Foundations of the Age-Area Hypothesis

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Background

- The economic basis for indigenous institutions:
    - Baker (2003, 2008), Baker and Miceli (2005), Baker and Jacobsen (2007, 2008).
- Exploring the relationship between environment, technology, and institutions.
- Interesting perhaps, but of limited larger interest...

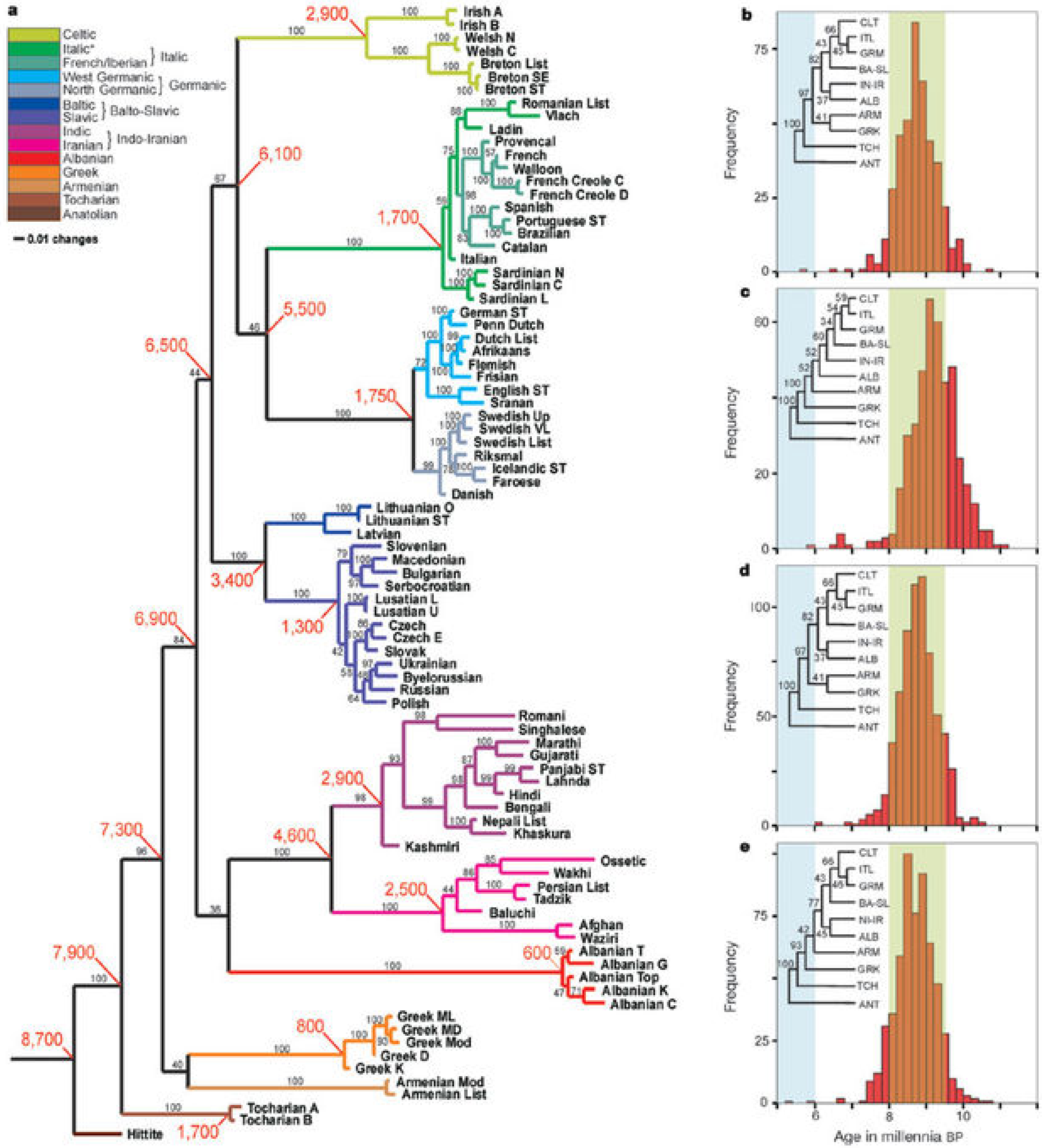
But recently...

- Applications in economic growth:
  - Alesina et. al. (2005), Spolaore and Wacziarg (2013), Michalopoulus (2012), Fenske (2012)
- Computational linguistics and Phylogenetic approaches to analyzing cultural diversity (Mace, 2006)
  - A computational field blending tools from biology, geographical data. Computing power!
- Incorporation of geographical data into analyses.

Deep Question: How did ethnic and geographic diversity that we observe today come about?

Cultural Phylogenetics

- Modeling cultural evolution using Phylogenetic tools.
- Attractive and novel approach - explicit consideration of path dependency. Typical Econometric treatment a hammer in search of a nail?
- Computational linguistics - direct means of phylogeny-building (Mace, 2006)
- Atkinson and Gray (2006) example: Indo-European Tree.


a
Celtic
Italic*
French/Iberian
Italic
West Germanic
North Germanic
Germanic
Baltic
Slavic
Balto-Slavic
Indic
Iranian
Indo-Iranian
Albanian
Greek
Armenian
Tocharian
Anatolian
0.01 changes
8,700
7,900
7,300
6,900
6,500
6,100
5,500
4,600
3,400
2,900
2,500
1,700
1,750
1,300
800
600
Irish A
Irish B
Welsh N
Welsh C
Breton List
Breton SE
Breton ST
Romanian List
Vlach
Ladin
Provencal
French
Walloon
French Creole C
French Creole D
Spanish
Portuguese ST
Brazilian
Catalan
Italian
Sardinian N
Sardinian C
Sardinian L
German ST
Penn Dutch
Dutch List
Afrikaans
Flemish
Frisian
English ST
Sranan
Swedish Up
Swedish VL
Swedish List
Riksmal
Icelandic ST
Faroese
Danish
Lithuanian O
Lithuanian ST
Latvian
Slovenian
Macedonian
Bulgarian
Serbocroatian
Lusatian L
Lusatian U
Czech
Czech E
Slovak
Ukrainian
Byelorussian
Russian
Polish
Romani
Singhalese
Marathi
Gujarati
Panjabi ST
Lahnda
Hindi
Bengali
Nepali List
Khaskura
Kashmiri
Ossetic
Wakhi
Persian List
Tadzik
Baluchi
Afghan
Waziri
Albanian T
Albanian G
Albanian Top
Albanian K
Albanian C
Greek ML
Greek MD
Greek Mod
Greek D
Greek K
Armenian Mod
Armenian List
Tocharian A
Tocharian B
Hittite
b
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d
e
CLT
ITL
GRM
BA-SL
IN-IR
ALB
ARM
GRK
TCH
ANT
Frequency
Age in millennia BP
6
8
10

