Brushless Direct-Current Motors

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Features Common to Rotating Magnetic Field Electromechanical Devices

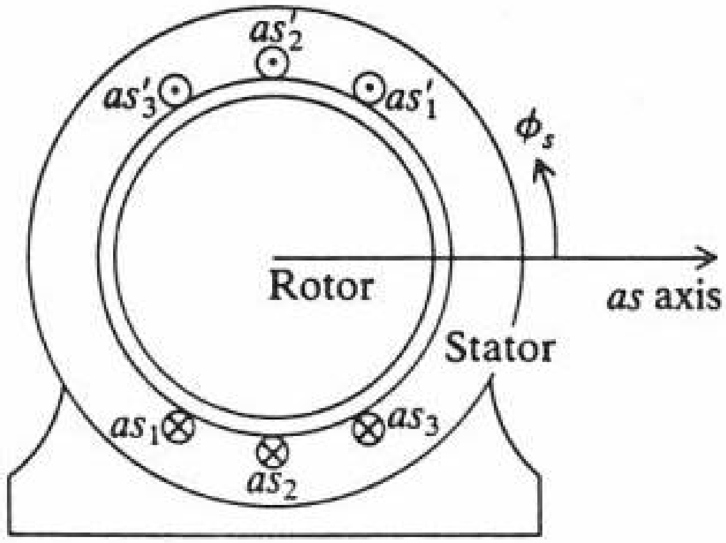
- Introduction
  - A *dc* machine has windings on both the stationary and rotating members, and these circuits are in relative motion whenever the armature (rotor) rotates. However, due to the action of the commutator, the resultant mmf produced by currents flowing in the rotor windings is stationary.
  - The rotor windings appear to be stationary, magnetically.
  - With constant current in the field (stator) winding, torque is produced and rotation results owing to the force established to align two stationary, orthogonal magnetic fields.

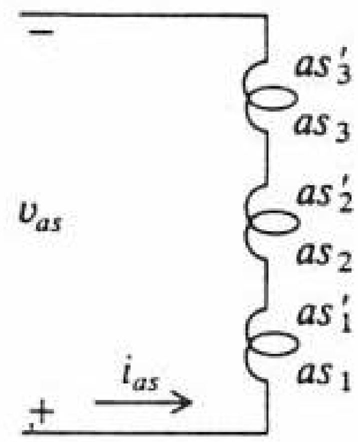
- In rotational electromechanical devices other than *dc* machines, torque is produced as a result of one or more magnetic fields which rotate about the air gap of the device.
- Reluctance machines, induction machines, synchronous machines, stepper motors, and brushless dc motors (permanent-magnet synchronous machines), all develop torque in this manner.
- There are features of these devices which are common to all, in particular:
  - Winding arrangement of the stator
  - Method of producing a rotating magnetic field due to stator currents
- Hence, we cover these common features now.

- Windings
  - Consider the diagram of the elementary two-pole, single-phase stator winding.
  - Winding *as* is assumed distributed in slots over the inner circumference of the stator, which is more characteristic of the stator winding than is a concentrated winding.
  - The winding is depicted as a series of individual coils. Each coil is placed in a slot in the stator steel.
  - Follow the path of positive current i_{as} flowing in the *as* winding.
  - Note that as_1 and as_1' are placed in stator slots which span π radians; this is characteristic of a two-pole machine.
  - as_1 around to as_1' is referred to as a coil; as_1 or as_1' is a coil side. In practice a coil will contain more than one conductor.

- The number of conductors in a coil side tells us the number of turns in this coil. This number is denoted as nc_s.
- Repeat this winding process to form the $as_2 - as_2'$ coil and the $as_3 - as_3'$ coil, assuming that the same number of turns, nc_s, make up each coil.
- With the same number of turns in each of these coils, the winding is said to be distributed over a span from as_1 to as_3 or 60°.
- The right-hand rule is used to give meaning to the *as* axis; it is the principal direction of magnetic flux established by positive current flowing in the *as* winding. It is said to denote the positive direction of the magnetic axis of the *as* winding.

Elementary Two-Pole, Single-Phase Stator Winding

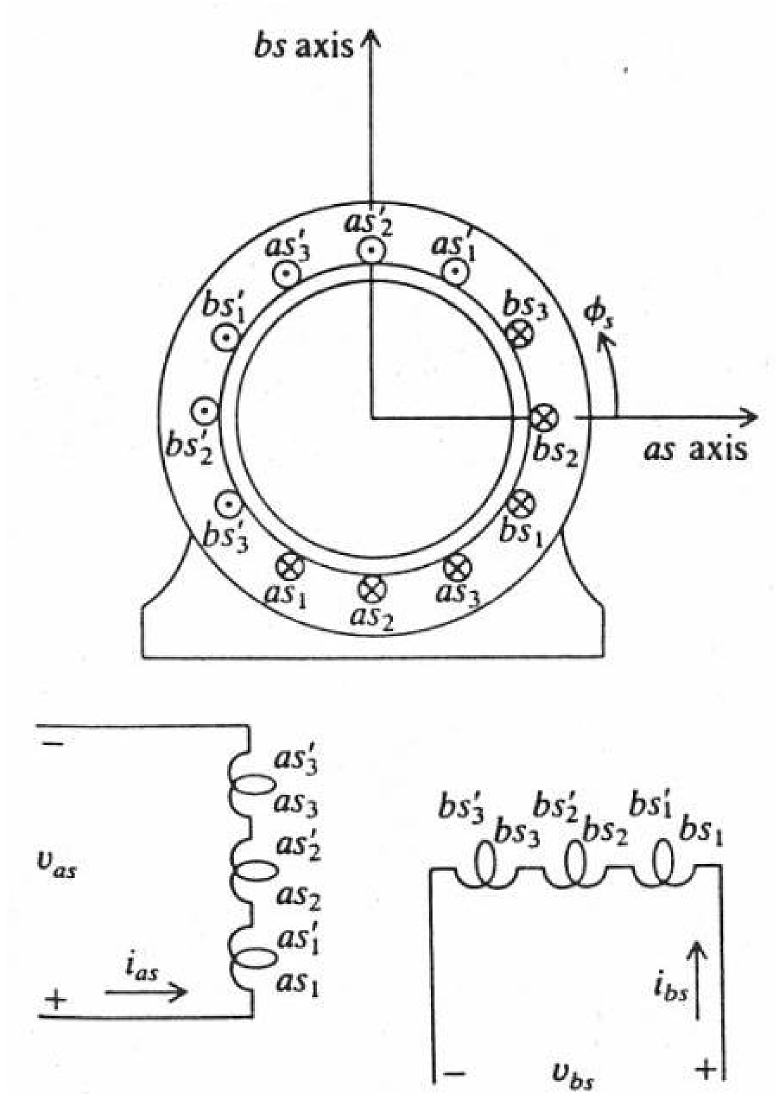




- Now, consider the diagram of the elementary two-pole, two-phase stator windings. Here we have added a second winding – the *bs* winding.
- The magnetic axis of the *bs* winding is displaced ½ π from that of the *as* winding.
- Assume that the positive direction of i_{bs} is such that the positive magnetic axis of the *bs* winding is at $\phi_s = \frac{1}{2}\pi$ where ϕ_s is the angular displacement about the stator referenced to the *as* axis.
- This is the stator configuration for a two-pole, two-phase electromechanical device.
- The stator windings are said to be *symmetrical* (as it is used in electromechanical devices) if the number of turns per coil and resistance of the *as* and *bs* windings are identical.

– For a two-pole, three-phase, symmetrical electromechanical device, there are three identical stator windings displaced 120° from each other. Essentially all multiphase electromechanical devices are equipped with symmetrical stators.

Elementary Two-Pole, Two-Phase Stator Windings



- Air Gap mmf – Sinusoidally-Distributed Windings
  - It is generally assumed that the stator windings (and in many cases the rotor windings) may be approximated as sinusoidally-distributed windings.
  - The distribution of a stator phase winding may be approximated as a sinusoidal function of ϕ_s, and the waveform of the resulting mmf dropped across the air gap (air gap mmf) of the device may also be approximated as a sinusoidal function of ϕ_s.
  - To establish a truly sinusoidal air gap mmf, the winding must also be distributed sinusoidally, and it is typically assumed that all windings may be approximated as sinusoidally-distributed windings.

– In the figure, we have added a few coils to the *as* winding, which now span 120°.
– For the purpose of establishing an expression for the air gap mmf, we employ the developed diagram of the cross-sectional view obtained by "flattening out" the rotor and stator.
– Note that displacement ϕ_s is defined to the left of the *as* axis since this allows us to position the stator above the rotor.
– The winding distributions may be approximated as:

$$N_{as} = N_p \sin\phi_s \qquad \text{for } 0 < \phi_s < \pi$$

$$N_{as} = -N_p \sin\phi_s \qquad \text{for } \pi < \phi_s < 2\pi$$

– N_p is the peak turns density in turns/radian.

– If N_s represents the number of turns of the equivalent sinusoidally distributed winding (not the total turns of the winding) that corresponds to the fundamental component of the actual winding distribution, then:

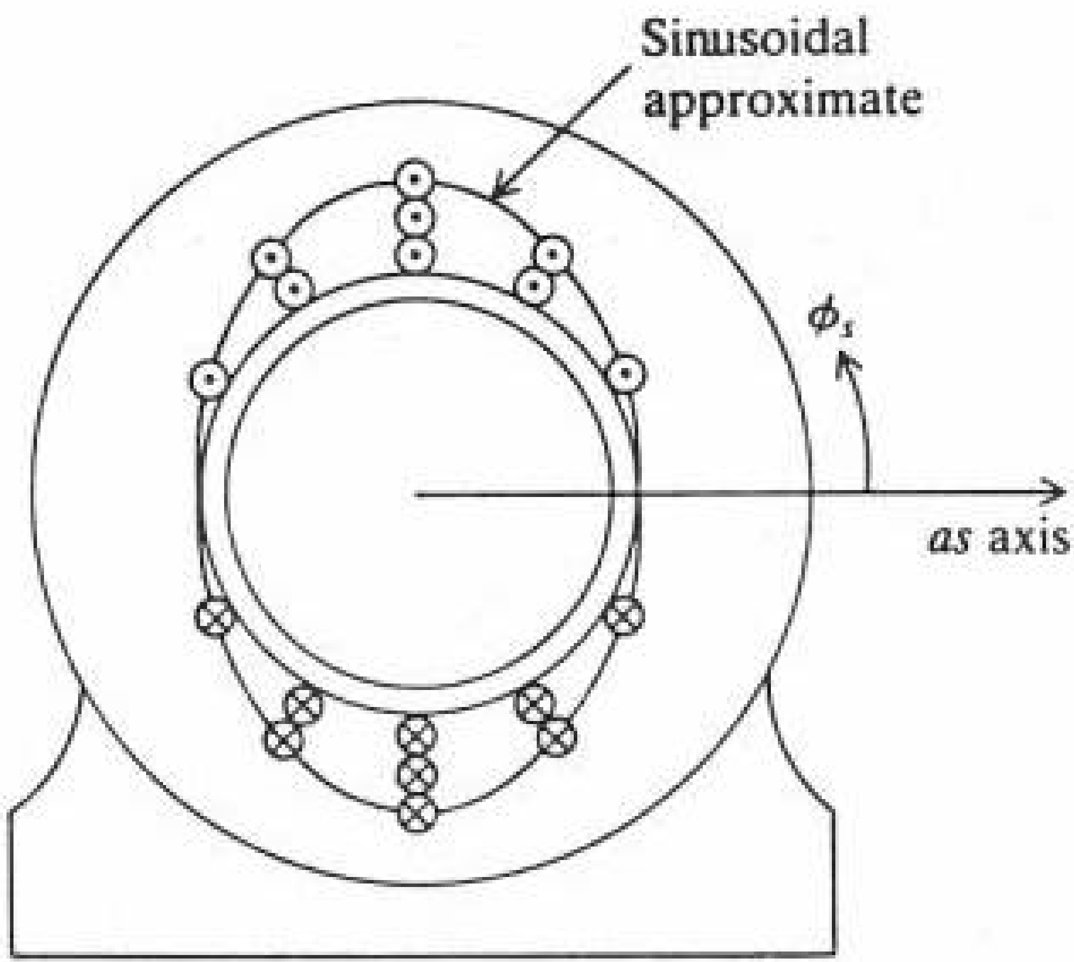
$$N_s = \int_0^{\pi} N_p \sin(\phi_s)\, d\phi_s = 2N_p$$

– The sinusoidally-distributed winding will produce a mmf that is positive in the direction of the *as* axis (to the right in the figure for positive i_{as}).

