

Chapter 3, Section 1. Exercise 1 only

MTH 594, Prof. Mikael Vejdemo-Johansson
Differential Geometry Independent Study

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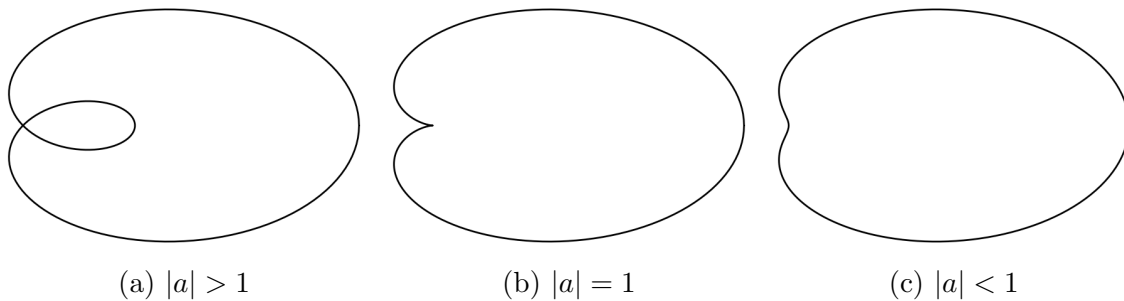
Exercise 3.1.1

Show that

$$\gamma(t) = ((1 + a \cos t)\cos t, (1 + a \cos t)\sin t)$$

where a is a constant, is a simple closed curve if $|a| < 1$, but that if $|a| > 1$ its complement is the disjoint union of three connected subsets of \mathbb{R}^2 , two of which are bounded and one is unbounded.

What happens if $a = \pm 1$?



A limaçon (a) will degrade to a cardioid (b).

When $|a| > 1$, γ will have a self intersection. By definition, a simple closed curve is not to have any self-intersections.

Proof of self-intersection:

For some t_0 and t_1 , γ will have the same position if γ has a self-intersection. Meaning,

$$\gamma(t_0) = \gamma(t_1)$$

but $t_0 \neq t_1$.