Practical Questions:

- A sophisticated statistical description of relationships between cultures.
- What can be said about the Geography of this the Tree?
- How did the cultures on the tree come to be where they are?
- Which related cultures have been in close proximity, and for how long?

The Age-Area Hypothesis (AAH)

- Sapir (1916) - *the root of the Phylogenetic tree is the most likely geographical point of origin.*
- Also: maximum divergence, maximum variety, maximum differentiation...
- Recursive application - migratory routes
- When coupled with a Phylogeny - times *and* places.
- Used to resolve historical debates, but also could be important in creating new theories

Old applications and continuing debates

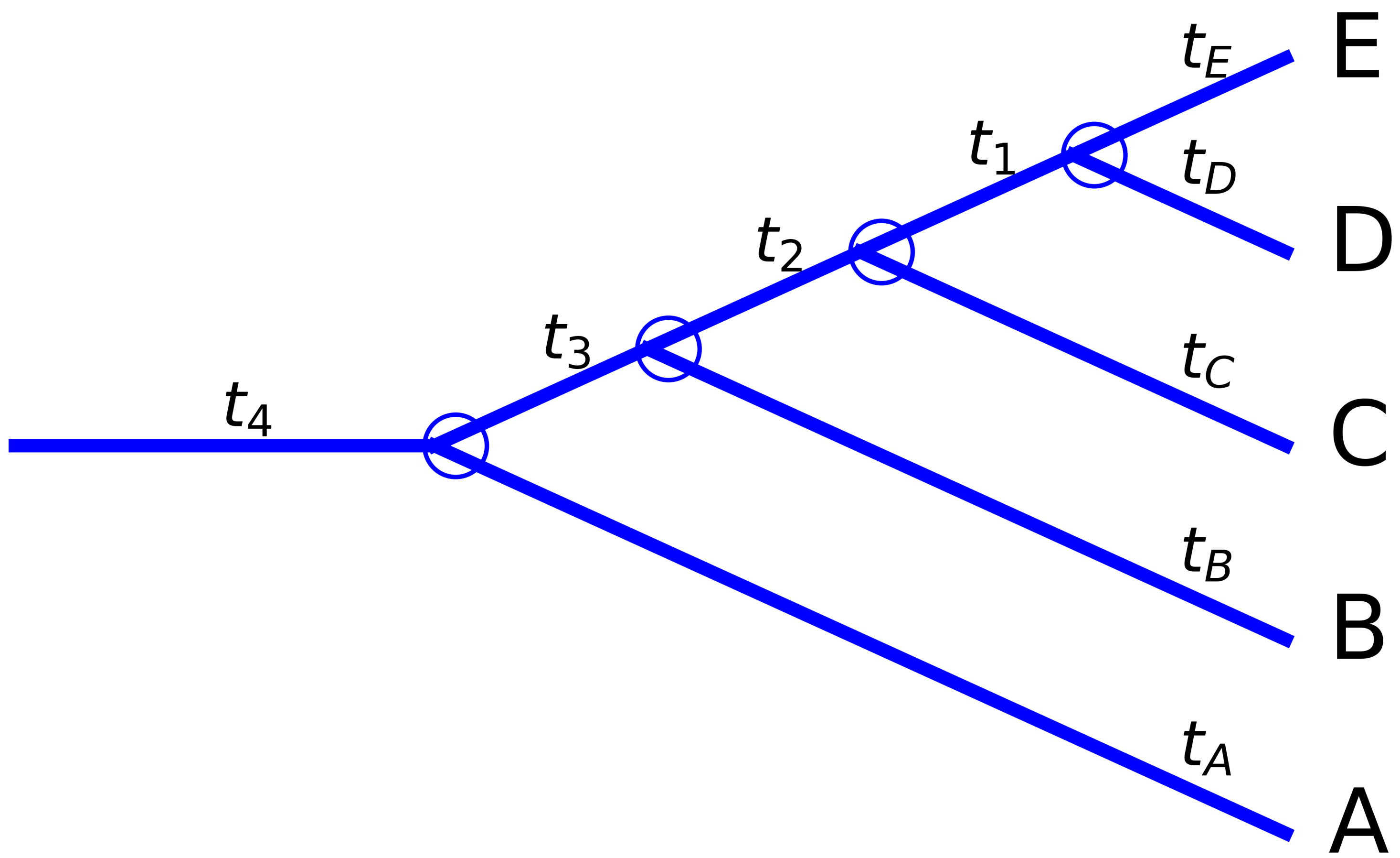
- Origins of Athabaskan/Na-Dene speakers
- Indo-European origins
- Afro-Asiatic origins
- Spread of Bantu peoples
- Native American population dispersal

On the need or theory...

- Greenhill and Gray (2005) write: "many expansion scenarios are little more than plausible narratives. A common feature of these narratives is the assertion that a particular lineof evidence (archaeological, linguistic, or genetic) is 'consistent with' the scenario. 'Consistent with' covers a multitude of sins.
- Regressions and Spatial Econometrics - Leave me wondering what the DGP is...

