Appendix to – Learning Through Hiring: Labor Mobility as a Mechanism for Endogenous Growth

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Careful consideration has been given to the privacy, security, and confidentiality issues associated with using administrative and survey data in the IDI. Further detail can be found in the Privacy impact assessment for the Integrated Data Infrastructure available from www.stats.govt.nz.

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A Proofs

A.1 Number of active firms

Following the approach of Luttmer (2012), first begin by integrating the KFE (equation 20) over all firm sizes l

$$\begin{aligned}\frac{\partial f(z,t)}{\partial t} =& -\gamma_I \frac{\partial f(z,t)}{\partial z} + \frac{1}{2}\sigma^2 \frac{\partial^2 f(z,t)}{\partial z \partial z} \\ &+ \int_{\tilde{z}=0}^{z} \int_l f(\tilde{z},l,t)\, \nu q(\theta) \int_s \left(\int_{\hat{z}} \int_{\hat{l}} \mathbf{1}_{accept} T(z,\tilde{z},\hat{z},s) h_\varepsilon(\hat{z},\hat{l},t;s)\, d\hat{l}\, d\hat{z} \right) d\tilde{z}\, ds\, dl \\ &- \int_l f(z,l,t) \nu q(\theta) \int_s \left(\int_{\hat{z}=z}^{\infty} \int_{\hat{l}} \mathbf{1}_{accept} h_\varepsilon(\hat{z},\hat{l},t;s)\, d\hat{l}\, d\hat{z} \right) ds\, dl \\ &+ \frac{\mathcal{F}^{inact}(t)}{\mathcal{F}(t)} f_{new}(z,t) 1_{\Pi(z,0,t)>E(t)} \end{aligned} \tag{1}$$

Integrate over productivity levels to obtain the CDF version of the KFE in productivity space

$$\begin{aligned}\frac{\partial F(\bar{z},t)}{\partial t} =& -\gamma_I \frac{\partial F(\bar{z},t)}{\partial \bar{z}} + \frac{1}{2}\sigma^2 \frac{\partial^2 F(\bar{z},t)}{\partial \bar{z} \partial \bar{z}} \\ &+ \int_{z=0}^{\bar{z}} \left(\int_{\tilde{z}=0}^{z} \int_l f(\tilde{z},l,t)\, \nu q(\theta) \int_s \left(\int_{\hat{z}} \int_{\hat{l}} \mathbf{1}_{accept} T(z,\tilde{z},\hat{z},s) h_\varepsilon(\hat{z},\hat{l},t;s)\, d\hat{l}\, d\hat{z} \right) d\tilde{z}\, ds\, dl \right) dz \\ &- \int_{z=0}^{\bar{z}} \left(\int_l f(z,l,t) \nu q(\theta) \int_s \left(\int_{\hat{z}=z}^{\infty} \int_{\hat{l}} \mathbf{1}_{accept} h_\varepsilon(\hat{z},\hat{l},t;s)\, d\hat{l}\, d\hat{z} \right) ds\, dl \right) dz \\ &+ \frac{\mathcal{F}^{inact}(t)}{\mathcal{F}(t)} F_{new}(\bar{z},t) 1_{\Pi(\bar{z},0,t)>E(t)} \end{aligned} \tag{2}$$

Do a change of variable. Define $G(\overline{z},t) = 1 - F(\overline{z},t)$ as the CDF integrating from right to left.

$$\begin{aligned}-\frac{\partial G(\bar{z},t)}{\partial t} =& \gamma_I \frac{\partial G(\bar{z},t)}{\partial \bar{z}} - \frac{1}{2}\sigma^2 \frac{\partial^2 G(\bar{z},t)}{\partial \bar{z} \partial \bar{z}} \\ &+ \int_{z=0}^{\bar{z}} \left(\int_{\tilde{z}=0}^{z} \int_l f(\tilde{z},l,t)\, \nu q(\theta) \int_s \left(\int_{\hat{z}} \int_{\hat{l}} \mathbf{1}_{accept} T(z,\tilde{z},\hat{z},s) h_\varepsilon(\hat{z},\hat{l},t;s)\, d\hat{l}\, d\hat{z} \right) d\tilde{z}\, ds\, dl \right) dz \\ &- \int_{z=0}^{\bar{z}} \left(\int_l f(z,l,t) \nu q(\theta) \int_s \left(\int_{\hat{z}=z}^{\infty} \int_{\hat{l}} 1_{accept} h_\varepsilon(\hat{z},\hat{l},t;s)\, d\hat{l}\, d\hat{z} \right) ds\, dl \right) dz \\ &- \frac{\mathcal{F}^{inact}(t)}{\mathcal{F}(t)} (1 - G_{Ent}(\bar{z},t)) 1_{\Pi(\bar{z},0,t)>E(t)} \end{aligned} \tag{3}$$

