Learning Through Hiring: Labor Mobility as a Channel for Endogenous Growth

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Abstract

Firm-level analysis suggests that labor mobility is one of the channels through which productive knowledge can spillover between firms. However, the macro-level implications of this channel for both aggregate growth and the distribution of firms has received less attention. This paper embeds a search-and-matching labor model within an endogenous growth framework so that the job-to-job transition of workers act as a channel through which knowledge diffuses between firms, generating long-run growth. Within the model, the rate at which firms are exposed to new knowledge and the distribution of new knowledge they learn from are both endogenously determined by the labor market. This model is calibrated to match firm-level and aggregate-data facts for New Zealand. Counterfactual analysis suggests that knowledge spillover through the labor mobility channel has significant effects on the long-run growth of an economy and on the dispersion of firm-level productivity. However, it appears to have little impact on the distribution of firm size within the economy.

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Disclaimer

The results in this paper are not official statistics, they have been created for research purposes from the Integrated Data Infrastructure (IDI), managed by Statistics New Zealand.

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Access to the anonymised data used in this study was provided by Statistics NZ in accordance with security and confidentiality provisions of the Statistics Act 1975. Only people authorised by the Statistics Act 1975 are allowed to see data about a particular person, household, business, or organisation, and the results in this paper have been confidentialised to protect these groups from identification.

Careful consideration has been given to the privacy, security, and confidentiality issues associated with using administrative and survey data in the IDI. Further detail can be found in the Privacy impact assessment for the Integrated Data Infrastructure available from www.stats.govt.nz.

The results are based in part on tax data supplied by Inland Revenue to Statistics NZ under the Tax Administration Act 1994. This tax data must be used only for statistical purposes, and no individual information may be published or disclosed in any other form, or provided to Inland Revenue for administrative or regulatory purposes.

Any person who has had access to the unit record data has certified that they have been shown, have read, and have understood section 81 of the Tax Administration Act 1994, which relates to secrecy. Any discussion of data limitations or weaknesses is in the context of using the IDI for statistical purposes, and is not related to the data's ability to support Inland Revenue's core operational requirements.

1 Introduction

New knowledge is the key driver of long-run economic growth. According to Klette and Kortum (2004), the distribution of research and development is highly skewed with many firms not investing in research and development. However, the non-rival nature of ideas means that the development of new knowledge in one firm can have positive benefits for the wider economy when this knowledge spills over into other firms.[1]

Labor mobility has often been thought of as one of the main channels that facilitates the spillover of knowledge between firms. By working within a firm, workers are able to absorb some of the knowledge that influences the level of productivity of the firm. When the workers move to another firm, they are able to take some of the productive knowledge and adapt it into their new employer's business practices. What's more, the positive externality of this initial spillover of knowledge can have a multiplied effect when workers at the hiring firm learn some of this new knowledge, and further diffuse it through the economy when they move to new jobs.

According to the 2013 Business Operations Survey (BOS), a nationally representative survey of New Zealand firms, 52 percent of businesses who reported undertaking some form of innovation in the previous two years stated that new staff were an important source of ideas for the innovation.[2] There is also a growing empirical literature that uses matched employer-employee data to estimate the effects of knowledge spillover at the firm level. These spillovers of knowledge between firms have been observed both within a country by domestic firms (see Stoyanov and Zubanov, 2012, and Serafinelli, 2015 as examples), as well as internationally by multi-national enterprises (see Fosfuri et al., 1998, and Balsvik, 2011 as examples).

Despite the firm-level evidence that labor mobility is an important channel for the spillover of productive knowledge, standard macroeconomic models of endogenous growth tend to treat knowledge spillover as an random and exogenous process.[3] The aim of this paper is to develop a model in which the macroeconomic implications of productive knowledge spillover through the labor mobility channel can be analyzed within an endogenous framework.

This paper begins its analysis by looking at New Zealand firm-level panel data to examine what, if any, support there is for knowledge spillover through a labor mobility channel. The New Zealand data provides a wider coverage of firms than the data used in previous papers. The data encompassing private-for-profit firms in all measured sectors of the economy. The worker data also covers all tax-paying employees, allowing the regression analysis to intro-

[1] see Griliches (1992) for a review of R&D spillovers.

[2] The types of innovation asked about were: (i) product innovation: "did this business introduce onto the market any new or significantly improved goods or services?"; (ii) process innovation: "did this business implement any new or significantly improved operational processes (i.e. methods of producing or distributing goods or services)?"; (iii) organizational innovation: "did this business implement any new or significantly improved organizational/managerial processes (i.e. significant changes in this businesses strategies, structures or routines)?"; and (iv) marketing innovation: "did this business implement any new or significantly improved sales or marketing methods which were intended to increase the appeal of goods or services for specific market segments or to gain entry to new markets?"

[3] See Luttmer (2012) and Luttmer (2015) as recent examples.

duce some control for knowledge spillover from sources for which we cannot measure the productivity (such as from non-profit firms).

While causal relationships cannot be inferred from the data, it is possible to compare the sign and size of the regression coefficients to check if they are at least consistent with the predictions implied by the spillover of knowledge through the labor mobility channel. The regression results show that a firm's productivity growth is significantly correlated with the productivity of the firms from which new workers are hired away from. More importantly, raising the average productivity of the set of more productive firms that workers are sourced from has a larger benefit for productivity growth at the hiring firm than raising the average productivity of the set of less productive firms that workers are sourced from.

This pattern matches the predictions implied by knowledge spillover through the labor mobility channel. According to the theory, hiring firm should adopt new knowledge if it is superior to their current knowledge. More productive firms should have, on average, more superior knowledge than less productive firms. Therefore, for a firm that hires new workers that have previous experience at other, more productive, firms, improving the productivity of the firms these workers are sources from should have result in greater knowledge spillover into the hiring firm. While for a firm that hires workers who have previous at less productive firms, improving the productivity of the firms these workers are sources from should have only a small, if any, effect on productivity growth. This is because any knowledge from the less productive firms workers are sourced from should still be, on average, inferior to the knowledge at the hiring firm, and therefore unlikely to be adopted at the hiring firm.

Further investigation suggests that the premium in productivity growth associated with hiring from more productive firms is related to knowledge regarding capital intensity. When using the capital-labor ratio instead of labor productivity, the regression results suggest firms that hire workers from more capital intensive firms subsequently raise their own capital intensity. While firms hiring workers from less capital intensive firms do not lower their capital intensity.[4]

Motivated by these empirical findings, this paper develops an endogenous growth model in which job-to-job transitions act as the channel through which knowledge diffuses between firms. Within the model, a search-and-matching labor market matches large heterogeneous firms with heterogeneous workers, some of whom are able to facilitate the spillover of knowledge from their previous employer to the hiring firm. This search-and-matching model is then encompassed within an endogenous growth framework to describe how the distribution of firms and workers evolves over time given the search choices made by firms and workers on both sides of the matching labor market.

The novel feature of this framework relative to traditional endogenous growth models is that the process by which knowledge spills over between firms is endogenized within the labor mobility channel of knowledge spillover. Firms optimize the rate at which they learn new productivity ideas through the labor mobility spillover channel by their choice of how

[4]See Kirker and Sanderson (2017) for a more detail analysis which also tests the predictions of knowledge spillover through labor mobility in terms of firm and worker characteristics.

many job vacancies to post, and the distribution of new knowledge each firm is exposed to is endogenously determined by the search intensity of workers on the other side of the labor market.

The model is calibrated to several important macroeconomic moments in the data as well as some of the empirical findings at the firm level. This allows us to assess some of the macro-level implications of knowledge spillover through the labor mobility channel. A counter-factual simulation that shuts down the knowledge spillover effect of the labor mobility channel reveals that knowledge spillover contributes around 0.7 percentage point to aggregate growth. In addition to affecting the aggregate growth rate, the knowledge spillover effect also makes the distribution of firm productivity somewhat more symmetrical by allowing smaller firms to improve productivity at a higher rate. Interestingly, the effect of knowledge spillover through the labor mobility channel does not have a significant effect on the distribution of firm size (number of employees) in the economy.

Because firms must split the benefits of any knowledge spillover with the workers they hire in order to attract them away from their current firm, the private benefit to seeking out knowledge spillovers is generally lower than the social benefit. As a result, the market-based equilibrium is inefficient for a social planner's point of view. Simulating the model under the socially optimal choices of search effort reveals that aggregate growth is not significantly higher than the market based equilibrium. Despite the higher search rates, congestion in the labor market also increases, limiting the scope for improving in aggregate productivity growth. However, the distribution of relative firm productivity is more skewed towards the right under the social planner indicating that there would be a larger share of firms with (relatively) high levels of productivity.

The remainder of the paper is structured as follows. Section 2 discusses in detail how this paper fits into the existing literature. Section 3 discusses the empirical analysis used to motivate the development of the model. Section 4 presents the structure of the theoretical model. Section 5 details the results of analysis of the model. Section 6 summarizes and concludes.

2 Literature Review

Since seminal works by the likes of Romer (1986) and Lucas (1988), endogenous growth models have often relied on the spillover of productive knowledge from more to less productive firms as a mechanism for generating long-run economic growth. Traditionally, these models have assumed that firms sample new ideas at an exogenous rate, and the distribution that firms sample from is identical to the distribution of firm productivity that exists in the economy (see Luttmer, 2015 as an example). Recently, some endogenous growth models have made progress in endogenizing the rate at which firms learn. Models by Perla and Tonetti (2014) and Lucas and Moll (2014) feature agents choosing the rate at which they receive new ideas by optimally choosing the amount of time they spend searching, trading off production today for search effort in the hopes of being able to produce at higher productivity levels in the future. However, in both these models, all firms that search are assumed to have equal probability of being exposed to the productive knowledge of all firms producing in the economy.

The model proposed in this paper looks to provide further structure to the learning process used in the previous literature by using a search-and-matching labor market to endogenize both the choice of search effort by firms and the distribution of productive knowledge each firm is exposed to, as well as taking the model to the data. As a result, firms optimally choose their vacancy postings rate not only to gain more labor for the production process, but to also acquire new knowledge that workers from more productive firms may bring with them.The distribution of knowledge firms are exposed to is endogenously determined on the other side of the labor market where the choice search effort by workers, and the willingness of workers to accept jobs at any particular firm, determines the probability of a searching firms being exposed to the various levels of productive knowledge.

Some previous papers in the literature have used an overlapping generations framework to study knowledge spillover via labor mobility in a dynamic context. For example Dasgupta (2011) and Monge-Naranjo (2012) both develop overlapping generations models in which agents learn on the job and use their acquired knowledge to start their own firms later in life. However, within these types of model, knowledge spillover is only from incumbent firms to new firms, not other incumbent firms. The incumbent firms in the model either do not improve their own productivity, or they shut down. These models are ill-suited for analyzing labor mobility's role in diffusing knowledge between incumbent firms, which, given the majority of job-to-job worker transitions are between incumbent firms, is the focus of this paper.

The firm-level analysis discussed later in this paper relates to a growing body of empirical papers in the literature that use linked employer-employee data to examine the relationship between firm-level productivity growth and the benefit of new workers (see Parrotta and Pozzoli, 2012, Serafinelli, 2015, and Stockinger and Wolf, 2016 as examples). The modeling approach I take in this paper is most closely related to the that of Stoyanov and Zubanov (2012). Stoyanov and Zubanov (2012) measure a firm's exposure to new productive knowledge through the notion of a 'productivity gap' — the difference between the hiring firm's productivity and the productivity of the new worker's previous employer. Their analysis shows that for Danish manufacturing firms, hiring new workers from more productive firms benefits the hiring firm's productivity, while hiring new workers from less productive firms does not have a significant effect on the hiring firm's productivity. These correlations match the predictions of the knowledge spillover hypothesis. Further support for the knowledge spillover channel comes from the finding that larger spillover coefficients are associated with worker characteristics expected to enhance workers' ability to transfer knowledge between firms (such as worker tenure and skill).

The analysis in this paper extends that of Stoyanov and Zubanov (2012) in a few key ways. Empirically, the New Zealand data set provides a broader coverage of industries, capturing all industries in the measured economy. In addition, employment data is observed monthly allowing it to capture all employment spells, and provide more accurate measures of the length of employment spells. On the modeling side, the main innovation is to introduce a proxy for the firm's exposure to knowledge from sources for which productivity cannot be measured, such as hiring from non-market firms. A firms decision to hire workers from sources for which productivity can be observed (and hence the productivity gap constructed) is likely to be correlated with the firms decision to hire workers from other sources for which the productivity gap cannot be measured. Therefore, failing to control for the knowledge spillover from these other sources could bias the estimate of the marginal effect of new productivity knowledge from the firms we can observe productivity data for.

3 Empirical Analysis

The following section uses individual firm and worker level data to investigate potential support for labor mobility to act as a channel for the spillover of productive knowledge between firms. The analysis is conducted using data for New Zealand businesses and workers taken from the Longitudinal Business Database (LBD) and Integrated Data Infrastructure (IDI) maintained by Statistics New Zealand, the national statistical agency. This section provides a relatively broad overview of the modeling approach and baseline results. For a more detailed discussion of the data, analysis, and additional extensions and robustness checks, the reader is referred to Kirker and Sanderson (2017).

3.1 Model

The general framework used to model the relationship between a firm's productivity growth and its exposure to external productive knowledge through the labor mobility channel is adapted from the approach taken by Stoyanov and Zubanov (2012). A proxy for the firm's exposure to outside knowledge is constructed based, in part, on the gap between the hiring firm's productivity and the productivity of the previous employer of new employees. A dynamic panel model is then used to relate a firm's productivity growth to this measure of knowledge exposure, while attempting to control for other productivity dynamics at the firm level. The model is estimated on linked employer-employee data for the population of private-for-profit firms in the measured sector of the New Zealand economy.[5]

3.1.1 Measure of a Firm's Exposure to Outside Knowledge

A firm's exposure to outside knowledge is assumed to occur through the process of hiring of new workers who have experience working at other firms. The baseline specification used to

[5]Private-for-profit businesses broadly covers private producer enterprises, central and local government enterprises, and private financial institutions. For more details see Fabling and Sanderson (2016). The measured sector is defined by Statistics New Zealand as "industries that mainly contain enterprises that are market producers. This means they sell their products for economically significant prices that affect the quantity that consumers are willing to purchase".

measure the hiring firm's (firm i's) exposure to outside knowledge is given by

$$\begin{aligned}
\text{Exposure}_{i,t} = {} & \beta_M \frac{\sum_{n \in \mathcal{N}_{i,t-1}} \mathbb{D}_n \left[\ln(A_{n,\tau-\delta}) - \ln(A_{i,t-1-\delta})\right]}{H_{i,\text{more prod},t-1}} \frac{H_{i,\text{more prod},t-1}}{L_{i,t-1}} \\
& + \beta_L \frac{\sum_{n \in \mathcal{N}_{i,t-1}} (1-\mathbb{D}_n) \left[\ln(A_{n,\tau-\delta}) - \ln(A_{i,t-1-\delta})\right]}{H_{i,\text{less prod},t-1}} \frac{H_{i,\text{less prod},t-1}}{L_{i,t-1}} \\
& + \sum_{s \in \mathcal{S}_{i,t-1}} \lambda_s \frac{H_{i,s,t-1}}{L_{i,t-1}}
\end{aligned} \tag{1}$$

where $\mathcal{N}_{i,t-1}$ is the set of workers hired by firm i in period $t-1$ for which we are able to observe the productivity of the previous employer, $\mathbb{D}_n$ is a dummy variable that takes on the value of 1 if worker n comes from a firm that is more productive than the hiring firm (firm i), and zero otherwise, $A_{i,t}$ is the hiring firm's productivity in period t, $L_{i,t}$ is the number of workers employed at firm i, and $H_{i,s,t}$ is the number of hires made by firm i for source $s \in \mathcal{S}_{i,t-1}$, where $\mathcal{S}_{i,t-1}$ is the set of all possible sources for new workers (detailed shortly).

The first two term in the right-hand side in equation 1 are what Stoyanov and Zubanov (2012) use to measure knowledge spillover in their model. They refer to these terms as 'productivity gaps', a term that will also be adopted throughout this paper. These two terms represent the average productivity difference between the hiring firm and the previous employer of new employees (separated into previous employers that are more and less productive), multiplied by the share of new employees in the firm for hires made from more and less productive firms.

