The College Application Problem

Max Kapur

Seoul National University
Department of Industrial Engineering
Management Science/Optimization Lab
maxkapur@snu.ac.kr

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Introduction

The optimal college application problem is a **novel combinatorial optimization problem**.

It involves **maximizing the expected maximum** of a portfolio of random variables subject to a budget constraint.

Methodological orientation:

- Investment with uncertain payoff, search for the efficient frontier recall **portfolio allocation** models.
- Generalizes the **knapsack problem**: Integral packing constraint, NP-completeness, approximation algorithms.
- Objective is a **submodular set function**.

Today's presentation: **define the problem** and briefly summarize our **solution algorithms**.

The admissions process

Consider a **single student's** college application decision.

Market contains m **schools**, indexed by $\mathcal{C} = \{1 \dots m\}$. School j is named c_j.

We know the student's **admissions probability** f_j at each school.

Let the independent **random variable** $Z_j \sim \mathrm{Bernoulli}(f_j) = 1$ if student is admitted, 0 otherwise.

Let $\mathcal{X} \subseteq \mathcal{C}$ denote the set of schools, or **application portfolio**, to which a student applies.

Application fees, time to write essays, and/or legal limits **constrain** applicant behavior. We consider a single knapsack constraint $\sum_{j \in \mathcal{X}} g_j \leq H$ where g_j is called c_j's **application cost**.

Utility model

Let $t_j \geq 0$ denote the **utility** the student receives if she attends c_j. Wlog, $t_j \leq t_{j+1}$.

Let t_0 denote her utility if she doesn't get into college. Wlog, $t_0 = 0$ (see paper).

The student's overall utility is the t_j-value associated with the **best** school she applies to and gets into:

$$\text{Utility} = \max\big\{t_0, \max\{t_j Z_j : j \in \mathcal{X}\}\big\}$$

The expected value of this quantity is called the **valuation** $v(\mathcal{X})$ of the portfolio $\mathcal{X}$.

Unpacking the portfolio valuation function

To get $v(\mathcal{X})$ into a tractable form, let $p_j(\mathcal{X})$ denote the probability that the student **attends** c_j.

This happens if and only if she **applies** to c_j, is **admitted** to c_j, and is **not admitted** to any school she prefers to c_j:

$$p_j(\mathcal{X}) = \begin{cases} f_j \prod_{\substack{i \in \mathcal{X}: \\ i > j}} (1 - f_i), & j \in \{0\} \cup \mathcal{X} \\ 0, & \text{otherwise.} \end{cases}$$

Therefore,

$$v(\mathcal{X}) = \sum_{j=1}^{m} t_j p_j(\mathcal{X}) = \sum_{j \in \mathcal{X}} \Bigl(f_j t_j \prod_{\substack{i \in \mathcal{X}: \\ i > j}} (1 - f_i) \Bigr).$$

Problem statement

Problem 1 (The college application problem)

$$\begin{aligned} \text{maximize} \quad & v(\mathcal{X}) = \sum_{j \in \mathcal{X}} \Bigl(f_j t_j \prod_{\substack{i \in \mathcal{X}: \\ i > j}} (1 - f_i) \Bigr) \\ \text{subject to} \quad & \mathcal{X} \subseteq \mathcal{C}, \quad \sum_{j \in \mathcal{X}} g_j \leq H \end{aligned}$$

Problem 2 (The college application problem, INLP form)

$$\begin{aligned} \text{maximize} \quad & v(x) = \sum_{j=1}^{m} \Bigl(f_j t_j x_j \prod_{i > j} (1 - f_i x_i) \Bigr) \\ \text{subject to} \quad & x_j \in \{0, 1\}, j \in \mathcal{C}; \quad \sum_{j=1}^{m} g_j x_j \leq H \end{aligned}$$

Timothy Peck — May 7, 2021 — 5 — 12th Grade, College Lists

Safety, Target, & Reach Schools: How to Find the Right Ones

What's Covered:

- What Are Reach, Target, and Safety Schools?
- Factors that Impact Your Chances
- Elements of a Balanced College List

Creating a school list is an important-yet-tricky step in the college application process. A strategically constructed school list weighs your desire to attend reach schools—the institutions you dream about going to—along with safety schools where you're very likely to secure admission. Consequently, the ideal school list is balanced between reach, target, and safety schools, allowing you to shoot for the stars while also ensuring admission into at least one school.