Evaluating this express at $\overline{z} = z_{lb}$ and noting that $G(z_{lb},t) = 1$ yields

$$-\frac{\partial G(z_{lb},t)}{\partial t} = \gamma_I \frac{\partial G(z_{lb},t)}{\partial \bar{z}} - \frac{1}{2}\sigma^2 \frac{\partial^2 G(z_{lb},t)}{\partial \bar{z}\partial \bar{z}} - \frac{\mathcal{F}^{inact}(t)}{\mathcal{F}(t)}(1 - G_{Ent}(z_{lb},t))1_{\Pi(z_{lb},0,t)>E(t)} \quad (4)$$

As Luttmer (2015) argues, $\partial G(z_{lb},t)/\partial z = 0$. Therefore

$$\frac{\partial G(z_{lb},t)}{\partial t} = \frac{1}{2}\sigma^2 \frac{\partial^2 G(z_{lb},t)}{\partial \bar{z}\partial \bar{z}} + \frac{\mathcal{F}(t)}{\mathcal{F}(t)}(1 - G_{Ent}(z_{lb},t))$$

A.2 Existence of a unique balanced growth path

Applying the functional form assumptions, and integrating the KFE over all firm sizes yields a KFE in productivity space of the form

$$\begin{aligned}
0 =& (\gamma - \gamma_I)\frac{\partial \phi_f(z)}{\partial z} + \frac{1}{2}\sigma^2 \frac{\partial^2 \phi_f(z)}{\partial z \partial z} \\
&+ \int_l \int_{\tilde{z}=0}^{z} \phi_f(\tilde{z},l)\nu(\tilde{z},l)q\left(\ddot{\theta}\right)\left(\int_{\hat{z}>z} \mathbf{1}_{accept} T(z;\tilde{z},l,\hat{z}) \int_{\hat{l}} \frac{\phi_{h\varepsilon}(\hat{z},\hat{l})}{\hat{l}}\, d\hat{l}\, d\hat{z}\right) d\tilde{z}\, dl \\
&- \int_l \phi_f(z,l)\nu(z,l)q(\ddot{\theta})\left(\int_{\hat{z}=z}^{\infty}\int_{\hat{l}}\left(\int_{y=z}^{\hat{z}} \mathbf{1}_{accept} T(y;\tilde{z},l,\hat{z}), dy\right)\frac{\phi_{h\varepsilon}(\hat{z},\hat{l})}{\hat{l}}\, d\hat{l}\, d\hat{z}\right) dl \\
&+ \xi\phi_f(z)\Phi_f(z) \\
&- \xi\phi_f(z)[1 - \Phi_f(z)] \\
&+ \frac{\ddot{\mathcal{F}}^{inact}}{\ddot{\mathcal{F}}}\ddot{f}_{new}(z)\mathbf{1}_{\ddot{\Pi}(z,0)>\ddot{E}}
\end{aligned}$$

We now consider the behavior of the KFE in the tail of the distribution when the productivity of a firm (z) is very high. Below, the behavior of the second and third lines of the KFE in the tail of the distribution above are analyzed separately. Recall that the second line refers to firms with productivity $\tilde{z} < z$ who advance to z through knowledge spillover, and the third line refers to firms with productivity z who improve their productivity.

Assuming that $\phi_f(z)$ has an exponential tail (implying the distribution of firm productivity level has a Pareto tail), with tail parameter ζ and scale parameter k implies that

$$\lim_{z\to\infty} \phi_f(z) = \lim_{z\to\infty} k\zeta e^{-\zeta z}$$

In addition:

$$\begin{aligned}
\lim_{z\to\infty} \frac{\partial \phi_f(z)/\partial z}{e^{-\zeta z}} &= -\zeta^2 k \\
\lim_{z\to\infty} \frac{\partial^2 \phi_f(z)/(\partial z)^2}{e^{-\zeta z}} &= \zeta^3 k
\end{aligned}$$