Because firms are likely to only implement new production ideas and practices that are superior to their current methods, the knowledge spillover through the labor mobility channel predicts we should see a different effect on productivity growth when the worker has knowledge from more productive firms compared to the case when the worker comes from a less productive firm. Disaggregating the productivity gaps into separate productivity gaps from more and less productive firms allows us to examine the difference in these effects by comparing the parameters β_M and β_L.[6]

In the data it is not possible to measure the productivity of the previous employer for every new worker. For example, some new hires may have never been previously employed, or a new hire may have previously worked for a firm for which productivity cannot be measured in the data (e.g. a non-profit firm). As a result, the productivity gap that the econometrician can observe represents only a fraction of the firm's potential exposure to outside knowledge coming from the labor mobility channel. Without controlling for the spillover of knowledge from these other sources, the estimates of β_M and β_L can potentially be biased.

Therefore, the productivity gaps in equation 1 are further augmented to include the share of labor hired from all the various sources within the data. More formally, the final term

[6]Note the in the productivity gap related to hires from *less* productive firms, the term for the average productivity difference is negative by construction (as $A_n < A_i$). Hence a positive value for the coefficient β_L implies that hiring from less productive firms hurts the hiring firm's productivity.

in equation 1 is defined as follows: $\mathcal{S}_{i,t-1}$ denote the set of sources from which the hiring firm obtains its new workers. $H_{i,s,t-1}/L_{i,t-1}$ represents the fraction of workers hired from source $s \in \mathcal{S}_{i,t-1}$, which measures the hiring intensity from source s. For sources for which it is not possible to measure the productivity gap, the parameter λ_s represents the average knowledge spillover from source s. For hires from sources for which it is possible to measure the productivity gap, the separate productivity gaps and hiring intensity terms allow for the distinction between the effects of the intensive (productivity gap) and extensive (hiring intensity) margins of knowledge exposure.[7]

3.1.2 The Relationship Between Productivity Growth and Knowledge Exposure

Productivity growth at the hiring firm is related to the firm's exposure to outside knowledge through labor mobility using a dynamic panel model, expressed in first-differences:

$$\begin{aligned}\Delta \ln A_{i,j,t} \quad = \quad & \text{Exposure}_{i,t} + \gamma \Delta Q_{i,t} + \delta \Delta \text{ExTurn}_{i,t} + \sum_{l=1}^{L} \alpha_{A,l} \Delta \ln A_{i,j,t-l} \\ & + \theta_{j,t} + \varepsilon_{i,t} \qquad (2)\end{aligned}$$

where $\Delta \ln A_{i,j,t}$ is the change in log productivity for firm i, in industry j, at time t, $\text{Exposure}_{i,t}$ is the exposure to new knowledge as defined previously, $\Delta Q_{i,t}$ is the change in the average (measurable) worker quality in the firm, $\text{ExTurn}_{i,t}$ is a measure of the excess turnover in the firm, $\sum_{l=1}^{L} \beta_{A,l} \Delta \ln A_{i,j,t-l}$ is a series of lagged auto-regressive terms, $\theta_{j,t}$ is an industry-year fixed effect, and $\varepsilon_{i,t}$ is the regression residual.

In equation 2, the exposure to outside knowledge through labor mobility is implicitly assumed to change the stock of productive knowledge used by the firm in production, and hence influence the productivity growth at a firm. The remaining terms in equation 2 are controls for other factors that are likely to have a significant influence on the firm's productivity growth.

Typically when estimating either labor productivity, or multi-factor productivity (MFP) for a firm, only the quantity of labor is used as the quality of the labor is often unmeasured. As a result, fluctuations in the quality of the workers will appear as fluctuations in the firm's productivity level. In the model, $\Delta Q_{i,t}$ is included as a control for the change in worker quality.

[7] For the empirical analysis to follow, the following sources of new hires ($\mathcal{S}$) are used: (i) new workers for whom we have not observed any work history (e.g. new college graduates, new immigrants, etc); (ii) hires from firms outside of the scope of productivity analysis (i.e. hires from non-market or not private-for-profit firms); (iii) hires from very small firms (for which the measure of productivity is likely to be particularly noisy); (iv) hires from private-for-profit firms that are within the scope of analysis but are missing some of the data required to compute productivity; and (v) hires from private-for-profit firms which are in scope and for which we have the data required to construct productivity gap measures (divided between firms that are more and less productive).

Excess labor turnover, a measure of the number of worker accessions and separations over and above that required to give effect to the firm's net change in employment, is included as a control because labor turnover can be disruptive to a firm when a significant amount of resources are needed to replace workers and/or train up new workers. Hence when labor turnover is high, these costs will lead to lower levels of output, and this will manifest in lower measured firm productivity.

The firm's past productivity performance can affect today's productivity through influencing the firm's investment and hiring decisions made both today and in the past. The inclusion of productivity lags in equation 2 aims to capture and control for the dynamics of firm productivity over time. Since we are not able to explicitly model all other potential sources of new knowledge, other factors that influence firm productivity are implicitly assumed to be time-invariant (and hence drop out of the model after first differencing), or random and independent of the other regressors (and captured by the random error term). Finally, the industry-year fixed effect soaks up any industry-wide trends in firm productivity that may remain in the data.

3.2 Data Sources

The data for this analysis is taken from two micro-level data sets from Statistics New Zealand. Information regarding firms comes from the Longitudinal Business Database (LBD) which combines a range of survey and administrative data sources for all economically significant businesses in New Zealand since 1999.[8] Information on workers comes from the Integrated Data Infrastructure (IDI). The IDI links employers to employees via Pay-As-You-Earn (PAYE) tax records for each job, and also contains a wide range of other survey and administrative data sources on individuals linked by anonymized individual identification numbers.

The regression analysis in this paper is conducted at the firm-year level. The sample period for the analysis in this paper is from 2001 to 2013. Below is a summary of the data and key variables used in the paper. For a more detailed discussion of the data, the reader is referred to Kirker and Sanderson (2017).

3.2.1 Firm Data

The unit of measurement for a firm is a Permanent Enterprise (PENT), as defined by Fabling (2011). The PENT identifier is based on the firm identifier within the LBD, and corrects for certain events such as the change in the legal status of a firm. The scope of this analysis is restricted to private-for-profit businesses within the measured sector identified by Statistics

[8]The term 'economically significant encompasses firms that meet *at least one* of the following criteria: (i) More than NZ$30,000 annual GST expenses or sales; (ii) more than three paid employees; (iii) in a GST exempt industry; (iv) part of a Business Register group of firms with ownership links; (v) a new GST registered firm. For more information on this, and a detailed discussion of the LBD, see Fabling and Sanderson (2016).

New Zealand. Only for these types of businesses are the revenue and cost data likely to provide a suitable measure of productivity.

For the measure of firm's multi-factor productivity, I use the trans-log based measure developed by Fabling and Maré (2015b). The measure of labor productivity used in this paper is estimated as real value-added (output less materials) per full time equivalent worker.[9]

3.2.2 Worker Data

Measures of individual labor supply are computed as full-time equivalent (FTE) workers using the approach of Fabling and Maré (2015a). The measure of average observed worker quality for each firm ($Q_{i,t}$) is computed as the FTE weighted average of individual worker quality, where the quality of each worker is constructed using the contribution of the worker fixed effect and the vector of worker-level observable characteristics from a two-way fixed effects regressions on wage data following the approach of Abowd et al. (1999).[10]

When workers transition between firms, it is assumed that any potential knowledge they bring is sourced from a single firm which will be referred to as their previous "main job". The main job prior to starting at a new firm is determined as follows. If the worker is employed at multiple firms in the three months prior to starting their new job, the previous main job is the one from which the worker received the highest real (CPI-adjusted) monthly income, for a full month's work, during this three month window.[11] In the cases where the new worker did not work at any job in the three months prior to starting at their new firm, the employment history of the worker is traced back in time to the last month in which they were employed for the full month. The main job is then assumed to be the job in this month that paid the highest.

Because worker movements are observed at a monthly frequency, but firm's productivity is only observed annually, when computing the productivity gap, if the worker leaves a firm in the first (last) six months of a financial year, their productivity knowledge is assumed to be the productivity of the firm during the previous (current) financial year. Similarly, if the worker arrives at a firm in the first (last) six months of the financial year, the hiring firm's productivity from the previous (current) financial year is used as the benchmark in

[9] According to Fabling and Maré (2015b), there are an average of 353,766 PENTs per year in the LBD with positive employment. Of these around 83 percent (292,978) are in the measured sector. Of the PENTs in the measured sector, around 32 percent are excluded because they lack the necessary production information to estimate productivity. Finally, I restrict the scope of analysis to only consider firms that employ an average of ten of more FTEs, as the productivity and labor input of small firms is likely to be imprecisely measured. For the construction of the productivity gap, the minimum firm size of the worker's previous employer is set to an average of five full time equivalent workers, as we are less concern with how precisely firm size in measured at these firms.

[10] Alternative measures of worker quality did not significantly affect the results. See Kirker and Sanderson (2017) for more details.

[11] The reason only months in which workers are employed for the full month are considered here is that the income for months in which the worker begins/ends a job are imprecisely measured due to any signing bonuses or final payouts.

computing the productivity gap.

3.3 Summary of Empirical Findings

Table 1 presents the regression results for the key parameters of the baseline model found when measuring firm productivity using: (i) trans-log multi-factor productivity; (ii) value-added per worker (labor productivity); and additionally (iii) the capital-to-labor ratio ($\ln(K_{i,j,t}/L_{i,j,t})$), a measure of input intensity. Some further high-level summary statistics and other results are presented in appendix C. For a more detailed examination of the results, along with a variety or robustness checks, the reader is referred to Kirker and Sanderson (2017).

Table 1: Baseline regression results for various productivity measures

	Firm productivity		**Capital-labor**
	Trans-log	**Value-added**	**ratio**
Productivity gap, hires from:			
More prod. Firms (β_M)	0.354***	0.480***	0.047***
	(0.068)	(0.098)	(0.017)
Less prod. Firms (β_L)	0.374***	0.153***	0.021
	(0.056)	(0.030)	(0.024)
Hire intensity:			
More prod. Firms (λ_M)	-0.037*	-0.200***	0.071**
	(0.021)	(0.057)	(0.031)
Less prod. Firms (λ_L)	0.004	-0.117***	-0.182***
	(0.019)	(0.027)	(0.035)
Parameter tests:			
$\Pr(\beta_M = \beta_L)$	0.808	0.001	0.369
$\Pr(\lambda_M = \lambda_L)$	0.174	0.237	0.000
Obs.	38037	37269	28260

Notes: For the firm productivity regressions, the dependent variable is the change in log productivity ($\Delta \ln(A_{i,j,t})$), and for the capital-labor ratio the dependent variable is the change in log capital-labor ratio ($\Delta \ln(K_{i,j,t}/L_{i,j,t})$). Standard errors are reported in parentheses. Each regression includes industry-year fixed effects, lagged productivity changes, hiring intensities from other sources, measured worker quality, and excess turnover as additional regressors. When included in the regression, $\Delta \ln(A_{i,j,t-1})$ and $\Delta \ln(K_{i,j,t-1}/L_{i,j,t-1})$ are instrumented for using $\Delta \ln(A_{i,j,t-2})$ and $\Delta \ln(K_{i,j,t-2}/L_{i,j,t-2})$ due to the presence of Nickell (1981) bias. The productivity lag length is chosen to minimize autocorrelation in the residual term. The regression standard errors are clustered at the firm level.
*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

In terms of the trans-log measure of multi-factor productivity, the results in table 1 show that the productivity gap is positively correlated with the subsequent productivity growth at the hiring firm. According to the coefficients, if a firm hires 10 percent of its workers from more (less) productive PFP firms, raising the average productivity of the PFP firms that workers are sourced from by one percent would be associated with an average 0.35 (0.37) percentage point increase in the hiring firm's subsequent productivity growth. Also, the effect of this productivity gain does not differ significantly between hires from more and less productive firms (p-value = 0.8). In terms of the hiring intensity (the extensive margin), there coefficients are not significantly different from zero.

These results for the trans-log measure of multi-factor productivity do not conform to the predictions implied by knowledge spillover through the labor mobility channel. Instead, the coefficient on the productivity gaps are more consistent with the predictions from an assortative matching based model. An assortative matching model would predict that a firm productivity should be correlated with the quality of workers. So hiring from more productive firms should, on average, raise the average quality of a firm's work force, and hiring from less productive firms should lower the average quality of a firm's work force, which will affect the hiring firm's productivity. Such an assortative matching could arise either as a result of more productive firms being better at screening workers of higher quality, or firm productivity correlates with the amount/quality of on-the-job training workers receive.[12]

Factors that affect a firm's multi-factor productivity will also affect the firm's measured labor productivity. Therefore, a priori, we would expect to see the same type of pattern when using value-added per worker as the measure of firm productivity. Column two of table 1 presents the relevant results. While we also see positive, and significant, coefficients on the productivity gaps when using value-added (labor productivity) to measure firm productivity, the coefficient on the productivity gap from more productive firms, $\beta_M = 0.48$, is significantly larger than the coefficient on the productivity gap from less productive firms, $\beta_L = 0.15$ (p-value = 0.001). This premium in the coefficient on the productivity gap is consistent with the predictions of the spillover of productive knowledge through the labor mobility channel. The fact that we see this difference in coefficients only for value-added and not multi-factor productivity data suggests that if the knowledge spillover channel is in operation, the knowledge would relate to knowledge regarding production technology (how to operate the firm at higher levels of input intensity per worker), rather than knowledge regarding multi-factor productivity (how to operate the current input ratios more efficiently).

This idea is further supported by the regression results using the capital-labor input ratio (column three of table 1). Only the coefficient on the productivity gap (capital-intensity gap) is significantly positive. This suggests that when firms hire from more capital intensive firms, raising the average capital intensity of the firms workers are sources form is associated with an increase capital intensity, on average, at the hiring. However, when hiring from less capital intensive firms, varying the average capital intensity of the firms workers are sourced from has no significant effect on the hiring firm's capital intensity. This suggests that if knowledge spillover does occur, the knowledge likely improves the ability of the hiring firm

[12]Because the regression controls for an estimate of observed worker quality based on wage data, the assortative matching story would imply this is a correlation between the unobserved component of worker quality and firm productivity.

to operate more capital intensive methods of production.

With the data set available, we cannot draw causal relationship between knowledge spillover through labor mobility and productivity growth at the hiring firm. The strongest statement possible is the results are consistent with the predictions from a spillover of productive knowledge related to technological knowledge (capital intensity). The appendix to this paper, as well as Kirker and Sanderson (2017), provide some further support for the existence of knowledge spillover by considering other dimensions of hiring which are likely to be correlated with the spillover of knowledge via labor mobility.

4 Theoretical Model

Having identified the firm-level data is consistent with the idea of labor mobility acting as a channel for knowledge spillover, attention now turns to the development of a modeling framework that can fit these empirical facts. The model is based upon an on-the-job search-and-matching labor model that featuring large, heterogeneous firms. When firms hire new workers, they not only receive additional labor, but are also exposed to new knowledge which the workers bring into the firm. Thus, the firm's choice of vacancy posting rate determined their exposure to outside knowledge through the labor mobility channel. The distribution of knowledge hiring firms are exposed to depends upon the distribution of worker search effort on the other side of the search and matching market.[13]

4.1 General Model Environment

Time is continuous in the model. Within the model there exists two types of agents, firms and workers. Both firms and workers are heterogeneous. Individual firms are collectively owned by the workers in the economy. Each individual firm produce a differentiated goods. Firms produce their output by combining units of labor, l, with a stock of productive knowledge x using the production function

$$y(i) = x(i)l(i)$$

where i indexes the firm.

The output of each firm is aggregated into a final consumption good using a CES Dixit-Stigliz aggregator. Because all firms enter the CES aggregator symmetrically, firms can be

[13] The model outlined in this section does not address the correlation between the hiring firm's productivity growth and the productivity gap that was identified as being consistent with positive assortative matching unmeasured worker quality and firm productivity. Appendix B outlines an extension to the model that features heterogeneity in worker skill where on-the-job training creates positive assortative matching between worker quality and firm productivity.

grouped by their level of productivity and labor, so aggregate output is given by

$$Y(t) = \left[\int_x \int_l y(x,l)^{(\rho-1)/\rho} \mathcal{F}^{act}(t) f(x,l,t)\, dl\, dx\right]^{\rho/(\rho-1)} \tag{3}$$

where ρ is the elasticity of substitution between individual goods, $\mathcal{F}^{act}(t)$ is the measure of active firms in the economy at time t, and $f(x,l,t)$ is the probability density function for the distribution of firms with productivity level x, and employing l units of labor at time t.

