COMS 3261: Computer Science Theory

Problem Set 6, due Wednesday, 12/4/13, at the beginning of the class

No late days allowed for this homework.

Please follow the Homework Guidelines.

Try to make your answers as precise, succinct, and clear as you can.

Part A: [30 points]   Do the problems posted at Gradiance.

Part B:

Problem 1. [20 points]
1. Show that the following problem is decidable: Given a Turing machine M, an input w to M and a positive integer k, does M on input w run for more than k steps?
2. Show that the following problem is decidable: Given a Turing machine M and a positive integer k, does there exist an input w that makes M run for more than k steps? (*Hint:* If there exists such an input w, how long does it need to be?)

Problem 2. [20 points]
Consider the language L = { < M> | Turing machine M accepts at least 100 different strings }.
1. Show that L is recursively enumerable.
(*Hint*: One simple way is to use nondeterminism.)
2. Show that L is not recursive.
3. Show that the complement L^c is not recursively enumerable.

Problem 3. [30 points]
Consider the following transformation f that maps each pair $\langle M,w\rangle$ consisting of a Turing machine M and input w to M to another Turing machine $N_{\langle M,w\rangle}$.
The TM $N_{\langle M,w\rangle}$ when given an input string x over its input alphabet Σ behaves as follows. $N_{\langle M,w\rangle}$ simulates the computation of M on w for $|x|$ steps (where $|x|$ is the length of x). If M does not accept within these steps, then $N_{\langle M,w\rangle}$ accepts its own input x and halts. If M accepts during these steps, then $N_{\langle M,w\rangle}$ rejects its input x and halts.

1. Suppose that *M* accepts *w*. What is $L(\, N_{\langle M,w \rangle} \,)$?

2. Suppose that *M* does not accept *w*. What is $L(\, N_{\langle M,w \rangle} \,)$?

3. Use the transformation *f* to show that the language { <*N*> | *N* is a Turing machine whose language *L*(*N*) is infinite } is not recursively enumerable.

4. If you carry out the reduction in the proof of Rice's theorem for the special case of the property *P*: "infinite language", does this reduction also show that the language L_P = { <*N*> | *N* is a Turing machine whose language *L*(*N*) is infinite } is not recursively enumerable? Explain why it does or it does not.