Dividing the KFE by $e^{-\zeta z}$ and taking the limit as $z \to \infty$yields:

$$\begin{aligned}
0 =&(\gamma - \gamma_I)(-k\zeta^2) + \frac{1}{2}\sigma^2(k\zeta^3) \\
&+ \lim_{z\to\infty} \int_l \int_{\tilde{z}=0}^{z} \phi_f(\tilde{z}, l)\nu(\tilde{z}, l) q\left(\ddot{\theta}\right) \frac{\int_{\hat{z}>z} \mathbf{1}_{accept} T(z; \tilde{z}, l, \hat{z}) \int_{\hat{l}} \frac{\phi_{h\varepsilon}(\hat{z},\hat{l})}{\hat{l}}\, d\hat{l}\, d\hat{z}}{e^{-\zeta z}}\, d\tilde{z}\, dl \\
&- \lim_{z\to\infty} \int_l \phi_f(z, l)\nu(z, l) q(\ddot{\theta}) \frac{\int_{\hat{z}=z}^{\infty} \int_{\hat{l}} \left(\int_{y=z}^{\hat{z}} \mathbf{1}_{accept} T(y; \tilde{z}, l, \hat{z})\, dy\right) \frac{\phi_{h\varepsilon}(\hat{z},\hat{l})}{\hat{l}}\, d\hat{l}\, d\hat{z}}{e^{-\zeta z}}\, dl
\end{aligned}$$

Limit of the term for firms with productivity $\tilde{z} < z$ advancing to z Using L'Hoptial's rule in combination with the fact that, by assumption, the probability that all knowledge can be transferred is τ, and the fact that for large enough values of z, all worker-firm matches with knowledge spillover of z will be accepted ($1_{accept} = 1$ for large enough z), yields the following limit for the second line of the above equation:

$$- \int_l \int_{\tilde{z}=0}^{\infty} \phi_f(\tilde{z}, l)\nu(\tilde{z}, l) q\left(\ddot{\theta}\right) \tau \lim_{z\to\infty} \frac{\left(\int_{\hat{l}} \frac{\phi_{h\varepsilon}(z,\hat{l})}{\hat{l}}\, d\hat{l}\right)}{e^{-\zeta z}}\, d\tilde{z}\, dl$$

The distribution of worker search effort is given by

$$\phi_{h\varepsilon}(z, \hat{l}) = \frac{\varepsilon(z, \hat{l}) h(z, \hat{l})}{\int_z \int_l \left[\varepsilon(z, l) h(z, l)\right]\, dz\, dl + \varepsilon(U) \frac{\mathcal{N}^{unemp}}{\mathcal{N}}}$$

Using the fact that the expected search effort in the economy is given as

$$E[\varepsilon] = \frac{\int_z \int_l \left[\varepsilon(z, l) h(z, l)\right]\, dz\, dl + \varepsilon(U) \frac{\mathcal{N}^{unemp}}{\mathcal{N}}}{\int_z \int_l \left[h(z, l)\right]\, dz\, dl + \frac{\mathcal{N}^{unemp}}{\mathcal{N}}}$$

The distribution of worker search effort is related to the distribution of labor via

$$\phi_{h\varepsilon}(z, \hat{l}) = \frac{\varepsilon(z, \hat{l})}{E[\varepsilon]} \frac{h(z, \hat{l})}{\int_z \int_l h(z, l)\, dz\, dl + \frac{\mathcal{N}^{unemp}}{\mathcal{N}}}$$

Similarly, the distribution of labor in the economy is related to the distribution of firms via

$$\phi_h(z, \hat{l}) = \frac{l(z, \hat{l})}{E[l(z, l)]} \phi_f(z, \hat{l})$$

where $E[l(z, l)]$ is the expected measure of workers at a firm.

By assumption, for very large values of z, all workers, independent of skill level, search with maximum effort. Therefore, $\phi_{h\varepsilon}(\cdot)$ is related to $\phi_f(\cdot)$ as

$$\phi_{h\varepsilon}(z,\hat{l}) = \frac{1}{E[\varepsilon]}\frac{l(z,\hat{l})}{E[l(z,l)]}\phi_f(z,\hat{l})$$

for large values of z.

Substituting this expression into the equation and taking the limit yields the following value for the second line of the KFE equation

$$-q\left(\ddot{\theta}\right)\tau\frac{1}{E[\varepsilon]}\frac{1}{E[l(z,l)]}k\zeta\int_l\int_{\tilde{z}=0}^{\infty}\phi_f(\tilde{z},l)\nu(\tilde{z},l)\,d\tilde{z}\,dl$$

where $E[\varepsilon]$ is the average search effort in the economy, and $E[l(z,l)] = \mathcal{N}/\mathcal{F}$ is the average number of workers per firm.