Each firm faces the standard inverse demand relationship for its output given by

$$p(x,l,t) = P(t)\left(\frac{y(x,l,t)}{Y(t)}\right)^{-1/\rho} \tag{4}$$

where $P(t)$ is the ideal price index of the Dixit-Stigliz aggregation defined as

$$P(t) = \left[\int_x \int_l p(y(x,l),t)^{1-\rho} \mathcal{F}^{act}(t) f(x,l,t)\, dl\, dx\right]^{\rho/(\rho-1)}$$

4.2 Firms

At every point in time there exists a continuum of firms of finite measure $\mathcal{F}$. This measure of firms is divided into two groups, active firms who are currently producing goods, and inactive firms who are trying to develop a product idea with which they can enter the market and become active. Our main point of interest with the firms is the endogenous choice of search intensity by active firms (described by the vacancy posting rate) which determines how they benefit from knowledge spillover.

4.2.1 Active Firms

Active firms are heterogeneous along two dimensions, the level of productive knowledge, x, and the measure of labor employed, l. For the model that will follow, it will be easier to work with the log of productivity, $z = \ln(x)$, rather than the productivity level x itself. In the remainder of this section, z will be referred to as the firm's productivity.

At each point in time, firms chooses how many vacancies to post in the search for new labor. This directly affects the rate at which they learn via the spillover of knowledge from the labor mobility channel. Even in the absence of this learning, firms can also improve their productivity through through two ways. First, following Luttmer (2012), noisy firm-level innovation is introduced in the form of Brownian motion shocks to z. These random productivity shocks have a mean of γ_I and variance of σ^2. In the context of this model, random firm-level innovations can be thought of as the engine of new ideas in the economy. All firms tinker with their current production process and discover new ideas. Labor mobility acts to transmit these new ideas (as well as older ideas) to the wider economy, amplifying the effect of new ideas beyond the scope of the individual firm.

The second outside way in which firms change their productivity is through exogenous learning. The labor mobility channel is likely only one way in which firms learn from other firms. To assist the model in matching the data, firms are also assumed to meet other firms at an exogenous rate ξ. If the firm meets a more productive firm, it is assumed to be able to replicate that firm's productivity.

Let $\Pi(z,l,t)$ denote the value of being an active firm with productivity z, firm size l, at time t. Firms (and workers) discount the future at rate r. The value of an active firm satisfies the following Hamilton-Jacobi-Bellman equation

$$
\begin{aligned}
r\Pi(z,l,t) &= \pi(z,l,t) + \gamma_I \frac{\partial \Pi(z,l,t)}{\partial z} + \frac{\sigma^2}{2}\frac{\partial^2 \Pi(z,l,t)}{\partial z \partial z} + \frac{\partial \Pi(z,l,t)}{\partial t} \\
&+ \max_{\nu \in [0,\nu_{max}]} \Bigg\{ -c_\nu(\nu) + \nu q(\theta) \frac{\partial \Pi(z,l,t)}{\partial l} \left(\int_{\tilde{z}=0}^{\infty} \int_{\tilde{l}} \mathbf{1}_{agree} h_\varepsilon(\tilde{z},\tilde{l},t)\, d\tilde{l}\, d\tilde{z} \right) \\
&+ \nu q(\theta) \left[\int_{\tilde{z}} \int_{\tilde{l}} \left(\int_y \mathbf{1}_{agree} \left[\Pi(y,l,t) - m(y;\tilde{z},z,t) - \Pi(z,l,t)\right] T(y;\tilde{z},z)\, dy \right) h_\varepsilon(\tilde{z},\tilde{l},t)\, d\tilde{l}\, d\tilde{z} \right] \Bigg\} \\
&+ \xi \int_{\tilde{z}=z}^{\infty} \left[\Pi(\tilde{z},l,t) - \Pi(z,l,t)\right] \left(\int_{\tilde{l}} f(\tilde{z},\tilde{l},t)\, d\tilde{l} \right) d\tilde{z} \\
&+ \Psi(z,l,t) \frac{\partial \Pi(z,l,t)}{\partial l}
\end{aligned} \tag{5}
$$

The first line of equation 5 has the standard form of a Hamilton-Jacobi-Bellman equation for a process with Brownian shocks. $\pi(z,l,t)$ denotes the flow of profit to the firm from production (revenue less expense). The next two terms relate to the capital gain/loss in value as a result of the Brownian motion process for firm innovation. The final term in the first line, $\partial\Pi(z,l,t)/\partial t$, relates to the capital gain/loss resulting from changing economic conditions over time (such as changes in the firm's relative productivity ranking within the economy).

The next two lines of the equation relate to the firm's choice vacancies posting rate, ν. Posting vacancies incurs a real cost to the firm, $c_\nu(\cdot)$, that is an increasing function of number of vacancy posted. Vacancies yield two sources of benefits to the firm, it allows for more labor to be employed, and it may also yield productive knowledge spillover through the labor mobility channel. The capital gains from each of these two components is presented separably in equation 5.

For each of the ν vacancies posted by the firm, the probability that the vacancy is matched with a searching worker is given by $q(\theta)$, where θ is a measure of the labor market tightness (the ratio of vacancies to worker search effort). From the firm's perspective, matches with workers are drawn randomly from the distribution of worker search effort which has the PDF denoted by $h_\varepsilon(\tilde{z},\tilde{l},t)$. The indicator function $\mathbf{1}_{agree}$ denotes the labor-market matches for which the net surplus of the match is positive, and hence the worker agrees to move to the firm.

The term in the third line denotes the expected capital gain resulting from productive knowledge spillovers through the labor mobility channel, less the the knowledge premium payment $m(\cdot)$ made to the worker required to attract the worker to the firm. The evidence from the empirical analysis suggests that productive knowledge spillover occurs only when the movement of the worker is from a more to less productive firm. In this model, the amount of productive knowledge the hiring firm has after the transfer, y, is drawn randomly from the distribution $T(y; \tilde{z}, z)$, and depends upon the productivity of the hiring firm (z) and the worker's previous employer ($\tilde{z}$).[14] By assumption, no knowledge spillover occurs when the worker is from a less productive firm (when $\tilde{z} < z$, $T(z; \tilde{z}, z) = 1$).

In addition to the wage a worker is paid for supplying their labor, they are also paid a one-off premium payment, $m(\cdot)$, negotiated when the employee first starts at the firm. The exact details of how this is determined is discussed in section 4.4. This premium payment takes into account a number of factors such as the worker's willingness to work for the firm, due to opportunities to learn from the firm, and the firm's willingness to hire the worker to obtain any knowledge spillovers.

The term in the fourth line relates to the capital gain from the exogenous learning process for firms. Firms are randomly matched with other active firms at rate ξ. It the firm is matched with a firm that has more productivity, $\tilde{z} > z$ (which occurs with probability $\int_{\tilde{l}} f(\tilde{z}, \tilde{l}, t)\, d\tilde{l}$), then the firm's productivity increases from z to $\tilde{z}$.

The final term in equation 5 relates to capital gain/loss from changes in the firm's incumbent labor force. $\Psi(z, l, t)$ is the probability of increasing (or decreasing) the measure of workers employed by the firm, and $\partial\Pi(z, l, t)/\partial l$ denotes the capital gain or loss for a marginal change in labor. More specifically, $\Psi(z, l, t)$ is given by

$$
\begin{aligned}
\Psi(z, L, t) \equiv\ & -(\lambda + \delta) l \\
& - l\varepsilon(z, l, t)\theta q(\theta) \int_{\tilde{z}} \int_{\tilde{l}} \mathbf{1}_{agree} f_\nu(\tilde{z}, \tilde{l}, t)\, d\tilde{l}\, d\tilde{z}
\end{aligned}
$$

where the first line on the right hand side relates to the loss of workers due to exogenous separation to unemployment at rate δ, and the random chance of death λ.

The second line on the right hand side relates to the loss of workers who leave the firm because they have accepted a job at another firm. Each of the l measure of workers at the firm searches for other job offers with search effort $\varepsilon(z, l, t)$. Each unit of worker search effort has a $\theta q(\theta)$ probability of being matched with a posted job vacancy. These matches are randomly drawn from the distribution of vacancy postings with a probability density function denoted by $f_\nu(\tilde{z}, \tilde{l}, t)$ where $\tilde{z}$ denote the productivity of the firm posting the vacancy, and $\tilde{l}$ is the measure of labor employed by the firm. Workers will only accept new jobs if it yields positive net surplus, denoted by the indicator function $\mathbf{1}_{agree}$.

[14]Notation wise, y will often be used to denote the productivity level that the firm goes to after knowledge spillover. It does not refer to firm-level output.

Given the value function defined by equation 5, the firm's choice of vacancies to post satisfies the first order condition (FOC) given by

$$\frac{\partial c_\nu\left(\nu\right)}{\partial \nu} = q(\theta)\frac{\partial \Pi(z,l,t)}{\partial l}\left(\int_{\tilde{z}=0}^{\infty}\int_{\tilde{l}}\mathbf{1}_{agree}h_\varepsilon(\tilde{z},\tilde{l},t)\,d\tilde{l}\,d\tilde{z}\right)$$
$$+\, q(\theta)\left[\int_{\tilde{z}}\int_{\tilde{l}}\left(\int_y \mathbf{1}_{agree}\left[\Pi(y,l,t)-m(y;\tilde{z},z,t)-\Pi(z,l,t)\right]T(y;\tilde{z},z)\,dy\right)\,h_\varepsilon(\tilde{z},\tilde{l},t)\,d\tilde{l}\,d\tilde{z}\right] \tag{6}$$

where the left hand side is the marginal cost of an additional vacancy posting, and the right hand side it the expected marginal benefit (both in terms of additional labor and knowledge spillover) of a vacancy posting.

Finally, the profit function, $\pi(z,l,t)$, for an active firm has the form

$$\pi(z,l,t) \;=\; p(z,l,t)y\left(z,l\right)-\omega(z,l,t)l \tag{7}$$

where $p(z,l,t)y\left(z,l\right)$ is the firm's revenue (price multiplied by output), and $\omega(z,l,t)l$ is the total wage cost to the firm for all units of labor employed, where $\omega(z,l,t)$ is the wage rate per unit of labor.

4.2.2 Inactive Firms

At time t, there is a measure $\mathcal{F}^{inact}(t)\equiv\mathcal{F}-\mathcal{F}^{act}(t)$ of inactive firms. Inactive firm pays a flow cost c_E to draw an idea for a new product, and an initial productivity level for producing the product. The productivity draws are made from a distribution with probability density function $f_{new}(z,t)$. The firm can then choose to enter the market and become an active firm, or remain an inactive firm. Following Acemoglu and Hawkins (2014), if the inactive firm decides to become active, it enters the market with zero employees.

Let $\Pi^I(t)$ denote the value of being an inactive firm at time t. The value satisfies the HJB given by

$$r\Pi^I(t) = \frac{\partial \Pi^I(t)}{\partial t} - c_I + \int_z \max\left\{\Pi(z,0,t)-\Pi^I(t)\,,\,0\right\} f_{new}(z,t)\,dz \tag{8}$$

where $\partial\Pi^I(t)/\partial t$ is the capital gain from changing market conditions, c_I is the cost of developing a new product, and $\int_z \max\left\{\Pi(z,0,t)-\Pi^I(t)\,,\,0\right\} f_{Ent}(z,t)\,dz$ is the expected capital gain from developing a new producing and becoming an active firm with productivity z, and labor force size zero.

4.3 Workers

Workers differ by their state, employed or unemployed, and the characteristics of their current employer. Each worker faces a constant probability of death λ, and throughout their life will transition between jobs, either moving up the productivity ladder to obtain better learning opportunities, or down the productivity ladder to benefit from spilling over their acquired knowledge.

The behavior of workers within the model determines the distribution of productivity ideas that searching firms are exposed to via the labor mobility channel of knowledge spillover. The total population of workers is assumed to be of a fixed measure $\mathcal{N}$. Each worker is infinitesimally small in size. When a worker dies, they are replaced by a new-born worker who begins life in the state of unemployment (as in Postel-Vinay and Robin, 2002). At each point in time, workers can choose how intensively they seek outside job offers.

4.3.1 Employed workers

Let $V(z, l, t)$ denote the value of being a worker who is employed at a firm with productivity z, and labor force size l at time t. The value of such a worker satisfies the following Hamilton-Jaccobi-Bellman equation

$$\begin{aligned}
(r+\lambda)V(z,l,t) &= \omega(z,l,t) + \gamma_I \frac{\partial V(z,l,t)}{\partial z} + \frac{\sigma^2}{2}\frac{\partial^2 V(z,l,t)}{\partial z \partial z} + \frac{\partial V(z,l,t)}{\partial t} \\
&+ \max_{\varepsilon\in[0,\varepsilon_{max}]} \left\{ -c_\varepsilon(\varepsilon) + \varepsilon\theta q(\theta) \int_{\tilde{z}=0}^{\infty} \int_{\tilde{l}} \left(\int_y \mathbf{1}_{agree} \left[V(y,\tilde{l},t,) + m(\cdot) - V(z,l,t) \right] T(y;\tilde{z},z)\, dy \right) f_\nu(\tilde{z},\tilde{l},t)\, d\tilde{l}\, d\tilde{z} \right\} \\
&- \delta\left[V^u(s,t) - V(s,z,l,t)\right] \\
&+ vq(\theta) \int_{\tilde{z}=z}^{\infty} \int_{\tilde{l}} \left(\int_{y=z}^{\tilde{z}} \mathbf{1}_{agree}[V(y,l,t) - V(z,l,t)] T(y;\tilde{z},z)\, dy \right) h_\varepsilon(\tilde{z},\tilde{l},t)\, d\tilde{l}\, d\tilde{z} \\
&+ \xi \int_{y=z}^{\infty} [V(y,l,t) - V(z,l,t)] \left(\int_{\tilde{l}} f(y,\tilde{l},t)\, d\tilde{l} \right) dy \\
&+ \Psi(z,l,t)\frac{\partial V(z,l,t)}{\partial l}
\end{aligned} \tag{9}$$

The first line of the Hamilton-Jacobi-Bellman is fairly standard. On the left hand side, the effective discount rate the worker faces comprises of the standard discount rate r, plus the probability of death λ. On the right hand side, $\omega(z, l, t)$ denotes the flow of wage income the worker receives. The next two terms are the expected capital gain/loss as a result of the Brownian motion shocks to their firm's productivity. The final term in the first line is the change in value due to changes in wider economy conditions over time (such as the firm's change in relative market position).

The second line of equation 9 relates to the worker's choice of search effort, ε. Given the choice of search effort ε, the worker incurs a real search cost of $c_\varepsilon(\varepsilon)$ which is increasing in the level of search effort. The other term in the second line is the expected capital gain from searching. For each unit of search effort, the worker has a $\theta q(\theta)$ probability of matching with a posted vacancy in the search-and-matching market. Matches are drawn at random from the distribution of firm vacancy postings with probability density function $f_\nu(\tilde{z}, \tilde{l}, t)$. The indicator function $\mathbf{1}_{agree}$ denotes matches for which the worker will agree to move firms. Given the possibility that the worker could facilitate some knowledge spillover to the hiring firm, moving to a new firm yields a capital gain (or loss) of $V(y, \tilde{l}, t,) - V(z, l, t)$ (rather than $V(\tilde{z}, \tilde{l}, t,) - V(z, l, t)$), and a one-off premium payment ($m(\cdot)$).

The term in the third line reflected the expected capital loss the worker faces from the exogenous probability δ that they separate into unemployment. $V^u(t)$ denotes the value of being an unemployed worker at time t (and is defined below).

The fourth line of equation 9 relates to the expected capital gain as a result of the worker's firm improving its productivity through the spillover of knowledge via labor mobility. The introduction of superior knowledge into the firm raises the firm's productivity which not only increases the workers wage, but also provides the worker with greater learning opportunities to acquire better knowledge. Similarly, the term in the fifth line represents the expected capital gain to the worker from their firm exogenously learning some new productive knowledge.

The final term in equation 9 relates to the expected capital gain/loss resulting from changes in the measure of other workers employed by the firm. From the point of view of an individual worker, this is taken as exogenous. $\Psi(z, l, t)$ denotes the probability that the number of workers employed at the firm increases, and is given by

$$\begin{aligned} \Psi(z, l, t) \quad \equiv \quad & -(\lambda + \delta) l \\ & - l \varepsilon \theta q(\theta) \int_{x=0}^{\infty} \int_{l_x} \mathbf{1}_{accept} f_\nu(x, l_x, t) \, dl_x \, dx \\ & + \nu q(\theta) \int_{x=z}^{\infty} \mathbf{1}_{accept} h_\varepsilon(x, t) \, dx \end{aligned}$$

where each of the terms have a similar interpretation to those in the firm's Hamiltonian-Jacobi-Bellman equation.