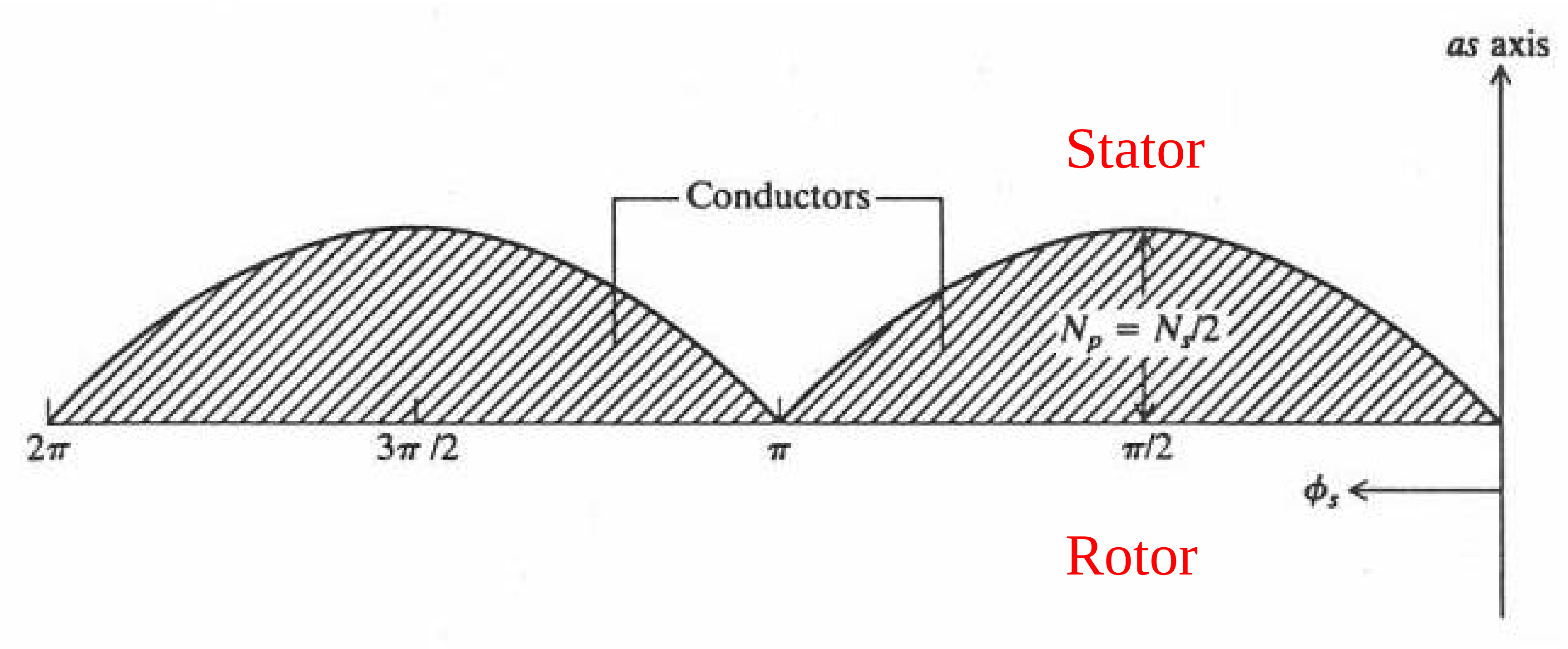
– We assume that all of the mmf is dropped across the air gap, as the reluctance of the steel is much smaller (neglecting saturation) than the reluctance of the air gap.

– So if the windings are sinusoidally-distributed in space, then the mmf dropped across the air gap will also be sinusoidal in space.

Approximate Sinusoidal Distribution of the *as* Winding



Developed Diagram with Sinusoidally-Distributed Stator Winding



- We need to develop an expression for the air gap mmf, mmf_{as}, associated with the *as* winding. We will apply Ampere's Law to two closed paths, shown in the diagram.
- For closed path (a), the total current enclosed is $N_s i_{as}$ and, by Ampere's Law, this is equal to the mmf drop around the given path ($\oint \vec{H} \cdot \vec{dL}$).
- If the reluctance of the rotor and stator steel is small compared with the air-gap reluctance, we can assume that ½ of the mmf is dropped across the air gap at $\phi_s = 0$ and ½ at $\phi_s = \pi$.
- By definition mmf_{as} is positive for a mmf drop across the air gap from the rotor to the stator. Thus mmf_{as} is positive at $\phi_s = 0$ and negative at $\phi_s = \pi$, assuming positive i_{as}.

– This suggests that for arbitrary ϕ_s, mmf_{as} might be expressed as:

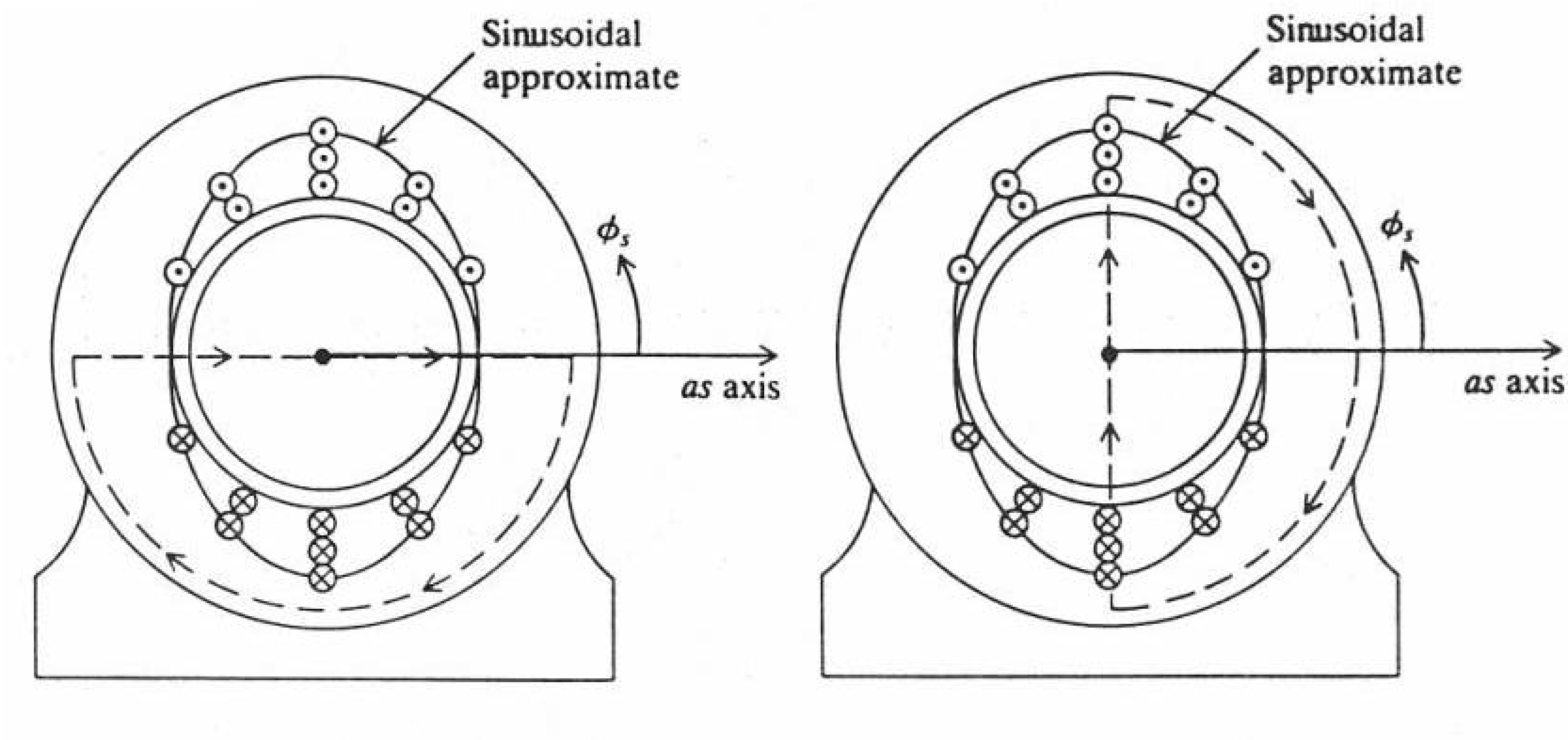
$$\left.\begin{aligned} mmf_{as}(0) &= \frac{N_s}{2} i_{as} \\ mmf_{as}(\pi) &= -\frac{N_s}{2} i_{as} \end{aligned}\right\} \quad mmf_{as} = \frac{N_s}{2} i_{as} \cos\phi_s$$

– This tells that the air gap mmf is zero at $\phi_s = \pm \frac{1}{2}\pi$. Check this by applying Ampere's Law to the second closed path in the figure, path (b). The net current enclosed is zero, and so the mmf drop is zero along the given path, implying that $mmf_{as} = 0$ at $\phi_s = \pm \frac{1}{2}\pi$.

– Let's consider the *bs* winding of a two-phase device. The air gap mmf due to a sinusoidally-distributed *bs* winding may be expressed as:

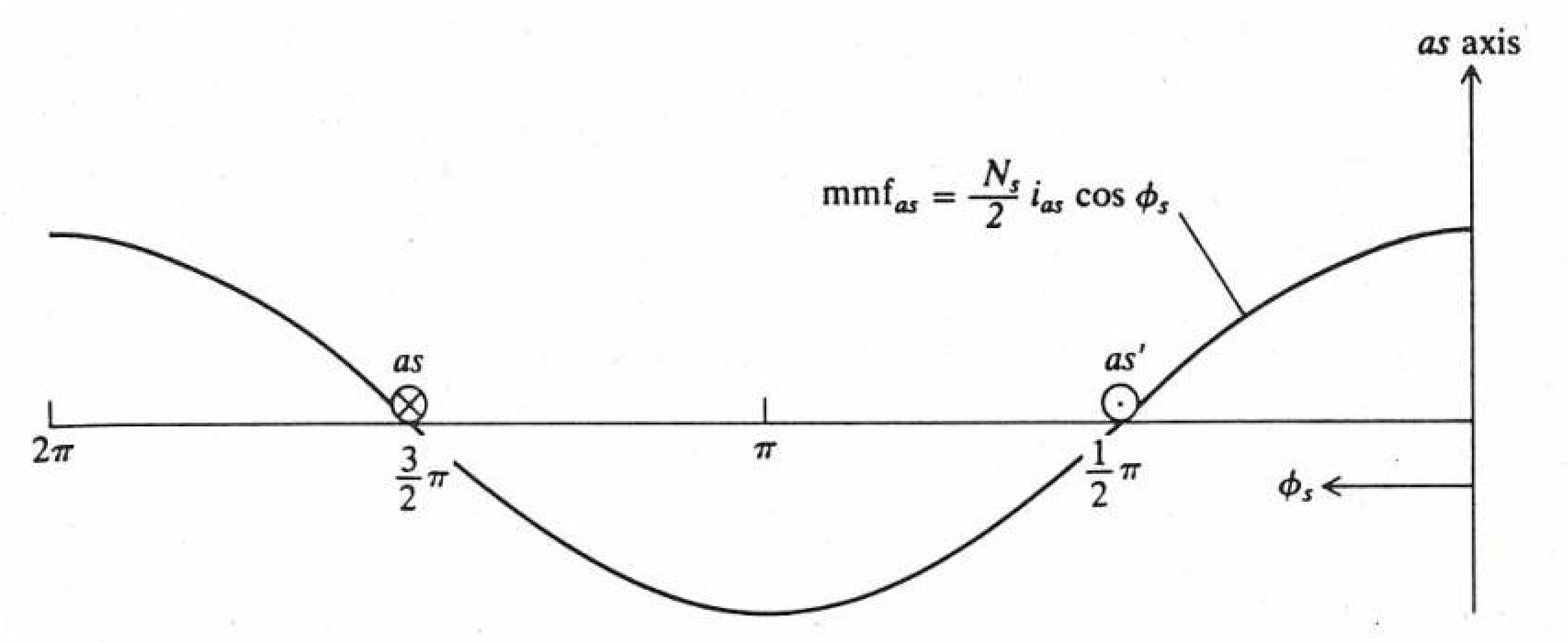
$$\text{mmf}_{bs} = \frac{N_s}{2} i_{bs} \sin\phi_s$$

Closed Paths used to Establish mmf_{as}



(a) (b)

