

# Introduction to Generative Models (and GANs)

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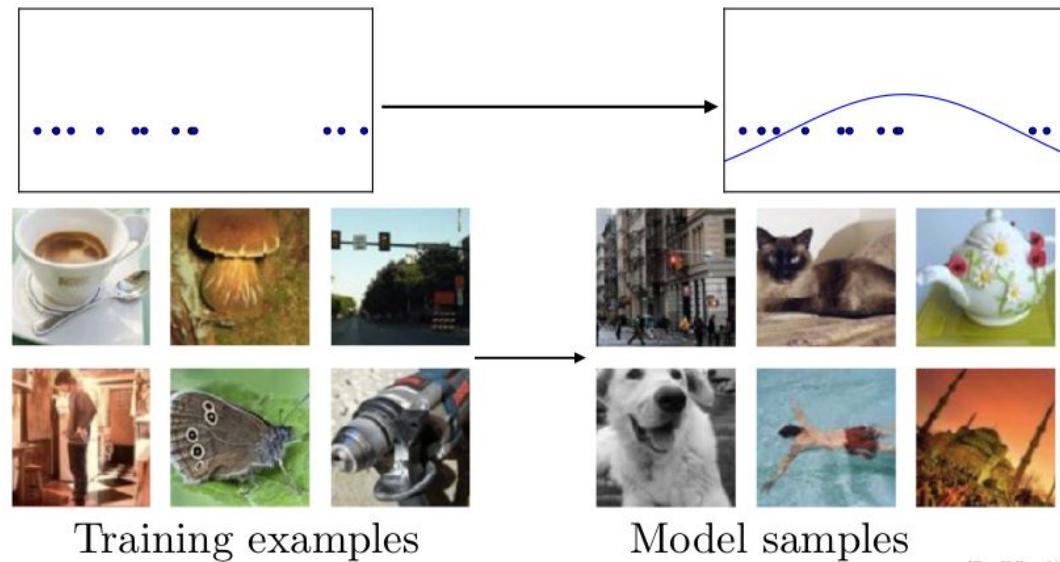
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Nov. 2017

# Generative Models: Learning the Distributions

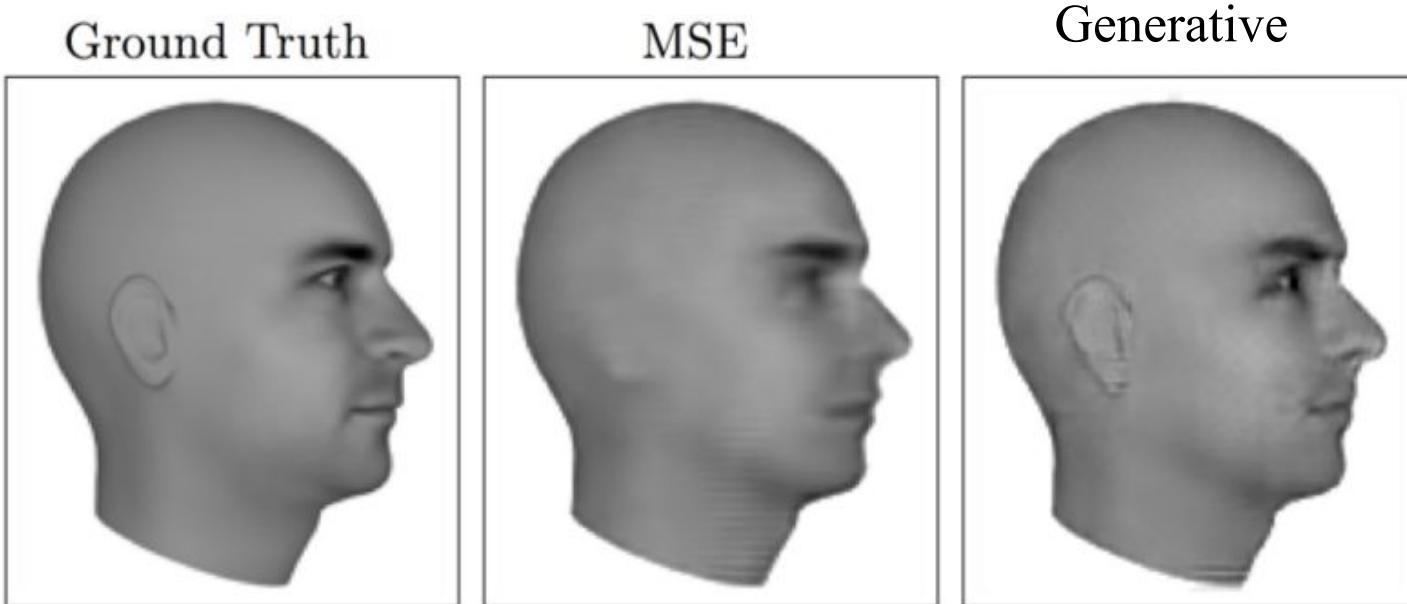
Discriminative: learns the likelihood

Generative: performs Density Estimation (learns the distribution) to allow sampling



# Loss function for distribution: Ambiguity and the “blur” effect

MSE: a Discriminative model just smoothes all possibilities.



# Ambiguity and the “blur” effect



Photo-Realistic Single Image Super-Resolution Using a Generative Adversarial Network

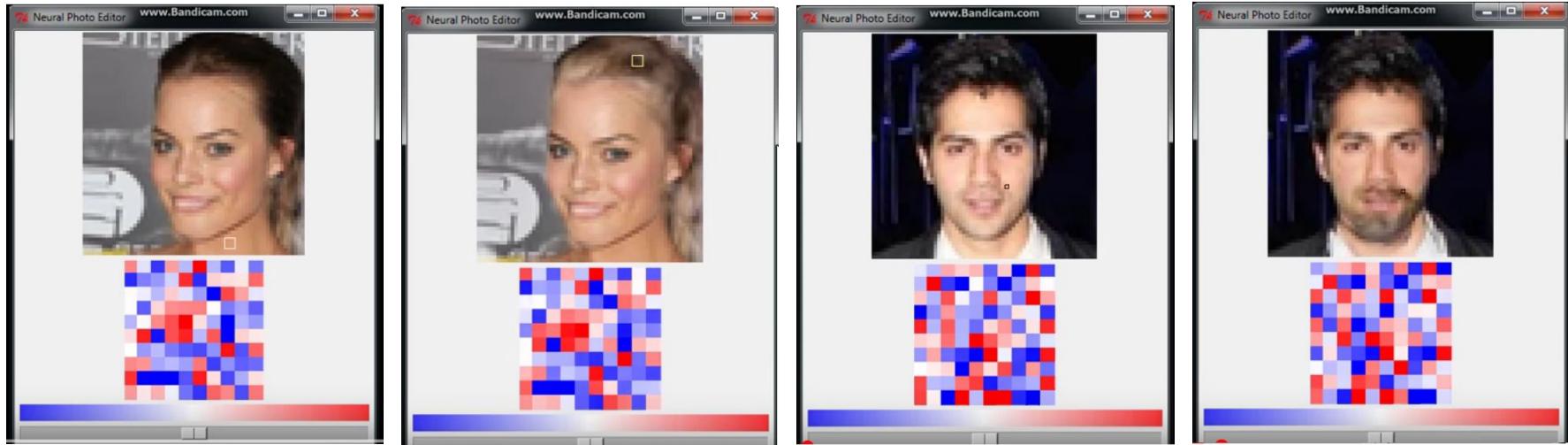
# Example Application of Generative Models

# Image Generation from Sketch



iGAN: Interactive Image Generation via Generative Adversarial Networks

# Interactive Editing



Neural Photo Editing with Introspective Adversarial Networks

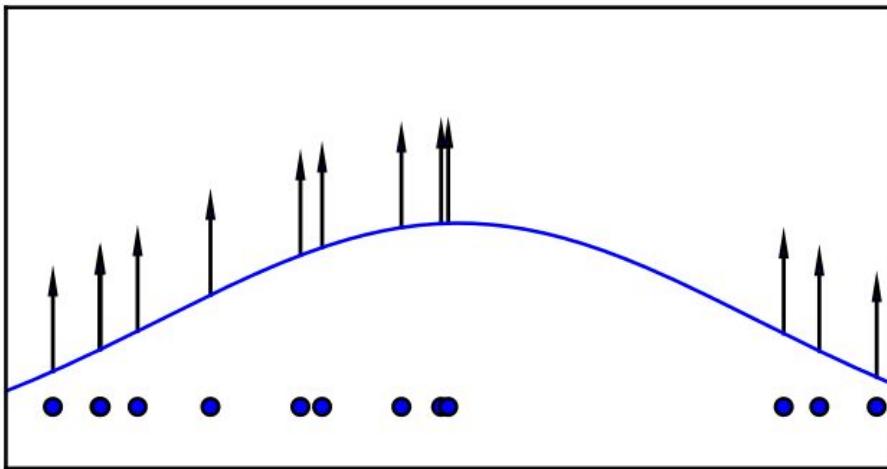
Figures adapted from NIPS 2016 Tutorial Generative Adversarial Networks

# Image to Image Translation



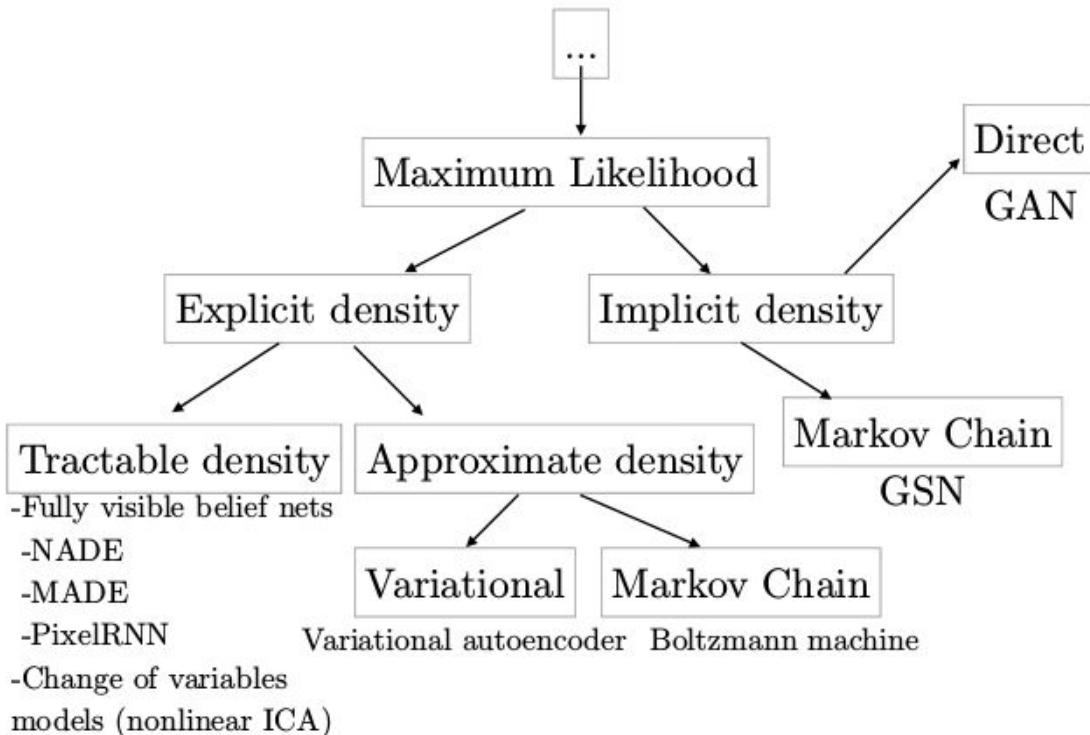
# How Generative Models are Trained

# Learning Generative Models

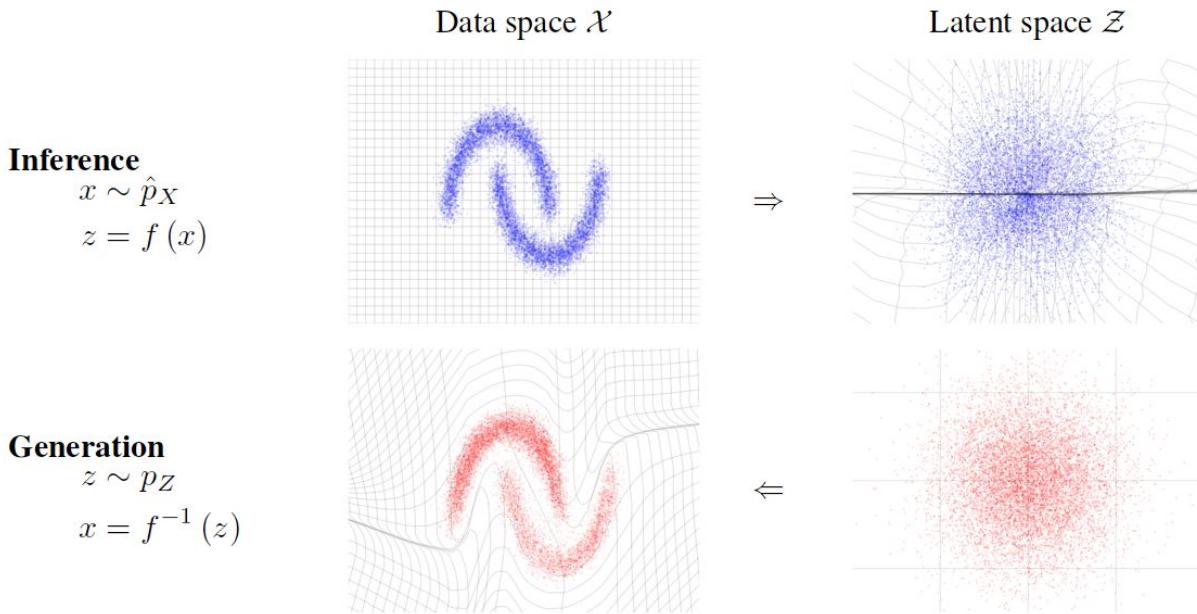


$$\theta^* = \arg \max_{\theta} \mathbb{E}_{x \sim p_{\text{data}}} \log p_{\text{model}}(x \mid \theta)$$

