COMS 3261: Computer Science Theory

Problem Set 2, due Monday, 10/14/13, at the beginning of the class

Please follow the Homework Guidelines.

Try to make your answers as precise, succinct, and clear as you can.

Part A: [30 points]   Do the problems posted at Gradiance.

Part B: Turn in the following problems.

Problem 1. [15 points] Give regular expressions for the following languages.
Provide brief justifications (you do not need to give a detailed proof).
a. The set of binary strings that contain at least three 1's.
b. The set of binary strings in which the number of 1's is divisible by 3.
c. The set of nonempty strings over the alphabet {0,1,2,3} such that the last digit does not appear earlier in the string.

Problem 2. [10 points]
a. Convert the regular expression 1+(10)* to an ε-NFA using the construction given in the class (and in the book). You may simplify the ε-NFA by collapsing or omitting some ε transitions if you think it is safe to do so (i.e., does not affect the language).
b. Give an NFA without ε-transitions.

Problem 3. [20 points]
a. [8 points] Prove that the language $L=\{\ 0^i10^i \mid i \geq 0\ \}$ is not regular. You can use either the pumping lemma or the Myhill-Nerode theorem.
b. [7 points] A *palindrome* is a string that is equal to its reverse (i.e. it reads the same forward and backward, for example 0110). Prove that the set of binary strings that are palindromes is not a regular language.
c. [5 points] Prove that the set of binary strings that are not palindromes is not a regular language.

Problem 4. [10 points]
A *proper prefix* of a string *w* is a prefix of *w* that is not equal to *w*.
Show that the regular languages are closed under the following operation:
min(*L*) = { *w* | *w* is in *L* and no proper prefix of *w* is in *L* }

In particular, show that if you are given a DFA A for the language *L*, then you can modify it to derive another DFA B for the language min(L). Give a justification for the correctness of your construction.

(*Hint:* Note that if *x* is any string in *L*, then every proper extension of it, i.e. every string of the form *xy* with $y \neq \varepsilon$, is not in min(*L*).)

Problem 5. [15 points] Consider the DFA given by its transition table below.
Give the minimum equivalent DFA.
Specify for each state of the minimized DFA the set of equivalent states of the original DFA.

	0	**1**
→A	B	J
B	H	C
***C**	D	G
D	E	C
***E**	F	G
F	C	E
***G**	J	C
H	B	G
I	H	E
J	H	A