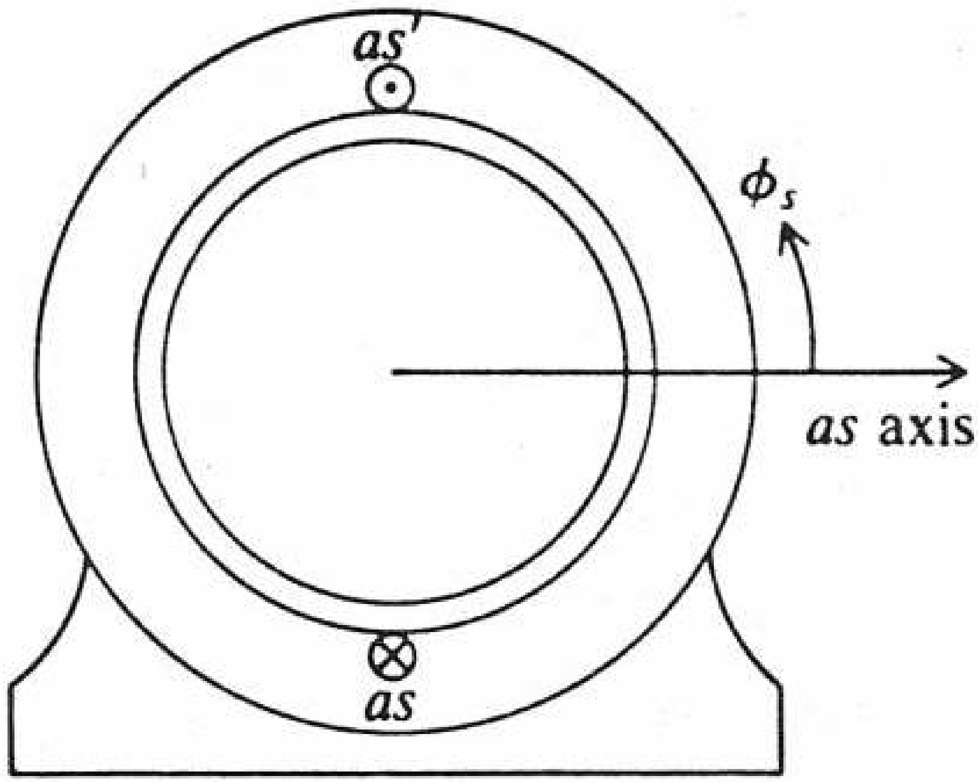
A mmf$_{as}$ Due to Sinusoidally Distributed *as* Winding



- Rotating Air Gap mmf – Two-Pole Devices
  - Considerable insight into the operation of electromechanical motion devices can be gained from an analysis of the air gap mmf produced by current flowing in the stator winding(s).
  - Let’s consider the rotating air gap mmf’s produced by currents flowing in the stator windings of single-, two-, and three-phase devices.

- *Single-Phase Devices*
  - Consider the device shown which illustrates a single-phase stator winding. Assume the *as* winding is sinusoidally distributed, with *as* and *as*′ placed at the point of maximum turns density.
  - Assume that the current flowing in the *as* winding is a constant. Then the *as* winding would establish a stationary magnetic system with a N pole from $0.5\pi < \phi_s < 1.5\pi$ and a S pole from $-0.5\pi < \phi_s < 0.5\pi$.
  - The air gap mmf is directly related to these poles; indeed, the flux flowing from the N pole and into the S pole is caused by the air gap mmf.

Elementary Two-Pole, Single-Phase Sinusoidally-Distributed Stator Winding



– What happens when the current flowing in the *as* winding is a sinusoidal function of time? Let's assume steady-state operation:

$$I_{as} = \sqrt{2} I_s \cos\left[\omega_e t + \theta_{esi}(0)\right]$$

– Capital letters denote steady-state instantaneous variables; I_s is the rms value of the current; ω_e is the electrical angular velocity; $\theta_{esi}(0)$ is the angular position corresponding to the time zero value of the instantaneous current.

– The air gap mmf expressed for the *as* winding is:

$$mmf_{as} = \frac{N_s}{2} i_{as} \cos\phi_s = \frac{N_s}{2} \sqrt{2} I_s \cos\left[\omega_e t + \theta_{esi}(0)\right] \cos\phi_s$$

- Consider this expression for a moment. It appears that all we have here is a stationary, pulsating magnetic field. Let us rewrite this expression using a trig identity:

$$\text{mmf}_{as} = \frac{N_s}{2}\sqrt{2}I_s\left\{\frac{1}{2}\cos\left[\omega_e t + \theta_{esi}(0) - \phi_s\right] + \frac{1}{2}\cos\left[\omega_e t + \theta_{esi}(0) + \phi_s\right]\right\}$$

- The arguments of the cosine terms are functions of time and displacement ϕ_s. If we can make an argument constant, then the cosine of this argument would be constant.

$$\left.\begin{aligned} \omega_e t + \theta_{esi}(0) - \phi_s &= C_1 \\ \omega_e t + \theta_{esi}(0) + \phi_s &= C_2 \end{aligned}\right\} \qquad \begin{aligned} \frac{d\phi_s}{dt} &= \omega_e \\ \frac{d\phi_s}{dt} &= -\omega_e \end{aligned}$$

- What does this mean?

- If you run around the air gap in CCW direction at an angular velocity ω_e, the first term in the expression for mmf_{as} will appear as a constant mmf and hence a constant set of N and S poles. On the other hand, if we run CW at ω_e, the second term in the expression for mmf_{as} will appear as a constant mmf.
- In other words, the pulsating air gap mmf we noted standing at $\phi_s = 0$ (or any fixed value of ϕ_s) can be thought of as two, one-half amplitude, oppositely-rotating air gap mmf's (magnetic fields), each rotating at the angular speed of ω_e, which is the electrical angular velocity of the current.
- Since we have two oppositely rotating sets of N and S poles (magnetic fields), it would seem that the single-phase machine could develop an average torque as a result of interacting with either.

– A single-phase electromechanical device with the stator winding as shown can develop an average torque in either direction of rotation.
– Note that this device is a two-pole device, even though there are two two-pole sets, as only one set interacts with the rotor to produce a torque with a nonzero average.

- *Two-Phase Devices*
  - Consider the two-pole, two-phase sinusoidally distributed stator windings shown.
  - For balanced (i.e., variables are equal-amplitude sinusoidal quantities and 90° out of phase) steady-state conditions, the stator currents may be expressed as:

$$I_{as} = \sqrt{2} I_s \cos\left[\omega_e t + \theta_{esi}(0)\right]$$

$$I_{bs} = \sqrt{2} I_s \sin\left[\omega_e t + \theta_{esi}(0)\right]$$

  - The reason for selecting this set of stator currents will become apparent.
  - The total air gap mmf due to both stator windings (assumed to be sinusoidally distributed) may be expressed by adding mmf_{as} and mmf_{bs} to give mmf_s.

– The total air gap mmf due to the stator windings is:

$$\left.\begin{aligned} \text{mmf}_{as} &= \frac{N_s}{2} i_{as} \cos\phi_s \\ \text{mmf}_{bs} &= \frac{N_s}{2} i_{bs} \sin\phi_s \end{aligned}\right\} \text{mmf}_s = \frac{N_s}{2}\left(i_{as}\cos\phi_s + i_{bs}\sin\phi_s\right)$$

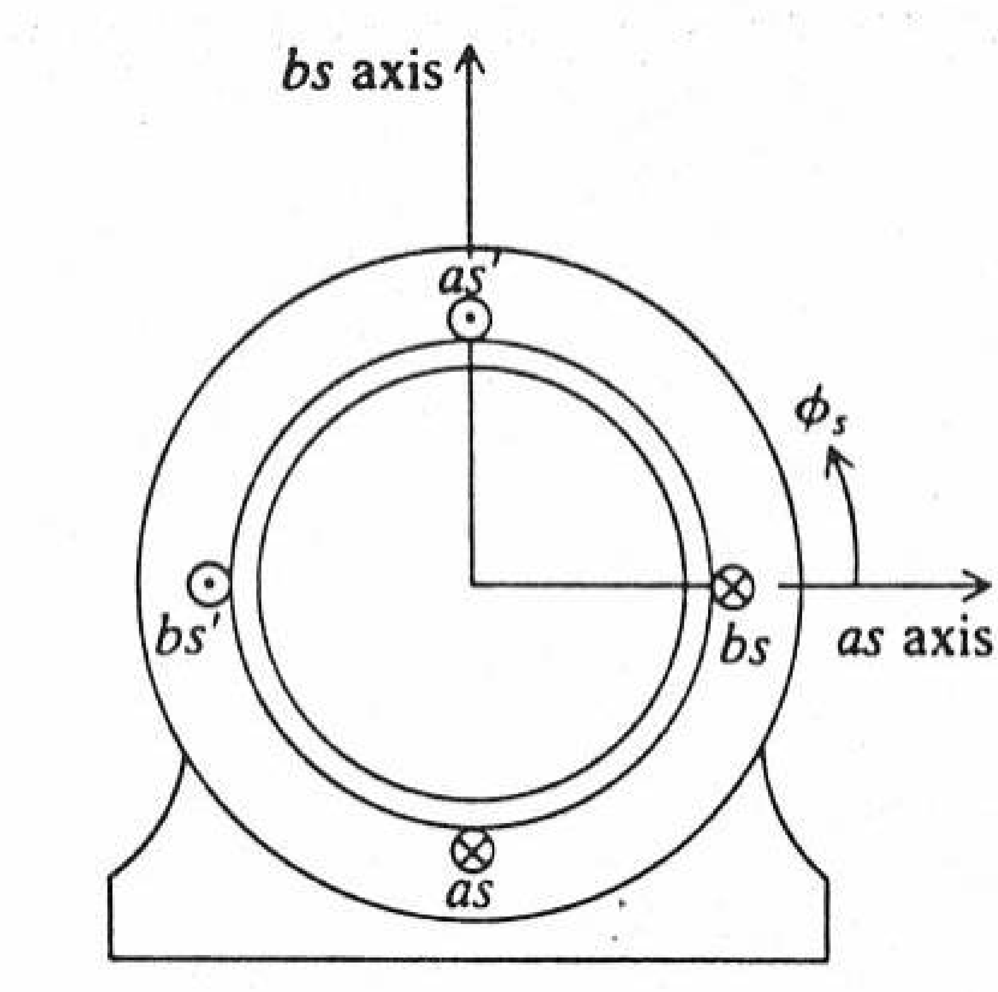
– Substitution:

$$\left.\begin{aligned} I_{as} &= \sqrt{2} I_s \cos\left[\omega_e t + \theta_{esi}(0)\right] \\ I_{bs} &= \sqrt{2} I_s \sin\left[\omega_e t + \theta_{esi}(0)\right] \end{aligned}\right\} \text{mmf}_s = \frac{N_s}{2}\left(i_{as}\cos\phi_s + i_{bs}\sin\phi_s\right)$$

– Result:

$$\text{mmf}_s = \frac{N_s}{2}\sqrt{2} I_s \cos\left[\omega_e t + \theta_{esi}(0) - \phi_s\right]$$

Elementary Two-Pole, Two-Phase Sinusoidally-Distributed Stator Winding



- It is interesting to note that we have only one rotating air gap mmf or rotating magnetic field.
- Set the argument equal to a constant, take the derivative with respect to time, and we find that the argument is constant if

$$\frac{d\phi_s}{dt} = \omega_e$$

- If we travel around the air gap in the CCW direction at ω_e, we will always see a constant mmf_s for the balanced set of currents

$$I_{as} = \sqrt{2} I_s \cos\left[\omega_e t + \theta_{esi}(0)\right]$$

$$I_{bs} = \sqrt{2} I_s \sin\left[\omega_e t + \theta_{esi}(0)\right]$$

- Hence a single rotating air gap mmf is produced. The actual value that we would see as we travel around the air gap at ω_e would depend upon the selection of time zero and our position on the stator at time zero.

– With the assigned positive direction of current in the given arrangement of the *as* and *bs* windings shown, the balanced set of stator currents produces a mmf_s that rotates CCW, which is desired for conventional purposes.

– In the case of the single-phase stator winding with a sinusoidal current, the air gap mmf can be thought of as two oppositely-rotating, constant-amplitude mmf's. However, the instantaneous air gap mmf is pulsating even when we are traveling with one of the rotating air gap mmf's. Unfortunately, this pulsating air gap mmf or set of poles gives rise to steady-state pulsating components of electromagnetic torque.

– In the case of the two-phase stator with balanced currents, only one rotating air gap mmf exists. Hence, the steady-state electromagnetic torque will not contain a pulsating or time-varying component; it will be a constant with the value determined by the operating conditions.

- *Three-Phase Devices*
  - The stator windings of a two-pole, three-phase device are shown in the figure.
  - The windings are identical, sinusoidally distributed with N_s equivalent turns and with their magnetic axes displaced 120°; the stator is symmetrical.
  - The positive direction of the magnetic axes is selected so as to achieve counterclockwise (CCW) rotation of the rotating air gap mmf with balanced stator currents of the *abc* sequence.