# Taxonomy of Generative Models

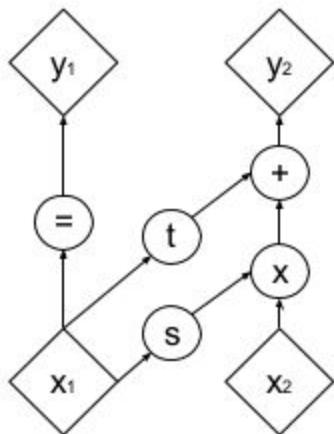


# Exact Model: NVP (non-volume preserving)

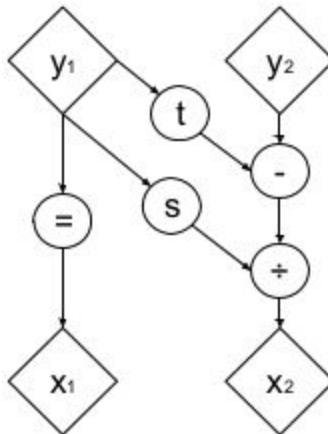


# Real NVP: Invertible Non-linear Transforms

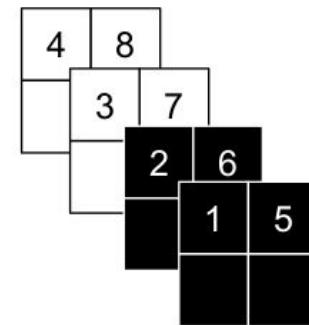
$$p_x(\mathbf{x}) = p_z(g^{-1}(\mathbf{x})) \left| \det \left( \frac{\partial g^{-1}(\mathbf{x})}{\partial \mathbf{x}} \right) \right|.$$



(a) Forward propagation



(b) Inverse propagation



# Real NVP: Examples

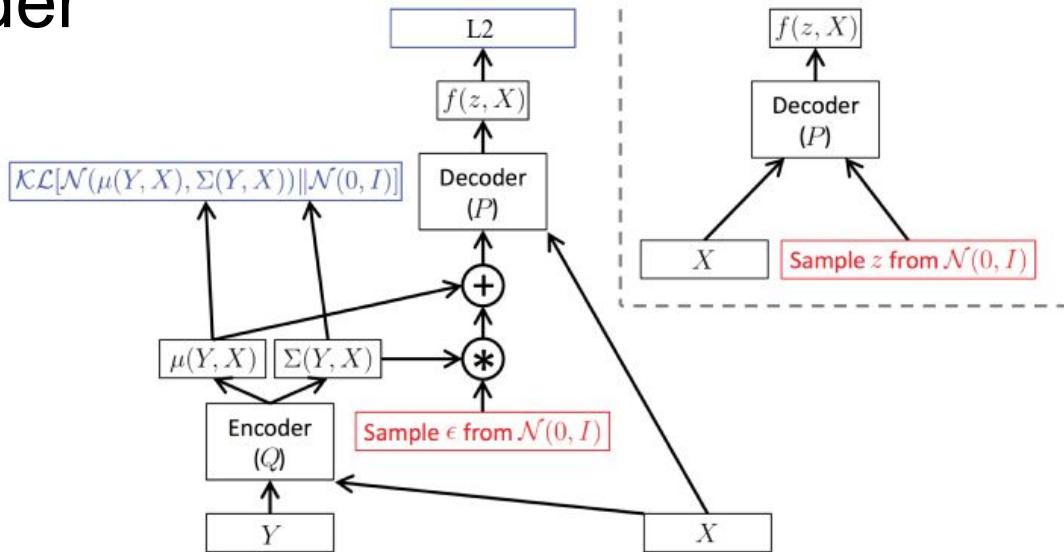


# Real NVP

Restriction on the source domain: must be of the same as the target.

# Variational Auto-Encoder

Auto-encoding with noise in hidden variable



$$\log p_{\theta}(\mathbf{x}^{(i)}) = D_{KL}(q_{\phi}(\mathbf{z}|\mathbf{x}^{(i)})||p_{\theta}(\mathbf{z}|\mathbf{x}^{(i)})) + \mathcal{L}(\boldsymbol{\theta}, \boldsymbol{\phi}; \mathbf{x}^{(i)})$$

$$\mathcal{L}(\boldsymbol{\theta}, \boldsymbol{\phi}; \mathbf{x}^{(i)}) \simeq \frac{1}{2} \sum_{j=1}^J \left( 1 + \log((\sigma_j^{(i)})^2) - (\mu_j^{(i)})^2 - (\sigma_j^{(i)})^2 \right) + \frac{1}{L} \sum_{l=1}^L \log p_{\theta}(\mathbf{x}^{(i)}|\mathbf{z}^{(i,l)})$$

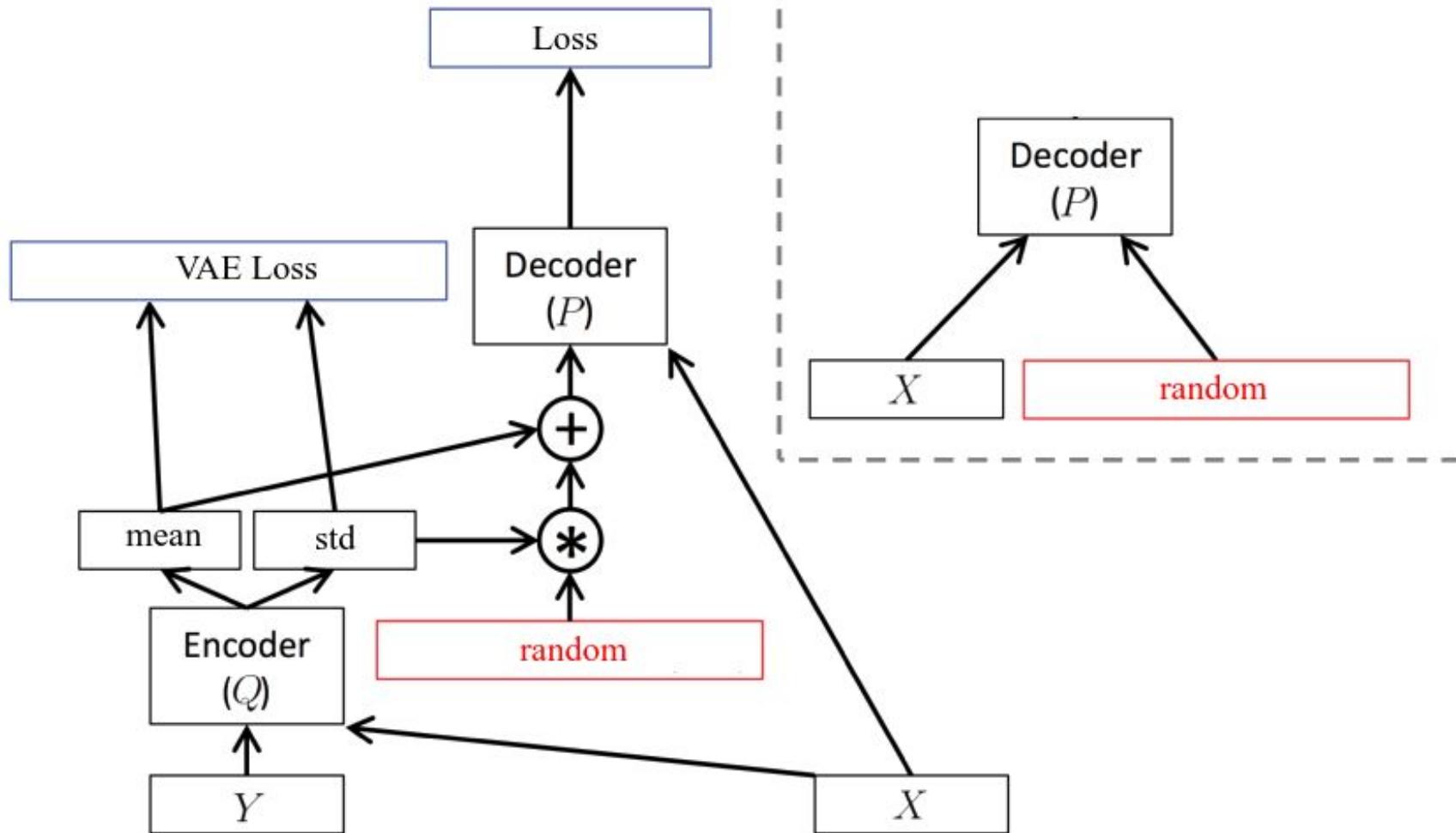
# Variational Auto-Encoder

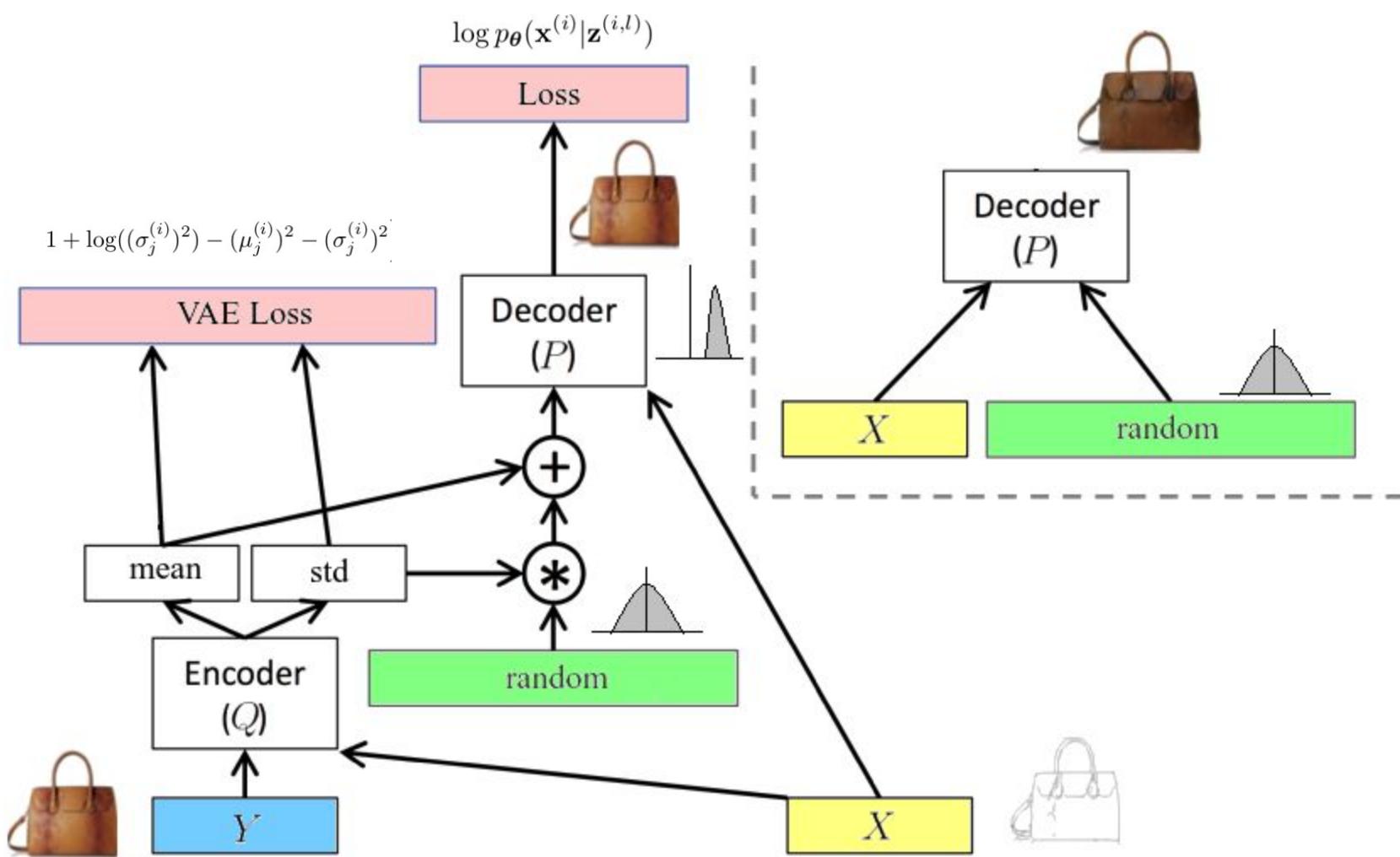
$$\mathcal{D} [Q(z) \| P(z|X)] = E_{z \sim Q} [\log Q(z) - \log P(z|X)] .$$

$$\mathcal{D} [Q(z) \| P(z|X)] = E_{z \sim Q} [\log Q(z) - \log P(X|z) - \log P(z)] + \log P(X).$$

$$\log P(X) - \mathcal{D} [Q(z) \| P(z|X)] = E_{z \sim Q} [\log P(X|z)] - \mathcal{D} [Q(z) \| P(z)]$$

$$\log P(X) - \mathcal{D} [Q(z|X) \| P(z|X)] = E_{z \sim Q} [\log P(X|z)] - \mathcal{D} [Q(z|X) \| P(z)]$$