Firms with productivity z who advance in productivity Similar to the case above, applying the fact that for large enough z, all worker-firm matched will be agreed upon, and that the probability of transferring all knowledge is τ, L'Hoptial's rule yields the following limit for the third line of the KFE

$$\lim_{z\to\infty}\int_l\frac{\partial\phi_f(z,l)}{\partial z}\nu(z,l)q(\ddot{\theta})\tau\frac{\int_{\hat{z}=z}^{\infty}\int_{\hat{l}}\frac{\phi_{h\varepsilon}(\hat{z},\hat{l})}{\hat{l}}\,d\hat{l}\,d\hat{z}}{e^{-\zeta z}}\,dl$$

Using the relationship between $\phi_{h\varepsilon}(\cdot)$ and $\phi_f(\cdot)$, for large values of z:

$$\begin{aligned}
&\lim_{z\to\infty}\int_l\frac{\partial\phi_f(z,l)}{\partial z}\nu(z,l)q(\ddot{\theta})\tau\frac{\int_{\hat{z}=z}^{\infty}\int_{\hat{l}}\frac{1}{E[\varepsilon]}\frac{1}{E[l(z,l)]}\phi_f(z,\hat{l})\,d\hat{l}\,d\hat{z}}{e^{-\zeta z}}\,dl\\
=&q(\ddot{\theta})\tau\frac{1}{E[\varepsilon]}\frac{l(z,\hat{l})}{E[l(z,l)]}\lim_{z\to\infty}\int_l\frac{\partial\phi_f(z,l)}{\partial z}\nu(z,l)\frac{[1-\Phi_f(z)]}{e^{-\zeta z}}\,dl\\
=&q(\ddot{\theta})\tau\frac{1}{E[\varepsilon]}\frac{l(z,\hat{l})}{E[l(z,l)]}\int_l\lim_{z\to\infty}\left(\frac{\partial\phi_f(z,l)}{\partial z}\nu(z,l)\right)\lim_{z\to\infty}\left(\frac{[1-\Phi_f(z)]}{e^{-\zeta z}}\right)\,dl\\
=&q(\ddot{\theta})\tau\frac{1}{E[\varepsilon]}\frac{l(z,\hat{l})}{E[l(z,l)]}\int_l\lim_{z\to\infty}0k\,dl\\
=&0
\end{aligned}$$

Therefore, after substituting in the limits for the two terms above and simplifying, the limit of the KFE equation is given by the expression

$$0 = -(\gamma-\gamma_I)\zeta+\frac{1}{2}\sigma^2\zeta^2+q\left(\ddot{\theta}\right)\tau\frac{1}{E[\varepsilon]}\frac{1}{E[l(z,l)]}\int_l\int_{\tilde{z}=0}^{\infty}\phi_f(\tilde{z},l)\nu(\tilde{z},l)\,d\tilde{z}\,dl+\xi$$

This equation is a quadratic equation for the tail parameter ζ:

$$\zeta = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

where

$$A = \frac{1}{2}\sigma^2$$
$$B = -(\gamma - \gamma_I)$$
$$C = q\left(\ddot{\theta}\right)\frac{1}{E[\varepsilon]}\frac{1}{E[l(z,l)]}\tau \int_l \int_{z=0}^{\infty} \phi_f(z,l)\nu(z,l)\,dz\,dl + \xi$$

So

$$\zeta = \frac{(\gamma - \gamma_I) \pm \sqrt{(\gamma - \gamma_I)^2 - 2\sigma^2\left(q\left(\ddot{\theta}\right)\frac{1}{E[\varepsilon]}\frac{1}{E[l(z,l)]}\tau \int_l \int_{z=0}^{\infty} \phi_f(z,l)\nu(z,l)\,dz\,dl + \xi\right)}}{\sigma^2}$$

Because $\phi_f(z)$ is a PDF, and must integrate to one, we can rule out the possibility of complex roots in the solution. This requires that

$$(\gamma_I - \gamma)^2 \geq 2\sigma^2 q\left(\ddot{\theta}\right)\frac{1}{E[\varepsilon]}\frac{1}{E[l(z,l)]}\tau \int_l \int_{z=0}^{\infty} \phi_f(z,l)\nu(z,l)\,dz\,dl + \xi$$

In addition, the CDF of the distribution must be an increasing function. This requires both roots to be positive. This is only the case when $\gamma > \gamma_I$. Applying these two restrictions to the equation for the tail parameter implies

$$\gamma \quad \geq \quad \gamma_I + \sigma\sqrt{2q\left(\ddot{\theta}\right)\frac{1}{E[\varepsilon]}\frac{1}{E[l(z,l)]}\tau \int_l \int_{z=0}^{\infty} \phi_f(z,l)\nu(z,l)\,dz\,dl + 2\xi}$$

Therefore a continuum of growth rates that satisfy the BGP exist. As long as the aggregate growth rate exceeds the value of the right-hand side, it will produce a balanced growth path, with a different tail parameter value for ζ.