- The air gap mmf's established by the stator windings may be expressed by inspection as:

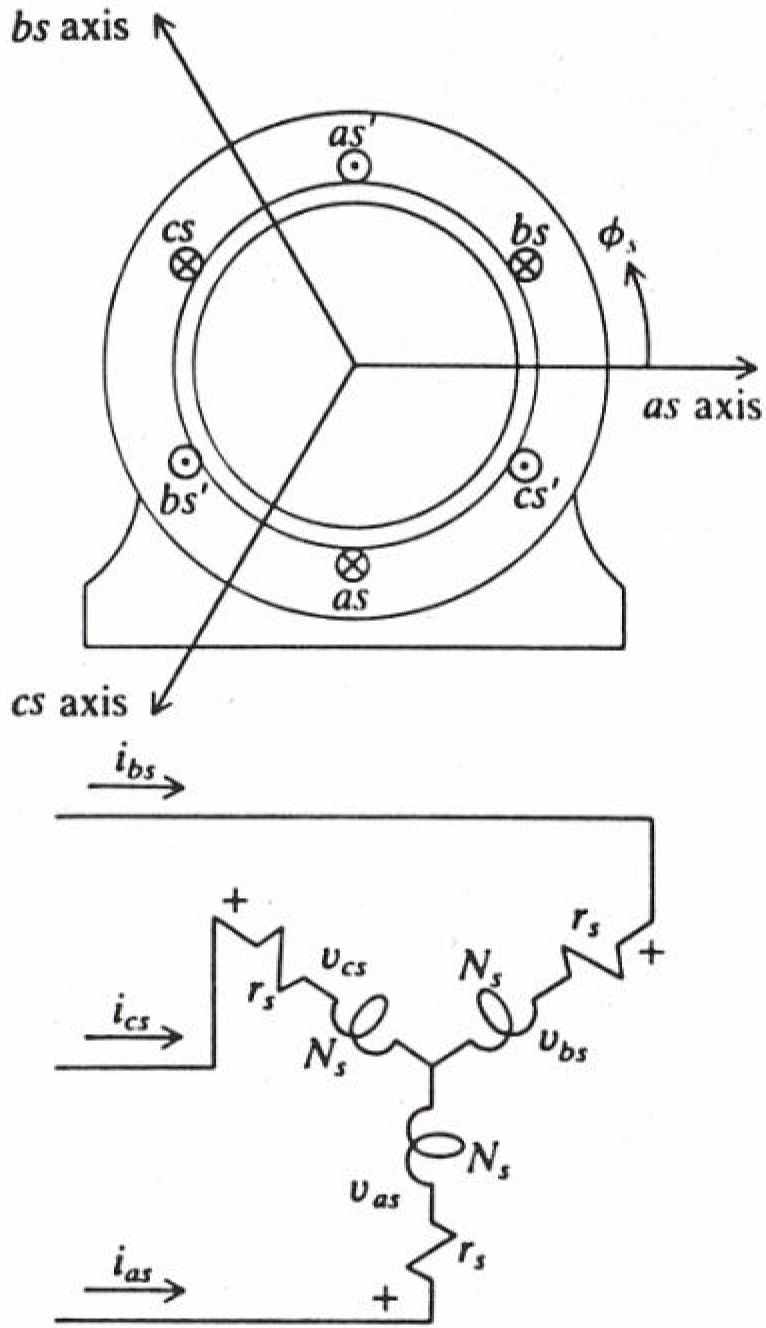
$$\text{mmf}_{as} = \frac{N_s}{2} i_{as} \cos\phi_s$$

$$\text{mmf}_{bs} = \frac{N_s}{2} i_{bs} \cos\left(\phi_s - \frac{2}{3}\pi\right)$$

$$\text{mmf}_{cs} = \frac{N_s}{2} i_{cs} \cos\left(\phi_s + \frac{2}{3}\pi\right)$$

- As before, N_s is the number of turns of the equivalent sinusoidally distributed stator windings and ϕ_s is the angular displacement about the stator.

Elementary Two-Pole, Three-Phase Sinusoidally-Distributed Stator Windings



- For balanced steady-state conditions, the stator currents for an *abc* sequence may be expressed as:

$$I_{as} = \sqrt{2} I_s \cos\left[\omega_e t + \theta_{esi}(0)\right]$$

$$I_{bs} = \sqrt{2} I_s \cos\left[\omega_e t - \frac{2}{3}\pi + \theta_{esi}(0)\right]$$

$$I_{cs} = \sqrt{2} I_s \cos\left[\omega_e t + \frac{2}{3}\pi + \theta_{esi}(0)\right]$$

- Substitution:

$$\left.\begin{aligned} I_{as} &= \sqrt{2} I_s \cos\left[\omega_e t + \theta_{esi}(0)\right] \\ I_{bs} &= \sqrt{2} I_s \cos\left[\omega_e t - \frac{2}{3}\pi + \theta_{esi}(0)\right] \\ I_{cs} &= \sqrt{2} I_s \cos\left[\omega_e t + \frac{2}{3}\pi + \theta_{esi}(0)\right] \end{aligned}\right\} \quad \begin{aligned} mmf_{as} &= \frac{N_s}{2} i_{as} \cos\phi_s \\ mmf_{bs} &= \frac{N_s}{2} i_{bs} \cos\left(\phi_s - \frac{2}{3}\pi\right) \\ mmf_{cs} &= \frac{N_s}{2} i_{as} \cos\left(\phi_s + \frac{2}{3}\pi\right) \end{aligned}$$

– Add the resulting expressions to yield an expression for the rotating air gap mmf established by balanced steady-state currents flowing in the stator windings:

$$\text{mmf}_s = \frac{N_s}{2}\sqrt{2}I_s\frac{3}{2}\cos\left[\omega_e t + \theta_{esi}(0) - \phi_s\right]$$

– Compare this with the mmf_s for a two-phase device:

$$\text{mmf}_s = \frac{N_s}{2}\sqrt{2}I_s\frac{3}{2}\cos\left[\omega_e t + \theta_{esi}(0) - \phi_s\right]$$ 3-Phase

$$\text{mmf}_s = \frac{N_s}{2}\sqrt{2}I_s\cos\left[\omega_e t + \theta_{esi}(0) - \phi_s\right]$$ 2-Phase

– They are identical except that the amplitude of the mmf for the 3-phase device is 3/2 times that of a 2-phase device.

- It can be shown that this amplitude for multiphase devices changes from that of a two-phase device in proportion to the number of phases divided by 2.
- It is important to note that with the selected positive directions of the magnetic axes a counterclockwise rotating air gap mmf is obtained with a three-phase set of balanced stator currents of the *abc* sequence.

Introduction to Several Electromechanical Motion Devices

- Rotational electromechanical devices fall into three general classes:
  - Direct-current
  - Synchronous
  - Induction
- We have already covered *dc* machines.
- Synchronous Machines
  - They are so called because they develop an average torque only when the rotor is rotating in synchronism (synchronous speed) with the rotating air gap mmf established by currents flowing in the stator windings.

- Examples are: reluctance machines, stepper motors, permanent-magnet machines, brushless dc machines, and the machine which has become known as simply the synchronous machine.

- Induction Machines
  - Induction is the principle means of converting energy from electrical to mechanical.
  - The induction machine cannot develop torque at synchronous speed in its normal mode of application.
  - The windings on the rotor are short-circuited and, in order to cause current to flow in these windings which produce torque by interacting with the air gap mmf established by the stator windings, the rotor must rotate at a speed other than synchronous speed.

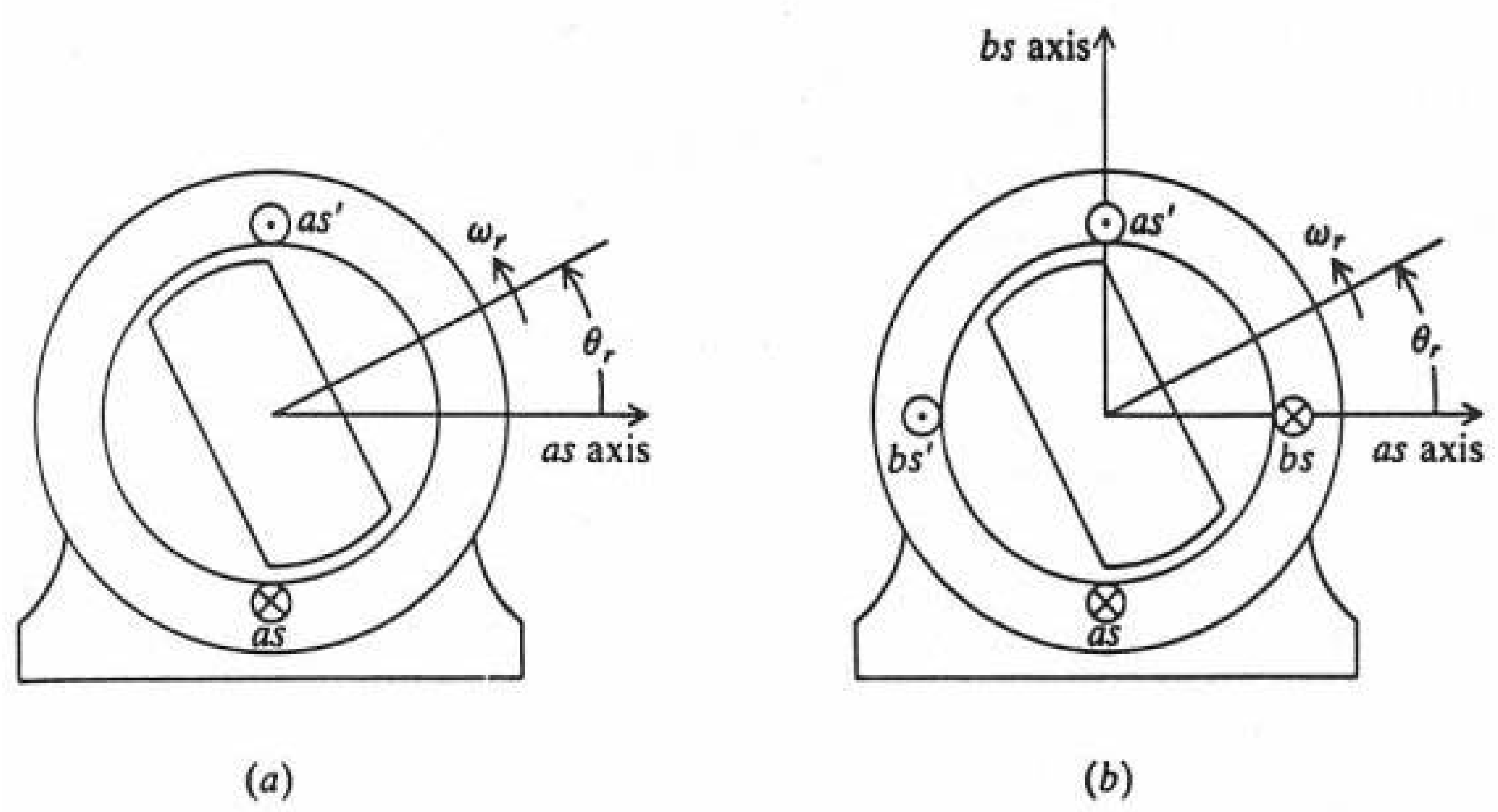
- Here we will show the winding arrangement for elementary versions of these electromechanical devices and describe briefly the principle of operation of each.

- Reluctance Drives
  - Elementary single-, and two-phase two-pole reluctance machines are shown in the figure.
  - Stator windings are assumed to be sinusoidally distributed.
  - The principal of operation is quite straightforward.
    - In an electromagnetic system a force (torque) is produced in an attempt to minimize the reluctance of the magnetic system.
    - We have established that, with an alternating current flowing in the winding of the single-phase stator, two oppositely-rotating mmf's are produced.
    - Therefore, once the rotor is rotating in synchronism with either of the two oppositely-rotating air gap mmf's, there is a force (torque) created by the magnetic system in an attempt to align the minimum-reluctance path of the rotor with the rotating air gap mmf.

- When there is no load torque on the rotor, the minimum-reluctance path of the rotor is in alignment with the rotating air gap mmf.
- When a load torque is applied, the rotor slows ever so slightly, thereby creating a misalignment of the minimum-reluctance path and the rotating air gap mmf.
- When the electromagnetic torque produced in an attempt to maintain alignment is equal and opposite to the load torque on the rotor, the rotor resumes synchronous speed.
- If the load torque is larger than the torque which can be produced to align, the rotor will fall out of synchronism and, since the machine cannot develop an average torque at a speed other than synchronous, it will slow to stall.
- The operation of a two-phase device differs from that of the single-phase device in that only one constant-amplitude rotating air gap mmf is produced during balanced steady-state conditions.
- Hence, a constant torque will be developed at synchronous speed rather than a torque which pulsates about an average value as is the case with the single-phase machine.