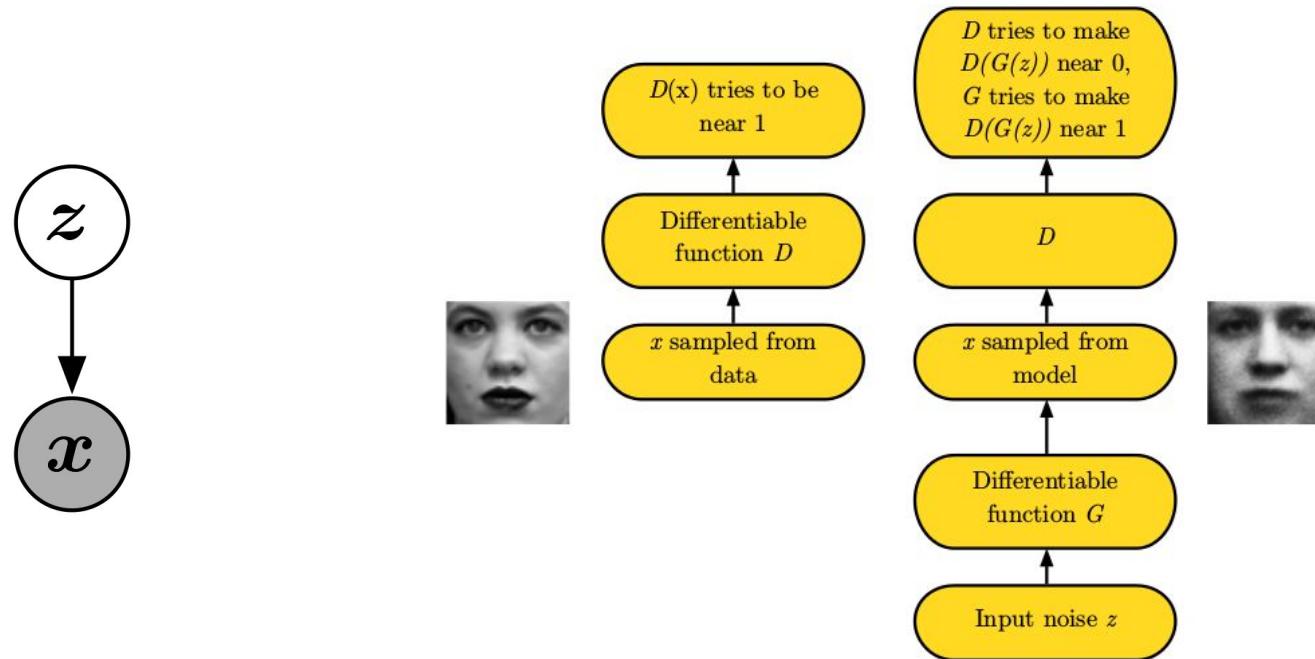




# VAE: Examples

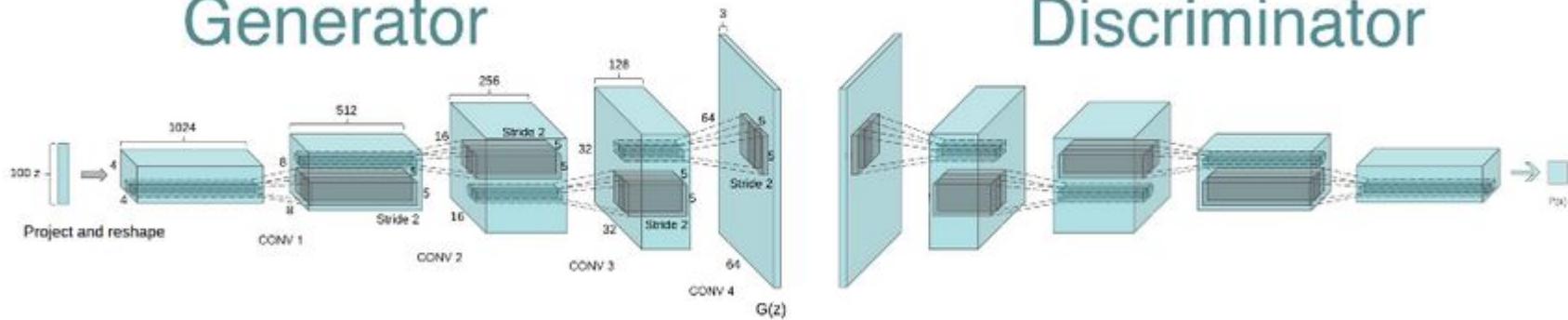


# Generative Adversarial Networks (GAN)



# DCGAN

## Generator

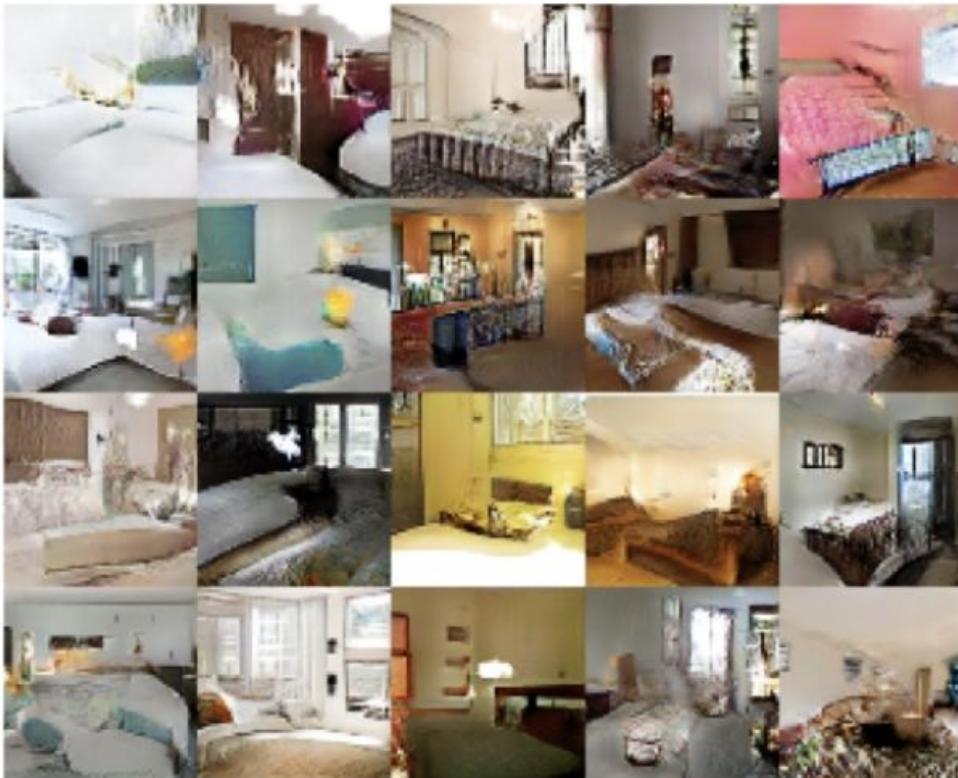


## Discriminator

Train D by  $\text{Loss}(D(\text{real}), 1), \text{Loss}(D(G(\text{random})), 0)$

Train G by  $\text{Loss}(D(G(\text{random}))), 1$

# DCGAN: Examples



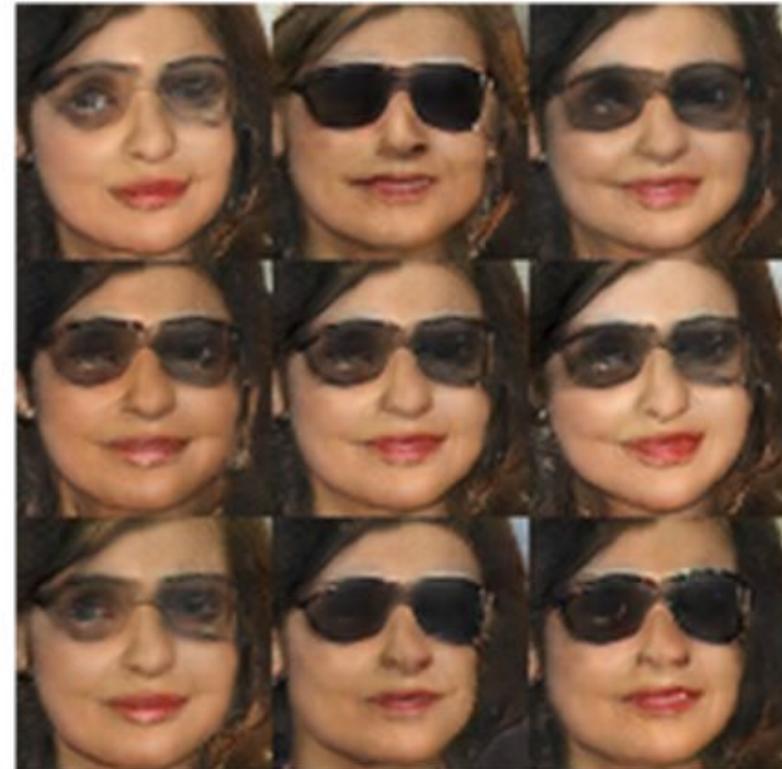
# DCGAN: Example of Feature Manipulation

Vector arithmetics in feature space



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# Conditional, Cross-domain Generation

## Generative adversarial text to image synthesis

this small bird has a pink breast and crown, and black primaries and secondaries.



this magnificent fellow is almost all black with a red crest, and white cheek patch.



