

Clausu Nicolita

$$I = 109$$

1.

$D+ \rightarrow$ "am boala"

$D- \rightarrow$ "nu am boala"

$T+ \rightarrow$ "test pozitiv"

$T- \rightarrow$ "test negativ"

$$P(D+) = \frac{109}{1000} = 0.109 \Rightarrow P(D-) = 0.891$$

$$P(\text{fals pozitiv}) = P(T+|D-) = \frac{3}{100} = 0.03$$

$$P(\text{fals negativ}) = P(T-|D+) = \frac{3}{100} = 0.03$$

$$P(T-|D-) = 1 - 0.03 = 0.97 \quad (1 - P(T+|D-)) \quad (\text{true neg})$$

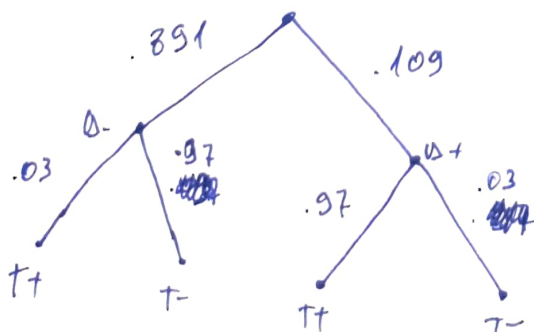
$$P(T+|D+) = 1 - P(T-|D+) = 1 - 0.03 = 0.97 \quad (\text{true pos})$$

$$P(D+|T+) = ?$$

Din legea lui Bayes:

$$P(D+|T+) = \frac{P(T+|D+) \cdot P(D+)}{P(T+)} = \frac{0.97 \times 0.109}{0.13246} \approx 0.79182$$

dar $P(T+) = ?$ Folosim arborele, din legea prob. totale:



$$P(T+) = 0.891 \cdot 0.03 + 0.109 \cdot 0.97 = 0.37303 = 0.13246$$

$$\Rightarrow P(D+|T+) \approx 0.79182 \approx 79\%$$

2. cu Discrete RV:

$$i \in 109$$

$$\lambda \leftarrow \text{RV}(\text{outcomes} = 1:3, \text{probs} = c((i/1000), (i/500), ((1000-3*i)/1000))$$

$$\text{print}(E(\lambda^2))$$

$$\text{rezultat} = 7.038$$

3.

$$F_X(x) = \frac{x-a}{b-a} = \frac{x}{109}$$

$$F_Y(y) = P(Y \leq y) = P(\sqrt{X} \leq y) = P(X \leq y^2) = \frac{y^2}{109}$$

$$f(y) = (F_Y(y))' = \frac{2y}{109}$$

$$f(y) = \frac{y^2}{109}$$

domeniul de valori pt $y = [0, \sqrt{109}]$.

$$4. P(109 \text{ averse}) = C_{200}^{109} p^{109} (1-p)^{91}$$

$$P(109 \text{ averse} | p) = C_{200}^{109} p^{109} (1-p)^{91}$$

$$(P(\text{date} | p))' = \frac{C_{200}^{109} (109 \cdot p^{90} (1-p) - p^{109} \cdot 91 (1-p)^{90})}{C_{200}^{109} (109 p^{108} (1-p)^{91} - p^{109} \cdot 91 (1-p)^{90})} = 0$$

$$\Rightarrow 109 p^{108} (1-p)^{91} = 91 p^{109} (1-p)^{90}$$

$$109(1-p) = 91p$$

$$109 - 109p = 91p$$

$$109 = 200p \Rightarrow p = \frac{109}{200}$$

5. a) 109 tip A ~~$P(A) = 0.5$~~

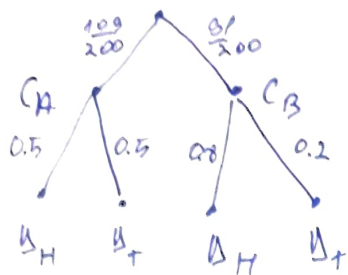
91 tip B ~~$P(A) = 0.8$~~

Fie $C_A \rightarrow$ „am ales moneda A”, $C_B \rightarrow$ „am ales moneda B”

A priori:

$$P(C_A) = \frac{109}{200}$$

$$P(C_B) = \frac{91}{200}$$



$$P(D_H) = P(D_H | C_A) P(C_A) + P(D_H | C_B) P(C_B)$$

$$= \frac{109}{200} \cdot 0.5 + \frac{91}{200} \cdot 0.8$$

$$= 0.6365$$

b) Fie $\Delta \rightarrow$ a avut loc o erorare si a fost aneas

H	$P(H)$	$P(\Delta H)$	$P(D H) \cdot P(H)$	$P(H \Delta)$
C_A	$\frac{109}{200}$	0.5	0.2725	0.4281
C_B	$\frac{91}{200}$	0.8	0.364	0.5719
Total	1		0.539 0.3089	1

$$P(D_H | \Delta) = P(D_H | C_A) \cdot P(C_A | \Delta) + P(D_H | C_B) \cdot P(C_B | \Delta)$$

$$P(H | \Delta) = \frac{P(\Delta | H) P(H)}{P(\Delta)}$$

$$\Rightarrow P(C_A | \Delta) = \frac{0.5 \cdot \frac{109}{200}}{0.6365} = 0.4281$$

$$P(C_B) = P(D | C_A) \cdot P(C_A) + P(D | C_B) \cdot P(C_B)$$

$$= 0.5 \cdot \frac{109}{200} + 0.8 \cdot \frac{91}{200} =$$

$$P(C_B | \Delta) = \frac{P(\Delta | C_B) \cdot P(C_B)}{P(\Delta)} = \frac{0.8 \cdot \frac{91}{200}}{0.6365} = 0.5719$$

$$P(D_H | \Delta) = 0.5 \cdot 0.4281 + 0.8 \cdot 0.5719 = 0.6715$$

6.

1090 prefera candidat A

510 prefera candidat B

Interval de încredere 95% normal conservator pt θ .

$$\bar{x} = \frac{1090}{1600} = 0.68125$$

$$\alpha = 0.05$$

$$z_{\alpha/2} = z_{0.025} \approx 1.96 \quad \text{! (din curs pg pagina 2)}$$

$$\text{Intervalul de încredere} =] \approx 0.68125 \pm \frac{1.96}{2 \cdot \sqrt{1600}}$$

$$] \approx 0.68125 \pm \frac{1.96}{2 \cdot 400}$$

$$] \approx 0.68125 \pm 0.0245$$

$$= \cancel{[0.43625, 0.92625]} \\ = [0.6768, 0.6837]$$

$$] \approx \bar{x} \pm z_{\alpha/2} \cdot \frac{1}{2\sqrt{n}}$$