But as argued by Luttmer (2012), The median of $\phi_f(z)$ always lags behind the trend γt. As a result γ represents the maximum long-run growth rate attainable by the economy where it converges to the growth rate from below, given an initial distribution with a compact support. Assumption 1 imposes that the initial distribution of firm productivity has bounded support.

Therefore, the model has a unique growth rate on the BGP of

$$\gamma \;=\; \gamma_I + \sigma\sqrt{2q\left(\ddot{\theta}\right)\tau\frac{1}{E[\varepsilon]}\left(\frac{\mathcal{F}}{\mathcal{N}}\right)\int_l\int_{\tilde{z}=0}^{\infty}\phi_f(\tilde{z},l)\nu(\tilde{z},l)\,d\tilde{z}\,dl+\xi}$$

B Theoretical Model

The model outlined in the main body of the paper focuses on addressing knowledge spillover through the labor mobility channel. The empirical firm-level work outlined in section 3 also identified a correlation between productivity growth at the hiring firm and the productivity of the firms new workers are sourced from that is consistent with the idea that there is positive assortative matching between worker quality and a firms productivity. Therefore, hiring from more productive firms will tend to raise the hiring firm's productivity growth (by raising the average quality of the labor force), while hiring from less productive firms will tend to lower the hiring firm's productivity growth (by lowering the average quality of the labor force).

In this appendix, an extension to the model is outlined that features heterogeneous worker skill/quality. On-the-job training ensures that a firm's productivity is positively correlated with worker skill. Such an extension is capable of replicating the worker quality effect seen in the firm-level data along side the knowledge spillover effect. However, the large degree of heterogeneity makes the model numerically challenging to simulate to any high degree of accuracy.

B.1 General model environment

Time is continuous in the model. Within the model there exists two types of agents, firms and workers. Both firms and workers are heterogeneous. Individual firms are collectively owned by the workers in the economy. Each individual firm produce a differentiated goods. Firms produce their output by combining units of effective labor, L, with a stock of productive knowledge x using the production function

$$y(i) = x(i)L(i)$$

where i indexes the firm. The measure of effective labor is found by integrating over all worker skill levels in the distribution of labor employed by the firm: $L(i) = \int_s sl(s)\,ds$ where s denotes the workers skill (measured in effective labor units).

The output of each firm is aggregated into a final consumption good using a CES Dixit-Stigliz aggregator. Because all firms enter the CES aggregator symmetrically, firms can be grouped by their level of productivity and labor, so aggregate output is given by

$$Y(t) = \left[\int_x \int_l y(x,l)^{(\rho-1)/\rho} \mathcal{F}^{act}(t) f(x,l,t)\,dl\,dx\right]^{\rho/(\rho-1)} \tag{5}$$

where ρ is the elasticity of substitution between individual goods, $\mathcal{F}^{act}(t)$ is the measure of active firms in the economy at time t, and $f(x,l,t)$ is the probability density function for the distribution of firms with productivity level x, and employing the (unnormalized) distribution of labor l at time t.

Each firm faces the standard inverse demand relationship for its output given by

$$p(x, l, t) = P(t) \left(\frac{y(x, l, t)}{Y(t)} \right)^{-1/\rho} \tag{6}$$

where $P(t)$ is the ideal price index of the Dixit-Stigliz aggregation:

$$P(t) = \left[\int_x \int_l p(y(x, l), t)^{1-\rho} \mathcal{F}^{act}(t) f(x, l, t)\, dl\, dx \right]^{\rho/(\rho-1)}$$

B.2 Firms

At any point in time there exists a continuum of firms of measure $\mathcal{F}$. This measure of firms is divided into two groups, active firms who are currently producing goods, and inactive firms (entrepreneurs) who are trying to develop a product with which they can enter the market. Our main point of interest with the firms is their endogenous choice of search intensity (described by the vacancy posting rate) which determines how they benefit from knowledge spillover.

B.2.1 Active firms

Active firms are heterogeneous along two dimensions, the level of productive knowledge, x, and the distribution of skilled labor employed $l = \{l(s)\}$. For the model's equation to follow it will be easier to work with the log of productivity, $z = \ln(x)$, rather than the productivity level x. In the remainder of this section, z will be referred to as the firm's productivity.