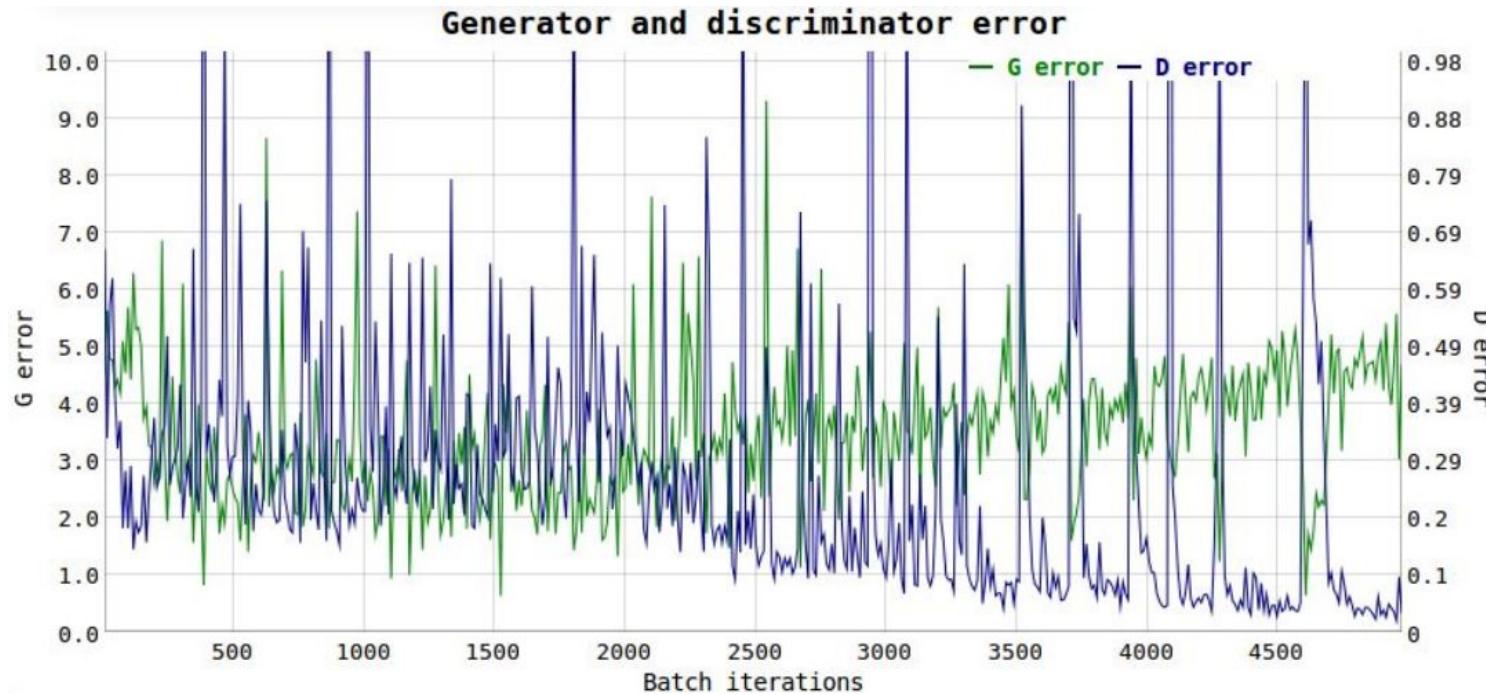
the flower has petals that are bright pinkish purple with white stigma



this white and yellow flower have thin white petals and a round yellow stamen



# GAN training problems: unstable losses



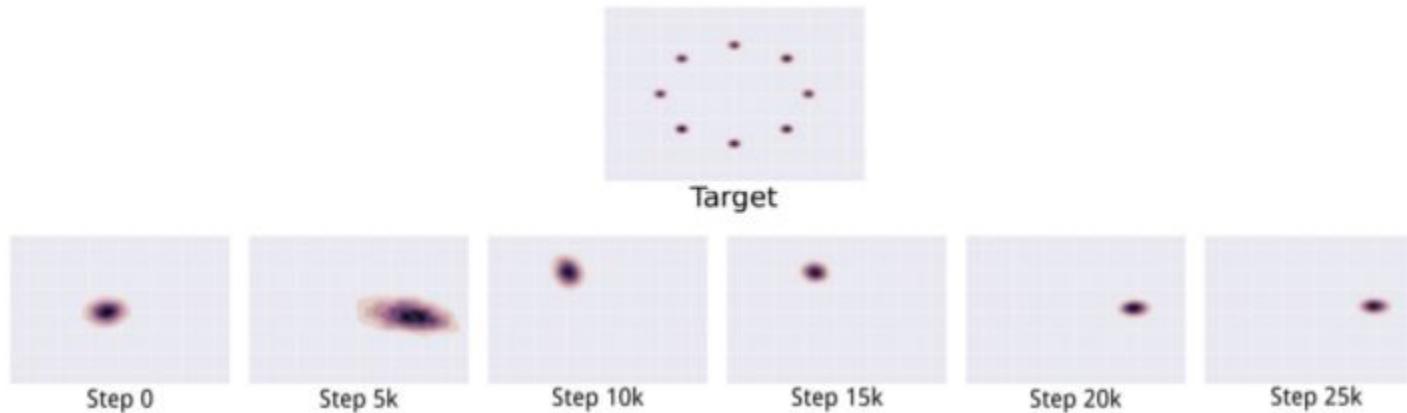
# GAN training problems: Mini-batch Fluctuation

Differs much even between consecutive minibatches.



# GAN training problems: Mode Collapse

Lack of diversity in generated results.



# Improve GAN training: Label Smoothing

Improves stability of training

```
d_on_data = discriminator_logits(data_minibatch)
d_on_samples = discriminator_logits(samples_minibatch)
loss = tf.nn.sigmoid_cross_entropy_with_logits(d_on_data, .9) +
       tf.nn.sigmoid_cross_entropy_with_logits(d_on_samples, 0.)
```

# Improve GAN training: Wasserstein GAN

Use linear instead of log

$$W(\mathbb{P}_r, \mathbb{P}_\theta) = \sup_{\|f\|_L \leq 1} \mathbb{E}_{x \sim \mathbb{P}_r}[f(x)] - \mathbb{E}_{x \sim \mathbb{P}_\theta}[f(x)]$$

**for**  $t = 0, \dots, n_{\text{critic}}$  **do**

    Sample  $\{x^{(i)}\}_{i=1}^m \sim \mathbb{P}_r$  a batch from the real data.

    Sample  $\{z^{(i)}\}_{i=1}^m \sim p(z)$  a batch of prior samples.

$g_w \leftarrow \nabla_w \left[ \frac{1}{m} \sum_{i=1}^m f_w(x^{(i)}) - \frac{1}{m} \sum_{i=1}^m f_w(g_\theta(z^{(i)})) \right]$

$w \leftarrow w + \alpha \cdot \text{RMSProp}(w, g_w)$

$w \leftarrow \underline{\text{clip}}(w, -c, c)$

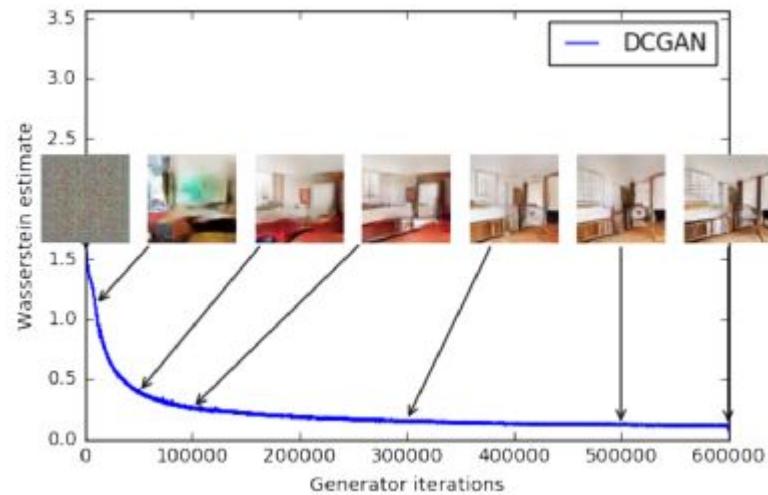
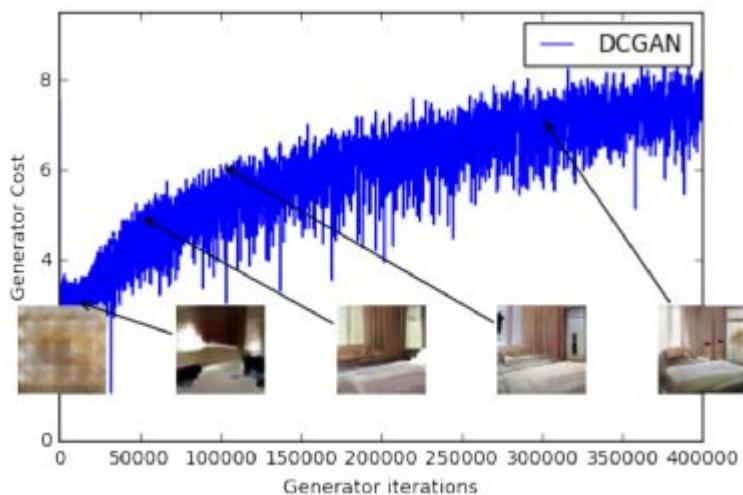
**end for**

    Sample  $\{z^{(i)}\}_{i=1}^m \sim p(z)$  a batch of prior samples.

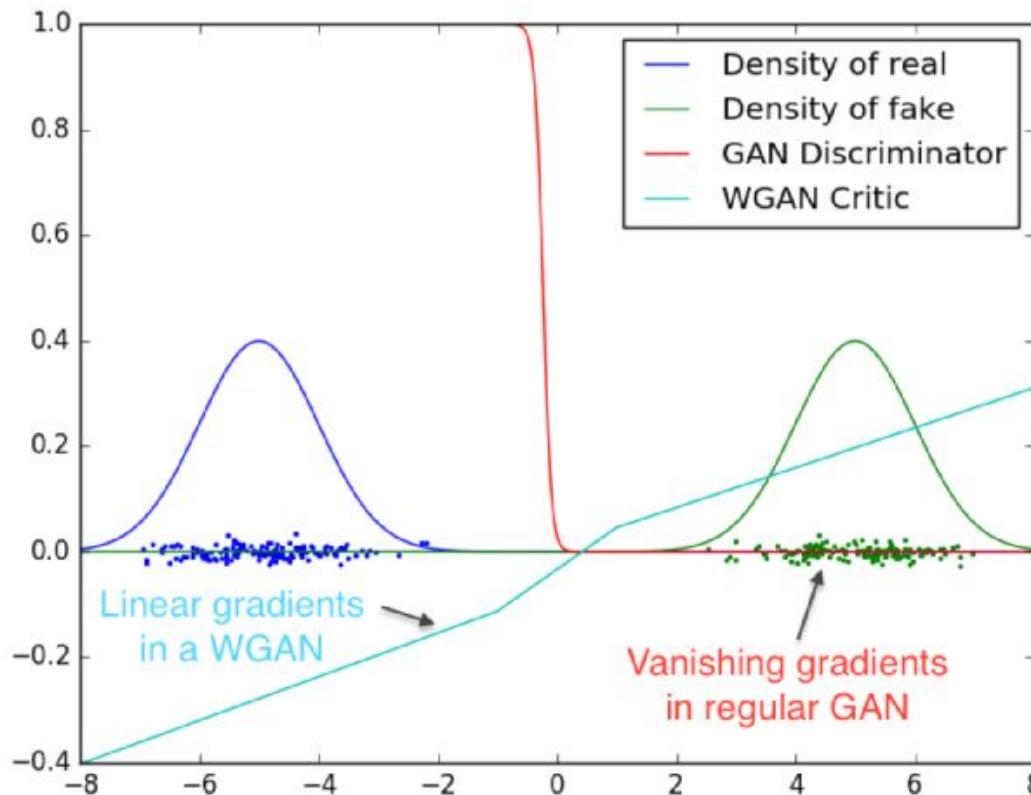
$g_\theta \leftarrow -\nabla_\theta \frac{1}{m} \sum_{i=1}^m f_w(g_\theta(z^{(i)}))$

$\theta \leftarrow \theta - \alpha \cdot \text{RMSProp}(\theta, g_\theta)$

# WGAN: Stabilized Training Curve



# WGAN: Non-vanishing Gradient

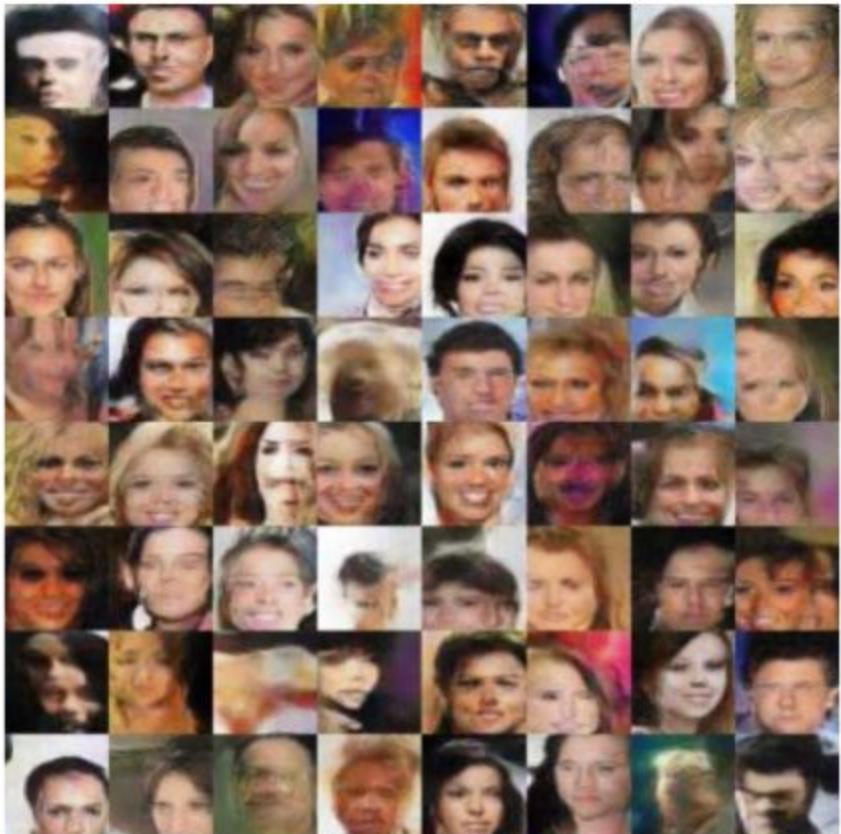


# Loss Sensitive GAN

$$\begin{aligned} \min_{\theta} S_{n,m}(\phi^*, \theta) &\triangleq \frac{1}{n} \sum_{i=1}^n L_{\theta}(\mathbf{x}_i) \\ &+ \frac{\lambda}{nm} \sum_{i,j=1}^{n,m} (\Delta(\mathbf{x}_i, G_{\phi^*}(\mathbf{z}_j)) + L_{\theta}(\mathbf{x}_i) - L_{\theta}(G_{\phi^*}(\mathbf{z}_j)))_+ \end{aligned} \quad (8)$$

and

$$\min_{\phi} T_k(\theta^*, \phi) = \frac{1}{k} \sum_{i=1}^k L_{\theta^*}(G_{\phi}(\mathbf{z}'_i)) \quad (9)$$