- Although the reluctance motor can be started from a source which can be switched at a frequency corresponding to the rotor speed as in the case of stepper or brushless dc motors, the devices cannot develop an average starting torque when plugged into a household power outlet.
- Many stepper motors are of the reluctance type. Some stepper motors are called variable-reluctance motors. Operation is easily explained. Assume that a constant current is flowing the *bs* winding of the figure with the *as* winding open-circuited. The minimum reluctance path of the rotor will be aligned with the *bs* axis, i.e., assume θ_r is zero. Now let's reduce the *bs* winding current to zero while increasing the current in the *as* winding to a constant value. There will be forces to align the minimum-reluctance path of the rotor with the *as* axis; however, this can be satisfied with $\theta_r = \pm \frac{1}{2} \pi$. There is a 50-50 chance as to which way it will rotate. We see that we need a device different from a single- or two-phase reluctance machine to accomplish controlled stepping. Two common techniques are single-stack and multistack variable-reluctance steppers.

Elementary Two-Pole Reluctance Machines: Single-Phase and Two-Phase



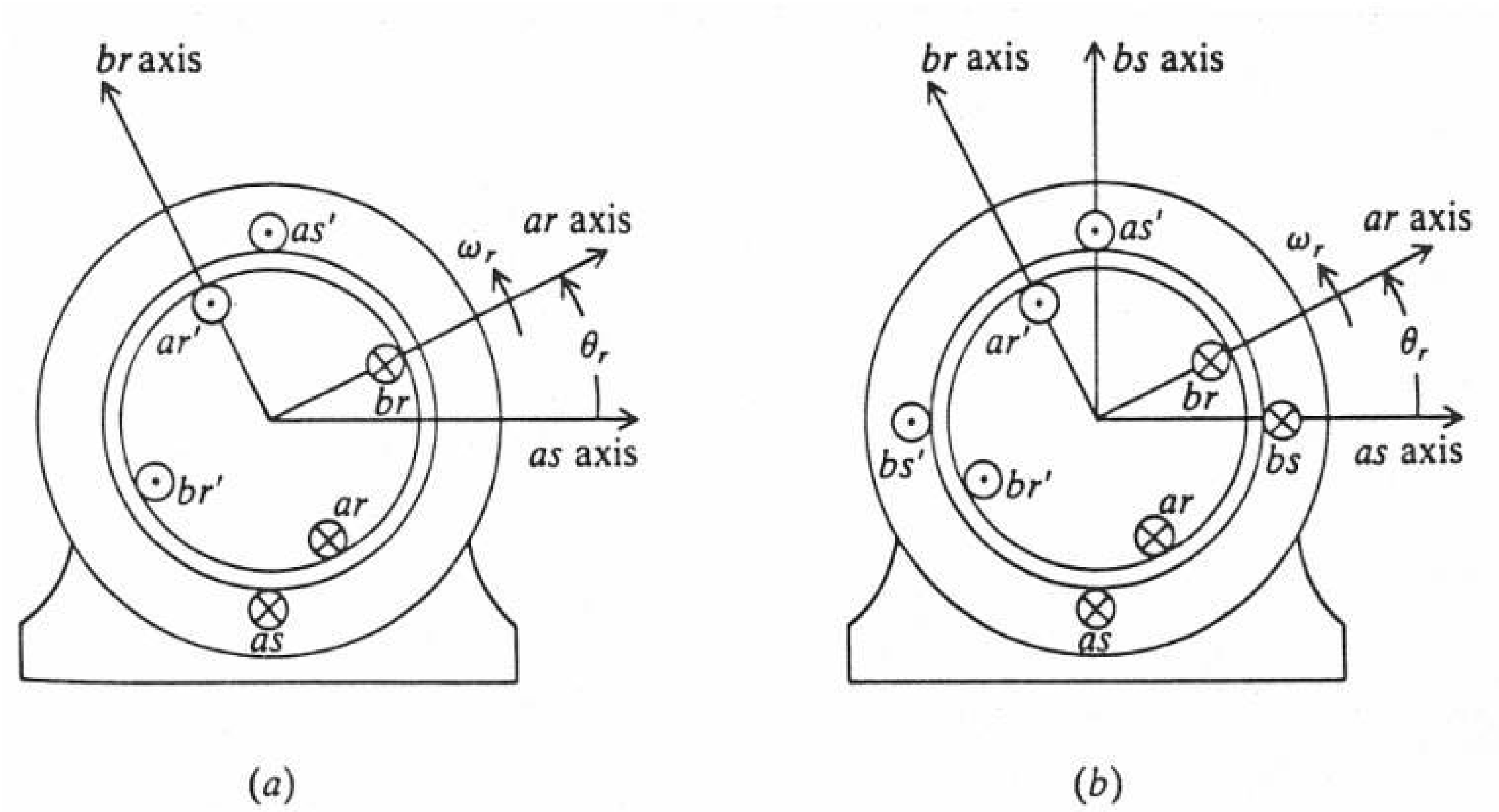
- Induction Machines
  - Elementary single- and two-phase induction machines are shown in the figure.
  - The rotors of both devices are identical in configuration; each has the equivalent of two orthogonal windings which are assumed to be sinusoidally distributed. The *ar* and *br* windings are equivalent to a symmetrical two-phase set of windings and, in the vast majority of applications, these rotor windings are short-circuited.
  - Let's look at the operation of the two-phase device first.
    - For balanced steady-state operation, the currents flowing in the stator windings produce an air gap mmf which rotates about the air gap at an angular velocity of ω_e.

- With the rotor windings short-circuited, which is the only mode of operation we will consider, a voltage is induced in each of the rotor windings only if the rotor speed ω_r is different from ω_e.
- The currents flowing in the rotor circuits due to induction will be a balanced set with a frequency equal to $\omega_e - \omega_r$, which will produce an air gap mmf that rotates at $\omega_e - \omega_r$ relative to the rotor or ω_e relative to a stationary observer.
- Hence, the rotating air gap mmf caused by the currents flowing in the stator windings induces currents in the short-circuited rotor windings which, in turn, establish an air gap mmf that rotates in unison with the stator rotating air gap mmf (mmf_s).
- Interaction of these magnetic systems (poles) rotating in unison provides the means of producing torque on the rotor.
- An induction machine can operate as a motor or a generator. However, it is normally operated as a motor.

- As a motor it can develop torque from $0 < \omega_r < \omega_e$. At $\omega_r = \omega_e$, the rotor currents are not present since the rotor is rotating at the speed of the stator rotating air gap mmf and, therefore, the rotor windings do not experience a change of flux linkages, which is, of course, necessary to induce a voltage in the rotor windings.
- The single-phase induction motor is perhaps the most widely used electromechanical device. The figure shown is not quite the whole picture of a single-phase induction motor. Recall that the single-phase stator winding produces oppositely rotating air gap mmf's of equal amplitude.
- If the single-phase induction motor is stalled, $\omega_r = 0$, and if a sinusoidal current is applied to the stator winding, the rotor will not move. This device does not develop a starting torque. Why?
- The rotor cannot follow either of the rotating mmf's since it develops as much torque to go with one as it does to go with the other. If, however, you manually turn the rotor in either direction, it will accelerate in that direction and operate normally.

- Although single-phase induction motors normally operate with only one stator winding, it is necessary to use a second stator winding to start the device. Actually single-phase induction motors we use are two-phase induction motors with provisions to switch out one of the windings once the rotor accelerates to between 60 and 80 percent of synchronous speed.
- How do we get two-phase voltages from a single-phase household supply? Well, we do not actually develop a two-phase supply, but we approximate one, as far as the two-phase motor is concerned, by placing a capacitor (start capacitor) in series with one of the stator windings. This shifts the phase of one current relative to the other, thereby producing a larger rotating air gap mmf in one direction than the other. Provisions to switch the capacitor out of the circuit is generally inside the housing of the motor.

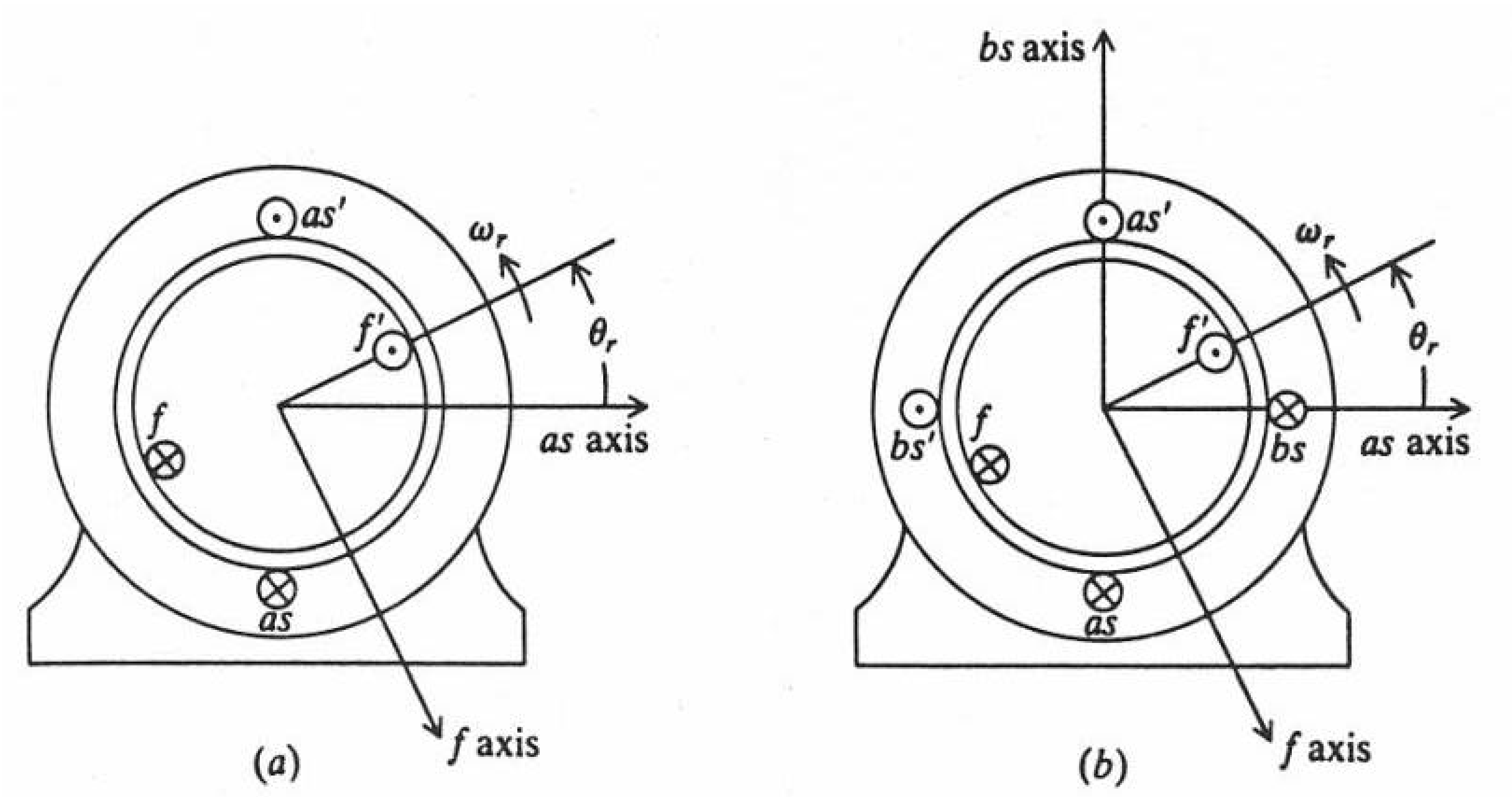
Elementary Two-Pole Induction Machines: Single-Phase and Two-Phase



- Synchronous Machines
  - Shown are elementary single- and two-phase two-pole synchronous machines. However, they are but one of several devices which fall into the synchronous machine category. We honor convention here and refer to these devices as synchronous machines.
  - The single-phase synchronous machine has limited application. The same can be said about the two-phase synchronous machine. It is the three-phase synchronous machine which is used to generate electric power in power systems such as in some automobiles, aircraft, utility systems, and ships. Nevertheless, the theory of operation of synchronous machines is adequately introduced by considering the two-phase version.

- The elementary devices shown have only one rotor winding – the field winding (*f* winding). In practical synchronous machines, the rotor is equipped with short-circuited windings in addition to the *f* winding which help to damp oscillations about synchronous speed and, in some cases, these windings are used to start the unloaded machine from stall as an induction motor.
- The principle of operation is apparent once we realize that the current flowing in the field winding is direct current. Although it may be changed in value by varying the applied field voltage, it is constant for steady-state operation of a balanced two-phase synchronous machine.
- If the stator windings are connected to a balanced system, the stator currents produce a constant-amplitude rotating air gap mmf. A rotor air gap mmf is produced by the direct current flowing in the field winding.
- To produce a torque or transmit power, the air gap mmf produced by the stator and that produced by the rotor must rotate in unison about the air gap of the machine. Hence, $\omega_r = \omega_e$.

