Chapter 4, Section 4. Exercises 1, 2, and 4.

MTH 594, Prof. Mikael Vejdemo-Johansson
Differential Geometry Independent Study

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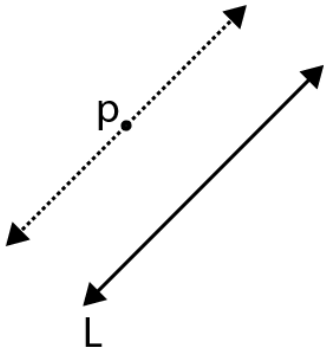
Exercise 11.1.1

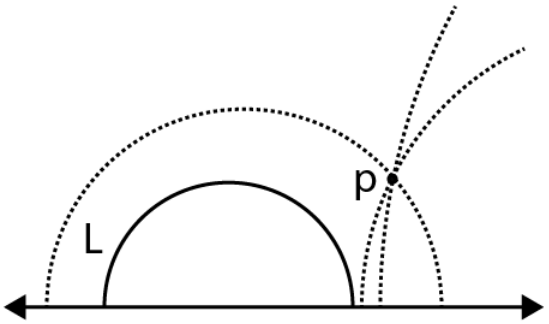
Show that, if l is a half-line geodesic in $\mathbb{H}$ and a is a point not on l, there are infinitely many hyperbolic lines passing through a that do not intersect l.

Preliminary Statements

Euclid's parallel postulate states that, in euclidean geometry, if there is a point p not on a line l, there is exactly one line parallel to l going through p.

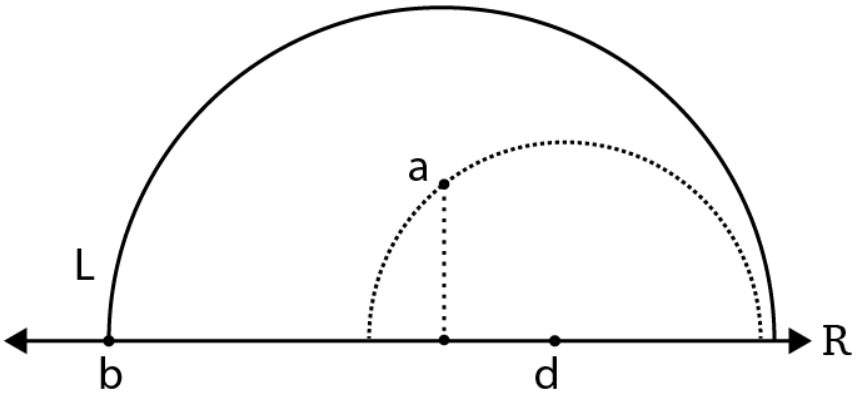


This is not true in hyperbolic geometry; rather, there are infinitely many lines parallel to l passing through p.



Solution

Let l meet the real axis at point b, and let the real component of $a > b$.



The semicircle with a center d on the real axis and radius $|a - d|$ passes through a and does not meet l if $|a - d| \leq |d - b|$.