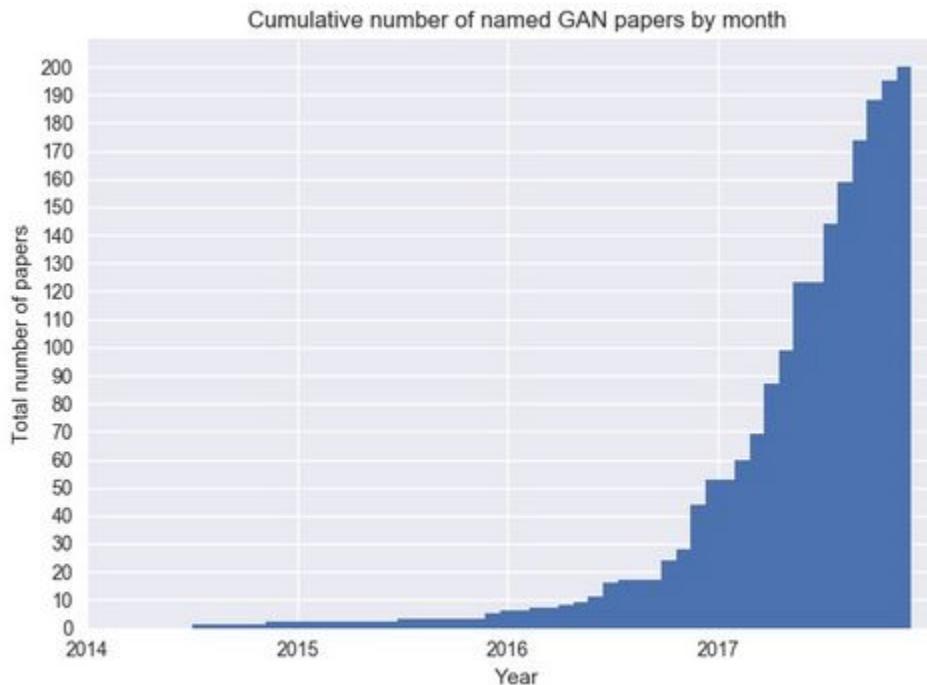
(a) DCGAN



(b) LS-GAN

# The GAN Zoo

<https://github.com/hindupuravinash/the-gan-zoo>



- 3D-GAN - [Learning a Probabilistic Latent Space of Object Shapes via 3D Generative-Adversarial Modeling](#) ([github](#))
- 3D-IWGAN - [Improved Adversarial Systems for 3D Object Generation and Reconstruction](#) ([github](#))
- 3D-RecGAN - [3D Object Reconstruction from a Single Depth View with Adversarial Learning](#) ([github](#))
- ABC-GAN - [ABC-GAN: Adaptive Blur and Control for improved training stability of Generative Adversarial Networks](#) ([github](#))
- AC-GAN - [Conditional Image Synthesis With Auxiliary Classifier GANs](#)
- acGAN - [Face Aging With Conditional Generative Adversarial Networks](#)
- AdaGAN - [AdaGAN: Boosting Generative Models](#)
- AE-GAN - [AE-GAN: adversarial eliminating with GAN](#)
- AEGAN - [Learning Inverse Mapping by Autoencoder based Generative Adversarial Nets](#)
- AffGAN - [Amortised MAP Inference for Image Super-resolution](#)
- AL-CGAN - [Learning to Generate Images of Outdoor Scenes from Attributes and Semantic Layouts](#)
- ALI - [Adversarially Learned Inference](#)
- AlignGAN - [AlignGAN: Learning to Align Cross-Domain Images with Conditional Generative Adversarial Networks](#)
- AM-GAN - [Activation Maximization Generative Adversarial Nets](#)
- AnoGAN - [Unsupervised Anomaly Detection with Generative Adversarial Networks to Guide Marker Discovery](#)
- ARAE - [Adversarially Regularized Autoencoders for Generating Discrete Structures](#) ([github](#))
- ARDA - [Adversarial Representation Learning for Domain Adaptation](#)
- ARIGAN - [ARIGAN: Synthetic Arabidopsis Plants using Generative Adversarial Network](#)
- ArtGAN - [ArtGAN: Artwork Synthesis with Conditional Categorical GANs](#)
- b-GAN - [Generative Adversarial Nets from a Density Ratio Estimation Perspective](#)
- Bayesian GAN - [Deep and Hierarchical Implicit Models](#)
- Bayesian GAN - [Bayesian GAN](#)
- BCGAN - [Bayesian Conditional Generative Adversarial Networks](#)
- BEGAN - [BEGAN: Boundary Equilibrium Generative Adversarial Networks](#)
- BGAN - [Binary Generative Adversarial Networks for Image Retrieval](#) ([github](#))

- BiGAN - [Adversarial Feature Learning](#)
- BS-GAN - [Boundary-Seeking Generative Adversarial Networks](#)
- C-RNN-GAN - [C-RNN-GAN: Continuous recurrent neural networks with adversarial training \(github\)](#)
- CaloGAN - [CaloGAN: Simulating 3D High Energy Particle Showers in Multi-Layer Electromagnetic Calorimeters with Generative Adversarial Networks \(github\)](#)
- CAN - [CAN: Creative Adversarial Networks, Generating Art by Learning About Styles and Deviating from Style Norms](#)
- CatGAN - [Unsupervised and Semi-supervised Learning with Categorical Generative Adversarial Networks](#)
- CausalGAN - [CausalGAN: Learning Causal Implicit Generative Models with Adversarial Training](#)
- CC-GAN - [Semi-Supervised Learning with Context-Conditional Generative Adversarial Networks \(github\)](#)
- CDcGAN - [Simultaneously Color-Depth Super-Resolution with Conditional Generative Adversarial Network](#)
- CGAN - [Conditional Generative Adversarial Nets](#)
- CGAN - [Controllable Generative Adversarial Network](#)
- Chekhov GAN - [An Online Learning Approach to Generative Adversarial Networks](#)
- CM-GAN - [CM-GANs: Cross-modal Generative Adversarial Networks for Common Representation Learning](#)
- CoGAN - [Coupled Generative Adversarial Networks](#)
- Conditional cycleGAN - [Conditional CycleGAN for Attribute Guided Face Image Generation](#)
- contrast-GAN - [Generative Semantic Manipulation with Contrasting GAN](#)
- Context-RNN-GAN - [Contextual RNN-GANs for Abstract Reasoning Diagram Generation](#)
- Coulomb GAN - [Coulomb GANs: Provably Optimal Nash Equilibria via Potential Fields](#)
- Cramèr GAN - [The Cramer Distance as a Solution to Biased Wasserstein Gradients](#)
- crVAE-GAN - [Channel-Recurrent Variational Autoencoders](#)
- CS-GAN - [Improving Neural Machine Translation with Conditional Sequence Generative Adversarial Nets](#)
- CVAE-GAN - [CVAE-GAN: Fine-Grained Image Generation through Asymmetric Training](#)
- CycleGAN - [Unpaired Image-to-Image Translation using Cycle-Consistent Adversarial Networks \(github\)](#)
- D2GAN - [Dual Discriminator Generative Adversarial Nets](#)

- DAN - [Distributional Adversarial Networks](#)
- DCGAN - [Unsupervised Representation Learning with Deep Convolutional Generative Adversarial Networks \(github\)](#)
- DelGAN - [DeLiGAN : Generative Adversarial Networks for Diverse and Limited Data \(github\)](#)
- DiscoGAN - [Learning to Discover Cross-Domain Relations with Generative Adversarial Networks](#)
- DistanceGAN - [One-Sided Unsupervised Domain Mapping](#)
- DM-GAN - [Dual Motion GAN for Future-Flow Embedded Video Prediction](#)
- DR-GAN - [Representation Learning by Rotating Your Faces](#)
- DRAGAN - [How to Train Your DRAGAN \(github\)](#)
- DSP-GAN - [Depth Structure Preserving Scene Image Generation](#)
- DTN - [Unsupervised Cross-Domain Image Generation](#)
- DualGAN - [DualGAN: Unsupervised Dual Learning for Image-to-Image Translation](#)
- Dualing GAN - [Dualing GANs](#)
- EBGAN - [Energy-based Generative Adversarial Network](#)
- ED//GAN - [Stabilizing Training of Generative Adversarial Networks through Regularization](#)
- EGAN - [Enhanced Experience Replay Generation for Efficient Reinforcement Learning](#)
- ExprGAN - [ExprGAN: Facial Expression Editing with Controllable Expression Intensity](#)
- f-GAN - [f-GAN: Training Generative Neural Samplers using Variational Divergence Minimization](#)
- FF-GAN - [Towards Large-Pose Face Frontalization in the Wild](#)
- Fila-GAN - [Synthesizing Filamentary Structured Images with GANs](#)
- Fisher GAN - [Fisher GAN](#)
- Flow-GAN - [Flow-GAN: Bridging implicit and prescribed learning in generative models](#)
- GAMN - [Generative Adversarial Mapping Networks](#)
- GAN - [Generative Adversarial Networks \(github\)](#)
- GAN-ATV - [A Novel Approach to Artistic Textual Visualization via GAN](#)
- GAN-CLS - [Generative Adversarial Text to Image Synthesis \(github\)](#)
- GAN-sep - [GANs for Biological Image Synthesis \(github\)](#)

- GAN-VFS - Generative Adversarial Network-based Synthesis of Visible Faces from Polarimetric Thermal Faces
- GANCS - Deep Generative Adversarial Networks for Compressed Sensing Automates MRI
- GANDI - Guiding the search in continuous state-action spaces by learning an action sampling distribution from off-target samples
- GAP - Context-Aware Generative Adversarial Privacy
- GAWWN - Learning What and Where to Draw ([github](#))
- GeneGAN - GeneGAN: Learning Object Transfiguration and Attribute Subspace from Unpaired Data ([github](#))
- Geometric GAN - Geometric GAN
- GMAN - Generative Multi-Adversarial Networks
- GMM-GAN - Towards Understanding the Dynamics of Generative Adversarial Networks
- GoGAN - Gang of GANs: Generative Adversarial Networks with Maximum Margin Ranking
- GP-GAN - GP-GAN: Towards Realistic High-Resolution Image Blending ([github](#))
- GP-GAN - GP-GAN: Gender Preserving GAN for Synthesizing Faces from Landmarks
- GRAN - Generating images with recurrent adversarial networks ([github](#))
- IAN - Neural Photo Editing with Introspective Adversarial Networks ([github](#))
- IcGAN - Invertible Conditional GANs for image editing ([github](#))
- ID-CGAN - Image De-raining Using a Conditional Generative Adversarial Network
- iGAN - Generative Visual Manipulation on the Natural Image Manifold ([github](#))
- Improved GAN - Improved Techniques for Training GANs ([github](#))
- InfoGAN - InfoGAN: Interpretable Representation Learning by Information Maximizing Generative Adversarial Nets ([github](#))
- IRGAN - IRGAN: A Minimax Game for Unifying Generative and Discriminative Information Retrieval models
- IWGAN - On Unifying Deep Generative Models
- KGAN - KGAN: How to Break The Minimax Game in GAN
- I-GAN - Representation Learning and Adversarial Generation of 3D Point Clouds
- LAGAN - Learning Particle Physics by Example: Location-Aware Generative Adversarial Networks for Physics Synthesis