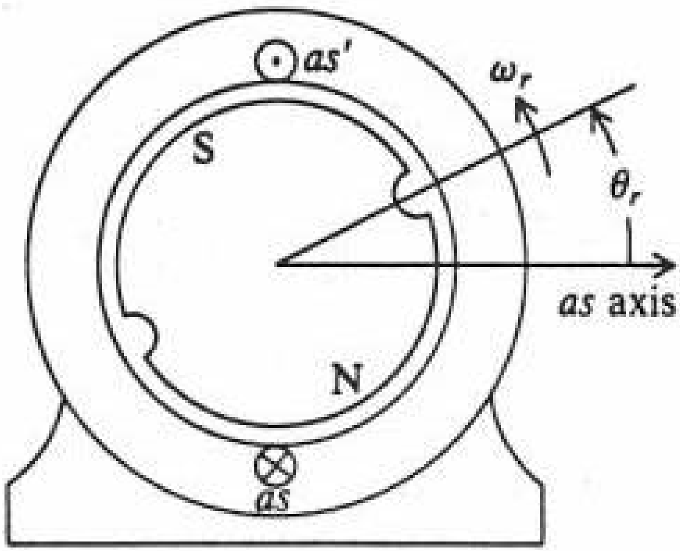
Elementary Two-Pole Synchronous Machines: Single-Phase and Two-Phase



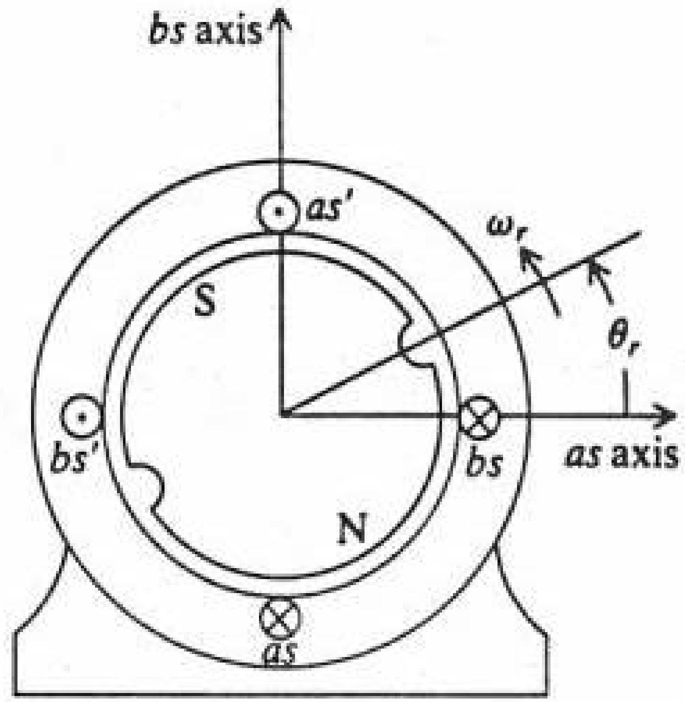
- Permanent-Magnet Devices
  - If we replace the rotor of the synchronous machines just considered with a permanent-magnet rotor, we have the so-called permanent-magnet devices shown in the figure.
  - The operation of these devices is identical to that of the synchronous machine.
  - Since the strength of the rotor field due to the permanent magnet cannot be controlled as in the case of the synchronous machine which has a field winding, it is not widely used as a means of generating power.
  - It is, however, used widely as a drive motor.

– In particular, permanent-magnet motors are used as stepper motors and, extensively, as brushless dc motors, wherein the voltages applied to the stator windings are switched electronically at a frequency corresponding to the speed of the rotor.

Elementary Two-Pole Permanent-Magnet Devices: Single-Phase and Two-Phase



(a)



(b)

Brushless DC Motors

- <u>Introduction</u>
  - Permanent magnet DC motors all have brushes to transmit power to the armature windings. Brush arcing causes electronic noise and maintenance problems from excessive wear.
  - A Brushless DC Motor has been developed to overcome these problems. It substitutes electronic commutation for the conventional mechanical brush commutation.
  - Because the electronic commutation exactly duplicates the brush commutation in conventional DC motors, the brushless DC motor exhibits the same linear torque-speed curve, has the same motor constants, and obeys the same performance equations.

- <u>Advantages of Brushless DC Motors</u>
  - High Reliability
    - The life of brushless DC motors is almost indefinite. Bearing failure is the most likely weak point.
  - Quiet
    - A lack of mechanical noise from brushes makes it ideal for a people environment. An added advantage is that there is no mechanical friction.
  - High Speed
    - Brush bounce limits DC motors to 10,000 RPM. Brushless DC motors have been developed for speeds up to 100,000 RPM, limited by the mechanical strength of the permanent magnet rotors.

- High Peak Torque
  - Brushless DC motors have windings on the stator housing. This gives efficient cooling and allows for high currents (torque) during low-duty-cycle, stop-start operation. Peak torques are more than 20 times their steady ratings compared to 10 times or less for conventional DC motors. Maximum power per unit volume can be 5 times conventional DC motors.

- <u>Disadvantages of Brushless DC Motors</u>
  - Cost
    - The relatively high cost of brushless DC motors is usually acceptable when considering complex machinery where normal downtime and maintenance are not only costly in itself, but often unacceptable.
  - Choice
    - Choice is restricted because there are few manufacturers.

- Types of Brushless DC Motors
  - Windings on the stator with the rotor on the inside; inside rotors have less inertia and are better suited for start-stop operation.
  - Windings on the stator with the rotor on the outside; outside rotors are better for constant load, high-speed applications.

- The brushless dc motor is becoming widely used as a small-horsepower control motor. It is a permanent-magnet synchronous machine. When it is supplied from a source, the frequency of which is always equal to the speed of its rotor, it becomes a brushless dc motor, not because it looks like a dc motor but because its operating characteristics can be made to resemble those of a dc shunt motor with a constant field current.
- How do we supply the permanent-magnet synchronous machine from a source the frequency of which always corresponds to the rotor speed?

- First we must be able to measure the rotor position. The rotor position is most often sensed by Hall-effect sensors which magnetically sense the position of the rotor poles.
- Next we must make the frequency of the source correspond to the rotor speed. This is generally accomplished with a dc-to-ac inverter, wherein the transistors are switched on and off at a frequency corresponding to the rotor speed. We are able to become quite familiar with the operating features of the brushless dc motor without getting involved with the actual inverter.
- If we assume that the stator variables (voltages and currents) are sinusoidal and balanced with the same angular velocity as the rotor speed, we are able to predict the predominant operating features of the brushless dc motor without becoming involved with the details of the inverter.

- Two-Phase Permanent-Magnet Synchronous Machine
  - A two-pole two-phase permanent-magnet synchronous machine is shown.
  - The stator windings are identical, sinusoidally distributed windings each with N_s equivalent turns and resistance r_s.
  - The magnetic axes of the stator windings are *as* and *bs* axes.
  - The angular displacement about the stator is denoted by ϕ_s, referenced to the *as* axis.
  - The angular displacement about the rotor is ϕ_r, referenced to the *q* axis.
  - The angular velocity of the rotor is ω_r and θ_r is the angular displacement of the rotor measured from the *as* axis to the *q* axis.
  - Thus $\phi_s = \phi_r + \theta_r$

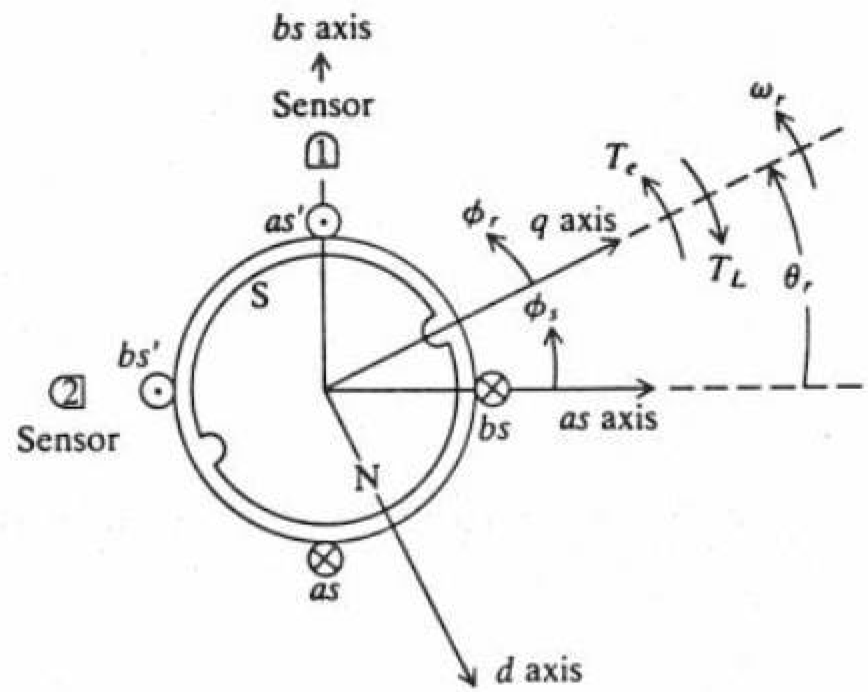
– The *d* axis (direct axis) is fixed at the center of the N pole of the permanent-magnet rotor and the *q* axis (quadrature axis) is displaced 90° CCW from the *d* axis.
– The electromechanical torque T_e is assumed positive in the direction of increasing θ_r and the load torque T_L is positive in the opposite direction.
– In the following analysis, it is assumed that:
  - The magnetic system is linear.
  - The open-circuit stator voltages induced by rotating the permanent-magnet rotor at a constant speed are sinusoidal.
  - Large stator currents can be tolerated without significant demagnetization of the permanent magnet.
  - Damper windings (short-circuited rotor windings) are not considered. Neglecting damper windings, in effect, neglects currents circulating in the surface of the rotor (eddy currents).

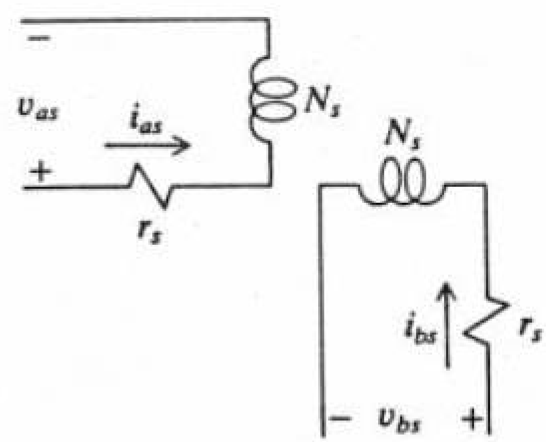
- The two sensors shown are Hall-effect sensors. When the N pole is under a sensor, its output is nonzero; with a S pole under the sensor, its output is zero.
- In the brushless dc motor applications, the stator is supplied from a dc-to-ac inverter the frequency of which corresponds to the rotor speed.
- The states of the sensors are used to determine the switching logic for the inverter which, in turn, determines the output frequency of the inverter.
- In the actual machine, the sensors are not positioned over the rotor. Instead, they are placed over a ring which is mounted on the shaft external to stator windings and which is magnetized by the rotor.

- The electromagnetic torque is produced by the interaction of the poles of the permanent-magnet rotor and the poles resulting from the rotating air gap mmf established by currents flowing in the stator windings.
- The rotating mmf (mmf_s) established by symmetrical two-phase stator windings carrying balanced two-phase currents is given by:

$$\text{mmf}_s = \frac{N_s}{2}\sqrt{2}I_s \cos\left[\omega_e t + \theta_{esi}(0) - \phi_s\right]$$

Two-Pole, Two-Phase Permanent-Magnet Synchronous Machine





- Voltage Equations and Winding Inductances
  - The voltage equations for the two-pole, two-phase permanent-magnet synchronous machine may be expressed as:

$$v_{as} = r_s i_{as} + \frac{d\lambda_{as}}{dt}$$

$$v_{bs} = r_s i_{bs} + \frac{d\lambda_{bs}}{dt}$$

  - The flux linkage equations may be expressed as:

$$\lambda_{as} = L_{asas} i_{as} + L_{asbs} i_{bs} + \lambda_{asm} \qquad \lambda_{asm} = \lambda'_m \sin\theta_r$$

$$\lambda_{bs} = L_{bsas} i_{as} + L_{bsbs} i_{bs} + \lambda_{bsm} \qquad \lambda_{bsm} = -\lambda'_m \cos\theta_r$$

  - λ'_m is the amplitude of the flux linkages established by the permanent magnet as viewed from the stator phase windings.

