1) $\frac{1}{3} + p + q = \frac{1}{3} + \frac{2}{3} - q + p \Rightarrow q = p - \frac{1}{3} \Rightarrow$

$x: \begin{pmatrix} a & a+1 & a-1 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$

$y: \begin{pmatrix} a+1 & a-1 & a-2 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$

$$x-y: \begin{pmatrix} -1 & -a & -a+1 & a-1 & 0 & 1 & a-2 \\ \frac{2}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{2}{9} & \frac{1}{9} & \frac{1}{9} \end{pmatrix}$$

$$E(x-y) = -\frac{2}{9} - \frac{a}{9} - \frac{a}{9} + \frac{1}{9} + \frac{a}{9} - \frac{1}{9} + \frac{1}{9} + \frac{a}{9} - \frac{2}{9}$$

$$= -\frac{1}{3}$$

$$E\left((x-y)^2\right) = \frac{2}{9} + \frac{a^2}{9} + \frac{a^2+1-2a}{9} + \frac{a^2+1-2a}{9}$$

$$+ \frac{1}{9} + \frac{a^2+4-4a}{9}$$

$$= \frac{4}{9}a^2 - \frac{8a}{9} + 1$$

$$Var(x-y) = \frac{4}{9}a^2 - \frac{8}{9}a + 1 = \frac{4}{9} \mid \cdot 9$$

$$4a^2 - 8a + 4 = 4 \mid :4 \Rightarrow a^2 - 2a + 1 = 1 \Rightarrow$$

$a^2 - 2a + 1 = 0 \Rightarrow (a-1)^2 = 0 \Rightarrow \boxed{a = 1}$

$Var(X+Y) = Var(X) + Var(Y) - 2\,Cov(X,Y)$ (1)

$Var(X) = \left(\frac{a^2}{3} + \frac{1}{3} + \frac{4}{3}\right) - \left(\frac{a}{3} + \frac{1}{3} + \frac{1}{3}\right)$

$= \frac{a^2+5}{3} - \frac{(a+3)^2}{9} = \frac{3a^2 + 15 - a^2 - 9}{9}$

$Var(Y) = \left(\frac{(a+1)^2}{3} + \frac{1}{3} + \frac{4}{3}\right) - \left(\frac{a+1}{3} + \frac{1}{3} + \frac{2}{3}\right)$

$= \frac{a^2+2a+6}{3} - \frac{(a+4)^2}{9} = \frac{3a^2 + 6a + 12 - a^2 - 8a - 16}{9}$

(1) $\Rightarrow \frac{4}{9}a^2 - \frac{8}{9}a + \frac{8}{9} = \frac{2a^2 - 6a + 6}{9} + \frac{2a^2 - 2a}{9}$

$= -2\,Cov(X,Y)$

$\Rightarrow Cov(X,Y)$ ne dépend de a

$2 \cdot 6p + 2p + 9p + p = 18p = 1 \Rightarrow p = \frac{1}{18} \Rightarrow$ +

-2	3	4	6
$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{2}$	$\frac{1}{18}$