- LAPGAN - Deep Generative Image Models using a Laplacian Pyramid of Adversarial Networks ([github](#))
- LD-GAN - Linear Discriminant Generative Adversarial Networks
- LDAN - Label Denoising Adversarial Network (LDAN) for Inverse Lighting of Face Images
- LeakGAN - Long Text Generation via Adversarial Training with Leaked Information
- LeGAN - Likelihood Estimation for Generative Adversarial Networks
- LR-GAN - LR-GAN: Layered Recursive Generative Adversarial Networks for Image Generation
- LS-GAN - Loss-Sensitive Generative Adversarial Networks on Lipschitz Densities
- LSGAN - Least Squares Generative Adversarial Networks
- MAD-GAN - Multi-Agent Diverse Generative Adversarial Networks
- MAGAN - MAGAN: Margin Adaptation for Generative Adversarial Networks
- MalGAN - Generating Adversarial Malware Examples for Black-Box Attacks Based on GAN
- MaliGAN - Maximum-Likelihood Augmented Discrete Generative Adversarial Networks
- MARTA-GAN - Deep Unsupervised Representation Learning for Remote Sensing Images
- McGAN - McGan: Mean and Covariance Feature Matching GAN
- MD-GAN - Learning to Generate Time-Lapse Videos Using Multi-Stage Dynamic Generative Adversarial Networks
- MDGAN - Mode Regularized Generative Adversarial Networks
- MedGAN - Generating Multi-label Discrete Electronic Health Records using Generative Adversarial Networks
- MGAN - Precomputed Real-Time Texture Synthesis with Markovian Generative Adversarial Networks ([github](#))
- MGGAN - Multi-Generator Generative Adversarial Nets
- MIX+GAN - Generalization and Equilibrium in Generative Adversarial Nets (GANs)
- MLGAN - Metric Learning-based Generative Adversarial Network
- MMD-GAN - MMD GAN: Towards Deeper Understanding of Moment Matching Network ([github](#))
- MMGAN - MMGAN: Manifold Matching Generative Adversarial Network for Generating Images
- MoCoGAN - MoCoGAN: Decomposing Motion and Content for Video Generation ([github](#))
- MPM-GAN - Message Passing Multi-Agent GANs
- MuseGAN - MuseGAN: Symbolic-domain Music Generation and Accompaniment with Multi-track Sequential Generative Adversarial Networks
- MV-BiGAN - Multi-view Generative Adversarial Networks

- OptionGAN - [OptionGAN: Learning Joint Reward-Policy Options using Generative Adversarial Inverse Reinforcement Learning](#)
- ORGAN - [Objective-Reinforced Generative Adversarial Networks \(ORGAN\) for Sequence Generation Models](#)
- PAN - [Perceptual Adversarial Networks for Image-to-Image Transformation](#)
- PassGAN - [PassGAN: A Deep Learning Approach for Password Guessing](#)
- Perceptual GAN - [Perceptual Generative Adversarial Networks for Small Object Detection](#)
- PGAN - [Probabilistic Generative Adversarial Networks](#)
- pix2pix - [Image-to-Image Translation with Conditional Adversarial Networks \(github\)](#)
- PixelGAN - [PixelGAN Autoencoders](#)
- Pose-GAN - [The Pose Knows: Video Forecasting by Generating Pose Futures](#)
- PPGN - [Plug & Play Generative Networks: Conditional Iterative Generation of Images in Latent Space](#)
- PrGAN - [3D Shape Induction from 2D Views of Multiple Objects](#)
- PSGAN - [Learning Texture Manifolds with the Periodic Spatial GAN](#)
- PS<sup>2</sup>-GAN - [High-Quality Facial Photo-Sketch Synthesis Using Multi-Adversarial Networks](#)
- RankGAN - [Adversarial Ranking for Language Generation](#)
- RCGAN - [Real-valued \(Medical\) Time Series Generation with Recurrent Conditional GANs](#)
- RefineGAN - [Compressed Sensing MRI Reconstruction with Cyclic Loss in Generative Adversarial Networks](#)
- RenderGAN - [RenderGAN: Generating Realistic Labeled Data](#)
- ResGAN - [Generative Adversarial Network based on Resnet for Conditional Image Restoration](#)
- RNN-WGAN - [Language Generation with Recurrent Generative Adversarial Networks without Pre-training \(github\)](#)
- RPGAN - [Stabilizing GAN Training with Multiple Random Projections \(github\)](#)
- RTT-GAN - [Recurrent Topic-Transition GAN for Visual Paragraph Generation](#)
- RWGAN - [Relaxed Wasserstein with Applications to GANs](#)
- SAD-GAN - [SAD-GAN: Synthetic Autonomous Driving using Generative Adversarial Networks](#)
- SalGAN - [SalGAN: Visual Saliency Prediction with Generative Adversarial Networks \(github\)](#)
- SBADA-GAN - [From source to target and back: symmetric bi-directional adaptive GAN](#)
- SD-GAN - [Semantically Decomposing the Latent Spaces of Generative Adversarial Networks](#)
- SEGAN - [SEGAN: Speech Enhancement Generative Adversarial Network](#)

- SeGAN - [SeGAN: Segmenting and Generating the Invisible](#)
- SegAN - [SegAN: Adversarial Network with Multi-scale L1 Loss for Medical Image Segmentation](#)
- SeqGAN - [SeqGAN: Sequence Generative Adversarial Nets with Policy Gradient \(github\)](#)
- SGAN - [Texture Synthesis with Spatial Generative Adversarial Networks](#)
- SGAN - [Stacked Generative Adversarial Networks \(github\)](#)
- SGAN - [Steganographic Generative Adversarial Networks](#)
- SimGAN - [Learning from Simulated and Unsupervised Images through Adversarial Training](#)
- SketchGAN - [Adversarial Training For Sketch Retrieval](#)
- SL-GAN - [Semi-Latent GAN: Learning to generate and modify facial images from attributes](#)
- SN-GAN - [Spectral Normalization for Generative Adversarial Networks \(github\)](#)
- Softmax-GAN - [Softmax GAN](#)
- Splitting GAN - [Class-Splitting Generative Adversarial Networks](#)
- SRGAN - [Photo-Realistic Single Image Super-Resolution Using a Generative Adversarial Network](#)
- SS-GAN - [Semi-supervised Conditional GANs](#)
- ss-InfoGAN - [Guiding InfoGAN with Semi-Supervision](#)
- SSGAN - [SSGAN: Secure Steganography Based on Generative Adversarial Networks](#)
- SSL-GAN - [Semi-Supervised Learning with Context-Conditional Generative Adversarial Networks](#)
- ST-GAN - [Style Transfer Generative Adversarial Networks: Learning to Play Chess Differently](#)
- StackGAN - [StackGAN: Text to Photo-realistic Image Synthesis with Stacked Generative Adversarial Networks](#)
- SteinGAN - [Learning Deep Energy Models: Contrastive Divergence vs. Amortized MLE](#)
- SVGAN - [SVGAN: Singing Voice Separation via Generative Adversarial Network](#)
- S<sup>A</sup>GAN - [Generative Image Modeling using Style and Structure Adversarial Networks](#)
- TAC-GAN - [TAC-GAN - Text Conditioned Auxiliary Classifier Generative Adversarial Network \(github\)](#)
- TAN - [Outline Colorization through Tandem Adversarial Networks](#)
- TextureGAN - [TextureGAN: Controlling Deep Image Synthesis with Texture Patches](#)
- TGAN - [Temporal Generative Adversarial Nets](#)
- TGAN - [Tensorizing Generative Adversarial Nets](#)

- TGAN - Tensor-Generative Adversarial Network with Two-dimensional Sparse Coding: Application to Real-time Indoor Localization
- TP-GAN - Beyond Face Rotation: Global and Local Perception GAN for Photorealistic and Identity Preserving Frontal View Synthesis
- Triple-GAN - Triple Generative Adversarial Nets
- Unrolled GAN - Unrolled Generative Adversarial Networks ([github](#))
- VAE-GAN - Autoencoding beyond pixels using a learned similarity metric
- VariGAN - Multi-View Image Generation from a Single-View
- VAW-GAN - Voice Conversion from Unaligned Corpora using Variational Autoencoding Wasserstein Generative Adversarial Networks
- VEEGAN - VEEGAN: Reducing Mode Collapse in GANs using Implicit Variational Learning ([github](#))
- VGAN - Generating Videos with Scene Dynamics ([github](#))
- VGAN - Generative Adversarial Networks as Variational Training of Energy Based Models ([github](#))
- ViGAN - Image Generation and Editing with Variational Info Generative Adversarial Networks
- VIGAN - VIGAN: Missing View Imputation with Generative Adversarial Networks
- VRAL - Variance Regularizing Adversarial Learning
- WaterGAN - WaterGAN: Unsupervised Generative Network to Enable Real-time Color Correction of Monocular Underwater Images
- WGAN - Wasserstein GAN ([github](#))
- WGAN-GP - Improved Training of Wasserstein GANs ([github](#))
- WS-GAN - Weakly Supervised Generative Adversarial Networks for 3D Reconstruction
- ZipNet-GAN - ZipNet-GAN: Inferring Fine-grained Mobile Traffic Patterns via a Generative Adversarial Neural Network
- $\alpha$ -GAN - Variational Approaches for Auto-Encoding Generative Adversarial Networks ([github](#))
- $\Delta$ -GAN - Triangle Generative Adversarial Networks