At each point in time, the firms chooses how many vacancies to post in the search for new labor. Even in the absence of knowledge spillover through the labor mobility channel, firms can also improve their productivity through random innovations. Following Luttmer (2012), noisy firm-level innovation is introduced in the form of Brownian motion shocks to z. These random productivity shocks have a mean of $\gamma_I > 0$ and variance of σ^2. In the context of this model, random firm-level innovations can be thought of as the engine of new ideas in the economy. All firms tinker with their current production process and discover new ideas. Labor mobility acts to transmit these new ideas (as well as older ideas) to the wider economy, amplifying the effect of new ideas beyond the scope of the individual firm.

Let $\Pi(z, l, t)$ denote the value of being an active firm with productivity z and distribution of labor $l = \{l(s)\}$ (a distribution over the various skill levels s) at time t. Firms (and workers) discount the future at rate r. The value of an active firm satisfies the following Hamilton-Jacobi-Bellman equation

$$
\begin{aligned}
r\Pi(z,l,t) &= \pi(z,l,t) + \gamma_I \frac{\partial \Pi(z,l,t)}{\partial z} + \frac{\sigma^2}{2}\frac{\partial^2 \Pi(z,l,t)}{\partial z \partial z} + \frac{\partial \Pi(z,l,t)}{\partial t} \\
&+ \max_{\nu \in [0,\nu_{max}]} \Bigg\{ -c_\nu(\nu) + \nu q(\theta) \int_s \frac{\partial \Pi(z,l,t)}{\partial l(s)} \left(\int_{x=0}^{\infty} \int_{l_x} \mathbf{1}_{agree} h_\varepsilon(x,l_x,t,s)\, dl_x\, dx \right) ds \\
&+ \nu q(\theta) \int_s \left[\int_x \int_{l_x} \left(\int_y \mathbf{1}_{agree} \left[\Pi(y,l,t) - m(x,y,s,t) - \Pi(z,l,t) \right] T(y;x,z,s)\, dy \right) h_\varepsilon(x,l_x,t,s)\, dl_x\, dx \right] ds \Bigg\} \\
&+ \int_s \Psi(s,z,l,t) \frac{\partial \Pi(z,l,t)}{\partial l(s)}\, ds
\end{aligned} \tag{7}
$$

The first line of equation 7 has the standard form of a Hamilton-Jacobi-Bellman equation for a process with Brownian shocks. $\pi(z,l,t)$ denotes the flow of profit to the firm from production (revenue less expense). The next two terms are a result of the Brownian motion process for firm innovation. The final term in the first line, $\partial \Pi(z,l,t)/\partial t$, relates to the capital gain/loss resulting from changing economic conditions over time (such as changes in the firm's relative productivity ranking within the economy).

The next two lines of the equation relate to the firm's choice vacancies posting rate ν. Posting vacancies incurs a real cost to the firm, $c_\nu(\cdot)$, that is an increasing function of number of vacancy posted. Vacancies yield two sources of benefits to the firm, it allows for more labor to be employed, and it may also yield productive knowledge spillover through the labor mobility channel. The capital gains from each of these two components is presented separably in equation 7.

For each of the ν vacancies posted by the firm, the probability that the vacancy is matched with a searching worker is given by $q(\theta)$, where θ is a measure of the labor market tightness (the ratio of vacancies to worker search effort). From the firm's perspective, matches with workers are drawn randomly from the distribution of worker search effort which has the PDF denoted by $h_\varepsilon(z,l_z,t,s)$ where s denotes the skill level of the worker. The indicator function $\mathbf{1}_{agree}$ denotes the labor-market matches for which the net surplus of the match is positive, and hence the worker agrees to move to the firm.

The term in the third line denotes the expected capital gain resulting from productive knowledge spillovers through the labor mobility channel, less the knowledge premium payment $m(\cdot)$ made to the worker. The evidence from the empirical analysis suggests that productive knowledge spillover occurs only when the movement of the worker is from a more to less productive firm. In this model, the amount of knowledge transferred is drawn randomly from the distribution $T(y;x,z,s)$, and depends upon the productivity of the firm (z) and the worker's previous employer (x) as well as the skill level of the worker (more skillful workers are more likely to transmit more knowledge). By assumption, no knowledge spillover occurs when the worker is from a less productive firm (when $x < z$, $T(z;x,z,s) = 1$).

