

## COMS 3261: Computer Science Theory

### Problem Set 3, due Wednesday, 10/30/13, at the beginning of the class

Please follow the Homework Guidelines.

*Try to make your answers as precise, succinct, and clear as you can.*

**Part A:** [30 points] Do the problems posted at Gradiance.

**Problem 1.** [15 points]

1. [10 points] Describe the languages generated by the following grammars with start symbol  $S$ . Provide brief justifications.

a.  $S \rightarrow 0S \mid S1 \mid \varepsilon$

b.  $S \rightarrow AB$

$A \rightarrow 0A1 \mid \varepsilon$

$B \rightarrow 1B0 \mid \varepsilon$

2. [5 points] For each of the above grammars, determine whether it is ambiguous. For each ambiguous grammar give two different parse trees for some string in the language.

**Problem 2.** [15 points] Give context-free grammars for the following languages. Provide brief explanations. The alphabet in all cases is  $\{0,1\}$ .

1. All strings that end in 001.

2. All strings of odd length whose middle symbol is 0.

3.  $\{0^i 1^j \mid i \neq j, i, j \geq 0\}$

**Problem 3.** [20 points] Do Exercise 5.1.4 in the textbook (page 182). We reproduce the exercise below for convenience.

A context-free grammar is said to be *right-linear* if each production body has at most one variable, and that variable is at the right end. That is, all productions of a right-linear grammar are of the form  $A \rightarrow wB$  or  $A \rightarrow w$ , where  $A$  and  $B$  are variables and  $w$  some string of zero or more terminals.

a. Show that every right-linear grammar generates a regular language.

*Hint:* Construct an  $\varepsilon$ -NFA that simulates leftmost derivations, using its state to represent the lone variable in the current left-sentential form.

b. Show that every regular language has a right-linear grammar.

*Hint:* Start with a DFA and let the variables of the grammar represent states.

**Problem 4.** [20 points]

1. Give a PDA that accepts the language  $\{ 0^i 1^{2i} \mid i \geq 0 \}$ .

2. Give a PDA that accepts the language  $\{ 0^i 1^j \mid 2i \geq j \geq i \geq 0 \}$ .

In each case, describe first in English informally how your PDA operates, and then give the precise specification.