Given the value function defined in equation 9, the employed worker's choice of search effort, ε, satisfies the FOC give by

$$\frac{\partial c_\varepsilon(\varepsilon)}{\partial \varepsilon} = \theta q(\theta) \int_{\tilde{z}=0}^{\infty} \int_{\tilde{l}} \left(\int_y \mathbf{1}_{agree} \left[V(y, \tilde{z}, t,) + m(\cdot) - V(z, l, t) \right] T(y; z, \tilde{z}) \, dy \right) f_\nu(\tilde{z}, \tilde{l}, t) \, d\tilde{l} \, d\tilde{z} \tag{10}$$

where the left hand side is the marginal cost of additional search effort, and the right hand side is the expected capital gain from finding a new job at another firm.

4.3.2 Unemployed Workers

Unemployed workers are assumed to undertake home production. The real flow value of their home production is $b(t)$ each period. Unemployed workers can leave the unemployment state by successfully searching for a job in the labor market. From the perspective of the firms, the main distinction between employed and unemployed workers is that unemployed workers do not have a stock of productive knowledge that from which the firm can learn from. The productive knowledge from an unemployed worker's last employer prior to becoming unemployed is assumed to fully depreciate upon entering unemployment.

Let $V^u(t)$ denote the value of being an unemployed worker at time t. The value function satisfies the following Hamilton-Jacobi-Bellman equation

$$[r+\lambda]V^u(t) = b(t) + \frac{\partial V^u(t)}{\partial t} + \max_{\varepsilon\in[0,\varepsilon_{max}]}\Big\{-c_\varepsilon(\varepsilon) + \varepsilon\theta q(\theta)\int_z\int_l \max\{V(z,l,t)+m(z,U,t)-V^u(t)\ 0\}f_\nu(z,l,t)\,dz\,dl\Big\} \tag{11}$$

where $r+\lambda$ is the effective discount rate, $b(t)$ is the flow payment of benefits (home production), $\partial V^u(t)/\partial t$ is the capital gain/loss due to changing labor market conditions over time, $c_\varepsilon(\varepsilon)$ is the cost of searching with effort ε, and the final term is the expected capital gain from matching with a searching firm and leaving unemployment.

The unemployed worker's choice of search effort satisfies the following first order condition

$$\frac{\partial c_\varepsilon(\varepsilon)}{\partial\varepsilon} = \theta q(\theta)\int_z\int_l \max\{V(z,l,t)+m(z,U,t)-V^u(t)\,,\,0\}f_\nu(z,l,t)\,dz\,dl \tag{12}$$

which balances the marginal cost of searching (the left hand side) against the expected marginal benefit of searching (the right hand side).

4.4 Worker's Compensation

A worker's total compensation is modeled in two parts. First there is the wage the worker receives for the labor they supply to the firm's production process. The second part is a premium they earn above, or below, the baseline wage. This premium nets out a number of factors that relates to the firms and workers incentivising the other to agree to the new match.

4.4.1 Wages

Wages are set independently at each firm. The wage setting mechanism used here follows the approach used by Mortensen (2010), which is in the spirit of Stole and Zwiebel (1996). The firm bargains individually with each worker in their labor force. Long-term wage contracts are not possible, and the worker and firm bargain over the marginal surplus from the worker's labor each period. In the bargaining between the firm and an individual worker, the firm's outside position is to not use that particular worker this period, and instead produce with the other workers who have agreed to contracts. The worker's outside option is to undertake home production, with payoff $b(t)$.

Let β_ω denote the worker's relative bargaining strength. The wage rate, $\omega(z, l, t)$, for each labor unit at a firm with productivity z and labor distribution l satisfies the Mortensen (2010) bargaining equation:

$$\beta_\omega \frac{\partial \pi(z, l, t)}{\partial l} = (1 - \beta_\omega)[\omega(z, l, t) - b(t)] \tag{13}$$

4.4.2 Knowledge Premium

When a worker is hired at a new firm, they negotiate the knowledge premium component of their compensation with the firm. It is through this premium that some workers are paid for their diffusion of productive knowledge, and other workers pay firms for the learning opportunities the firm provides. The knowledge premium negotiation is assumed to occur after the firm and worker observe the amount of knowledge spillover that will occur if the match was to go ahead.

To drastically simplify the numerical computation of the model, the knowledge premium is assumed to be a fixed, one-off payment when the worker starts. [15] In this sense, the knowledge premium can either be thought of in terms of a signing bonus, or as the net present value of a series of premium payments that will occurred over the worker's employment spell.

Let z denote the firm's productivity, and $\tilde{z}$ denote the productivity of the worker's previous employer. The expected capital gain for the worker of accepting the new job offer from firm (z, l_z) is given by

$$V(z', l, t) - V(\tilde{z}, \tilde{l}, t) + m(z'; z, l, \tilde{z}, \tilde{l}, t)$$

where $m(\cdot)$ denotes the knowledge premium payment, and z' denote the productivity of the hiring firm after the occurrence of any knowledge transfer ($z' \geq z$).

From the perspective of the firm, the gains from hiring the new worker comprise of the expected capital gain due to the knowledge spillover (if any), and the capital gain from having another worker employed

$$\Pi(z', l, t) - \Pi(z, l, t) - m(z', z, l, \tilde{z}, \tilde{l}, t) + \frac{\partial \Pi(z, l, t)}{\partial l}$$

[15] If the knowledge premium was modeled as an ongoing payment, we would have to not only keep track of the worker's premium at each point in time, but also the entire distribution of premiums within each firm's pool of labor.

A worker's relative bargaining strength when first joining a job (and potentially facilitating the spillover of knowledge) may be very different to their relative bargaining strength once they have assimilated into the work force. Therefore, the worker's relative bargaining strength for the knowledge premium payment is allowed to differ from that of the wage negotiations. β denotes the worker's relative bargaining strength for the knowledge premium. The value of m that satisfies the following Nash-bargaining equation

$$\beta\left(\Pi(z',l,t)-\Pi(z,l,t)-m+\frac{\partial\Pi(z,l,t)}{\partial l}\right)=(1-\beta)\left(V(z',l,t)+m-V(\tilde{z},\tilde{l},t)\right) \quad (14)$$

Similarly, for an unemployed worker (who has no knowledge spillover), the knowledge premium payment is the value of m that satisfies

$$\beta\left(-m+\frac{\partial\Pi(z,l,t)}{\partial l}\right)=(1-\beta)\left(V(z,l,t)+m-V^{u}(t)\right) \quad (15)$$

Because workers will be willing to offer a negative knowledge premium to work at the most productive firms (pay the firms for the learning opportunities), for some matches, the total worker compensation (wages plus knowledge premium) will be negative. In effect, the most productive firms will be paid by the factor inputs. To avoid this situation, the premium payment is bounded to be non-negative, so that workers must receive some positive compensation from each job.[16]

4.5 Labor Market

As is standard for search-and-matching models, the labor market tightness, θ, is characteristics by the ratio of vacancy postings to worker search effort and is defined as

$$\theta = \frac{\mathcal{F}^{act}(t)\int_{z=0}^{\infty}\int_{l} v(z,l,t)f(z,l,t)\,dl\,dz}{\mathcal{N}^{unemp}(t)\varepsilon(U,t)+\mathcal{N}^{emp}(t)\int_{z}\int_{l}\varepsilon(z,l,t)h^{emp}(z,l,t)\,dl\,dz} \quad (16)$$

where $\mathcal{N}^{unemp}(t)$ and $\mathcal{N}^{emp}(t)$ are the measures of unemployed and employed workers at time t.

Searching firms and workers are matched at random according to a matching function. Let $Q(\cdot)$ denote the constant returns to scale, homogeneous of degree one, matching function. Then the probability that a firm's vacancy is matched with a worker is given by

$$q(\theta)=Q(\theta,1)$$

Given the distribution of firms, $f(z,l,t)$, and their vacancy posting choices, $\nu(z,l,t)$, workers

[16] In the cases where the premium payment is at the boundary, a match only occurs if both the firm and worker receive some positive surplus from the match.

sample from the distribution of vacancy postings has the probability density function given by

$$f_\nu(z,l,t) = \frac{\nu(z,l,t)f(z,l,t)}{\int_{\tilde{z}}\int_l \nu(\tilde{z},l,t)f(\tilde{z},l,t)\,dl\,d\tilde{z}} \tag{17}$$

On the other side of the market, firms sample from the distribution of worker search effort with a probability density function given by

$$h_\varepsilon(z,l,t) \;\; = \;\; \frac{\varepsilon(z,l,t)h(z,l,t)}{\int_{\tilde{z}}\int_l \varepsilon(\tilde{z},l,t)h(\tilde{z},l,t)\,dl\,d\tilde{z} + \frac{\mathcal{N}^{unemp}(t)}{\mathcal{N}^{emp}(t)}\varepsilon^{unemp}(t)} \tag{18}$$

for all pairs of (z,l) corresponding to productivity-labor points in which firms are active, and

$$h_\varepsilon(0,0,t) \;\; = \;\; \frac{\frac{\mathcal{N}^{unemp}(t)}{\mathcal{N}^{emp}(t)}\varepsilon^{unemp}(t)h_{unemp}(t)}{\int_{\tilde{z}}\int_l \varepsilon(\tilde{z},l,t)h(\tilde{z},l,t)\,dl\,d\tilde{z} + \frac{\mathcal{N}^{unemp}(t)}{\mathcal{N}^{emp}(t)}\varepsilon^{unemp}(t)} \tag{19}$$

corresponding to unemployed workers search effort. In the above equations $h(z,l,t)$ refers to the PDF of the distribution of labor employed by firms with productivity z and firm size l, and $h_{unemp}(t)$ refers to the PDF of the labor distribution that is unemployed.

4.6 Kolmogorov Forward Equation

The search-and-matching labor model described above determines the search effort (policy choice) for both worker and firms. Given the policy choices of the workers and firms in the economy, the distribution of firms (and hence also workers) across the productivity-labor space evolves over time according to a Kolmogorov Forward Equation (KFE).

Let $f(z,l,t)$ denote the PDF for the distribution of firms who have productivity z, and employs a measure l of workers.[17] The change in the mass of firms at (z,l,t) over time is given by

[17] Inactive firms can be considered a point mass below the lower bound of productivity level for active firms.

$$
\begin{aligned}
\frac{\partial f(z,l,t)}{\partial t} &= -\gamma_I \frac{\partial f(z,l,t)}{\partial z} + \frac{1}{2}\sigma^2 \frac{\partial^2 f(z,l,t)}{\partial z \partial z} \\
&+ \int_{\tilde{z}=0}^{z} f\left(\tilde{z},l,t\right) \nu(\tilde{z},l,t) q\left(\theta\right) \left(\int_{\hat{z}=z}^{\infty} \int_{\hat{l}} \mathbf{1}_{accept} T(z,\tilde{z},\hat{z}) h_{\varepsilon}(\hat{z},\hat{l},t)\, d\hat{l}\, d\hat{z} \right) d\tilde{z} \\
&- f(z,l,t)\nu(z,l,t)q(\theta) \left(\int_{\hat{z}=z}^{\infty} \int_{\hat{l}} \mathbf{1}_{accept} h_{\varepsilon}(\hat{z},\hat{l},t)\, d\hat{l}\, d\hat{z} \right) \\
&+ \xi \int_{\tilde{z}=0}^{z} f(\tilde{z},l,t) \left[\int_{\hat{l}} f(z,\hat{l},t)\, d\hat{l} \right] d\tilde{z} \\
&- \xi f(z,l,t) \int_{\tilde{z}=z}^{\infty} \int_{\tilde{l}} f(\tilde{z},\tilde{l},t)\, d\tilde{l}\, d\tilde{z} \\
&+ \mathbf{1}_{l=0} \frac{\mathcal{F}^{inact}(t)}{\mathcal{F}(t)} f_{new}(z,t) \mathbf{1}_{\Pi(z,0,t)>E(t)} \\
&+ \frac{\partial \Psi(z,l,t)}{\partial l}
\end{aligned} \tag{20}
$$

The first line on the right hand side contains terms related to the net outflow of firms as a result of the Brownian motion process for productivity shock innovations.

The second line represents the inflow of firms with productivity $\tilde{z} < z$ and are of size l, who hire a new worker who previously worked at a firm with productivity $\hat{z} > z$, and the amount of knowledge that spills over is the exact amount needed to boost the hiring firm's productivity to z (which occurs with probability $T(z, \hat{z}, \tilde{z})$).

The third line represents firms who currently have productivity z and firm size l, who hire a new worker from a move productive firm and receive any amount of knowledge spillover which will improve their productivity above z.

Similarly, the fourth line represents and inflow of firms of size l who improve their productivity to z through exogenous learning (at rate ξ). And the fifth line represents the outflow of firms who improve their productivity through exogenous learning.

The term in the sixth line relates to the entry of inactive firms. $F^{inact}(t)/\mathcal{F}(t)$ is the relative share of inactive firms in the economy. The probability that the inactive firms entering the market with productivity z is given by the probability density function for the distribution of initial ideas $f_{new}(z,t)$. Finally, when an entrepreneur becomes an active firm, it starts with an initial firm size of zero. Hence the indicator function $\mathbf{1}_{l=0}$ so this term only applies in cases when the firm size is zero.

As in the case of the Hamilton-Jacobi-Bellman equations for the firm and workers, the final term in the KFE relates to changes in the mass of firms due to changes in the incumbent labor being employed by the firms. Where $\Psi(z,l,t)$ is the probability of a firm losing a worker, and is defined as:

$$\begin{aligned}
\Psi(z,l,t) \;\; \equiv \;\; & (\delta+\lambda)l(z)f(z,l,t) \\
& + \left[l\varepsilon(z,l,t)\theta q\left(\theta\right) \int_{\tilde{z}} \int_{\tilde{l}} 1_{accept} f_{\nu}(\tilde{z},\tilde{l},t)\, d\tilde{l}\, d\tilde{z} \right] f(z,l,t) \\
& - \left[\nu(z,l,t) q\left(\theta\right) \int_{\tilde{z}=0}^{z} \int_{\tilde{l}} h_{\varepsilon}(\tilde{z},\tilde{l},t)\, d\tilde{l}\, d\tilde{z} \right] f(z,l,t)
\end{aligned}$$

where the interpretations of these terms is the same as in the case of the Hamilton-Jacobi-Bellman equations.

4.6.1 Number of active and inactive firms

The value of a firm is increasing in the firm's productivity level. Within the economy there will exist some lower bound on productivity, z_{lb}, such that inactive firms will choose not to enter the market if they draw an initial productivity below this amount. I.e. $\Pi(z,0,t) \leq \Pi^{I}(t)$ for all $z \leq z_{lb}$. In addition, active firms who happen to have a series of bad productivity shocks such that their productivity falls below this bound will also choose to shut down and become inactive firms (since the value of being inactive is greater than being active).

Following the approach of Luttmer (2012) the measure of active firms (or equivalently the measure of inactive firms) varies according to the flow of firms over the lower bound z_{lb} given by the expression:[18]

$$\frac{\partial(1-F(z_{lb},t))}{\partial t} \;\; = \;\; \frac{1}{2}\sigma^2 \frac{\partial^2(1-F(z_{lb},t))}{\partial z \partial z} + \frac{\mathcal{F}^{inact}(t)}{\mathcal{F}^{act}(t)}[1-F_{Ent}(z_{lb},t)] \tag{21}$$

where $F(z,t) = \int_l \int_{x=z_{lb}}^{z} f(x,l)\, dx\, dl$ is the CDF of distribution of firms in productivity space. The left hand side of equation 21 denotes the change in mass of firms above the lower bound which corresponds to the measure of active firms. The final term on the right hand side denotes the inflow of inactive firms who draw an initial productivity above the level z_{lb}. Therefore, the first term on the right hand side denotes the flow of active firms who cross the lower bound and choose to become inactive as a result of the Brownian motion shocks.

4.6.2 Number of unemployed workers

The change in the measure of unemployed workers is found using the PDE:

$$\frac{\partial \mathcal{N}^{unemp}(t)}{\partial t} = \delta \mathcal{N}^{emp}(t) + \lambda \mathcal{N}^{emp}(-\mathcal{N}^{unemp}(t)\varepsilon(U,t)\theta q(\theta) \int_z \int_l \mathbf{1}_{accept} f_{\nu}(z,l,t)\, dl\, dz \tag{22}$$

[18]For more details, see appendix A.1

The first term on the right hand side in the inflow of employed workers who are separated from their employers at rate δ. The second term relates to the inflow of new worker to replace the employed workers who died (keeping the overall population constant). The final term on the right hand side is the outflow of unemployed workers who successfully find a job.