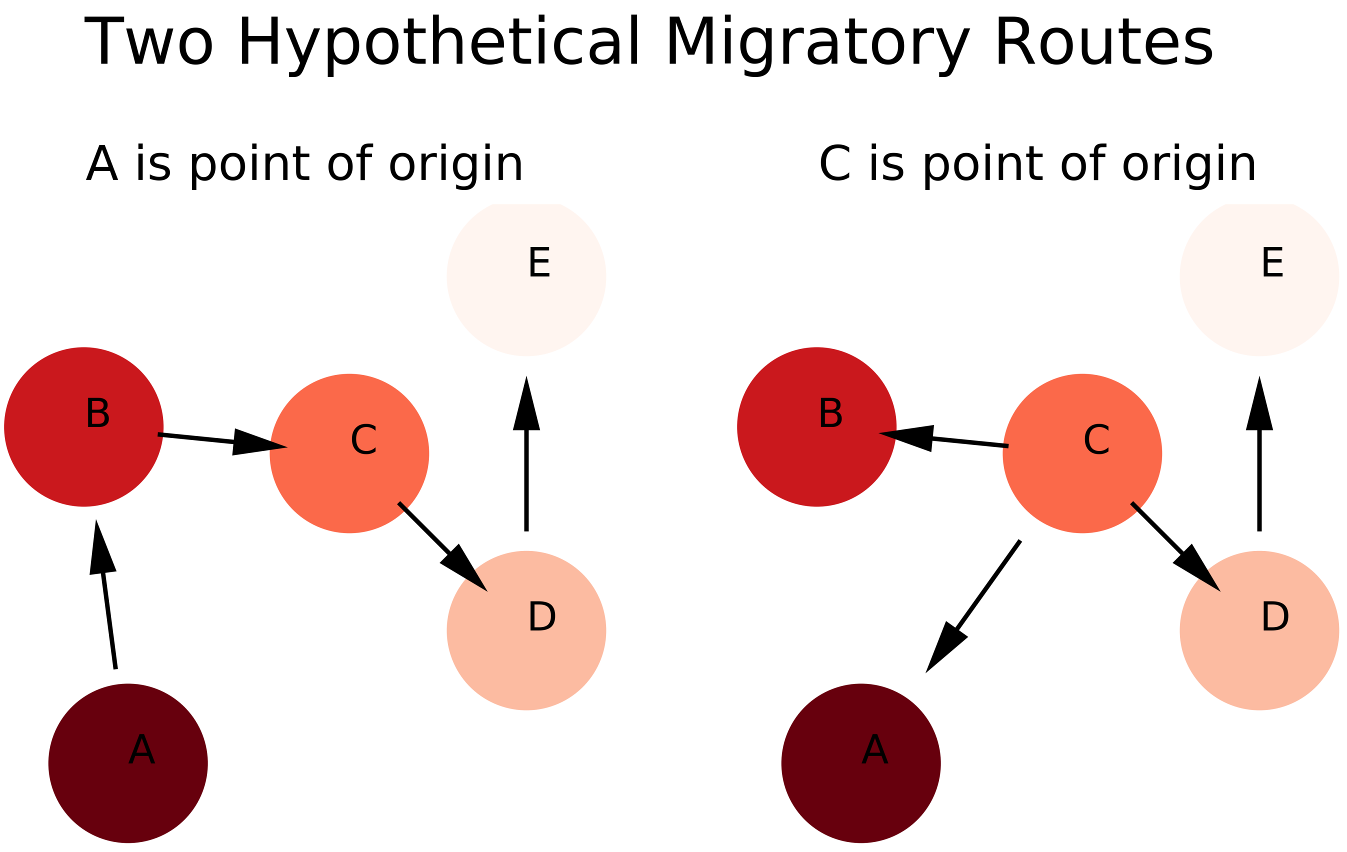
So why believe the AAH (or not)?

- Occam's Razor
- Minimum effort or # of moves
- Dyer (1956, p. 613) hits upon the idea of conserving moves of a particular sort: "...the probabilities of different reconstructed migrations are in inverse relation to the number of language movements required."



E
D
C
B
A
t_E
t_D
t_C
t_B
t_A
t_1
t_2
t_3
t_4

Problem Preview



Candidate Migratory Histories:

- A is point of origin - A to B to C to D to E
- C is point of origin - C to A, C to B, C to D to E
- Both are consistent with observed phylogenetic difference or drift. **The tree tells us which migrations happened first!**
- Note "minimum moves" doesn't get us very far. Both have four moves!
- Actually - example approximates the debate between Ehret (2004) and Bellwood and Diamond (2003) about Afrasan or Afroasiatic cultures/languages.

Basic Model:

- Assume a full, rooted binary tree
  - k terminal nodes/taxa/cultures, $k-1$ internal nodes.

    $k-1$ moves needed to span the tree.
- Current locations coincide with historic locations
- All constituents of the tree observed

Definitions

Migratory Event

A location jump from one location to a new, unoccupied one.

Migratory Chain

A chronological sequence of jumps through connected nodes that end at a terminal node/taxa/culture.

Migratory History

A collection of chains spanning the whole tree.

Basic assumptions

1. A migratory chain occupies one location at a time ("propensity to migrate" passed location to location).
2. A chain corresponds with a population movement. When a chain moves from its location to a new one, a new chain starts in its place.
3. Migratory chains move to new locations at random times, according to an Exponential/Poisson density.
4. Each migratory chain is unique in that it has its own parameters.

Chain One:

- requires a chain from A to B to C to D to E (or D to E)
- By the previous rules, new chains start at A, B, C, and D. Let T denote the length of the tree.
- Likelihood:

$$L_A = \frac{(\lambda_1 T)^4 e^{-\lambda_1 T}}{4!} \times \frac{(\lambda_A t_A)^0 e^{-\lambda_A t_A}}{0!} \frac{(\lambda_B t_B)^0 e^{-\lambda_3 t_B}}{0!} \frac{(\lambda_C t_C)^0 e^{-\lambda_C t_C}}{0!} \frac{(\lambda_D t_D)^0 e^{-\lambda_D t_D}}{0!}$$

- Seems like overkill, but the degeneracies are important!