What Are Reach, Target, and Safety Schools?

"Reach," "safety," and "target" are common terms used in college applications to describe the odds a student has of getting accepted at a particular institution. Understanding these terms, and which categories colleges fall into, is a critical step in the application process.

What is a Reach School?

Reach schools are colleges where you have less than a 25% chance of admission (this is your

The JoongAng

[대입 수시 전략] 총 6번의 기회 …'상향·소신·안정' 분산 지원하라

중앙일보 | 업데이트 2015.08.26 10:15 지면보기

전민희 기자 구독

대학 최저학력기준 고려해 전략 지원
지난해 같은 전형 합격한 선배 내신 참고
수능 전 대학별고사 보는 곳은 최소화

'지피지기 백전불태(知彼知己百戰不殆).' 적을 알고 나를 알면 백 번 싸워도 위태롭지가 않다는 뜻이다. 고대 중국의 병법서인 『손자』에 나온 말이지만 현대사회에서도 여러 가지 분야에서 회자된다. 그중 하나가 대학입시다. 특히 2주 앞으로 다가온 수시모집은 전형 종류가 다양해 '적'(모집전형) 을 알고, '나'(학생)에 대해 파악하는 게 무엇보다 중요하다.

자신의 학교생활기록부, 교과성적, 대학별고사 준비 상황, 예상 수능점수, 최저학력기준 통과 가능성에 대해 자세히 살핀 후 지원해야 합격률을 높일 수 있다. 수시모집 마무리 전략을 알아봤다.

논술전형도 학생부 성적 기준으로 지원

Do you trust the admissions consultant's advice?

Existing solutions

Admissions consultants recommend a **distributive heuristics** that splits applications evenly among reach, target, and safety schools. Turns out to be a **risk-averse** approach.

Another intuitive idea is the **linearization heuristic**: Since the expected utility associated with applying to c_j (alone) is $f_j t_j$, solve the knapsack problem

$$\text{maximize} \quad \sum_{j \in \mathcal{X}} f_j t_j \qquad \text{subject to} \quad \sum_{j \in \mathcal{X}} g_j \leq H$$

as a surrogate. This solution can be **arbitrarily bad**.

Fu (2014) solved a similar problem by **enumeration**, which is intractable for $m \geq 20$ or so.

Our algorithms provide both **time and accuracy guarantees**.

Our algorithms

In the **special case** where each $g_j = 1$, we provide an $O(m^2)$ algorithm.

The general problem is **NP-complete** (reduction from knapsack). We offer four algorithms:

- A linear relaxation and **branch-and-bound scheme**. Primarily of theoretical interest.
- A **dynamic program based on total expenditures**. Exact solution in $O(Hm + m \log m)$ time (pseudopolynomial). Very fast for "typical" instances in which g_j are small integers.
- A different DP based on truncated portfolio valuations. $(1-\varepsilon)$-optimal solution in $O(m^3/\varepsilon)$ time: an **FPTAS!**
- A **simulated annealing heuristic**. Fast, typically within 2% of optimality.

Conclusion

"Maximax" form, integrality constraints make the college application problem **theoretically interesting**. Formally, it is a submodular maximization problem, but its approximability is more like knapsack (cf. Fisher et al. 1978; Kulik et al. 2013; Kellerer et al. 2004).

Solutions to the college application problem have **practical value**: US admissions consultants charge an average of $200/hr (Sklarow 2018)!

$\Rightarrow$ Open-sourcing our code for public benefit (Kapur 2022).

Lots of extensions to consider: parametric risk aversion, distribution constraints, FPTAS memory-usage improvements.

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