– In other words, the magnitude of λ'_m is proportional to the magnitude of the open-circuit sinusoidal voltage induced in each stator phase winding. Visualize the permanent-magnet rotor as a rotor with a winding carrying a constant current and in such a position to cause the N and S poles to appear as shown in the diagram.
– Assume that the air gap of the permanent-magnet synchronous machine is uniform. This may be an oversimplification.
– With this assumption of uniform air gap, the mutual inductance between the *as* and bs windings is zero.
– Since the windings are identical, the self-inductances L_{asas} and L_{bsbs} are equal and denoted as L_{ss}.

- The self-inductance is made up of a leakage and a magnetizing inductance:

$$L_{ss} = L_{\ell s} + L_{ms}$$

- The machine is designed to minimize the leakage inductance; it generally makes up approximately 10% of L_{ss}. The magnetizing inductances may be expressed in terms of turns and reluctance:

$$L_{ms} = \frac{N_s^2}{\Re_m}$$

- The magnetizing reluctance $\Re_m$ is an equivalent reluctance due to the stator steel, the permanent magnet, and the air gap. Assume that it is independent of rotor position θ_r.

- <u>Torque</u>
  - An expression for the electromagnetic torque may be obtained from:

$$T_e\left(\vec{i},\theta\right) = \frac{\partial W_c\left(\vec{i},\theta\right)}{\partial \theta}$$

  - The co-energy W_c may be expressed as:

$$W_c = \frac{1}{2}L_{ss}\left(i_{as}^2 + i_{bs}^2\right) + \lambda_m^{'} i_{as}\sin\theta_r - \lambda_m^{'} i_{bs}\cos\theta_r + W_{pm}$$

  - Where W_{pm} relates to the energy associated with the permanent magnet, which is constant for the device under consideration.

- Taking the partial derivative with respect to θ_r yields:

$$T_e\left(\vec{i},\theta\right) = \frac{\partial W_c\left(\vec{i},\theta\right)}{\partial \theta} = \frac{P}{2}\lambda'_m\left(i_{as}\cos\theta_r + i_{bs}\sin\theta_r\right)$$

- This expression is positive for motor action. The torque and speed may be related as:

$$T_e = J\left(\frac{2}{P}\right)\frac{d\omega_r}{dt} + B_m\left(\frac{2}{P}\right)\omega_r + T_L$$

- ## Machine Equations in the Rotor Reference Frame
  - A change of variables is helpful in the analysis of the permanent-magnet synchronous machine.
  - The objective of a change of variables is to transform all machine variables to a common reference frame, thereby eliminating θ_r from the inductance equations.
  - But the inductance L_{ss} is not a function of θ_r. However the flux linkages L_{asm} and L_{bsm} are functions of θ_r. In other words, the magnetic system of the permanent magnet is viewed as a time-varying flux linkage by the stator windings.
  - For a two-phase system the transformation is:

$$\begin{bmatrix} f_{qs}^{r} \\ f_{ds}^{r} \end{bmatrix} = \begin{bmatrix} \cos\theta_r & \sin\theta_r \\ \sin\theta_r & -\cos\theta_r \end{bmatrix} \begin{bmatrix} f_{as} \\ f_{bs} \end{bmatrix}$$

$$f_{qds}^{r} = K_s^r f_{abs}$$

$$f_{abs} = \left[K_s^r \right]^{-1} f_{qds}^{r} = K_s^r f_{qds}^{r}$$

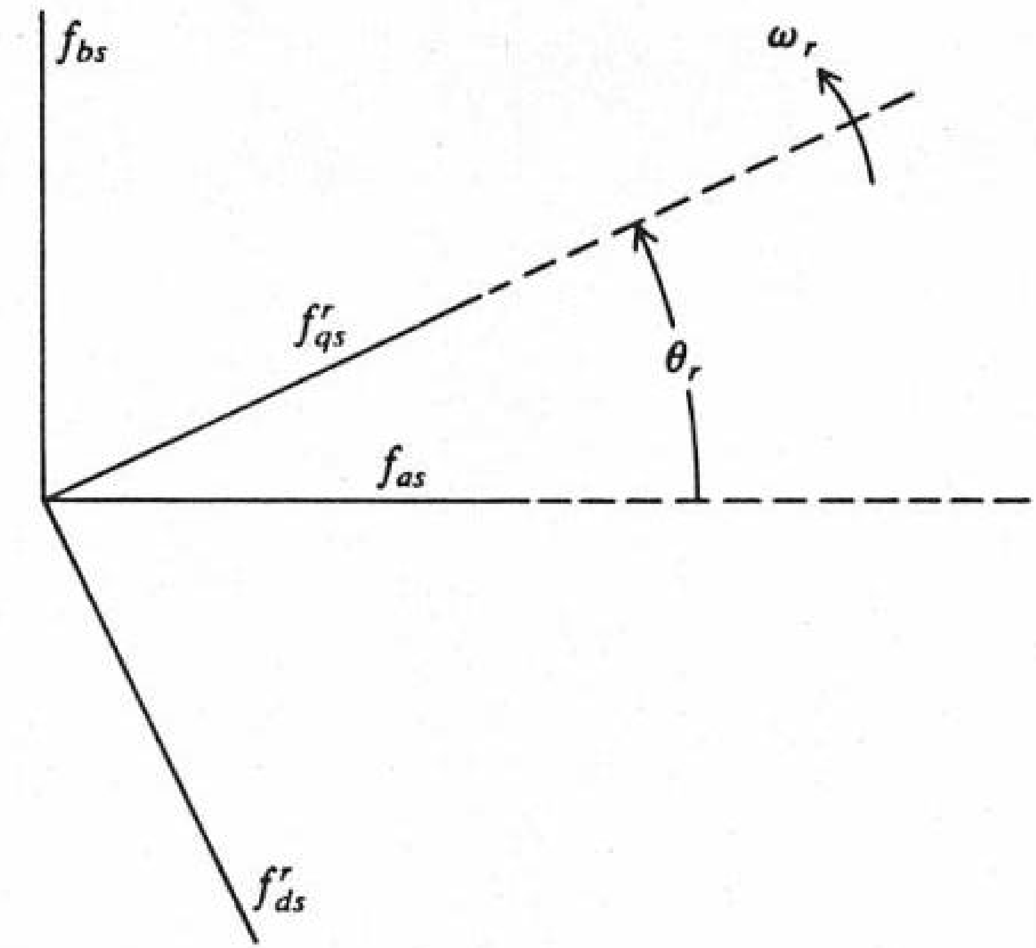
– *f* can represent either voltage, current, or flux linkage and θ_r is the rotor displacement.

– The *s* subscript denotes stator variables and the *r* superscript indicates that the transformation is to a reference frame fixed in the rotor.

– Shown in the figure is a trigonometric interpretation of the change of stator variables.

- The direction of f_{as} and f_{bs} variables is the positive direction of the magnetic axes of the *as* and *bs* windings, respectively.
- The f^r_{qs} and f^r_{ds} variables are associated with fictitious windings the positive magnetic axes of which are in the same direction as the direction of f^r_{qs} and f^r_{ds}.
- The *s* subscript denotes association with the stator variables.
- The superscript *r* indicates that the transformation is to the rotor reference frame, which is the only reference frame used in the analysis of synchronous machines.

Trigonometric Interpretation of the Change of Stator Variables



– Transformation

$$v_{as} = r_s i_{as} + \frac{d\lambda_{as}}{dt}$$

$$v_{bs} = r_s i_{bs} + \frac{d\lambda_{bs}}{dt}$$

Matrix Form

$$\begin{bmatrix} v_{as} \\ v_{bs} \end{bmatrix} = \begin{bmatrix} r_s & 0 \\ 0 & r_s \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \end{bmatrix}$$

$$v_{abs} = r_s i_{abs} + p\lambda_{abs}$$

$$f_{abs} = \begin{bmatrix} f_{as} \\ f_{bs} \end{bmatrix} \qquad r_s = \begin{bmatrix} r_s & 0 \\ 0 & r_s \end{bmatrix}$$

– Transformation

$$\lambda_{as} = L_{asas} i_{as} + L_{asbs} i_{bs} + \lambda_{asm}$$

$$\lambda_{bs} = L_{bsas} i_{as} + L_{bsbs} i_{bs} + \lambda_{bsm}$$

$$\lambda_{abs} = L_s i_{abs} + \lambda'_m$$

$$\begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \end{bmatrix} = \begin{bmatrix} L_{asas} & L_{asbs} \\ L_{bsas} & L_{bsbs} \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \end{bmatrix} + \lambda'_m \begin{bmatrix} \sin\theta_r \\ -\cos\theta_r \end{bmatrix}$$

$$\lambda'_m = \begin{bmatrix} \lambda_{asm} \\ \lambda_{bsm} \end{bmatrix} = \lambda'_m \begin{bmatrix} \sin\theta_r \\ \cos\theta_r \end{bmatrix}$$

$$\begin{bmatrix} L_{asas} & L_{asbs} \\ L_{bsas} & L_{bsbs} \end{bmatrix} = \begin{bmatrix} L_{ss} & 0 \\ 0 & L_{ss} \end{bmatrix} = L_s$$

– Transformation

$$f_{qds}^{r} = K_s^r f_{abs}$$

$$\begin{bmatrix} f_{qs}^{r} \\ f_{ds}^{r} \end{bmatrix} = \begin{bmatrix} \cos\theta_r & \sin\theta_r \\ \sin\theta_r & -\cos\theta_r \end{bmatrix} \begin{bmatrix} f_{as} \\ f_{bs} \end{bmatrix}$$

$$f_{abs} = \left[K_s^r\right]^{-1} f_{qds}^{r} = K_s^r f_{qds}^{r}$$

$$f_{abs} = \left[K_s^r\right]^{-1} f_{qds}^{r} \quad \xrightarrow{\text{substitute}} \quad v_{abs} = r_s i_{abs} + p\lambda_{abs}$$

$$\left(K_s^r\right)^{-1} v_{qds}^{r} = r_s \left(K_s^r\right)^{-1} i_{qds}^{r} + p\left[\left(K_s^r\right)^{-1} \lambda_{qds}^{r}\right]$$

Multiply by K_s^r

$$\left(K_s^r\right)\left(K_s^r\right)^{-1} v_{qds}^{r} = \left(K_s^r\right) r_s \left(K_s^r\right)^{-1} i_{qds}^{r} + \left(K_s^r\right) p\left[\left(K_s^r\right)^{-1} \lambda_{qds}^{r}\right]$$

$$v_{qds}^{r} = r_s i_{qds}^{r} + \left(K_s^r\right) p\left[\left(K_s^r\right)^{-1} \lambda_{qds}^{r}\right]$$

– Transformation

$$v_{qds}^{r} = r_s i_{qds}^{r} + \left(K_s^r\right) p\left[\left(K_s^r\right)^{-1} \lambda_{qds}^{r}\right]$$

$$v_{qds}^{r} = r_s i_{qds}^{r} + \left(K_s^r\right)\left[p\left(K_s^r\right)^{-1}\right]\lambda_{qds}^{r} + \left(K_s^r\right)\left(K_s^r\right)^{-1} p\lambda_{qds}^{r}$$

$$= r_s i_{qds}^{r} + \omega_r \lambda_{dqs}^{r} + p\lambda_{qds}^{r}$$

$$\lambda_{dqs}^{r} = \begin{bmatrix} \lambda_{ds}^{r} \\ -\lambda_{qs}^{r} \end{bmatrix}$$

For a magnetically linear system: $\lambda_{abs} = L_s i_{abs} + \lambda_m^{'}$

$$\left(K_s^r\right)^{-1} \lambda_{qds}^{r} = L_s \left(K_s^r\right)^{-1} i_{qds}^{r} + \lambda_m^{'}$$

– Transformation

$$K_s^r \left(K_s^r\right)^{-1} \lambda_{qds}^r = K_s^r L_s \left(K_s^r\right)^{-1} i_{qds}^r + K_s^r \lambda_m'$$

$$\lambda_{qds}^r = L_s i_{qds}^r + K_s^r \lambda_m'$$

$$\lambda_{qds}^r = \begin{bmatrix} L_{\ell s} + L_{ms} & 0 \\ 0 & L_{\ell s} + L_{ms} \end{bmatrix} \begin{bmatrix} i_{qs}^r \\ i_{ds}^r \end{bmatrix} + \lambda_m^{'r} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

– In our new system of variables, the flux linkage created by the permanent magnet appears constant. Hence, our fictitious circuits are fixed relative to the permanent magnet and, therefore, fixed in the rotor. We have accomplished the goal of eliminating flux linkages which vary with θ_r.

