Section 2. Exercises 1 and 2

MTH 594, Prof. Mikael Vejdemo-Johansson
Differential Geometry Independent Study

Matthew Connelly

October 2, 2018

Exercise 3.2.1

Show that the length $l(\gamma)$ and the area $A(\gamma)$ are unchanged by applying an isometry to γ.

By definition, $l(\gamma)$ cannot be changed by an isometry, for an isometric transformation must preserve length.

Following this rule, the isoperimetric inequality will remain intact, based on its dependence on $l(\gamma)$:

$$A(\gamma) \leq \frac{l(\gamma)^2}{4\pi}$$

Where length will be equal to:

$$l(\gamma) = \int_0^T ||\gamma'|| \, dt$$

T being the period of the curve.