Log-Likelihood:

$$\begin{aligned}\ln L_A = 4\ln(\lambda_1 T) - 4\lambda_1 T - \ln(4!) \\ - \lambda_A t_A - \lambda_B t_B - \lambda_C t_C - \lambda_D t_D\end{aligned}$$

Optimized with $\lambda_A = \lambda_B = \lambda_C = \lambda_D = 0$, and then:

$$\lambda_1 = \frac{4}{T}$$

Substituting this all back into the original likelihood gives "Profile" or "Concentrated" likelihood:

$$L_A = \frac{4^4 e^{-4}}{4!}$$

Chain Two:

Log-Likelihood

$$L_C = \frac{(\lambda_1(t_4+t_A)^1 e^{-\lambda_1(t_4+t_A)}}{1!} \frac{(\lambda_2(t_3+t_B))^1 e^{-\lambda_B(t_3+t_B)}}{1!} \\ \times \frac{(\lambda_3(t_2+t_1+t_E))^2 e^{-\lambda_3(t_2+t_1+t_E)}}{2!} \\ \times \frac{(\lambda_C t_C)^0 e^{-\lambda_C t_C}}{0!} \frac{(\lambda_D t_D)^0 e^{-\lambda_D t_D}}{0!}$$

Highlight: fewer degenerate chains, and more active chains!

Chain Two: profile/concentrated-likelihood:

$$L_C = \frac{1^1 e^{-}}{1!} \frac{1^1 e^{-1}}{1!} \frac{2^2 e^{-2}}{2!} = \frac{2^2 e^{-4}}{2!}$$

Comparison of L_A and L_C is a race between $\frac{4^4}{4!}$ and $\frac{2^2}{2!}$.

Relative likelihood: $P(A|A \text{ or } C) = \frac{\frac{4^4}{4!}}{\frac{4^4}{4!}+\frac{2^2}{2!}} = .84$

Key Feature:

$$h(n) = \frac{n^n}{n!}$$

This kernel is *convex*. Breaking it up into smaller chunks reduces likelihood. That is:

$$h(n) > h(n-k)h(k)$$

Continuing on...

- Probabilistic explanation of Occam's Razor in a way that captures Dyen's notion of simplicity.
- How can these ideas be related to a notion of distance or divergence?
- How can divergence and probability be tied together formally, as the AAH posits?

A Start:

- With each location/culture k, there are a family of possible migratory histories $\mathcal{H}_k$ that explain the underlying geography of the phylogeny.
- For $H_k \in \mathcal{H}_k$, define $N(H_k)$ as a count of the number of (non-degenerate) migratory chains in the history.
- Define $n(C)$ as the number of events in a non-degenerate migratory chain, and then define:
- $n^*_{H_k} = \max_{C_{ik} \in H_k}[n(C_{1k}), n(C_{2k}), ..., n(C_{N(H_k)k})]$ - The maximum node count for the chains in H_k.

Definition: Dyen Divergence

Start with a function $D_{H_k} = m(n^*_{H_k}, N(H_k))$, where m is increasing in its first argument, and decreasing in the second. Define now the *Dyen Divergence* as

$$D_k = \max[D_{H_{1k}}, D_{H_{2k}}, ..., D_{H_{Ik}}]$$

A family of divergence measures. Examples:

- $D^1_k = n^*_{H_k} - N(H_k)$
- $D^2_k = \frac{n^*_{H_k}}{N(H_k)}$

Age-Area Theorem

Suppose model assumptions hold, and define a Dyen Divergence measure. Then:

$$D_k \geq D_j \implies L_k \geq L_j$$

Further

$$\begin{aligned} & k = \arg\max\left[D_1, D_2, D_3, ..., D_n\right] \\ \implies & k = \arg\max\left[L_1, L_2, L_3, ..., L_n\right] \end{aligned}$$

Proof (sketch)

Note likelihood obeys

$$L_k \propto \prod_{j=1}^{N(H_k^*)} h(n_j), \quad \sum n_j = I$$

Because of convexity of $h(n)$, pile up as many n's in as few chains as possible. Analogy: a risk-loving investor with fixed assets and a bunch of investment choices.

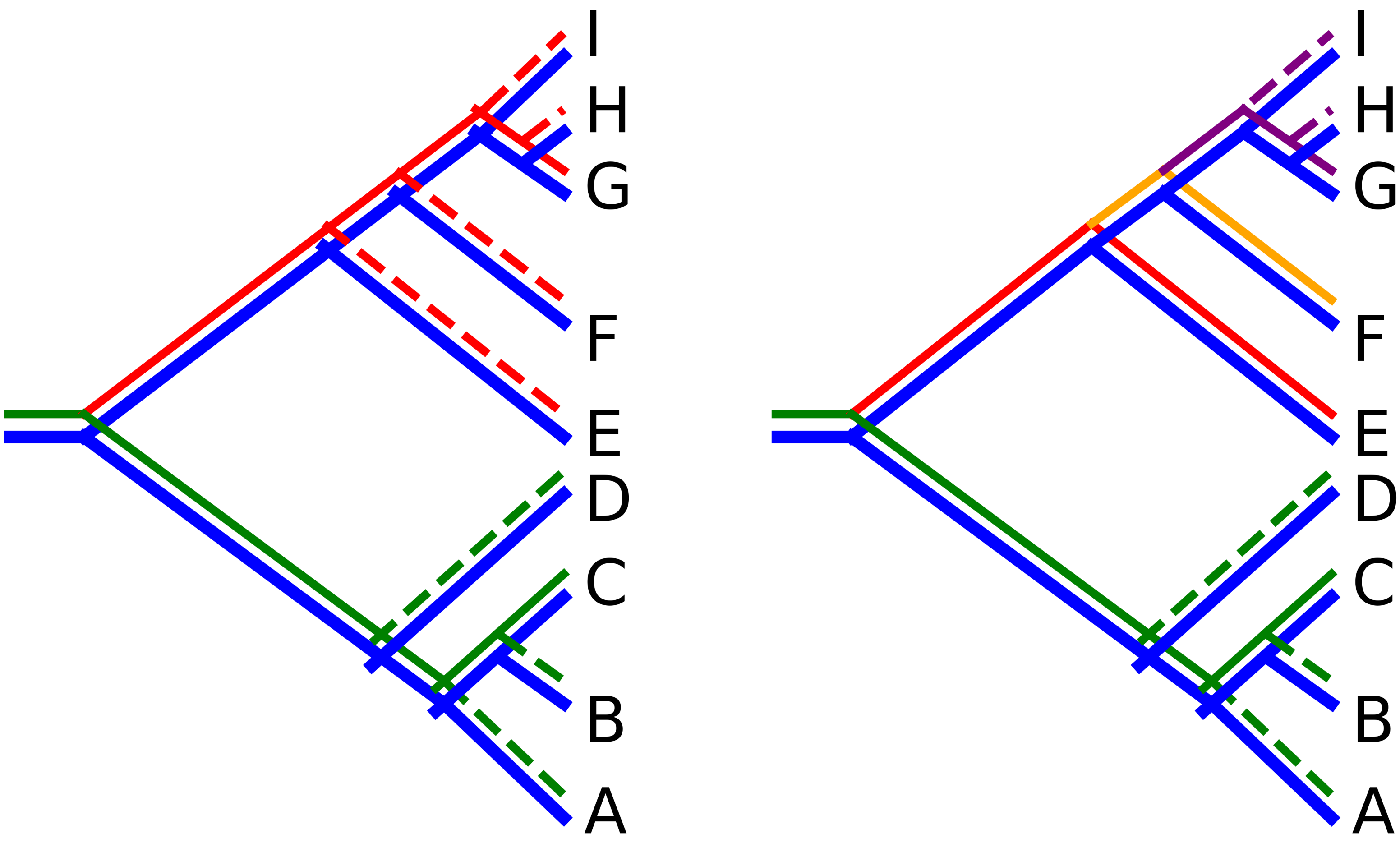
Also, finite ordered sets have a maximum.