The measure of employed workers is found by integrating over the distribution of labor across active firms:

$$\mathcal{N}^{emp}(t) \quad = \quad \int_z \int_l l \mathcal{F}_{active}(t) f(z,l,t)\, dl\, dz \tag{23}$$

Finally, the probability density function for the distribution of employed labor is given by

$$h(z,l,t) = \frac{l f(z,l,t)}{\int_z \int_l l f(z,l,t)\, dl\, dz} \tag{24}$$

4.7 Equilibrium and the balanced growth path

Having outlined the model equations, attention is now turned to the equilibrium and balanced growth path of the model. The focus of the analysis to follow with be solely on a balanced growth path (BGP) in which the economy grows at a constant rate, and the relative distribution of firms and labor are constant. Below, definition 1 specifies the requirements for an equilibrium in the model, and definition 2 specifies the conditions needed for that equilibrium to have a balanced growth path.

Definition 1. *Given an initial distribution of firms $f(z,l,0)$, the set of parameters $\{r,\ \lambda,\ \delta,\ \alpha,\ \rho,\ \gamma_I,\ \sigma,\ \mathcal{N}^{total},\ \mathcal{F}^{total}\}$, and the set of exogenous functions $\{c_\nu(\nu),\ c_\varepsilon(\varepsilon),\ y(z,l,t),\ q(\theta),\ T(x,z),\ f_{Ent}(z,t)\}$, an* equilibrium *of the model is describe by: (i) the set of value functions $\{\Pi(z,l,t),\ \Pi^I(t),\ V(z,l,t),\ V^u(t)\}$; (ii) choice variables $\{\nu(z,l,t),\ \varepsilon(z,l,t)\}$; (iii) payments/prices $\{\pi(z,l,t),\ m(z,l_z,x,l_x,t),\ p(z,l,t),\ \omega(z,l,t),\ P(t)=1\}$; (iv) the labor market condition $\{\theta\}$; (v) aggregate variables $\{\mathcal{F}^{inact}(t),\ \mathcal{N}^{unemp}(t),\ \mathcal{N}^{emp}(t),\ Y(t),\ \mathcal{F}^{act}(t),\ z_{lb}\}$; and (vi) the distributions $\{h_\varepsilon(z,l,t),\ f_\nu(z,l,t),\ f(z,l,t),\ h(z,l,t)\}$ such that:*

1. *Given the value functions $\Pi(z,l,t)$, $\Pi^I(t)$, $V(z,l,t)$, and $V^u(t)$ (defined by equations 5, 8, 9, and 11), firms and workers choose their search efforts $v(z,l,t)$, $\varepsilon(z,l,t)$, and $\varepsilon(U,t)$ to satisfy the first order conditions given in equations 6, 10, and 12.*

2. *For each firm, the wage $\omega(z,l,t)$, and the knowledge premium $m(z,l,\tilde{z},\tilde{l},t)$, are determined by the surplus sharing rules given by equations 13, 14, and 15.*

3. *The goods market clears and each firm maximizes its profits by choosing $p(z,l,t)$ to satisfy equation 4.*

4. *The labor market tightness, $\theta(t)$, is determined by equation 16.*

5. *The number of active firms $\mathcal{F}^{act}(t)$ (and hence also the number of inactive firms $\mathcal{F}^{inact}(t)$) is determined by the boundary condition 21.*

6. *The number of employed and unemployed workers, $\mathcal{N}^{emp}(t)$ and $\mathcal{N}^{unemp}(t)$, satisfies equations 22 and 23.*

7. *The productivity lower bound , $z_{lb}(t)$, satisfies the identity: $\Pi(z_{lb}, 0, t) = \Pi^I(t)$.*

8. *The distribution of worker search effort, $h_\varepsilon(z, l, t)$, satisfies equation 18, the distribution of vacancy postings $f_\nu(z, l, t)$ satisfies equation 17, the distribution of firms $f(z, l, t)$ satisfies equation 20, and the distribution of labor $h(z, l, t)$ satisfies equation 24.*

Definition 2. *A* Balanced Growth Path *(BGP) for the model is defined by a scalar γ (the aggregate growth rate), and tuple of functions $\{\ddot{\Pi}(\cdot)$, $\ddot{\Pi}^I(\cdot)$, $\ddot{V}(\cdot)$, $\ddot{V}^u(\cdot)$, $\ddot{v}(\cdot)$, $\ddot{\varepsilon}(\cdot)$, $\ddot{\pi}(\cdot)$, $\ddot{m}(\cdot)$, $\ddot{p}(\cdot)$, $\ddot{\omega}(\cdot)$, $\ddot{\theta}(\cdot)$, $\ddot{\mathcal{R}}(\cdot)$, $\ddot{\mathcal{N}}^{unemp}(\cdot)$, $\ddot{\mathcal{N}}^{emp}(\cdot)$, $\ddot{Y}(\cdot)$, $\ddot{\mathcal{F}}^{act}(\cdot)$, z_{lb}, $\phi_{h\varepsilon}(\cdot)$, $\phi_{f\nu}(\cdot)$, $\phi_f(\cdot)$, $\phi_h(\cdot)\}$ such that:*

1. *For the various value functions: $g(\cdot) \in \{\Pi(\cdot), \Pi^I(\cdot), V(\cdot), V^u(\cdot)\}$, $g(z, \cdot, t) = e^{\gamma t}\ddot{g}(e^{-\gamma t}z, \cdot)$*

2. *For the policy rules: $g(\cdot) \in \{\nu(\cdot), \varepsilon(\cdot)\}$, $g(z, \cdot, t) = \ddot{g}(e^{-\gamma t}z, \cdot)$*

3. *For the prices and payments: $g(\cdot) \in \{\pi(\cdot), m(\cdot), \omega(\cdot)\}$, $g(z, \cdot, t) = e^{\gamma t}\ddot{g}(e^{-\gamma t}z, \cdot)$*

4. *For the output prices, $p(z, l, t) = \ddot{p}(ze^{-\gamma t}, l)$ and $P(t) = 1$*

5. *Labor market tightness and the measure of various firms and workers are constant: $g(t) \in \{\theta(t), \mathcal{R}(t), \mathcal{F}^{act}(t), \mathcal{N}^{unemp}(t), \mathcal{N}^{emp}(t)\}$, $g(t) = g$*

6. *For the levels: $g \in \{Y, z_{lb}\}$, $g(t) = e^{\gamma t}\ddot{g}$*

7. *For the PDFs of distributions: $g \in \{h_\varepsilon, f_\nu, f, h\}$, $g(z, \cdot, t) = e^{-\gamma t}\phi_g(e^{-\gamma t}z, \cdot)$*

and the model is in equilibrium (as defined previously).

4.7.1 Existence of a Unique Balanced Growth Path

The model presented above is too generalized for a single proof of the existence of a unique balanced growth path. The shape and characteristics of both the equilibrium and the balanced growth path depend on the properties of key components of the model, such as the knowledge spillover function $T(\cdot)$, which have not be specified.

To illustrate that at least under certain conditions the model can be solved for a unique balanced growth path, I outline a set of assumptions below that are sufficient, but not necessary, for the existence of a unique balanced growth path.

Assumption 1. *Assume that the following set of conditions hold in the model*

1. *The initial distribution of firms, $f(z, l, t = 0)$, has a bounded support for productivity. I.e. there exists some z^* such that $f(z, l, 0) = 0$ for all $z \geq z^*$.*

2. *Assumptions regarding the knowledge transfer function ($T(\cdot)$):*

    (a) *Conditional upon firm (firm i) meeting a worker from a more productive firm (denote the more productive firm by j), there is a $1/l_j$ probability the worker is able to facilitate knowledge spillover, and a $1 - 1/l_j$ probability that the worker cannot.*

    (b) *Conditional upon the worker being able to facilitate knowledge spillover, the probability density function for the amount of knowledge transferred has some positive probability on the full amount of knowledge being transferred. I.e. there is some chance (denoted by τ), that the a worker from a firm with productivity $\tilde{z}$ can raise the productivity of a firm with productivity $z < \tilde{z}$ up to $\tilde{z}$.*

3. *There is an upper bound of the search effort of workers that binds for workers at highly productive firms.*

4. *The distribution of initial productivity draws by inactive firms, $f_{new}(z, t)$, has a sufficiently thin tail.*

Assumption 1 is not required for the existence of the BGP. However, as discussed by Luttmer (2012), A bounded initial distribution of productivity combined with the Brownian innovation shocks and learning process ensure that the balanced growth path of the model is unique.

Assumption 2a can be thought of in the following way. To produce output for the firm, there is a continuum of job/tasks distributed over the range $[0, 1]$ that the workers must do. These jobs are distributed proportionally among all the workers at the firm. So if the firm has a measurel_j of employees, each is responsible for $1/l_j$ of the total jobs within the firm. When a firm posts a vacancy in the market, it must advertise a specific job with the posting. When workers are matched with posted vacancies, knowledge transfer can only occur when the worker's current set of tasks matches with the advertised job. Workers who move to a firm when there is no job match can be thought of as changing career, and will not be in a position to utilize their productive knowledge at the new firm. The motivation for this assumption is that not all workers within the firm will be a suitable match in terms of skills and knowledge for every advertised job. Additionally, the knowledge that a worker has about what makes their current employer productive is likely to be restricted by the scope of their role within the firm. The larger the firm, the less likely each individual worker is to understand the full structure of the firm and what makes it productive.[19]

Assumption 2b ensures that all firms in the economy has some positive probability of advancing to the productivity frontier when meeting a worker with knowledge from the frontier. Without some positive probability of each firm reaching the frontier, only the Brownian innovation shocks and exogenous learning would drive long run growth in the economy .

[19]For example, Walmart employs a very large number of relatively low skilled workers as cashiers and floor staff. These workers are unlikely to have an intricate understanding of what actually makes Walmart as productive as it is, and are unlikely to be able to transfer this knowledge to any new employer.

Assumption 3, limiting the maximum search effort of individual workers, could be imposed either through the rate at which search costs increase with search effort (the shape of $c_\varepsilon(\cdot)$), or a physical time limit workers can devote to search. This assumption ensures that workers at the most productive firms do not search with unbounded effort, which would allow all firms to instantly jump to the productivity frontier.

Finally, assumption 4 ensures that long-run growth on the BGP is driven by the knowledge spillover channel (and exogenous learning), and not driven by the productivity draws of new entrants. As Lucas and Moll (2014) show, when firms sample productivity draws from two distributions, the distribution of agents along the BGP will be affected by whichever sample distribution has the thickest tail. By ensuring that the distribution of initial productivity draws for inactive firms is sufficiently thin, it will not play a role in determining the long run balance growth path.

Proposition 1. *Under the above assumptions, there exists a unique balanced growth path for the model where the distribution of firm productivity has a thick tail, and the aggregate growth rate, γ, is given as*

$$\gamma \;=\; \gamma_I + \sigma\sqrt{2q\left(\ddot{\theta}\right)\frac{1}{E[\ddot{\varepsilon}]}\tau\frac{\mathcal{F}}{\mathcal{N}}\int_l\int_{z=0}^{\infty}\phi_f(z,l)\nu(z,l)\,dz\,dl + 2\xi} \tag{25}$$

where $E[\ddot{\varepsilon}]$ denotes the expected (average) search effort in the whole economy.

The proof to this proposition is provided in appendix A.2. According to above expression, the aggregate growth rate in the economy exceeds the rate of average innovation, γ_I. The second term on the RHS measures the contribution of the knowledge spillover, both through the labor mobility channel and the exogenous learning channel, to aggregate growth.

The benefit to aggregate growth from the spillover of knowledge through the labor mobility channel is large when: (i) the probability of matching in the labor market is high ($q(\ddot{\theta})$); (ii) the average worker search effort in the economy is low, making it more likely a given match is with a worker from the productivity frontier; (iii) the probability of transferring the full amount of knowledge (τ) is high; (iv) the average number of workers employed by a firm ($\mathcal{N}/\mathcal{F}$) is low, exposing each worker to more jobs and hence more knowledge; and (v) firm post a high number of vacancies ($\nu(z,l)$).

4.8 Optimal Policy

Lucas and Moll (2014) found that in their model, where agents endogenously choose the rate of their learning, the market based equilibrium was inefficient. Improving one's own productivity has spillover effects for the wide economy by improving the distribution of productivity ideas from which other firms sample from. However, agents receive no direct benefit from this spillover, and hence invest less in learning than would be socially optimal.

In this paper's model, the market-based equilibrium described above also sub-optimal from the perspective of a social planner who attempts to maximize aggregate output net of total search costs. However, it is sub-optimal for different reasons than in the case of Lucas and Moll (2014). In the model above, the benefit of a firm's increase in productivity for other firms in the economy is inherent in the value of workers at the firm as workers receive a share of this knowledge increase when they diffuse the knowledge to other firms. Therefore, the social benefits of a firm's productivity are (partially) internalized by the worker's choice of search effort. Instead, the social inefficiencies within this model are a result of the fact that when firms and workers meet in a situation where knowledge transfer is possible, they must divide the gains from the knowledge surplus between themselves. Therefore, the private returns to either firms or workers are generally below the social returns. As a result, both firms and workers will tend to search with less intensity that would be optimal from the social planner's perspective.

In this section the value function and the policy choices of the social planner are derived and compared to the market equilibrium choices. Let $W(f(z,l,t))$ denote the value function of the social planner given the state of the economy described by the distribution of firms in the productivity-labor space, $f(z,l,t)$. The social planner's problem is to choose the vacancy posting rates and worker search efforts in order to maximize aggregate output net of total search costs.[20] More formally, the social planner maximizes

$$
\begin{aligned}
W\left(f(z,l,t)\right) = \max_{\{\nu(\cdot),\varepsilon(\cdot)\}} \int_{\tau=t}^{\infty} e^{-r(\tau-t)} \Bigg[& Y(f(z,l,\tau)) - \int_z\int_l c_\nu(\nu(z,l,\tau)) f(z,l,\tau)\, dl\, dz \\
& - \int_z\int_l (c_\varepsilon(\varepsilon(z,l,\tau)l)\, f(z,l,\tau)\, dl\, dz \Bigg]
\end{aligned}
$$

subject to the law of motion for the distribution of $f(z,l,t)$ given by the KFE specified in equation 20.

In the above equation, $Y(f(z,l,t))$ denotes the aggregate output (GDP) of the economy given the distribution of active firms $f(z,l,\tau)$, and $\int_z\int_l (c_\nu(\nu(z,l,\tau)) + c_\varepsilon(\varepsilon(z,l,\tau)l)\, f(z,l,\tau)\, dl\, dz$ denotes the total search cost incurred by all firms and workers in the economy at time τ.

Below, I follow the approach of Lucas and Moll (2014) and focus on the optimizing the 'marginal social value' of a firm with productivity z and labor input l, denoted as $w(z,l,t)$ where

$$
w(z,l,t) = \frac{\partial W\left(f(z,l,t)\right)}{\partial f(z,l,t)}
$$

The marginal social value can be interpreted as the marginal value to the social planner of

[20] By assumption, the social planner can only choose the search intensity of workers and firms. It cannot force firms and workers to accept particular matches against their will. Nor can the social planner affect which firms match with which workers in the labor market.

having one more firm operating with productivity z, and size l. Because the distribution of firms is defined in productivity-labor space, if we know the distribution of firms, we know the distribution of labor as well. So we can equivalently think of each firm as a collection of l workers at the point in productivity-labor space.