# Cycle GAN: Correspondence from Unpaired Data

Monet ↪ Photos



Monet → photo

Zebras ↪ Horses



zebra → horse

Summer ↪ Winter

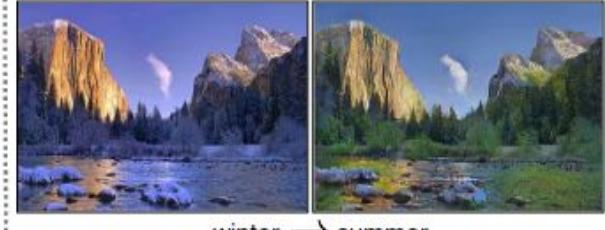


summer → winter

photo → Monet



horse → zebra



winter → summer

Photograph



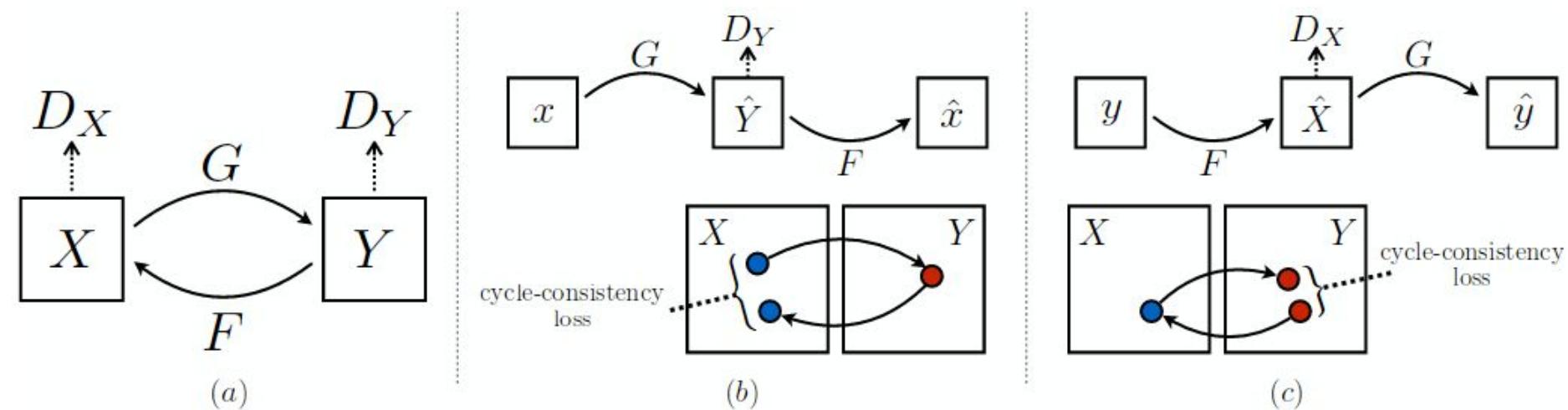
Monet

Van Gogh

Cezanne

Ukiyo-e

# Cycle GAN

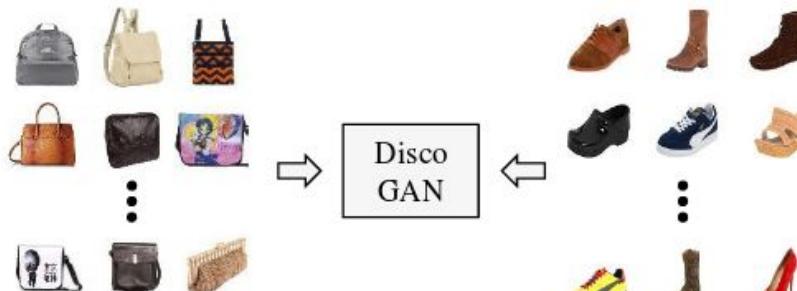


# Cycle GAN: Bad Cases



# DiscoGAN

Cross-domain relation



(a) Learning cross-domain relations **without any extra label**

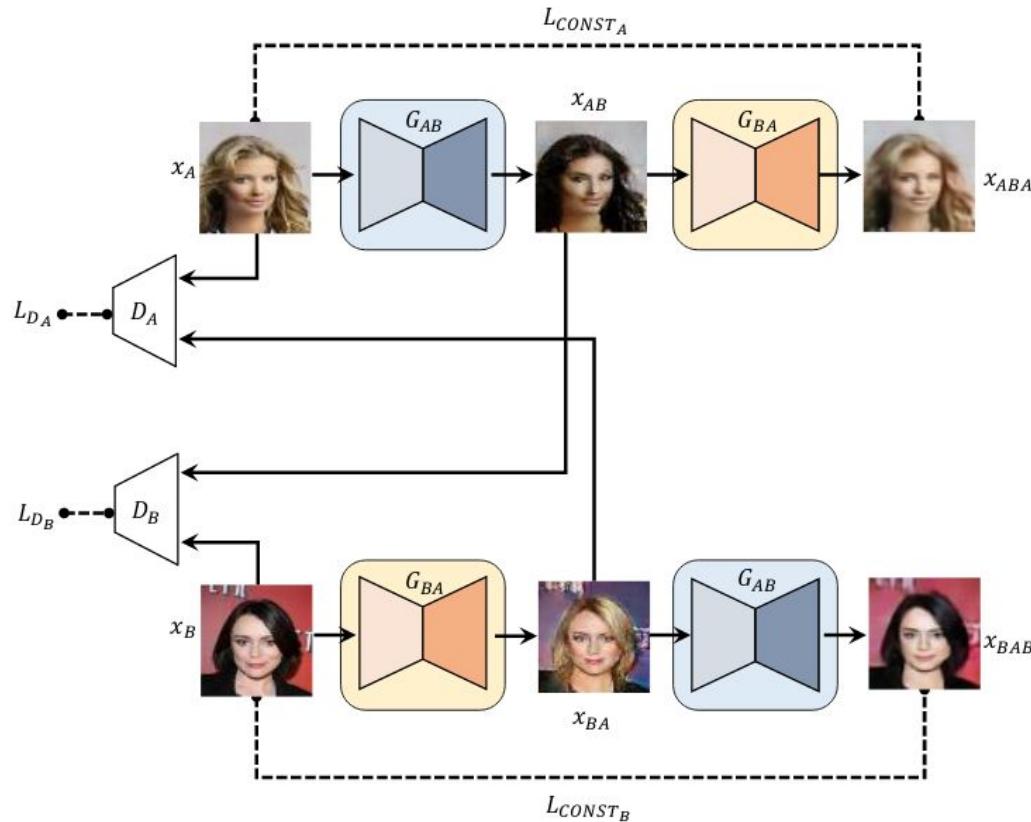


(b) Handbag images (input) & **Generated** shoe images (output)

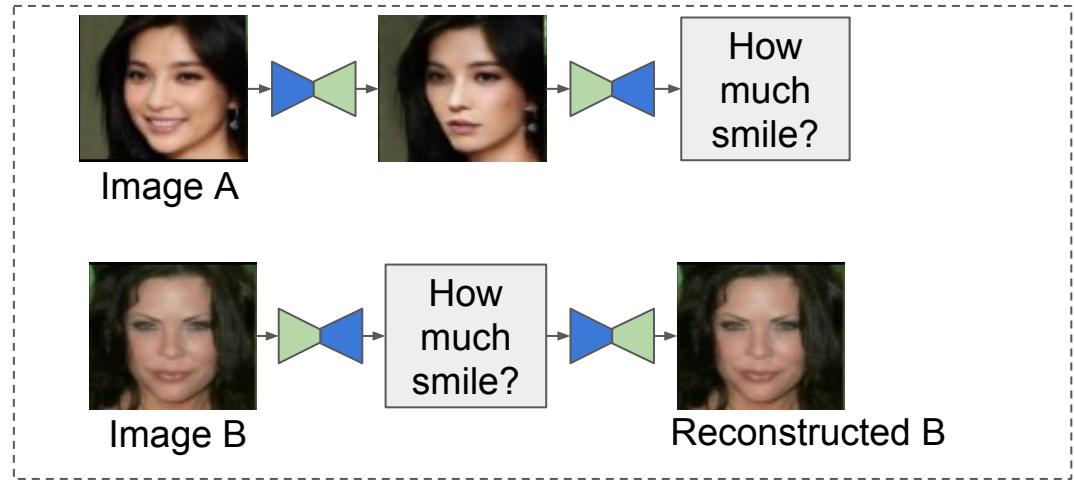


(c) Shoe images (input) & **Generated** handbag images (output)

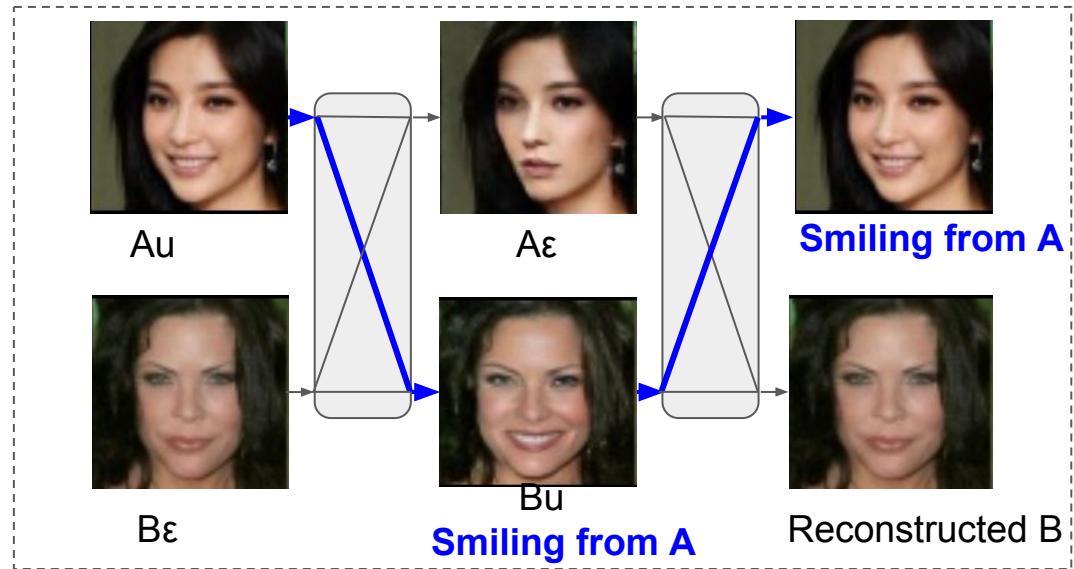
# DiscoGAN



# Underdetermined CycleGAN pattern

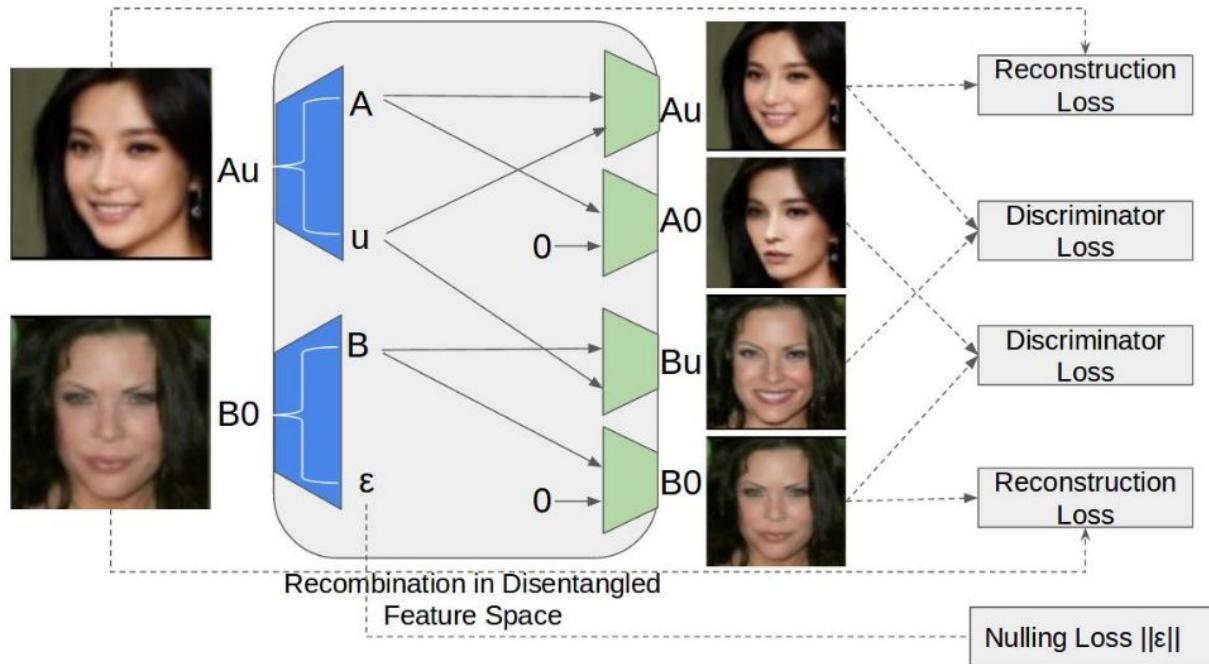


# Information Preserving GeneGAN pattern



# GeneGAN: shorter pathway improves training

Cross breeds and reproductions



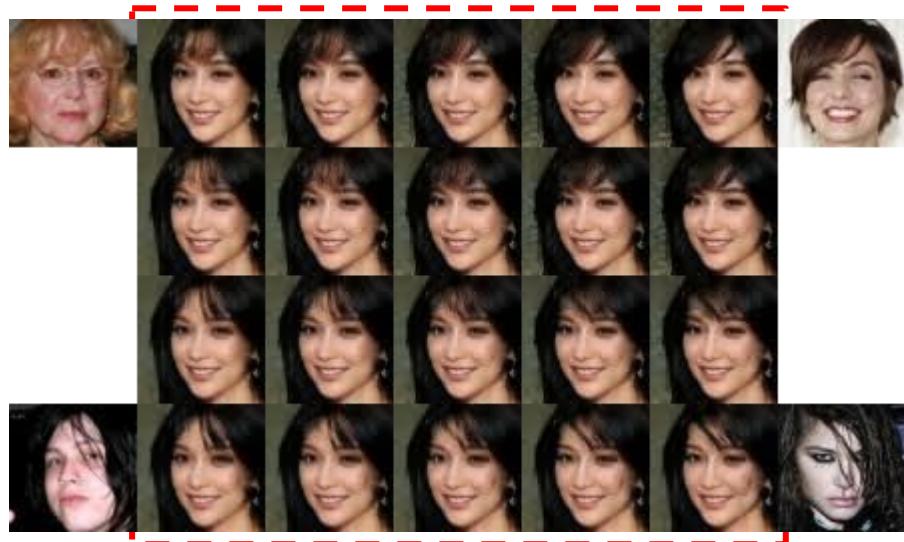
# GeneGAN: Object Transfiguration

Transfer "*my*" hairstyle to him, not just *a* hairstyle.



# GeneGAN: Interpolation in Object Subspace

Check the directions of the hairs.



Bi-linearly interpolated

$\epsilon$  instance



# Math behind Generative Models

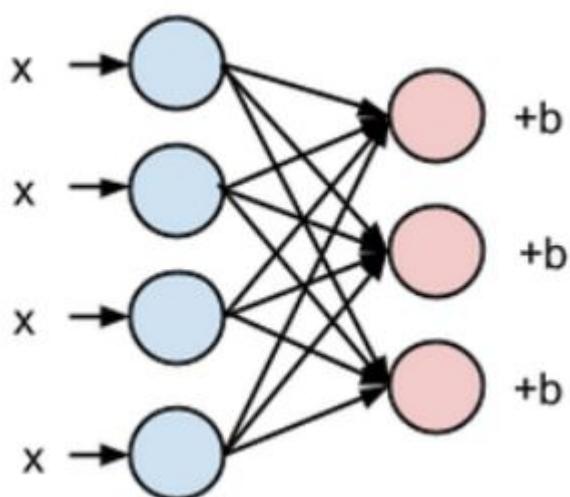
Those who don't care about math or theory can open their PyTorch now...