Illustration of Dyen Divergences:

$$D_i^1 = \frac{n^*_{H_k^*}}{N(H_k^*)}$$

$$D_i^2 = n^*_{H_k^*} - N(H_k^*)$$

Additional Example: E versus I



Comparison of divergence measures

- If E if the point of origin:
  - chain from E to D to A to B to C
  - chain from E to F to I to G to H
  - Dyen Divergence - 2 chains, 4 events each. $D^1 = 2$, $D^2 = 2$.
- If I is the point of origin:
  - chain from I to D to A to B to C
  - chain from I to E, chain from I to F
  - chain from I to H to G.
  - Dyen Divergence - 4 chains, with 4, 1, 1, and 2 events. $D^1 = 0$, $D^2 = 1$.

Likelihoods

- For E, calculating it out gives relative likelihood as .84.
- A better contender to E, however, is D. Two chains, one with 5 events, and one with 3.
  - Dyen Divergence - $D^1 = 3$, $D^2 = 2.5$
- Oftentimes, this isn't obvious from looking at the Tree...

Bells and Whistles

- Known branch lengths -
  - Poisson becomes Exponential distribution.
  - Important case for including migration model with tree estimation.
- Algorithm for calculating probabilities/divergences -
  - one can traverse the tree backwards, using dynamic programming to pick out the most likely continuation path
- Include other information in the decision
  - physical distance
  - prior knowledge

Micro foundations

- Why would one believe the exponential/Poisson arrival rate story?
- Idea: stochastic population growth model
    - Chain is propelled by a positive resource/technology shock.
    - If population in a location reaches a barrier, the shock dissipates.
    - Population splits and some fraction moves on to a new area.
    - A new shock parameter is drawn in the location.

Formal approach (Baker, 2008):

- Utility, children, and (net) income are equal in the current location:
- Income has a fixed component, a congestion component, and a stochastic component: $y_t = 1 + r(1 - \frac{p_t}{K}) + \sigma(\epsilon_{t+\Delta} - \epsilon_t)$
- Total population next period, $p_{t+\Delta}$ is $p_t k_t$ or:

$$p_{t+\Delta} = p_t y_t = p_t \left[1 + r\left(1 - \frac{p_t}{K}\right) + \sigma(\epsilon_{t+\Delta} - \epsilon_t)\right]$$

As Δ gets small...

$$dp = rp\left(1 - \frac{p}{K}\right) + \sigma p dz$$

Stochastic Logistic population growth model: drift $rp\left(1 - \frac{p}{K}\right)$ and variance $\sigma^2 p^2$.

Crucial property: *stationary distribution*.

Theorem

Nobile, Ricciardi & Sacerdote (1985)

If a sde has a stationary distribution, the first passage time to a barrier B given initial population p_0, **is approximately exponential**:

$$g(B, t|p_0) \approx \frac{1}{t_1(B|p_0)} e^{-\frac{t}{t_1(B|p_0)}}$$

where t_1 is the first moment of the distribution of the first passage time distribution.

Rounding out the Story

1. Moving to a new location involves a cost c and requires a minimum population M to move.
2. If $p = B$ is achieved K falls immediately to $K - D$ for the current generation.
3. Current generation: Can split and move to a new location?
4. Upon exit, a new B, K, D combination is drawn.

Current Parameterization

- Utility is $1 + r(1 - \frac{p_t}{K})$.
- Assume that $B = K$.
- Costs of moving are $c = r$, so utility in a new location is maximally 1, while utility before the barrier is hit in the original always greater than one.
- When $p = B = K$, arbitrage condition dictates size of staying population and emigrating population:

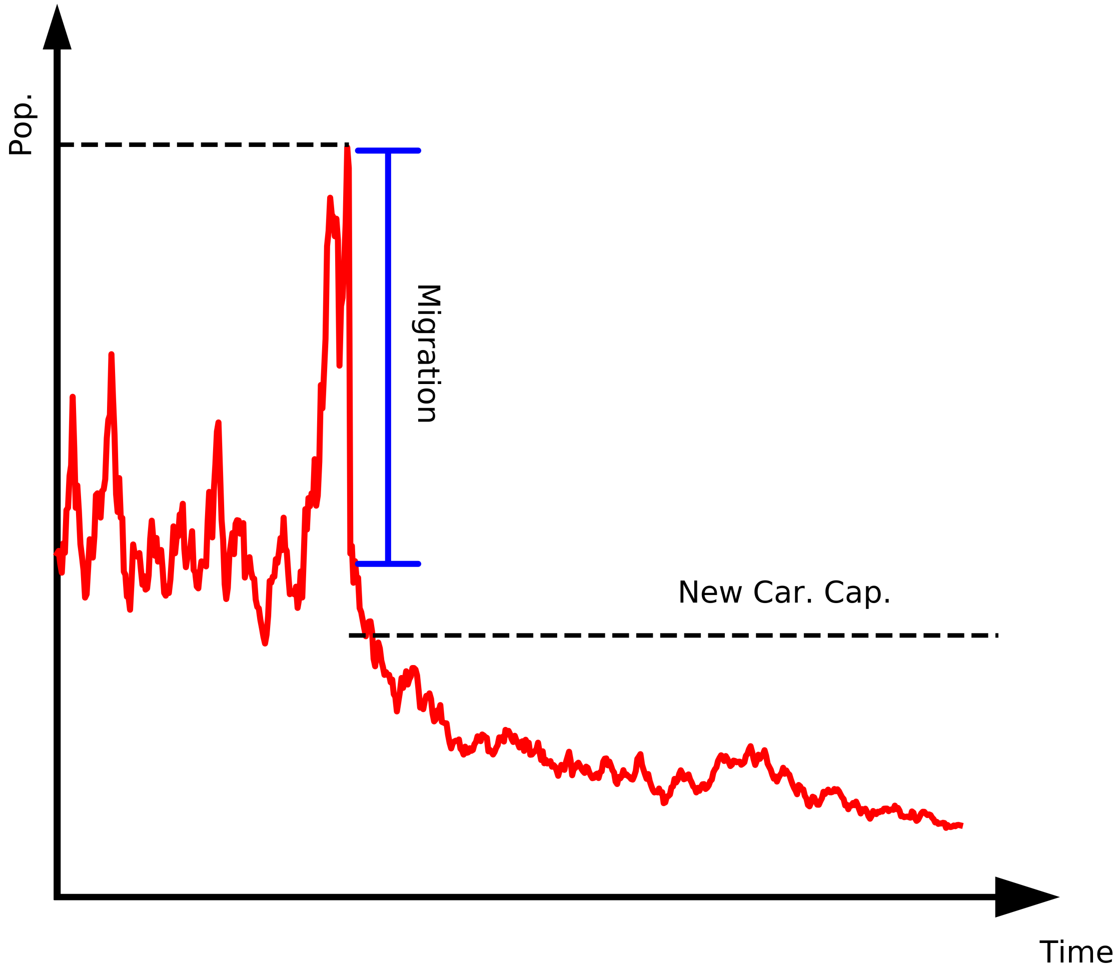
Arbitrage

If m is the emigrating group, then:

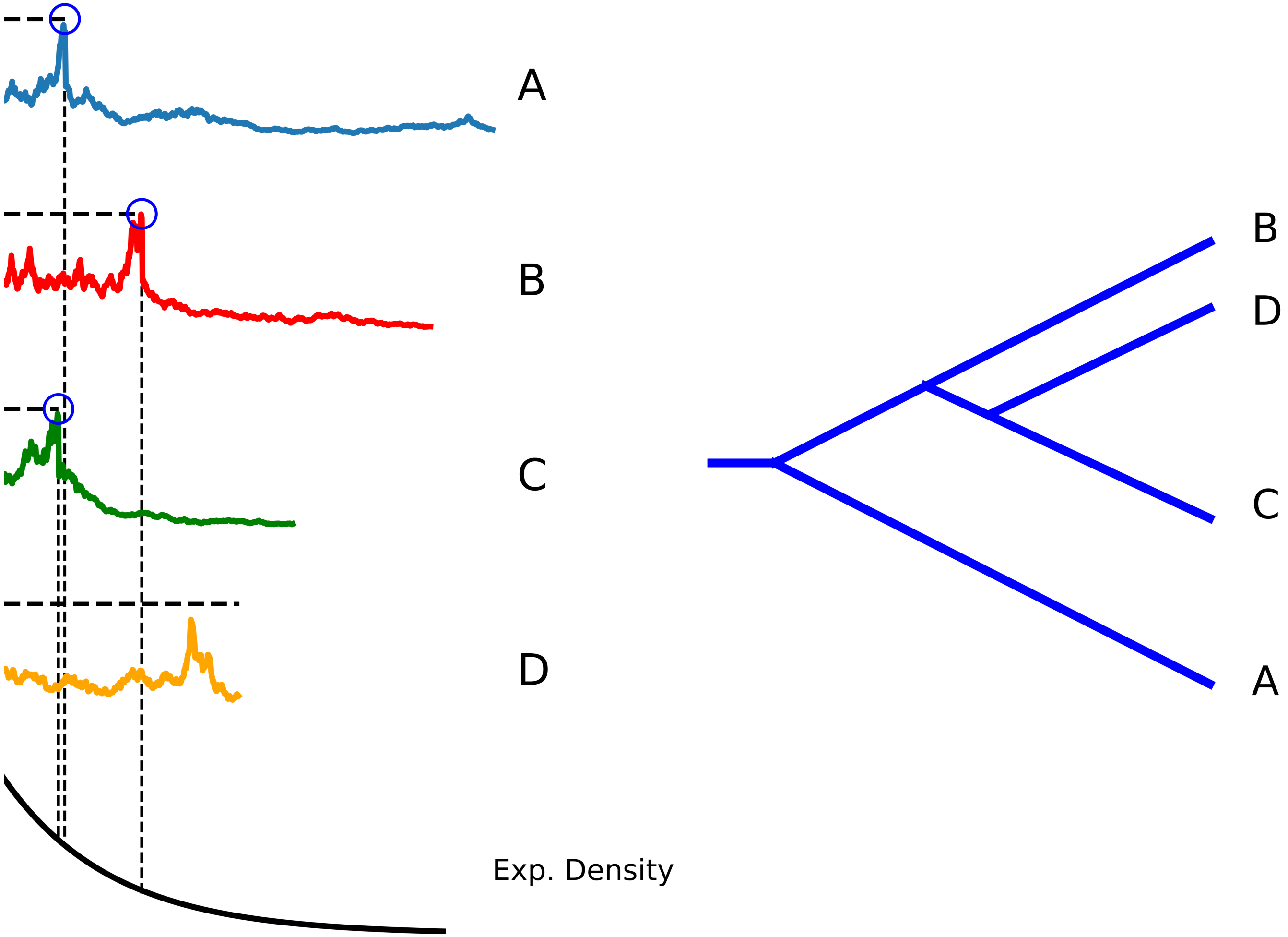
$$1 + r\left(1 - \frac{B - m}{K - D}\right) = 1 - c + r\left(1 - \frac{m}{K}\right)$$