The marginal social value of a type (z, l) firm satisfies the following Bellman equation

$$\begin{aligned}
rw(z,l,t) = \frac{\partial Y(f(z,l,t))}{\partial f(z,l,t)} + \max_{\nu,\varepsilon} \Bigg\{ & -c_\nu(\nu) - c_\nu(\varepsilon) l \\
& + \gamma_I \frac{\partial w(z,l,t)}{\partial z} + \frac{\sigma^2}{2} \frac{\partial^2 w(z,l,t)}{\partial z \partial z} + \frac{\partial w(z,l,t)}{\partial t} \\
& + \nu q(\theta) \frac{\partial w(z,l,t)}{\partial l} \left(\int_{\tilde{z}=0}^{\infty} \int_{\tilde{l}} \mathbf{1}_{agree} h_\varepsilon(\tilde{z},\tilde{l},t)\, d\tilde{l}\, d\tilde{z} \right) \\
& + \nu q(\theta) \int_{\tilde{z}} \int_{\tilde{l}} \left(\int_y \mathbf{1}_{agree} \left[w(y,l,t) - w(z,l,t) \right] T(y;\tilde{z},z)\, dy \right) h_\varepsilon(\tilde{z},\tilde{l},t)\, d\tilde{l}\, d\tilde{z} \Bigg\} \\
& + \xi \int_{y=z}^{\infty} \left[w(y,l,t) - w(z,l,t) \right] \left(\int_{\tilde{l}} f(y,\tilde{l},t)\, d\tilde{l} \right) dy \\
& - \left[(\lambda+\delta) l + l\varepsilon\theta q(\theta) \int_{\tilde{z}=0}^{\infty} \int_{\tilde{l}} \mathbf{1}_{accept} f_\nu(\tilde{z},\tilde{l},t)\, d\tilde{l}\, d\tilde{z} \right] \frac{\partial w(z,l,t)}{\partial l} \\
& + \Psi(z,l,t)
\end{aligned} \tag{26}$$

The first line on the right hand side denotes the flow contribution of the firm (and its workers) to aggregate output net of search costs in the economy. The second line on the right hand side denotes the capital gain/loss resulting from the Brownian motion productivity shocks, and other changes to broader economic conditions. The term in the third line denotes the expected capital gain from the firm hiring more workers. The fourth and fifth lines denotes the expected capital gain from knowledge spillover through the labor mobility channel and exogenous learning. The sixth line denotes the capital loss from workers departing the firm. The final term, $\Psi(z, l, t)$, refers to the value that the additional firm (or equivalently the collection of workers at this additional firm) has for other firms in the economy. This term is defined as:

$$\begin{aligned}
\Psi(z,l,t) \equiv & \int_{\tilde{z}} \int_{\tilde{l}} \frac{\partial w(\tilde{z},\tilde{l},t)}{\partial \tilde{l}} f(\tilde{z},\tilde{l},t) \nu(\tilde{z},\tilde{l},t) q(\theta) \mathbf{1}_{agree} Dh_\varepsilon(z,l,t)\, d\tilde{l}\, d\tilde{z} \\
& + \left(\int_{\tilde{z}=0}^{z} \int_{\tilde{l}} \int_{x=\tilde{z}}^{z} \mathbf{1}_{agree} f(\tilde{z},\tilde{l},t) \nu(\tilde{z},\tilde{l},t) q(\theta) \left[w(x,\tilde{l},t) - w(y,\tilde{l},t) \right] T(x;y,z)\, dx\, d\tilde{l}\, d\tilde{z} \right) Dh_\varepsilon(z,l,t) \\
& + \frac{\partial f(z,t)}{\partial f(z,l,t)} \int_{\tilde{z}}^{z} \int_{\tilde{l}} f(\tilde{z},\tilde{l},t) \left[w(z,\tilde{l},t) - w(\tilde{z},\tilde{l},t) \right] d\tilde{l}, d\tilde{z}
\end{aligned}$$

where

$$Dh_\varepsilon(z,l,t) = \frac{\varepsilon l}{\int_x \int_{l_x} \varepsilon(x,l_x,t) l(x,l_x,t) \mathcal{F}^{act}(t) f(z,l,t)\, dl_x\, dx + \varepsilon(U,t) N^{unemp}(t)}$$

is the marginal increase in search effort at point (z, l) as a result of the additional firm's labor.

The first term on the RHS of the expression for $\Psi(z, l, t)$ denotes the expected capital gain to other firms (where $(\tilde{z}, \tilde{l})$ denotes the point in productivity-labor space of the other firm) from meeting and hiring one of the additional workers at the additional firm (z, l). The second term in the RHS denotes the expected capital gain to other (less productive) firms from having l additional workers who are capable of transferring productive knowledge from the additional firm. The final term denotes the improved exogenous learning opportunities for less productive firms due to the marginal increase in firms with productivity z.

The social planner's optimal choice of vacancies to post satisfies

$$\begin{aligned}\frac{\partial c_\nu(\nu)}{\partial \nu} &= q(\theta)\frac{\partial w(z,l,t)}{\partial l}\int_{\tilde{z}}\int_{\tilde{l}} \mathbf{1}_{agree} h_\varepsilon(\tilde{z},\tilde{l},t)\,d\tilde{l}\,d\tilde{z} \\ &\quad + q(\theta)\int_{\tilde{z}}\int_{\tilde{l}}\left(\int_y \mathbf{1}_{agree}\left[w(y,l,t) - w(z,l,t)\right] T(y;\tilde{z},z)\,dy\right) h_\varepsilon(\tilde{z},\tilde{l},t)\,d\tilde{l}\,d\tilde{z}\end{aligned}$$

where the left-hand side denotes the marginal cost of additional search effort, and the right-hand side denotes the marginal benefit in terms of improving the firm's labor supply and expected knowledge spillover. Note that the vacancy posting rate of firms in the market-based equilibrium (equation 6) differs from the social planner's choice of vacancy by the expected premium payment to the worker. Because in the market equilibrium, firms must share a fraction of the net benefit of knowledge spillover with the worker, the expected private benefit to the firm is lower than the socially benefit. As a result the vacancy posting rate in the market-based equilibrium with differ from the socially optimal rate.

The social planner's optimal choice of search effort for a worker satisfies

$$\begin{aligned}\frac{\partial c_\varepsilon(\varepsilon)}{\partial \varepsilon} &= -\frac{\partial w(z,l,t)}{\partial l}\theta q(\theta)\int_{\tilde{z}}\int_{\tilde{l}} \mathbf{1}_{agree} f_\nu(\tilde{z},\tilde{l},t)\,d\tilde{l}\,d\tilde{z} \\ &\quad + \int_{\tilde{z}}\int_{\tilde{l}} \frac{\partial w(\tilde{z},\tilde{l},t)}{\partial \tilde{l}} f(\tilde{z},\tilde{l},t)\nu(\tilde{z},\tilde{l},t)q(\theta)\mathbf{1}_{agree}\frac{1}{X}\,d\tilde{l}\,d\tilde{z} \\ &\quad + \left(\int_{\tilde{z}=0}^{z}\int_{\tilde{l}}\int_{x=\tilde{z}}^{z} \mathbf{1}_{agree} f(\tilde{z},\tilde{l},t)\nu(\tilde{z},\tilde{l},t)q(\theta)\left[w(x,\tilde{l},t) - w(y,\tilde{l},t)\right] T(x;y,z)\,dx\,d\tilde{l}\,d\tilde{z}\right)\frac{1}{X} \\ &= -\frac{\partial w(z,l,t)}{\partial l}\theta q(\theta)\int_{\tilde{z}}\int_{\tilde{l}} \mathbf{1}_{agree} f_\nu(\tilde{z},\tilde{l},t)\,d\tilde{l}\,d\tilde{z} \\ &\quad + \int_{\tilde{z}}\int_{\tilde{l}} \frac{\partial w(\tilde{z},\tilde{l},t)}{\partial \tilde{l}}\theta q(\theta)\mathbf{1}_{agree} f_\nu(\tilde{z},\tilde{l},t)\,d\tilde{l}\,d\tilde{z} \\ &\quad + \int_{\tilde{z}=0}^{z}\int_{\tilde{l}}\left(\int_{x=\tilde{z}}^{z} \mathbf{1}_{agree}\theta q(\theta)\left[w(x,\tilde{l},t) - w(y,\tilde{l},t)\right] T(x;y,z)\,dx\right) f_\nu(\tilde{z},\tilde{l},t)\,d\tilde{l}\,d\tilde{z}\end{aligned}$$

where $X = \int_x\int_{l_x} \varepsilon(x,l_x,t)l(x,l_x,t)\mathcal{F}^{act}(t)f(z,l,t)\,dl_x\,dx + \varepsilon(U,t)N^{unemp}(t)$ is the total search effort in the economy. The left-hand side denotes the marginal social cost of the extra search effort of a worker, the first term on the right-hand side denotes the expected loss to the (z, l) firm from losing the worker to another firm, the second term on the right-hand side denotes the expected capital gain from having the worker at another firm, and the final term on the right-hand side denotes the expected capital gain from knowledge spillover from the additional worker moving to a less productive firm. All together, we can interpret the right-hand side as the expected net capital gain, both in terms of labor and knowledge spillover, from reallocating the worker from one firm to another.

Similar to the case for firms, the choice of worker search effort from the social planner differs from the market-based choice of workers given by equation 10 as in the market equilibrium, workers choose their search effort based on the assumption they receive only part of the total returns of knowledge spillover. They do not take into account the broader social effects of the value of the firm to society.

5 Analysis

The following sections discusses the calibration and simulation of the theoretical model outlined above.

5.1 Calibration

Before the model can be simulated, function forms for several parts of the model must be chosen and parameter values must be defined. The parameters used in the simulation model are selected using a mixture of two different approaches. The values of some parameters are take directly from the literature or from various data sources. Other parameters are calibrated so that the balanced growth path of the model approximates several important moments in the New Zealand data.

5.1.1 Functional Forms:

Search costs: The search cost function for both firms and workers are assumed to be quadratic in form. I.e.

$$c_\nu(\nu) = \frac{\psi_\nu}{2}\nu^2$$
$$c_\varepsilon(\varepsilon) = \frac{\psi_\varepsilon}{2}\varepsilon^2$$

where ψ_ν and ψ_ε are the marginal search costs for firms and workers respectively.

Distribution of initial productivity draws: The distribution of initial productivity draws by inactive firms, $f_{new}(z,t)$, is assumed to be a truncated normal distribution. The mean of the normal distribution is centered on the lower bound of productivity (z_{lb}), and variance of the distribution will be chosen to match the average productive of new entrants relative to the average incumbent firm. In addition I truncate the right-hand tail so that new entrants can only enter with productivity in the lower half of the productivity grid. Truncating the right hand tail is an extreme assumption. However, it makes it clear that new entrants will not be the engine of long-run growth in the model.

Transfer friction (knowledge spillover function): Similar to the previous proof for the existence of a BGP in the model, I assume that workers can only transfer knowledge to a less productive firm when their current job responsibilities match the advertised job description. Therefore, if a firm matches with an employed from a firm with $\tilde{l}$ amount of labor, there is a $1/\tilde{l}$ probability that the worker is a match for the job type, and knowledge transfer can take place. I assume that there is a τ probability that worker transfers all of their productive knowledge, and a $1-\tau$ probability that the worker can only transfer some of the productive knowledge. In these cases on intermediate knowledge transfer, the amount of knowledge transferred is drawn from a Beta$(2,\kappa)$ distribution.[21]

The empirical evidence regarding the productivity gap suggests that the size of the receiving firm appears to be a factor in how much the hiring firm benefits from any knowledge spillover from more productive firms. I therefore include it in the distribution of knowledge spillover. More specifically, the transfer function $T(\cdot)$ takes on the following form when a worker from a firm with productivity $\tilde{z} > z$ moves to a firm with productivity z (and hence transfers knowledge),

$$T(y;z,l,\tilde{z},\tilde{l}) = \frac{1}{\tilde{l}} \times \begin{cases} \left(1-\frac{1}{l}\right) & y = z \\ (1-\tau)\frac{\text{Beta}(\frac{y-z}{\tilde{z}-z},2,\kappa)}{l} & z < y < \tilde{z} \\ \frac{\tau}{l} & y = \tilde{z} \end{cases}$$

and when the worker comes from a less productive firm, no knowledge transfer takes place.

Matching function: Given the total number of vacancy postings $\hat{V}$, and total worker search effort $\hat{E}$, the number of matches is given by a Cobb-Douglas equation

$$Q(\hat{V},\hat{E}) \quad = \quad q_{norm}\hat{V}^{q_{cd}}\hat{E}^{1-q_{cd}}$$

[21]The Beta distribution is chosen as it provides a flexible distribution that is bounded between the productivity levels of the hiring firm and the workers previous employer.

Table 2: Pre-defined parameter values

	Parameter	Model value
ρ	Elasticity of substitution between goods	5
λ	Probability of death	0.0069
$F^{act}(t)$	Number of active firms	1
r	Real discount rate	0.04
q_{cd}	The Cobb-Douglas share parameter on vacancies in the matching function	0.5
v_{max}	Maximum rate of vacancy postings	1
σ	Standard deviation of Brownian innovation shocks	0.2
ξ	Rate of exogenous learning	0.05

5.1.2 Pre-defined Parameter Values

Table 2 summarizes the values for parameters in the model that are take from the literature and other data sources.

The elasticity of substitutions between goods is set to $\rho = 5$ which is around the mid-point of the range of values commonly used in the literature. The probability of a worker dying, $\lambda = 0.0069$, is found simulating a Markov process using the quarterly worker transition values reported by Silverstone and Bell (2011) based on the nationally representative Household Labour Force Survey. This value includes external migration in the model's definition of death. The number of active firms is normalized to one. The real discount rate is set to $r = 0.04$, a value used by the Reserve Bank of New Zealand in their policy model (see Kamber et al., 2016). The relative weighting on vacancies in the matching function (the Cobb-Douglas coefficient) is set to $q_{cd} = 0.5$. In the steady state, this parameter is not separately identifiable from q_{norm}, so the value will not influence the BGP analysis of this paper. Vacancy postings are capped at the rate $v_{max} = 1$ for firms. The standard deviation of the Brownian motion innovation shocks, σ, is a difficult parameter to calibrate in the model. While there are several moments in the data that could be used to calibrate the parameter, these moments often imply very different values for σ, and are not always consistent with the other moments used to calibrate the model. Therefore, the value of σ is set to 0.2, a value that seems reasonable given the constraints of the other target moments. Finally, the rate of exogenous learning is set to five percent. Given the target moments and the implied cross-equation restrictions, only values in the range of around 3% to 6% are plausible for the model.[22]

[22] Below 3%, the model implies that the economy would shrink along the BGP without the labor mobility channel. Given that the BOS results indicate only half of innovating firms state new workers were an important source of innovation ideas, this seems to be an implausible scenario. Above 6%, and the implied tail thickness for the distribution of firm productivity would be thicker than that actually observed in the data.

Table 3: Calibrated parameters and target moments

	Parameter	Value	Target moment	Moment value Data	Moment value Model
N	Number of workers in the economy	16.5	Average firm size	13.7	14.7
ψ_ν	Marginal search cost for firms	47.6	Share of firms posting vacancies	0.83	0.81
ψ_ε	Marginal search cost for workers	13.4	Share of workers searching with more than half effort	0.5	0.50
q_{norm}	Matching efficiency parameter	2.48	Share of labor supplied by new workers	0.194	0.21
γ_I	Growth rate of innovation shocks	-0.049	Aggregate growth rate (γ)	0.021	0.021
σ^2_{new}	Variance of inactive firm productivity draws	2.74	Relative productivity of new entrants	0.33	0.33
τ	Probability of transferring all knowledge	0.347	Tail parameter in firm productivity data	1.7554	1.755
$\frac{2}{2+\kappa}$	Mean of beta distribution for knowledge spillover	0.4719	Avg. productivity gain from knowledge spillover	0.327	0.48
β	Worker's relative bargaining strength	0.1	Ratio of average wage changes for workers move to more/less productive firms	0.436	0.438
δ	Probability of becoming unemployed	0.050	Unemployment rate	0.12	0.13

5.1.3 Calibrated Parameters

Table 3 summarizes the remaining parameters in the simulation model that are calibrated by jointly matching important moments of the New Zealand data. While the parameter values are calibrated jointly, and hence influence the model's fit to all moments, the target moments were chosen with specific parameters in mind. In the table, the target moment corresponding to each parameter is presented next to that specific parameter.

The total number of workers in the economy is set to $N = 16.5$. This value is chosen to match the average firm size in the economy. According to Mills and Timmins (2004), the average New Zealand firm employs 13.7 employees.

The marginal search costs for firms (ψ_ν) and workers (ψ_ε) are difficult parameters to calibrate to as in the data search effort choices are not directly observed. According to the 2008 and 2013 Business Operations Surveys, around 83 percent of firms report having vacancies.

Because the choice of vacancy postings is a continuous variable in the model, the vacancy posting rate cannot be directly mapped into this value. I choose to set the marginal search cost of firms is set to $\psi_\nu = 47.6$ which implies that 83 percent of firms post more than half the maximum allowed vacancy posting rate. The search effort choices for workers are more difficult to observe. Therefore, I choose $\psi_\varepsilon = 13.4$ to imply that 50 percent employed workers search with more than half the maximum search effort.

The matching efficiency parameter, q_{norm}, is set to 2.48 targeting the 19.4 percent average share of labor supplied by new hires seen in table 1 in appendix C.

The trend growth rate in the Brownian motion innovation shocks is set to $\gamma_I = -4.9\%$ to imply an aggregate (real) growth rate of $\gamma = 2.1\%$, which is the average real growth rate observed in New Zealand since 1992.