In addition to the wage a worker is paid for supplying their labor, they are also paid a one-off premium payment, $m(\cdot)$, negotiated when the employee first starts at the firm. The exact details of how this is determined is discussed in section B.4. This premium payment takes into account a number of factors such as the worker's willingness to work for the firm, due to opportunities to learn from the firm, and the firm's willingness to hire the worker to obtain any knowledge spillovers.

The final term in equation 7 relates to capital gains/losses from changes in the distribution of the firm's incumbent labor. For each skill level (s) employed by the firm, $\Psi(s,z,l,t)$ is the probability of increasing (or decreasing) the number of s skilled workers employed by the firm, and $\partial\Pi(z,l,t)/\partial l(s)$ denotes the capital gain or loss for a marginal change in skill labor s. More specifically, $\Psi(s,z,l,t)$ is given by

$$\begin{aligned}\Psi(s,z,L,t) \quad \equiv \quad & -(\lambda+\delta)l(s) \\ & -l(s)\varepsilon(s,z,l,t)\theta q(\theta)\int_x\int_{l_x}\mathbf{1}_{agree}f_\nu(x,l_x,t)\,dl_x\,dx \\ & +i(s,z)l(s)\end{aligned}$$

where the first line on the right hand side relates to the loss of workers due to exogenous separation to unemployment at rate δ, and the random chance of death λ.

The second line on the right hand side relates to the loss of workers who leave the firm because they have accepted a job at another firm. Each of the $l(s)$ workers searches for other job offers with search effort $\varepsilon(s,z,l,t)$. Each unit of worker search effort has a $\theta q(\theta)$ probability of being matched with a posted job vacancy. These matches are randomly drawn from the distribution of vacancy postings with a probability density function denoted by $f_\nu(x,l_x,t)$ (where x denote the productivity of the firm posting the vacancy, and l_x is the distribution of labor employed by the firm). Workers will only accept new jobs if it yields positive net surplus, denoted by the indicator function $\mathbf{1}_{agree}$.

The final line relates to incumbent workers improving their skill through on-the-job learning. By working at a firm with productivity z, workers of skill s improve their inherit skill at rate $i(s,z)$. This rate is assumed to be increasing in z such that more productive firms provide better opportunities to improve the skill level of the worker.

This assumption creates positive assortative matching between firm productivity and worker skill. The empirical analysis suggested that hiring from more productive firms tends to raise multi-factor productivity growth at the hiring firm, while hiring from less productive firms will tend to lower a firm's productivity growth. By creating positive assortative matching between firm productivity and worker skill, we create such a pattern in the hiring firm's productivity growth when hiring from more and less productive firms.[1]

Given the value function defined by equation 7, the firm's choice of vacancies to post satisfies the first order condition (FOC) given by

$$\begin{aligned}\frac{\partial c_\nu(\nu)}{\partial\nu} &= q(\theta)\int_s\frac{\partial\Pi(z,l,t)}{\partial l(s)}\left(\int_{x=0}^{\infty}\int_{l_x}\mathbf{1}_{agree}h_\varepsilon(x,l_x,t,s)\,dl_x\,dx\right)ds \\ &+q(\theta)\int_s\left[\int_x\int_{l_x}\left(\int_y\mathbf{1}_{agree}\left[\Pi(y,l,t)-m(x,y,s,t)-\Pi(z,l,t)\right]T(y;x,z,s)\,dy\right)h_\varepsilon(x,l_x,t,s)\,dl_x\,dx\right]ds\end{aligned} \tag{8}$$

[1] On-the-job learning also provides the model with an internal career ladder, such that workers may choose to advance their careers within the firm, rather than seeking outside jobs.

where the left hand side is the marginal cost of an additional vacancy posting, and the right hand side it the expected marginal benefit (both in terms of additional labor and knowledge spillover) of a vacancy posting.

Finally, the profit function, $\pi(z,l,t)$, for an active firm has the form

$$\pi(z,l,t) \;\; = \;\; p(z,l,t)y\left(z,l\right) - \omega(z,l,t)\int_s sl(s)\,ds \tag{9}$$

where $p(z,l,t)y\left(z,l\right)$ is the firm's revenue (price multiplied by output), and $\omega(z,l,t)\int_s sl(s)\,ds$ is the total wage cost to the firm for all effective labor units employed, where $\omega(z,l,t)$ is the wage rate per effective unit of labor.

B.2.2 Inactive firms

At time t, there is a measure $\mathcal{F}^{inact}(t) \equiv \mathcal{F} - \mathcal{F}^{act}(t)$ of inactive firms. Inactive firm pays a flow cost c_E to draw an idea for a new product, and an initial productivity level for producing the product. The productivity draws are made from a distribution with probability density function $f_{new}(z,t)$. The firm can then choose to enter the market and become an active firm, or remain an inactive firm. Following Acemoglu and Hawkins (2014), if the inactive firm decides to become active, it enters the market with zero employees.