Results in $B = K$, $p_0 = m = \frac{DK}{2K - D}$

Illustration:



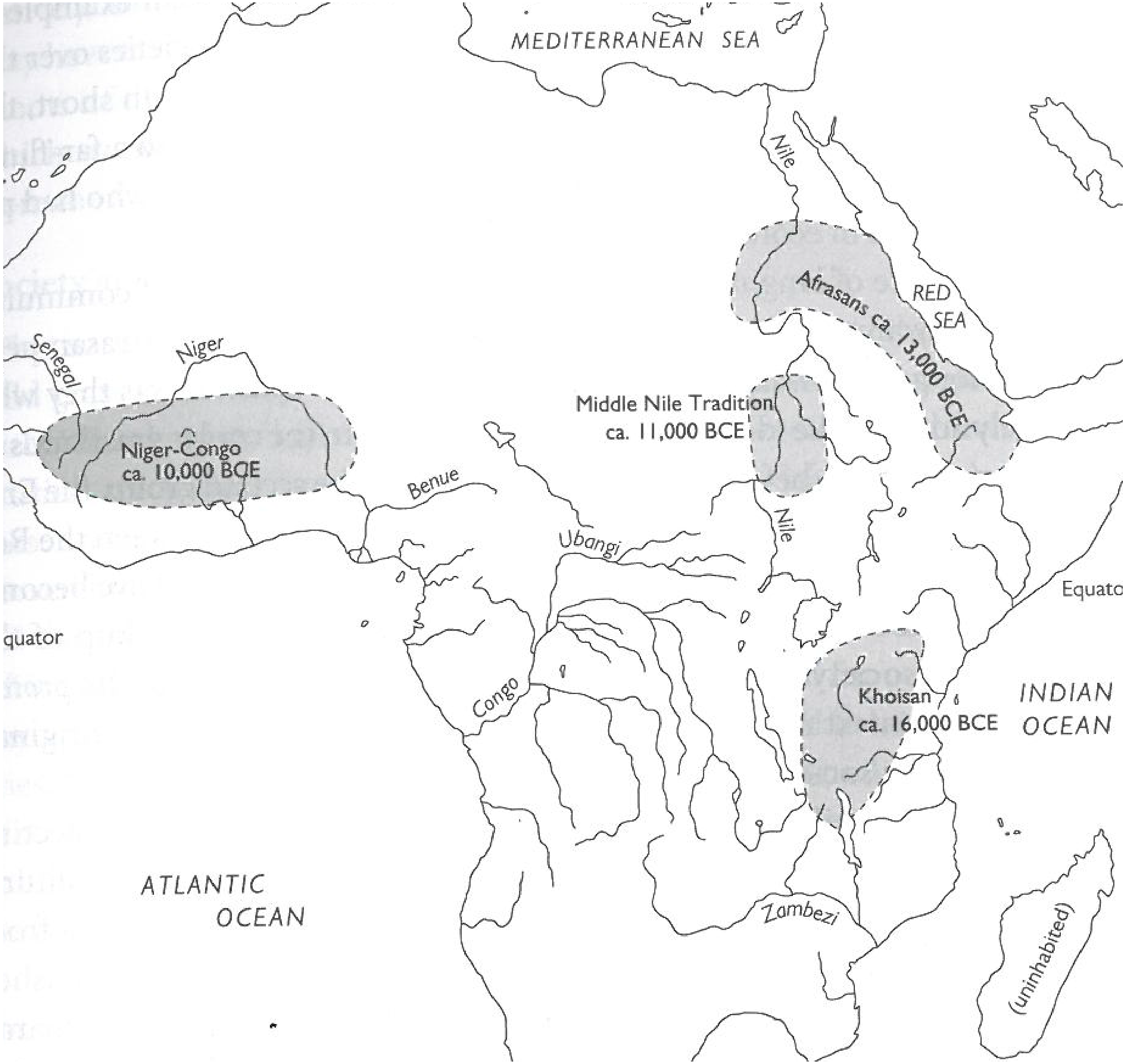
Further illustration:



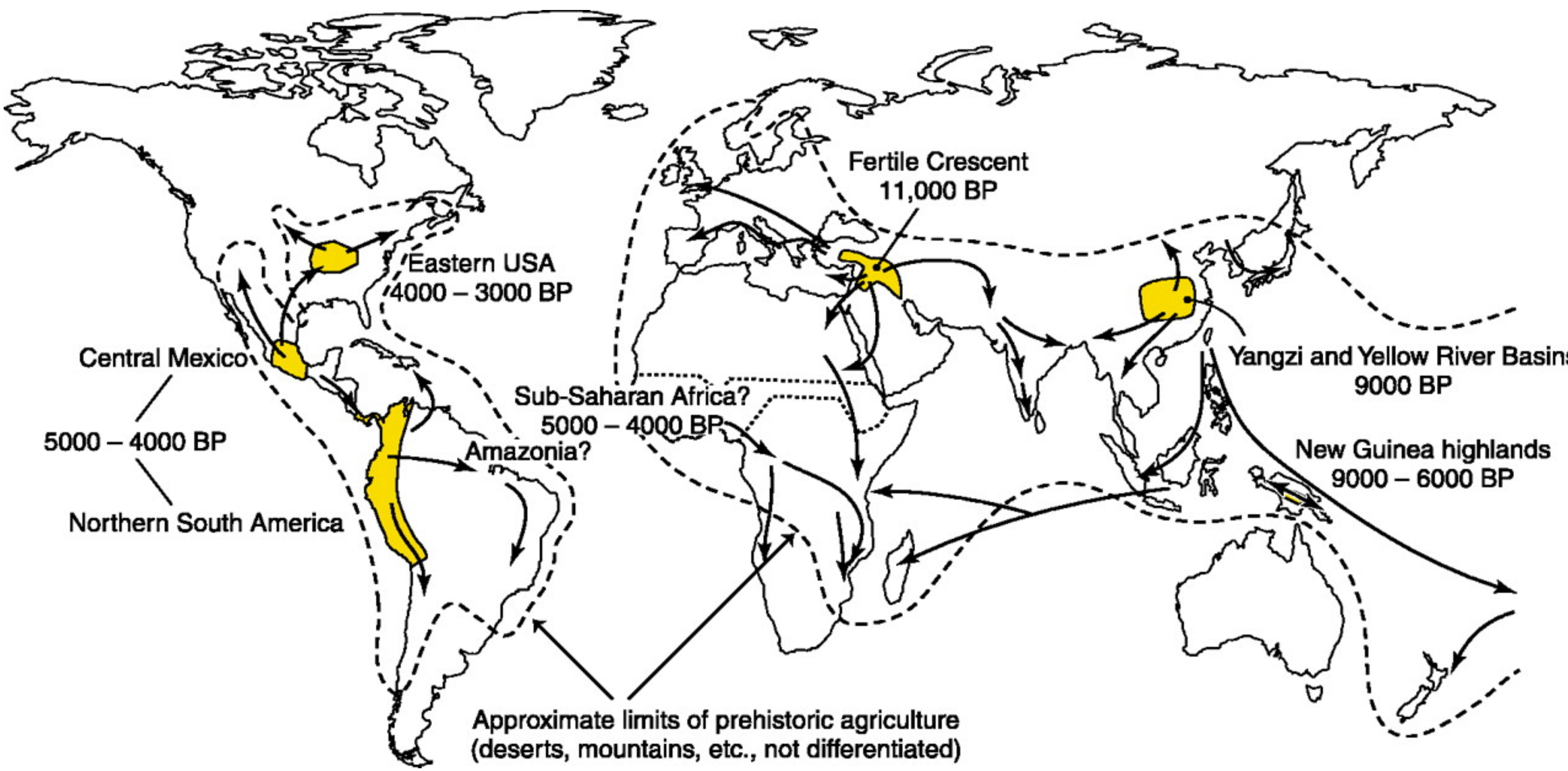
Applications

- Afrasan or Afroasiatic and its point of origin. Arabic and Semitic languages, Ancient Egyptian, and Ethiopiac languages as well. Where did it all begin?
- NaDene phylogeny and its point of origin. Simulating *spatial and temporal points of origin*.

From Ehret (2000) Figure - origins of Afrasans



Diamond and Bellwood (2003)- origins



Probabilities of Origin Points

- Link to Afrasan Map
- Link to Na Dene Map
- Link to Khoisan Map

Sampling NaDene History

- Idea: blend known branch lengths, migratory distances, and estimation of a linguistic phylogeny using standard methods:
  - Lexicostatistics/glottochronology
  - Bayesian Priors on some dates, MCMC sampling (Baker, 2015)
- Create a probability distribution over histories.
- Idea follows Baxter and Ramer (1993), Mace and Pagel (1993) - use the type of first letter for Swadesh lists.

```
cwittl-cw-ctssscwlilcclttyiNycciiccsts-s  HAIDA
citllyyccltttctclNcliccssicsycccccticrci  CHIRICAHUA
sNNcltllsccsttcsssssssssiiiiissttcttttcs  NAVAJO
sNtlNtllyct-tctstmc-clc-Nicc-sytsct-lm-i  JICARILLA
sNNlNNllyc--sctcNiwscl--Nict-sstsct-lp-y  JICARILLA_APACHE
sNNlNclNyct-cctcNNcsclc-Nict-cctcct-cp-c  SAN_CARLOS
wNNlNcllyit-cctcNNc-clc-Nicc-sytsct-cm-i  LIPAN
sNNlitllyii-tctcNccccl c-Nicc-cstcci-cc-i  BEAVER
sNNlNtllytt-tttcNNctcNt-titt-stttct-cp-i  CARRIER
sNNiNtllyti-iiiiiiiiilt-iicp-sptccN-ti-i  C_CARRIER
sNNiNtllsti-ttiiiiiicNt-citt-sstccN-ti-i  KUTCHIN
iiiiNttl-tiiiiiiiiiiiiiii-tt-sstcctcit-p  HARE
sNciNtllyct-tttcNNciclt-Nicc-ssttct-cp-i  CHIPEWYAN
sNNlitllytt-tctctNwc-N--ticc-sstcct-c--y  SARCEE
sNNlNtlNyct-tctcNicsclp-licc-ysccci-lp-i  HUPA_2
sNNlNNllyct-tctcNccsclp-licc-yctcci-lp-i  MATTOLE
stttNilt-t--tityttwsclpstc-c--sc-cictc-s  KATO
sNNlNtllyNt-tctcNccsclp-licc-yctcci-lp-i  GALICE
sNNlNtllyciitciiiiiiiiiiiyttysstccittiii  TANACROSS
sNNlNNlcyct-tctcNicsclp-licc-cstcci-lp-i  EYAK
ittssNcc-ccccsNcccctcsttNccciccccccctsscc  TLINGIT
```

Words: I, you, we, one, two, person, fish, dog, louse, tree, leaf, skin, blood, bone, horn, ear, eye, nose, tooth, tongue, knee, hand, breast, liver, drink, see, hear, die, come, sun,

Estimation using MCMC methods

- Density is $P(H|T)P(T)$, coupled with prior on certain split dates and on tree structure.
- Simulation from distribution estimated using linguistics

In Conclusion:

- Recent literature on diversity is getting more sophisticated and multidisciplinary
- Doesn't mean it should sacrifice rigor
- An effort to formalize and operationalize some of this
- **In the future:** formal models of borrowing, interaction, and cultural evolution.