Doan et al. (2012) find that during the period 2001 to 2006, entering cohorts of firms had an average labor productivity of around one third of that of incumbent firms. I set the variance of the distribution of initial productivity draws ($f_{new}(z,t)$) to $\sigma^2_{new} = 2.74$ so the model matches this fact.

The value of $\tau = 0.347$ (probability of full knowledge transfer when the job is a match) is chosen to match the observed tail thickness observed in the New Zealand firm data. The value of κ is set so that the average amount of knowledge spillover when hiring workers from more productive firms matches that observed in table 1.

The value of the worker's relative bargaining strength, $\beta = 0.1$, is chosen to match the ratio of income changes for workers who move to more productive firms relative to those who move to less productive firms, as seen in table 2 of appendix C.

Finally, the rate of exogenous separation into unemployment is set to $\delta = 0.050$, targeting an overall unemployment rate of 12 percent in the economy. This unemployment rate is the same value used by Albertini et al. (2012) and is higher than the official unemployment rate seen in the data to account for individuals classified as "not in the labor force" but who may be loosely attached to the labor market.[23]

5.2 Simulation results

The model is simulated using a discrete grid of productivity values and firm sizes. The model is solved by iterating between the value functions, policy rules, and Kolmogorov Forward Equation until the numerical values of each have converged. Presented below are the results of the simulated model.

Figure 1 plots the distribution of active firms in the economy. Sub-figure (a) shows the PDF of the distribution in the productivity-labor space. The plot shows that more productive firms tend to be, on average, larger than less productive firms. Sub-figure (b) plots the

[23]Silverstone and Bell (2011) show that around 0.8 percent of people employed during the average quarter were classified as not in the labor force in the previous quarter.

distribution of firms aggregated into the productivity space. The distribution features a thick tail for higher levels of productivity, matching the Pareto tail seen in the data. The mode of the distribution is at a productivity around seven times (exp(2)) the minimum productivity level needed to operate in the market. Sub-figure (c) shows the distribution of firms aggregated in terms of the firm size (number of workers). The distribution features a thick tail, with a tail parameter of 1.5, which is somewhat higher than the value of 1 that is implied by Zipf's law.

Figure 2 plots the policy choices of firms (the vacancy posting rate) and workers (search effort) along the balanced growth path. According to the estimation of the model, firms with higher levels of productivity post the most vacancies. Firms with larger levels of productivity have a larger desired firm size as a result of the imperfect competition in the goods market. As a result, highly productive firms that are also relatively small (upper-right of the plot) post vacancies at a high rate in order to increase their firm size. In addition, for all highly productive firms, the loss of a worker, either through the worker exogenously separating or choosing to move to another firm, has a relatively large impact on the firm's value. Therefore, larger firms are willing to post higher levels of vacancies to replace their higher worker attrition levels.

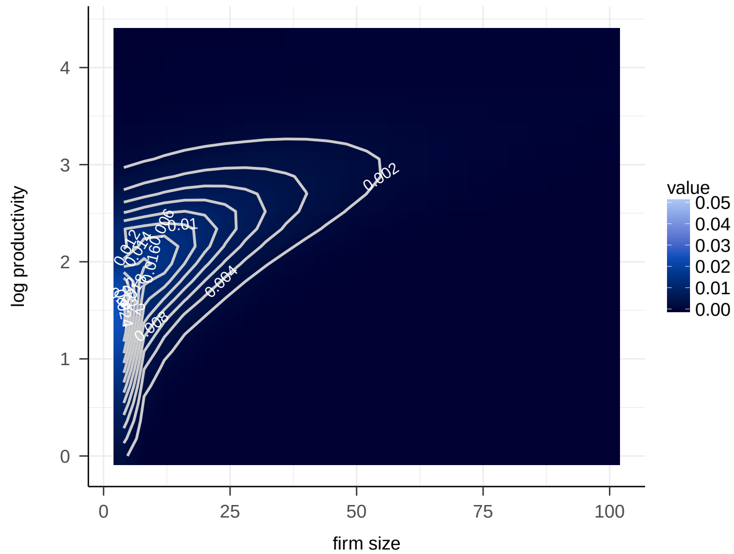
For large firms with very low productivity (bottom-right of the plot), the vacancy posting rate is relatively small, around 15 percent of the maximum rate allowed. While the benefit of acquiring new knowledge through the labor mobility channel is high for these firms, their large size makes absorbing and implementing new knowledge difficult. As a result, the expected returns from knowledge spillover are lower. Firms in this position instead tend to follow a strategy of letting their incumbent workforce naturally depreciate. This will allow the firm to become smaller and more able to adapt to new technology later on. For the small unproductive firms (bottom-left of the plot), who tend to be younger than the average firm, the expected returns from posting vacancies are high. Not only is the firm likely to acquire new knowledge through the labor mobility channel (which it can easily adapt due to the firm's size), but even if the match does not yield knowledge spillover, the addition of more labor at the firm is still valuable to a firm of a small size. As a result, the smallest firms with low productivity also post at a high vacancy rate in the model.[24]

With regards to worker search effort, workers at the least productive firms have a strong incentive to search for jobs at other firms and search with a very high intensity. For workers at the least productive firms (bottom of the plot), potential new job matches are likely to yield a job at a more productive firm, which not only pays better, but also has greater opportunities to learn productive knowledge (yielding higher income in the future). Workers at small, highly productive firms (top-left of the plot) have the strongest incentive to search for new jobs. Their productive knowledge is highly valuable to other firms, yielding large premium knowledge payments when the worker moves. In addition, the small size of their current firm means they preform a wide range of the jobs within a firm, giving them a high probability of being able to transfer knowledge.

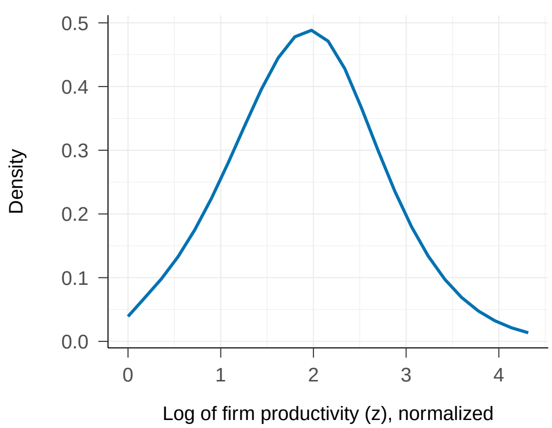
[24] Out of all the firms in the productivity-labor space, the expected returns from posting vacancies is highest for small, highly productive firms.

Figure 1: Distribution of firms - Market equilibrium

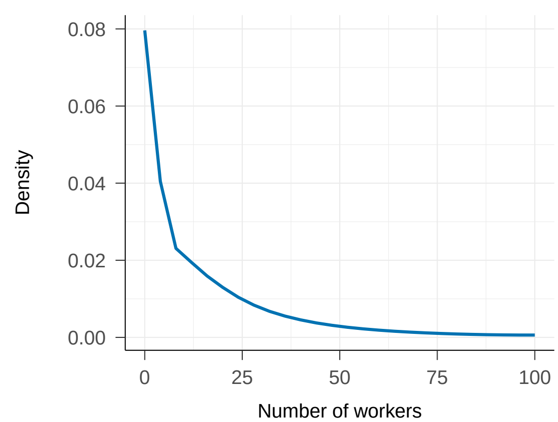
(a) Joint distribution of firm productivity and size



(b) Distribution of firm productivity $\phi_f(z)$

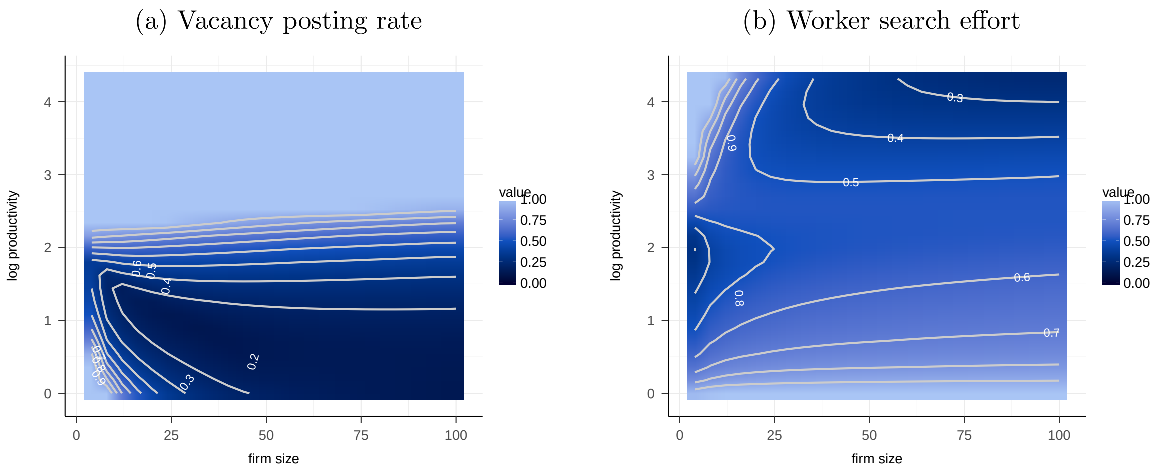


(c) Distribution of firm size $\phi_f(l)$



Notes: (a): Shading represents the density of the distribution of active firms with lighter (darker) colors corresponding the a higher (lower) density value. The while lines represent contours of the same density value.
(b): The PDF of the distribution has been normalized so that zero (ln(1)) represents the minimum productivity level needed for an inactive firm to become active.

Figure 2: Policy choices — Market equilibrium



Notes: Graphs show the policy choice (vacancy posting rate and worker search effort) for firms and workers at each point in the productivity-labor space. Shading represents the value of policy choice with lighter (darker) colors corresponding the higher (lower) levels of search intensity. The while lines represent contours of similar policy choices.

For workers at larger firms with high levels of productivity (top-right of the plot), the large firm size means each worker is very specialized in their job role. As a result, when searching for a new job, it is unlikely that they will match the job description of advertised jobs. Thus, the expected returns to spilling over knowledge are low for these workers, even though their productive knowledge is high. Furthermore, because their current employer has is highly productive, the workers receive a relatively high wage rate for supplying their labor to the firm. This lowers the worker's incentive to search for a new job as moving to a less productive firm without the diffusion of knowledge will result in lower wages.

Figure 3 shows how the distribution of firms in productivity-labor space ($\phi_f(z,l)$) in panel (a) differs from the distribution of worker search effort ($\phi_{h\varepsilon}(z,l)$) in panel (b), which is the distribution of new knowledge that can spillover through labor mobility. Because more productive firms tend to be larger, the distribution of potential new knowledge, $\phi_{h\varepsilon}(z,l)$, skews towards higher productivity levels than the distribution of firm productivity. However, the frictions inherent in transferring knowledge means that the distribution of firm productivity will always lag behind the distribution of potential new knowledge.

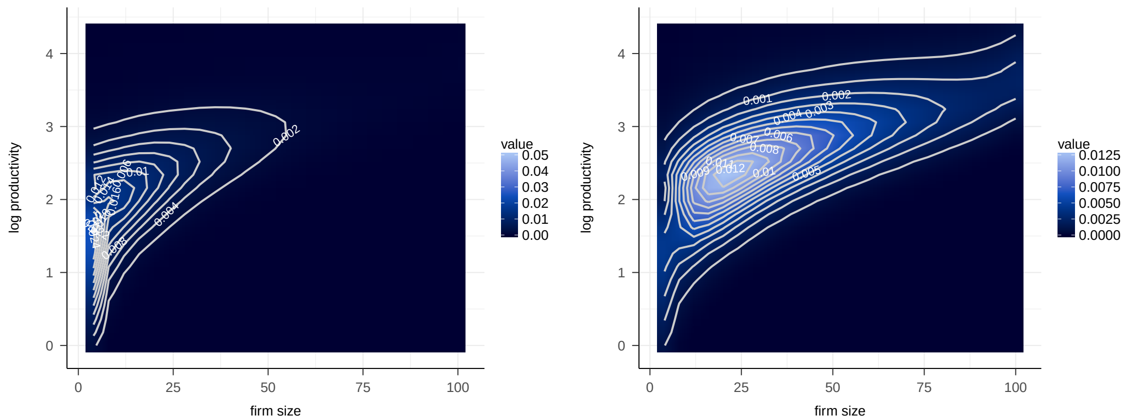
5.2.1 Additional Dimensions of Fit

While the model does a reasonable job of matching the target moments related to the diffusion of knowledge, the results do not tell us if knowledge spillover through the labor mobility channel can explain other labor market moments.

Table 4 plots the fraction of hires made by each productivity decile of firms from each of the productivity deciles, both from the data (see Kirker and Sanderson (2017) for more details), and from the model. Part (a) of table 4 shows that firms in all productivity deciles have a broad access to hire workers from all other productivity deciles in the economy. Firms show

Figure 3: Distribution of knowledge firms sample from — Market equilibrium

(a) Distribution of firm productivity ($\phi_f(z, l)$) (b) Distribution of worker search effort ($\phi_{h\varepsilon}(z, l)$)



Notes: Shading represents the value of policy choice with lighter (darker) colors corresponding the higher (lower) levels of search intensity. The while lines represent contours of similar policy choices.

some weak bias in their hiring towards firms around their own productivity decile, mainly at the higher and lower ends of the productivity distribution. In part (b) of table 4, we are able to see that in the model, there is a strong bias towards hiring new workers from similar, or slightly less productive, declies.

Because labor is more valuable at more productive firms, and workers have greater learning opportunities at more productive firms, firms find it relatively easy to lure away workers from slightly less productive firms. This generates the strong hiring pattern seen along the diagonal of the table in part (b) of table 4. The same effects make it difficult for firms to hire away workers from more productive firms without the existence of some knowledge spillover. As a result, we do not see many hires that are sourced from other firms that are slightly more productive than the hiring firm. However, in the top-right part of the table, we do see that the least productive firms do hire from other firms that are significantly more productive than they are. These matches, have the largest expected surplus from knowledge spillover.

Overall the spillover of productive knowledge through the labor mobility channel, and the incentives this implies for workers to acquire knowledge, cannot explain much of the pattern of labor market flows observed in the data. However, it is important to keep in mind that the structure of the model fails to capture other reasons that workers may have for moving jobs. These other factors could play a large role in determining the overall hiring patterns seen in the data.