$Var(aX+b) = a^2 Var(X) = a^2(E(X^2) - E^2(X)) = 75$

$E(aX+b) = aE(X) + b = 57$

$E(X) = \frac{-12+6+36+6}{18} = 2$

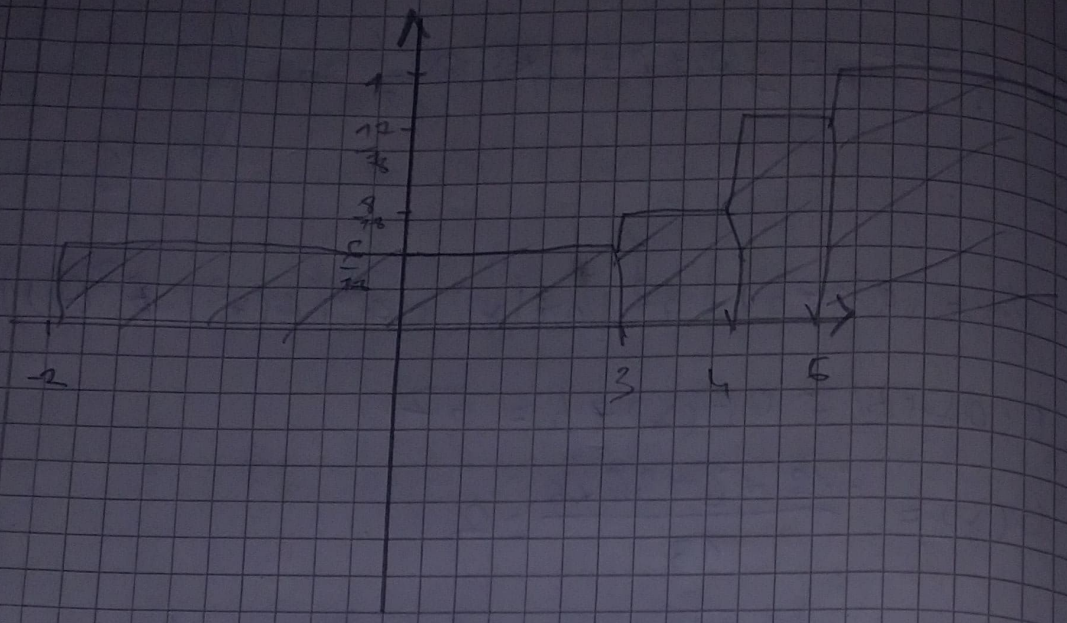
$E(X^2) = \frac{24+12+144+36}{18} = \frac{24}{18} + 11 = \frac{4}{3} + 11 = 12 + \frac{1}{3}$

$2a + b = 57$

$a^2\left(\frac{5}{9} + \frac{1}{3}\right) = 75 \Leftrightarrow a^2 = \frac{3}{25} \cdot 9 \Rightarrow a = \pm 3 \Rightarrow$

$(a,b) = \{(-3, 63), (3, 51)\}$

$$f(x) = \begin{cases} 0, \text{ daca } x \leq 2 \\ \frac{6}{18}, \text{ daca } 2 < x \leq 3 \\ \frac{8}{18}, \text{ daca } 3 < x \leq 4 \\ \frac{12}{18}, \text{ daca } 4 < x \leq 6 \\ 1, \text{ daca } x > 6 \end{cases}$$



3) $X \begin{pmatrix} -2 & 1 \\ 0.4 & 0.6 \end{pmatrix}$ $Y: \begin{pmatrix} -1 & 3 \\ 0.3 & 0.7 \end{pmatrix}$

a)

~~X~~ Y	-2	1	
-1	$0.4-k$	$k-0.1$	0.3
3	~~$0.7-k$~~ k	$0.7-k$	0.7
	0.4	0.6	1

b) $Cov(X,Y) = 0$

$\Rightarrow E(XY) = E(X)E(Y)$

$(-0.2)(1.8) = -1(-0.8 + 2k - k - 0,1)$

$+ 3(-2k + 0.7 - k)$

~~$(0.2)(1.8) \quad 3k \quad (2k \quad k)$~~

$(-0.2)(1.8) = -3k + 0,3 - 3k + 2$

$-0.36 = 12k + 3$

$\Rightarrow k = 0,28$

c) X, Y indépendante $\Leftrightarrow P(X = x_i ; Y = y_i)$

$= P(X = x_i) \cdot P(Y = y_i)$

$\Rightarrow X, Y$ sont indépendante

4) a) $3p + 4p + 2p + p + p = 1 \Rightarrow p = \frac{1}{11}$

c) $E(X) = -6p + 3p - 4p = -7p$

$E(X^2) = 12p + 4p + p + 4p = 21p$

$E(3X - 2) = 3 \cdot E(X) - 2 = 21p$

$Var(6X - 3) = 36 \cdot [E(X^2) - E^2(X)] = 1008p$

$E(X + X^2) = E(X) + E(X^2) = 14p$

d) $P(-0.5 < X < 0.5 \mid -1.25 < X < 0.75)$

$= \frac{P(-0.5 < X < 0.5)}{P(-1.25 < X < 0.75)}$

$= \frac{2p}{6p} = \frac{1}{3}$