# Formulation of Generative Models

sampling v.s. density estimation

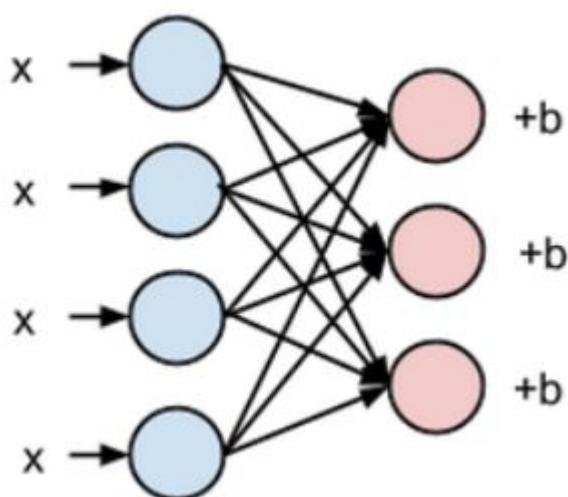
# RBM

$$p(x) = \frac{1}{Z} \sum_{y \in \{0,1\}^n} e^{x^T A y + x^T b + y^T c}$$



# RBM

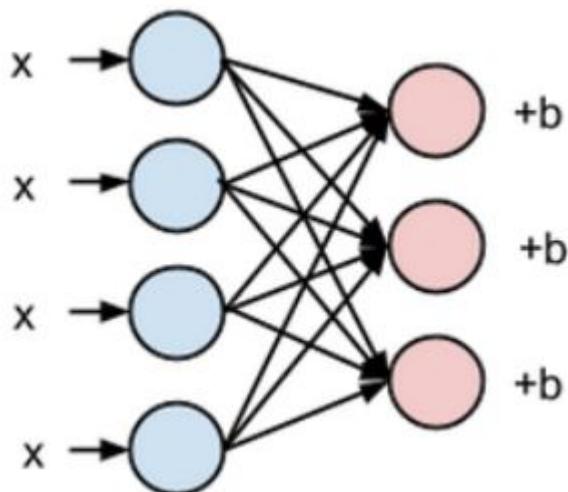
$$p(x) = \frac{1}{Z} \sum_{y \in \{0,1\}^n} e^{x^T A y + x^T b + y^T c}$$



It is NP-Hard to estimate Z

# RBM

$$p(x) = \frac{1}{Z} \sum_{y \in \{0,1\}^n} e^{x^T A y + x^T b + y^T c}$$



It is NP-Hard to sample from P

# Score Matching

Let  $L$  be the likelihood function, score  $V$  is:

$$V \equiv V(\theta, X) = \frac{\partial}{\partial \theta} \log L(\theta; X) = \frac{1}{L(\theta; X)} \frac{\partial L(\theta; X)}{\partial \theta}$$

If two distribution's scores match, they also match.

## A Connection Between Score Matching and Denoising Autoencoders

**Pascal Vincent**

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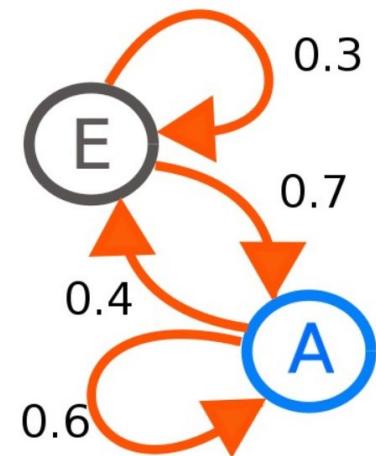
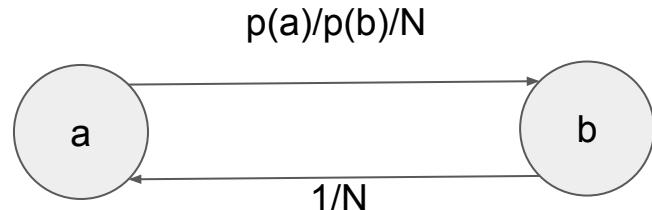
# Markov Chain Monte Carlo

From each node  $a$ ,

walk to “neighbor”  $b$  with probability **proportional to  $p(b)$** .

Neighbors must be reciprocal:  $a \leftrightarrow b$

Walk for long enough time to reach equilibrium



# MCMC in RBM

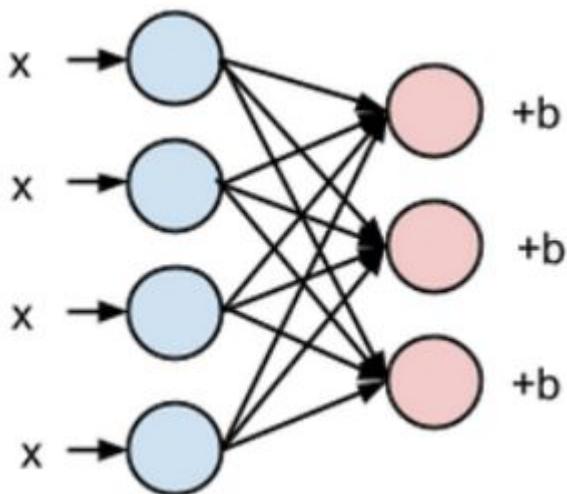
$$p(x) = \frac{1}{Z} \sum_{y \in \{0,1\}^n} e^{x^T A y + x^T b + y^T c}$$

Sample x given y

Sample y given x

Sample x given y

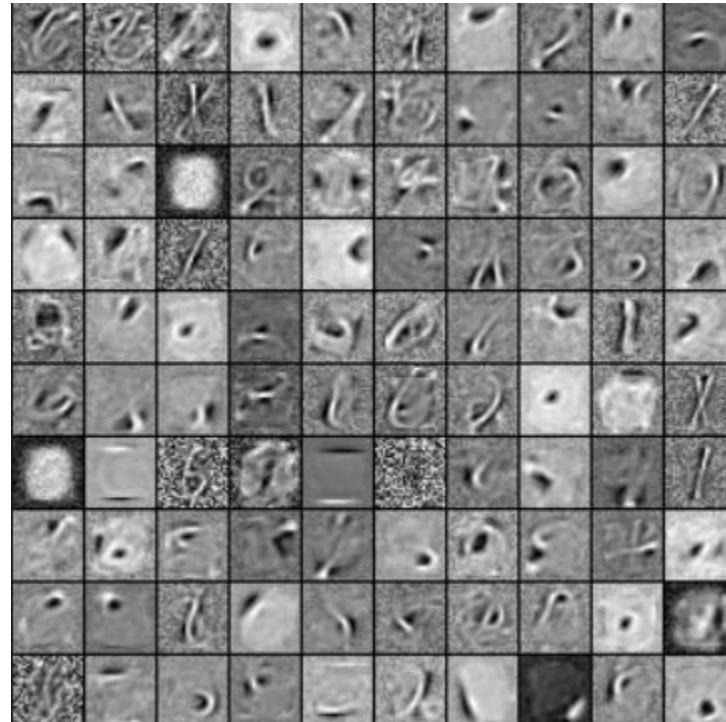
....



In theory, repeat for long enough time.

In practice, repeat a few times. ("burnin")

# RBM: Learned “Filters”



# From Density to Sample

Given density function  $p(x)$ , can we efficiently black-box sample from it?

No!     $p(x) = \text{MD5}(x) == 0$

Unless query  $\Omega(N)$  samples, it is hard to determine.

# From Sample to Density

Given black-box sampler G, can we efficiently estimate the density (frequency) of x?

Naive bound:  $\Omega(\varepsilon^{-2})$  absolute,  $\Omega(1/p(x) \varepsilon^{-2})$  relative

Cannot essentially do better.

Example: Sample x randomly. Retry iff  $x=0$ .

# What can be done if only samples are available?

Problem: Given black box sampler G, decide if:

- (1) it is uniform
- (2) it is  $\varepsilon$ -far from uniform

How to define distance between distributions?

Statistical distance:  $\frac{1}{2} \sum |p(x)-q(x)|$       p:G    q:Uniform

L2 distance:  $\sum (p(x)-q(x))^2$

KL divergence:  $\sum q(x)\log(q(x)/p(x))$

# Uniformity Check using $q(x)\log(q(x)/p(x))$

Impossible to check unless  $\Omega(N)$  samples are obtained.

Consider  $\{1,2,\dots,N\}^T$  and  $\{1,2,\dots,N-1\}^T$ . Unbound KL.

Statistical distance = sum  $\max(p(x)-q(x), 0)$

$((N-1)/N)^T = 1-o(1)$  if  $T=o(N)$

**Statistical distance is the best distinguisher's advantage over random guess!**

advantage =  $2*|\Pr(\text{guess correct})-0.5|$

# Uniformity Check using L2 Distance

$$\sum (p(x)-q(x))^2 = \sum p(x)^2 + q(x)^2 - 2p(x)q(x) = \sum p(x)^2 - 1/N$$

$p(x)^2$  : seeing two  $x$  in a row

$\sum p(x)^2$ : counting collisions

Algorithm: Get  $T$  samples, count the number of  $x[i]==x[j]$  for  $i < j$ , divide by  $C(T,2)$

variance calculation:  $O(\varepsilon^2)$  is enough!

# Uniformity Check using L1 Distance

Estimate collision probability to  $1 \pm O(\varepsilon^2)$

$O(\varepsilon^{-4} \sqrt{N})$  samples are enough.

# Lessons Learned: What We Can Get From Samples

Given samples, some properties of the distribution can be learned, while others cannot.

# Discriminator based distances

$$\max_D E(D(x))_{x \sim p} - E(D(y))_{y \sim q}$$

$0 \leq D \leq 1$  : Statistical Distance

D is Lipschitz Continuous: Wasserstein Distance

# Wasserstein Distance

Duality

Earth Mover Distance:

$$W_p(\mu, \nu) := \left( \inf_{\gamma \in \Gamma(\mu, \nu)} \int_{M \times M} d(x, y)^p \, d\gamma(x, y) \right)^{1/p}$$

Definition using Discriminator:

$$W_1(\mu, \nu) = \sup \left\{ \int_M f(x) \, d(\mu - \nu)(x) \middle| \text{continuous } f : M \rightarrow \mathbb{R}, \text{Lip}(f) \leq 1 \right\}$$

# Estimating Wasserstein Distance in High Dimension

The curse of dimensionality

There is no algorithm that, for any two distributions  $P$  and  $Q$  in an  $n$ -dimensional space with radius  $r$ ,

takes  $\text{poly}(n)$  samples from  $P$  and  $Q$  and estimates  $W(P, Q)$  to precision  $o(1)*r$  w.h.p.

# Finite Sample Version of EMD

Let  $W_N(P, Q)$  be the expected EMD between  $N$  samples from  $P$  and  $Q$ .

$$W_N(P, Q) \geq W(P, Q)$$

$$W(P, Q) \geq W_N(P, Q) - \min(W_N(P, P), W_N(Q, Q))$$

# Projected Wasserstein Distance

The k-dimensional projected EMD: let  $\sigma$  be a random k-dim subspace

$$W^k(P, Q) = \mathbb{E}_\sigma W(\sigma(P), \sigma(Q))$$

As a lower bounding approach

$$W(P, Q) \geq \sqrt{n}W^1(P, Q) \geq \sqrt{n}(W_N^1(P, Q) - W_N^1(P, P))$$

# Game Theory: The Generator - Discriminator Game

Stackelberg Game:

min. D max. G

min. G max. D

Nash equilibrium

(G,D) where both G and D will not deviate

Which is the largest?

# Linear Model

## minimax theorem

Let  $X \subset \mathbb{R}^n$  and  $Y \subset \mathbb{R}^m$  be compact convex sets. If  $f : X \times Y \rightarrow \mathbb{R}$  is a continuous function that is convex-concave, i.e.

$f(\cdot, y) : X \rightarrow \mathbb{R}$  is convex for fixed  $y$ , and

$f(x, \cdot) : Y \rightarrow \mathbb{R}$  is concave for fixed  $x$ .

Then we have that

$$\min_{x \in X} \max_{y \in Y} f(x, y) = \max_{y \in Y} \min_{x \in X} f(x, y).$$

# The Future of GANs

Guaranteed stabilization: new distance

Broader application: apply adversarial loss in XX / different type of data

# References

GAN Tutorial: <https://arxiv.org/pdf/1701.00160.pdf>

Slides: <https://media.nips.cc/Conferences/2016/Slides/6202-Slides.pdf>