5.2.2 Effects of Knowledge Spillover Through the Labor Mobility Channel

To better understand how the economy is affected by knowledge spillover through the labor mobility channel, a counter-factual exercise is conducted. The balanced growth path of the model is re-computed under the assumption that workers cannot diffuse knowledge between

Table 4: Worker transitions between different labor productivity deciles

(a) Data

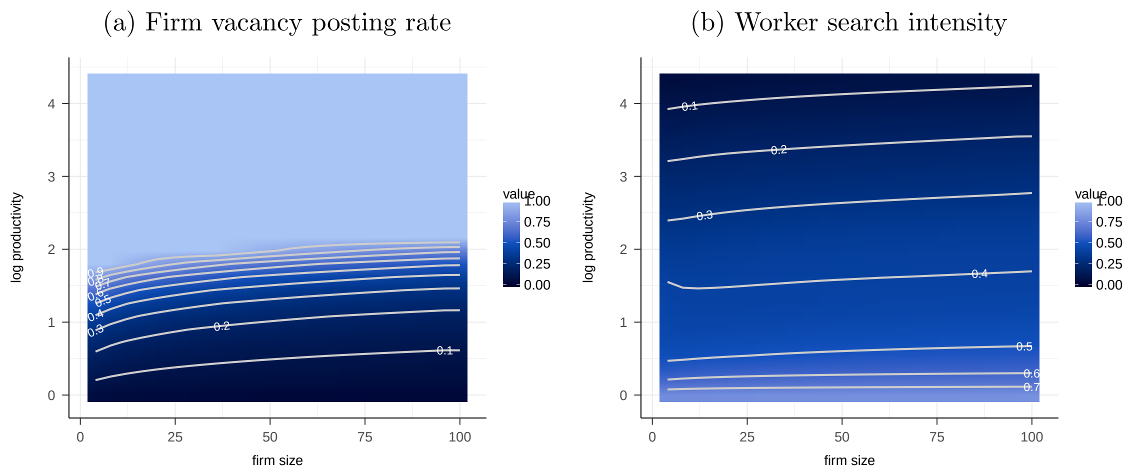
Hiring firm's prod. decile	Source of new employee hires: PFP productivity decile 1	2	3	4	5	6	7	8	9	10
1	0.14	0.17	0.1	0.1	0.09	0.08	0.08	0.1	0.07	0.08
2	0.13	0.17	0.11	0.11	0.09	0.08	0.08	0.09	0.06	0.06
3	0.1	0.15	0.18	0.12	0.09	0.08	0.07	0.08	0.06	0.06
4	0.11	0.15	0.12	0.12	0.11	0.09	0.08	0.1	0.07	0.07
5	0.1	0.14	0.11	0.11	0.1	0.09	0.09	0.11	0.07	0.07
6	0.1	0.14	0.09	0.1	0.1	0.09	0.09	0.11	0.08	0.09
7	0.1	0.13	0.09	0.1	0.1	0.09	0.11	0.11	0.09	0.09
8	0.09	0.12	0.08	0.09	0.09	0.09	0.11	0.11	0.1	0.11
9	0.1	0.1	0.07	0.08	0.08	0.08	0.09	0.12	0.12	0.14
10	0.09	0.09	0.06	0.07	0.07	0.07	0.08	0.1	0.13	0.24

(b) Model

Hiring firm's prod. decile	Source of new employee hires: Productivity decile 1	2	3	4	5	6	7	8	9	10
1	0.09	0.01	0.03	0.05	0.07	0.1	0.26	0.12	0.19	0.08
2	0.39	0.04	0	0.02	0.04	0.06	0.18	0.08	0.13	0.06
3	0.3	0.3	0.08	0	0.02	0.03	0.1	0.05	0.08	0.04
4	0.19	0.19	0.31	0.18	0	0.01	0.04	0.02	0.04	0.02
5	0.11	0.11	0.17	0.29	0.28	0	0.01	0.01	0.01	0.01
6	0.06	0.06	0.1	0.16	0.26	0.35	0	0	0.01	0
7	0.03	0.03	0.05	0.08	0.12	0.2	0.5	0	0	0
8	0.02	0.02	0.03	0.05	0.08	0.13	0.42	0.25	0	0
9	0.01	0.01	0.02	0.03	0.06	0.09	0.29	0.17	0.3	0.01
10	0.01	0.01	0.02	0.02	0.04	0.07	0.22	0.13	0.3	0.19

Notes: Each cell shows the fraction of total hires made by all firms in each productivity decile (row) from each of the productivity deciles in the model. Each row sums to one. Cells are shaded based upon the fraction of hires, with darker shades corresponding to a higher fraction of total hires. Deciles correspond to the firm's productivity ranking within each year, with decile 10 referring to the most productive firms.

Figure 4: Search policy choices — No knowledge spillover through labor mobility



Notes: Shading represents the density of the distribution of active firms with lighter (darker) colors corresponding the a higher (lower) density value. The while lines represent contours of the same density value.

firms. This is accomplished by setting τ to zero and eliminating the possibility of workers transmitting intermediate levels of productivity. All other parameters remain at the same value as in the market-based balanced growth path described above. As a result, the only benefit of new hires to firms is the additional labor they supply to the firm's production process.

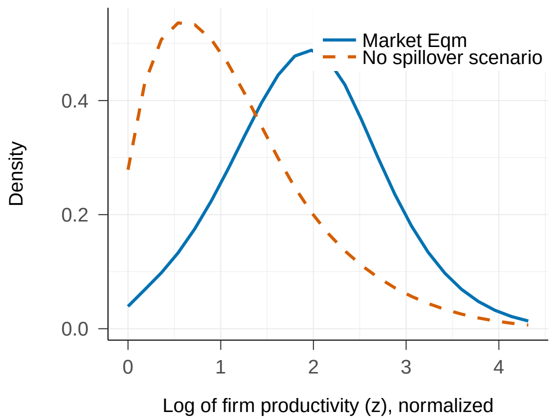
Figure 4 plots the vacancy posting rate and worker search effort choices along the new balanced growth path without knowledge spillover through the labor mobility channel. Similar to the case where there was knowledge spillover, it is the largest least productive firms (bottom-right of the plot) that have the smallest incentive to post vacancies. However, with the removal of knowledge spillover through labor mobility, now relatively small unproductive firms (bottom-left of the plot) also have a much lower incentive to search for new workers. With the low level of productivity at these firms, the wages offered by these firms are relatively low. As a result, all less productive firms find it harder to attract any of the new potential hires they are matched with. Therefore, with lower expected benefits from searching, these firms choose to search with less effort than more productive firms.

Firms that have relatively high productivity levels (upper part of the plot) have a strong incentive to post vacancies. As described below, the incentive of workers to leave these firms is reduced, which increases the expected employment spell at the firms. This makes investing in searching for new workers more valuable to the productive firms.

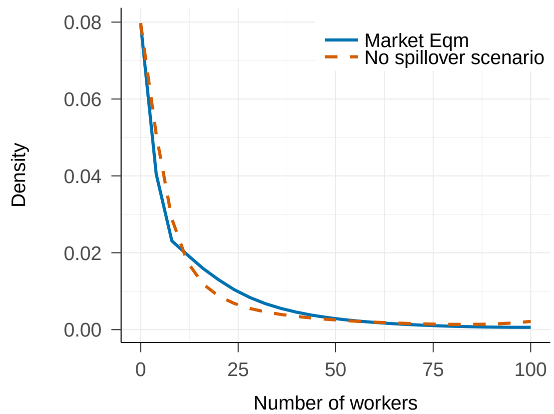
Without the knowledge premium earned from diffusing knowledge from more to less productive firms, the incentive for workers at the most productive firms to search for a new job is dramatically reduced. Workers at firms with high levels of productivity (top part of the plot) now have a stronger incentive to stay at these firms as wages are positively correlated with firm productivity. At the other end of the productivity spectrum, workers at less productive firms have a (relatively) stronger incentive to move firms in order to improve their wage prospects at a more productive firm. Therefore, they still search with high intensity.

Figure 5: Distribution of firms assuming no knowledge spillover through labor mobility

(a) Distribution of firm productivity $\phi_f(z)$



(b) Distribution of firm size $\phi_f(l)$



Notes: (a): The PDF of the distribution has been normalized so that zero (ln(1)) represents the minimum productivity level needed for an inactive firm to become active.

Along this new balanced growth path without the spillover of knowledge through the labor mobility channel, the aggregate growth rate of the economy falls to 1.4 percent (from the 2.1 percent with knowledge spillover through labor mobility). Figure 1 shows the distributions of relative firm productivity and size along the balanced growth path without knowledge spillover through the labor mobility channel, against the distributions in the previous market equilibrium.

Sub-figure (a) shows that the distribution of firm productivity when knowledge spillover through labor mobility is removed skews dramatically lower. Because less productive firms now have less opportunities to improve their productivity (relying only on innovation shocks and exogenous learning), firms find it more difficult to escape from the area around the lower bound on productivity, leading to an increase in the number of (relatively) less productive firms in equilibrium.

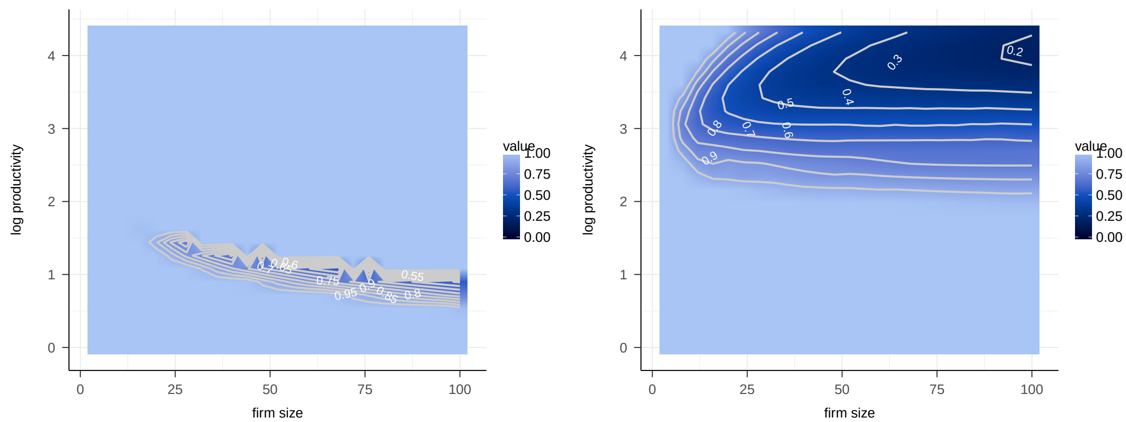
Interestingly, despite the large effect the knowledge spillover channel has the distribution of firm productivity, it has very little effect on the distribution of firm size. Sub-figure (b) of figure 1 shows that the knowledge spillover through the labor mobility channel has virtually no effect on the distribution of firm size in the economy. Overall there are slightly more larger sized firms, and fewer medium sized firms, once the spillover of knowledge through the labor mobility channel is removed. This is driven by the search behavior of workers. Because workers at more productive firms are less likely to leave, productive firms retain more of their incumbent workers, allowing them to grow larger in size.

5.2.3 Optimal Social Allocation

Figure 6 plots the socially optimal choices of search effort by firms and workers in the model from the perspective of a social planner who is focused on maximizing aggregate output

Figure 6: Search policy choices — Socially optimal search efforts

(a) Firm vacancy posting rate (b) Worker search intensity



Notes: Shading represents the density of the distribution of active firms with lighter (darker) colors corresponding the a higher (lower) density value. The while lines represent contours of the same density value.

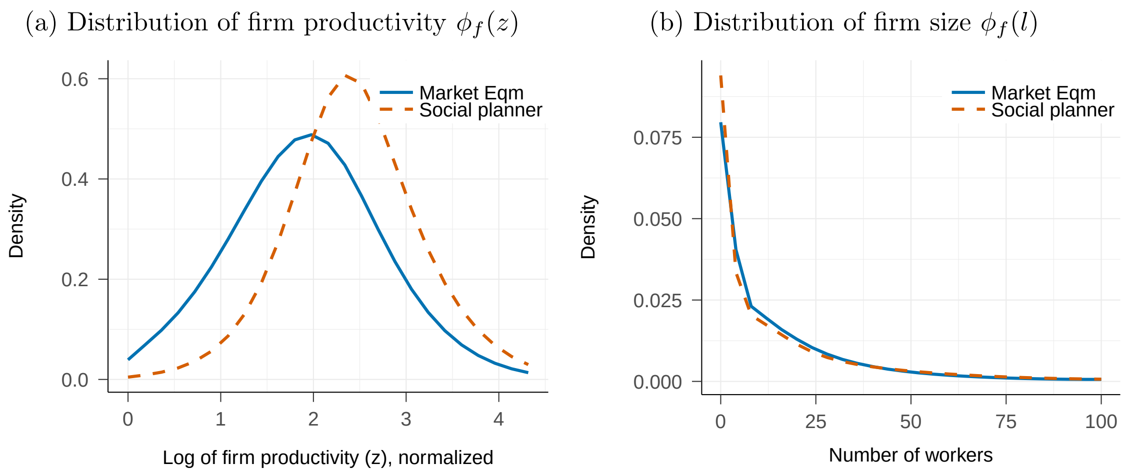
net of search costs. Relative to the market-based equilibrium, the socially optimal vacancy posting rate for all firms is higher. For the vast majority of points in the productivity-firm size space, this means workers search with the maximum amount of search effort. Sub-figure (a) shows that for a small wedge of values in the productivity-labor space, the optimal search effort is less than the maximum. However, the minimum search effort is still little above 0.5, and higher than in the market-based equilbrium.[25]

In terms of worker search effort, the socially optimal choice of search effort is generally high for workers at firms with low productivity, and lower for workers at large very productive firms. From the perspective of the social planner, it is optimal for workers to be assigned to firms that are already large, and very productive. Since workers at large productive firms are less likely to facilitate knowledge spillover, the optimal search effort of these workers is low. For workers at small, highly productive firms, the loss of output at the productive firm when the worker leaves is out-weighted by the social benefit of that worker's ability to raise the productivity of a larger, less productive firm. Finally, for workers at the least productive firms, the social value of reallocating these workers to more productive firms is large, encouraging the social planner to choose high levels of search effort for the workers.

Along the balanced growth path of the economy under the social planner, the economy grows at a rate of 2.11 percent, very close to the the 2.1 percent in the market equilibrium. Despite the higher vacancy posting rates by firms, the increased congestion in the labor market limits

[25]The shape and size of this wedge relates to the calibration choice of the model. The social value of adding a worker to a firm is generally increasing in the firm's productivity. In the current calibration of the model, the social value of adding labor to large, unproductive firms is actually negative. This effect encourages the social planner to choose a vacancy posting rate that is increasing with firm productivity. On the other hand, the expected returns from knowledge spillover is decreasing in firm productivity. This effect, encourages the social planner to post high vacancy rates at less productive effects. These two effects working in opposite directions explains why we see a small band of firms posting less than the maximum vacancy posting rate. At these points, the net effect of the two effects is at its weakest.

Figure 7: Distribution of firms assuming socially optimal policy choices



Notes: (a): The PDF of the distribution has been normalized so that zero (ln(1)) represents the minimum productivity level needed for an inactive firm to become active.

the ability of the social planner to squeeze out significantly more productivity growth.

Panel (a) of figure 7 shows that the distribution of firm productivity in the socially optimal equilibrium is skewed slightly more to the right when compared to the distribution under the market place equilibrium. The higher vacancy posting rate by less productive firms sees these firms leave the area around the lower bound at a faster rate, shifting the mass of the distribution higher. Panel (b), shows that the distribution of firm size is once again relatively unaffected by changes in the spillover of knowledge through the labor mobility channel.

6 Conclusion

There is growing empirical support for the important role that the labor mobility channel plays in facilitating the spill over of productive knowledge between firms. This paper develops a modeling framework in which the macro-level implications of knowledge spillover through the labor mobility channel can be assessed. Within the model, workers absorb some of the productive knowledge of their current employer, and are able to transmit this knowledge to less productive firms when they move jobs. Through this process, less productive firms are able to gain access to some of the productive knowledge closer to the frontier. The mechanism by which firms and workers are matched is embedded within an on-the-job search-and-matching model. Through the lens of this structure, the rate at which firms are exposed to new knowledge can be viewed in terms of their endogenous choice of vacancy posting rate. In addition, the distribution of new knowledge the firms can sample from is determined by the search effort choices of workers on the other side of the labor market.

When the model is calibrated to key macroeconomic and firm-level moments in New Zealand data, the model does a reasonable job at matching the target moments. However, there are areas in which the spillover mechanism does not do a good job in matching the patterns observed in the data. The most notable being the share of workers hired from different productivity deciles. In the model, there is a stronger incentive to hire from other firms with similar levels of productivity than what is observed in the data.

To assess the importance of knowledge spillover through the labor mobility channel on the model, a counter-factual exercise is performed in which the ability of workers to transmit knowledge through this channel is turned off. The results show that under this scenario, the search effort of workers declines, especially for those at the most productive firms (who no longer can receive compensation for diffusing knowledge to less productive firms). When the knowledge spillover effects are turned off, aggregate growth is around 1.4 percent, lower than the 2.1 percent observed in the data (where knowledge spillover is assumed to occur). When examining the distribution of firms, knowledge spillover through the labor mobility channel tends to skew the distribution of firm productivity higher, making the overall distribution look more symmetrical by allowing the less productive firms greater opportunities to leave the lower levels of productivity at a faster rate. In terms of the distribution of firm size, knowledge spillover through the labor mobility channel has very little effect.

Because firms and workers split the net surplus from knowledge spillover, the private returns to firms from searching for labor are generally lower than the social returns. As a thought experiment, the model is simulated under the assumption a social planner, who is interested in maximizing aggregate output net of search costs, chooses the search intensities of workers and firms. The results show that under the social planner, agents in the economy generally search with higher intensity (especially firms). However, because of the increased congestion in the labor market, aggregate growth under the social planner is not significantly higher than the market-based equilibrium (2.11 vs 2.1 percent). However, the distribution of productivity (and hence incomes) is different, with a larger share of more productive firms under the social planner.

Traditional in macroeconomic models, the effect of labor on firm productivity has typically either focused on the miss-allocation of worker skill, which has no roll for labor mobility to affect the long-run growth of an economy, or focused on the spillover of knowledge only to new firms, ignoring job-to-job transitions which are by far the largest flows observed in the labor market. This model shows that knowledge spillover through the labor mobility channel can have an effect on both the aggregate growth rate as well as the relative distribution of productivity within an economy. While the model is not able to explain all of the pattern of labor movements seen in the data, it does provide an interesting framework in which labor market policy interventions can have an effect both on the on the long-run growth and the distribution of relative firm productivity of an economy.

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