Chapter 4, Section 2. Exercises 1, 2, and 4 through 9

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Differential Geometry Independent Study

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Exercise 4.2.9

Show that every open subset of a smooth surface is a smooth surface.

A surface $S \in \mathbb{R}^3$ is defined in the context of an open $\mathbb{R}^2$ set U and an open $\mathbb{R}^3$ set W where $S \cap W$ is homeomorphic to U; this homeomorphism can be written as the following smooth bijective mapping:

$$\sigma : U \to S \cap W$$

S will be an atlas consisting of these homeomorphisms which are the patches of the atlas.

$\therefore$, taking an open subset of S consisting of one or more patch (a partial atlas) will produce another surface.

If S is smooth, then it is continuously differentiable all over; there should be no subset of S then that is *not* smooth.

Lastly, the patches of a surface are defined as being open; the subset of a surface will then be a surface itself, provided it is open, much in the way a patch must be open.