COMS 3261: Computer Science Theory

Problem Set 5, due Monday, 11/25/13, at the beginning of the class

Please follow the Homework Guidelines.

Try to make your answers as precise, succinct, and clear as you can.

Part A: [20 points] Do the problems posted at Gradiance.

Part B:

Problem 1. [20 points] Do Exercise 8.2.3 in HMU (page 336). We reproduce the exercise below for convenience. The Turing machine is deterministic and has one tape.

> **Exercise 8.2.3:** Design a Turing machine that takes as input a number N and adds 1 to it in binary. To be precise, the tape initially contains a \$ followed by N in binary. The tape head is initially scanning the \$ in state q_0. Your TM should halt with N+1, in binary, on its tape, scanning the leftmost symbol of $N+1$, in state q_f. You may destroy the \$ in creating $N+1$, if necessary. For instance, $q_0\$10011 \vdash^* \$q_f10100$, and $q_0\$11111 \vdash^* q_f100000$.
>
> a) Give the transitions of your Turing machine, and explain the purpose of each state.
>
> b) Show the sequence of ID's of your TM when given input \$111.

Problem 2. [20 points] Describe an 1-tape, deterministic Turing machine that accepts the language L={ w | w is a binary string with an equal number of 0's and 1's }. Provide enough details in clear, precise English that describe the operation of the machine: how it moves its head, changes state, writes data on the tape etc. You do not have to give a formal definition (although you may do so if you prefer).

Problem 3. [20 points]. Describe a Turing machine that accepts all strings of the form $w_1 \# w_2 \# \cdots \# w_n$, for any n, such that each w_i is a string of 0's and 1's, and for some j, w_j is the integer j in binary. You may use nondeterminism and multiple tapes to simplify the design of your Turing machine. Describe the operation of your Turing machine in clear, precise English. You do not have to give a formal definition.

Problem 4. [20 points]. Show that the set of recursively enumerable languages and the set of recursive languages are each closed under the operations of (a) union, and (b) concatenation .You may give informal, but clear constructions with sufficient information to show the closures.
(Note: Both sets of languages are also closed under Kleene star, and intersection, but you do not have to show these.)