– In expanded form, the voltage equations are:

$$v_{qs}^{r} = r_s i_{qs}^{r} + \omega_r \lambda_{ds}^{r} + p\lambda_{qs}^{r}$$

$$v_{ds}^{r} = r_s i_{ds}^{r} - \omega_r \lambda_{qs}^{r} + p\lambda_{ds}^{r}$$

– where

$$\lambda_{qs}^{r} = L_{ss} i_{qs}^{r} = \left(L_{\ell s} + L_{ms}\right) i_{qs}^{r}$$

$$\lambda_{ds}^{r} = L_{ss} i_{ds}^{r} + \lambda_{m}^{'r} = \left(L_{\ell s} + L_{ms}\right) i_{ds}^{r} + \lambda_{m}^{'r}$$

– Substitution

$$v_{qs}^{r} = (r_s + pL_{ss})i_{qs}^{r} + \omega_r L_{ss} i_{ds}^{r} + \omega_r \lambda_m^{'r}$$

$$v_{ds}^{r} = (r_s + pL_{ss})i_{ds}^{r} - \omega_r L_{ss} i_{qs}^{r}$$

$$\begin{bmatrix} v_{qs}^{r} \\ v_{ds}^{r} \end{bmatrix} = \begin{bmatrix} r_s + pL_{ss} & \omega_r L_{ss} \\ -\omega_r L_{ss} & r_s + pL_{ss} \end{bmatrix} \begin{bmatrix} i_{qs}^{r} \\ i_{ds}^{r} \end{bmatrix} + \begin{bmatrix} \omega_r \lambda_m^{'r} \\ 0 \end{bmatrix}$$

– The electromagnetic torque is obtained by expressing i_{as} and i_{bs} in terms of i^{r}_{qs} and i^{r}_{ds}. In particular:

$$T_e = \frac{P}{2} \lambda_m^{'r} i_{qs}^{r}$$

(positive for motor action)

- Time-Domain Block Diagrams and State Equations
  - Nonlinear System Equations
  - Voltage Equations

$$\begin{bmatrix} v_{qs}^r \\ v_{ds}^r \end{bmatrix} = \begin{bmatrix} r_s + pL_{ss} & \omega_r L_{ss} \\ -\omega_r L_{ss} & r_s + pL_{ss} \end{bmatrix} \begin{bmatrix} i_{qs}^r \\ i_{ds}^r \end{bmatrix} + \begin{bmatrix} \omega_r \lambda_m^{'r} \\ 0 \end{bmatrix}$$

  - Relationship between torque and rotor speed

$$T_e = J\left(\frac{2}{P}\right)\frac{d\omega_r}{dt} + B_m\left(\frac{2}{P}\right)\omega_r + T_L$$

– Rewriting the equations:

$$v_{qs}^{r} = r_s\left(1+\tau_s p\right)i_{qs}^{r} + r_s\tau_s\omega_r i_{ds}^{r} + \lambda_m^{\prime r}\omega_r$$

$$v_{ds}^{r} = r_s\left(1+\tau_s p\right)i_{ds}^{r} - r_s\tau_s\omega_r i_{qs}^{r}$$

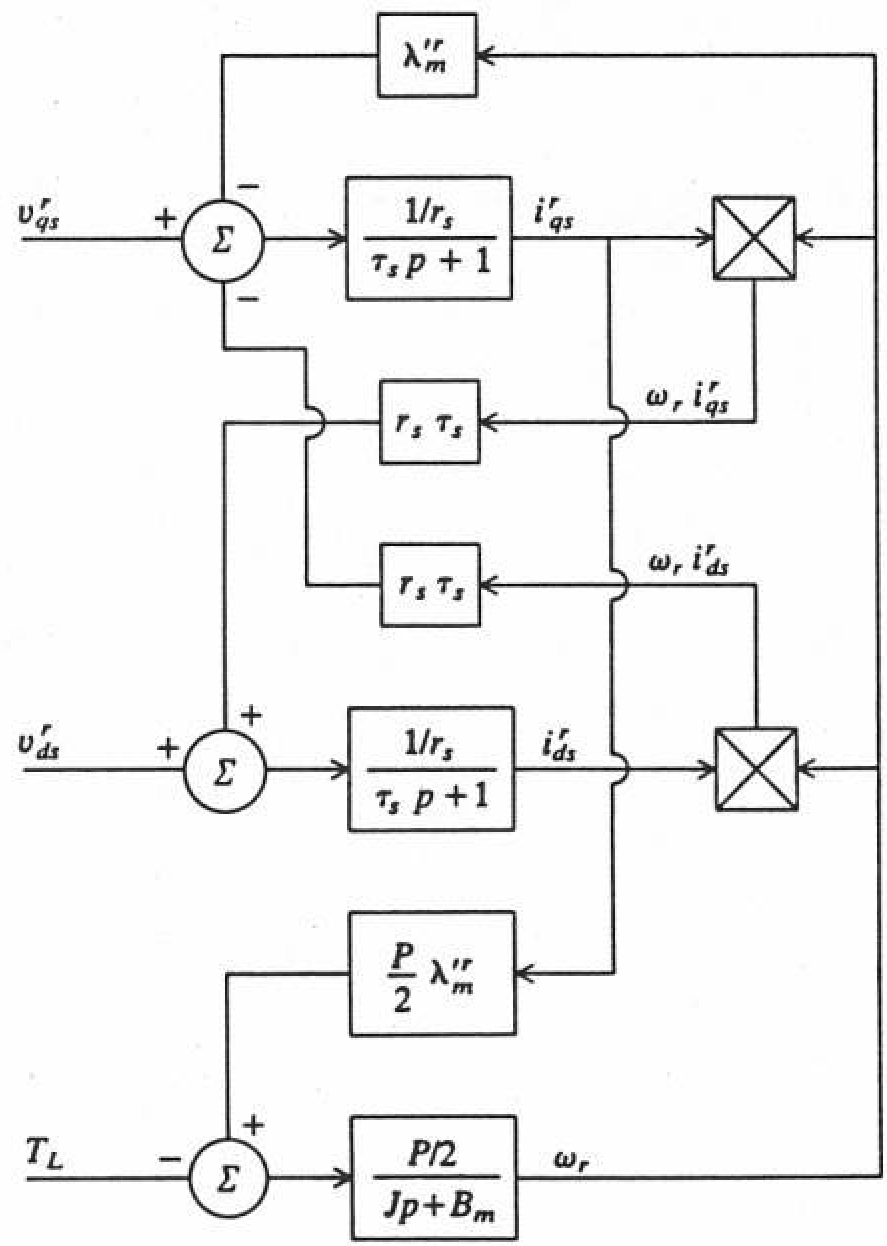
$$\tau_s = \frac{L_{ss}}{r_s}$$

$$T_e - T_L = \frac{2}{P}\left(B_m + Jp\right)\omega_r$$

$$T_e = \frac{P}{2}\lambda_m^{\prime r} i_{qs}^{r}$$

$$\boxed{\begin{aligned} i_{qs}^{r} &= \frac{1/r_s}{\tau_s p + 1}\left(v_{qs}^{r} - r_s\tau_s\omega_r i_{ds}^{r} - \lambda_m^{\prime r}\omega_r\right) \\ i_{ds}^{r} &= \frac{1/r_s}{\tau_s p + 1}\left(v_{ds}^{r} + r_s\tau_s\omega_r i_{qs}^{r}\right) \\ \omega_r &= \frac{P/2}{Jp + B_m}\left(T_e - T_L\right) \end{aligned}}$$

Time-Domain Block Diagram of a Brushless DC Machine

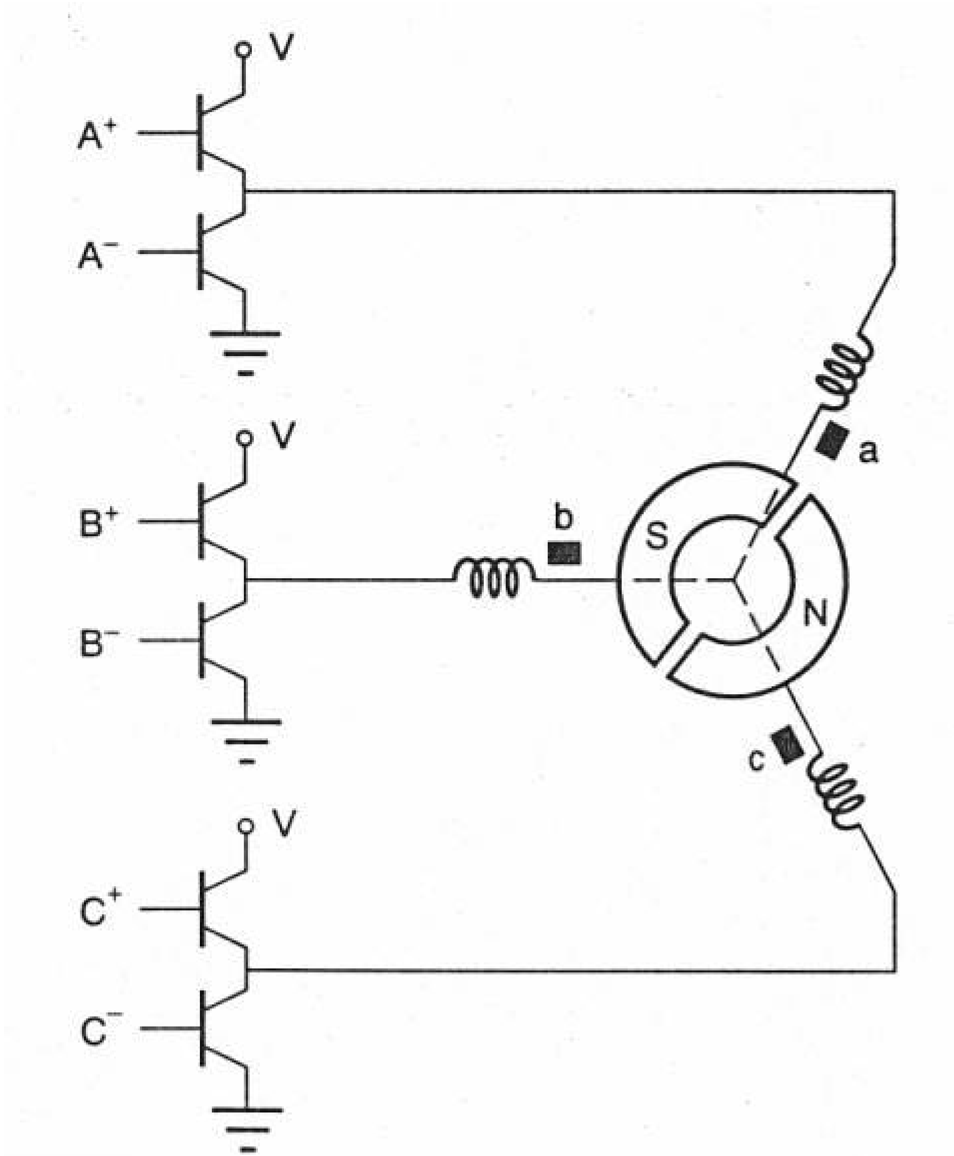


– <u>State Variables</u>

– The state variables are the stator currents i^r_{qs} and i^r_{ds}, the rotor speed ω_r, and the rotor position θ_r. Here we will omit θ_r since it is considered a state variable only when shaft position is a controlled variable. Also it can be established from ω_r.

– In Matrix Form the state-variable equations are:

$$\frac{d}{dt}\begin{bmatrix} i^r_{qs} \\ i^r_{ds} \\ \omega_r \end{bmatrix} = \begin{bmatrix} -\dfrac{r_s}{L_{ss}} & 0 & -\dfrac{\lambda'^r_m}{L_{ss}} \\ 0 & -\dfrac{r_s}{L_{ss}} & 0 \\ \left(\dfrac{P}{2}\right)^2 \dfrac{\lambda'^r_m}{J} & 0 & -\dfrac{B_m}{J} \end{bmatrix} \begin{bmatrix} i^r_{qs} \\ i^r_{ds} \\ \omega_r \end{bmatrix} + \begin{bmatrix} -\omega_r i^r_{ds} \\ \omega_r i^r_{qs} \\ 0 \end{bmatrix} + \begin{bmatrix} \dfrac{1}{L_{ss}} & 0 & 0 \\ 0 & \dfrac{1}{L_{ss}} & 0 \\ 0 & 0 & -\dfrac{P}{2}\dfrac{1}{J} \end{bmatrix} \begin{bmatrix} v^r_{qs} \\ v^r_{ds} \\ T_L \end{bmatrix}$$



V
A+
A−
V
B+
B−
V
C+
C−
a
b
c
S
N