Chapter 4, Section 1. Exercises 1, 2, 3 and 5

MTH 594, Prof. Mikael Vejdemo-Johansson
Differential Geometry Independent Study

Matthew Connelly

October 8, 2018

Exercise 4.1.3

The *hyperboloid of one sheet* is

$$S = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 - z^2 = 1\}.$$

Show that, for every θ, the straight line

$$(x - z)cos\theta = (1 - y)sin\theta,\ (x + z)sin\theta = (1 + y)cos\theta$$

is contained in S, and that every point of the hyperboloid lies on one of these lines. Deduce that S can be covered by a single surface patch, and hence is a surface. (Compare the case of the cylinder in Example 4.1.3.)