Let $\Pi^I(t)$ denote the value of being an inactive firm at time t. The value satisfies the HJB given by

$$r\Pi^I(t) = \frac{\partial \Pi^I(t)}{\partial t} - c_I + \int_z \max\left\{\Pi(z,0,t) - \Pi^I(t)\,,\,0\right\} f_{new}(z,t)\,dz \tag{10}$$

where $\partial\Pi^I(t)/\partial t$ is the capital gain from changing market conditions, c_I is the cost of developing a new product, and $\int_z \max\left\{\Pi(z,0,t) - \Pi^I(t)\,,\,0\right\} f_{Ent}(z,t)\,dz$ is the expected capital gain from developing a new producing and becoming an active firm with productivity z, and labor force size zero.

B.3 Workers

The behavior of workers within the model determines the distribution of productivity ideas that searching firms are exposed to via the labor mobility channel of knowledge spillover. Because worker face a constant probability of death, λ, workers can be viewed as having a career path in which they move between firms, and also in and out of employment, before eventually dying and exiting the market.

The total population of workers is assumed to be of a fixed measure $\mathcal{N}$. Each worker is infinitesimally small in size. When a worker dies, they are replaced by a new-born worker who begin life with the minimum skill level, s_{min}, and also in the state of unemployment (as in Postel-Vinay and Robin, 2002). At each point in time, workers can choose how intensively they seek outside job offers. In doing so, workers trade off the rate of on-the-job training they receive with their current firm against learning opportunities at other firms or the payment the worker can receive from diffusing their current knowledge to a less productive firm.

B.3.1 Employed workers

Let $V(s,z,l,t)$ denote the value of being an employed worker of skill level s, employed at a firm with productivity z, and labor distribution l at time t. The value of such a worker satisfies the following Hamilton-Jaccobi-Bellman equation

$$
\begin{aligned}
(r+\lambda)V(s,z,l,t) &= \omega(z,l,t)s + \gamma_I \frac{\partial V(s,z,l,t)}{\partial z} + \frac{\sigma^2}{2}\frac{\partial^2 V(s,z,l,t)}{\partial z \partial z} + \frac{\partial V(s,z,l,t)}{\partial t} \\
&+ \max_{\varepsilon\in[0,\varepsilon_{max}]}\left\{-c_\varepsilon(\varepsilon) + \varepsilon\theta q(\theta)\int_{x=0}^{\infty}\int_{l_x}\left(\int_y \mathbf{1}_{agree}\left[V(s,y,l_x,t,) + m(\cdot) - V(s,z,l,t)\right]T(y;z,x,s)\,dy\right) f_\nu(x,l,t)\,dl_x\,dx\right\} \\
&+ i(s,z)\frac{\partial V(s,z,l,t)}{\partial s} - \delta\left[V^u(s,t) - V(s,z,l,t)\right] \\
&+ vq(\theta)\int_{\hat{s}}\int_{x=z}^{\infty}\int_{l_x}\left(\int_{y=z}^{x}\mathbf{1}_{agree}[V(s,y,l,t) - V(s,z,l,t)]T(y;x,z,\hat{s})\,dy\right) h_\varepsilon(x,l_x,t,\hat{s})\,dl_x\,dx\,d\hat{s} \\
&+ \int_{\hat{s}}\Psi(\hat{s},z,l,t)\frac{\partial V(s,z,l,t)}{\partial l(\hat{s})}\,d\hat{s} \qquad (11)
\end{aligned}
$$

The first line of the Hamilton-Jacobi-Bellman is fairly standard. On the left hand side, the effective discount rate the worker faces comprises of the standard discount rate r, plus the probability of death λ. On the right hand side, $\omega(z,l,t)s$ denotes the flow of wage income the worker receives. The next two terms are a result of the Brownian motion shocks to their firm's productivity. The final term in the first line is the change in value due to changes in wider economy conditions over time (such as the firm's change in relative market position).

The second line of equation 11 relates to the worker's choice of search effort, ε. Given the choice of search effort ε, the worker incurs a (real) search cost of $c_\varepsilon(\varepsilon)$ which is increasing in the level of search effort. The other term in the second line is the expected capital gain from searching. For each unit of search effort, the worker has a $\theta q(\theta)$ probability of matching with a posted vacancy in the search-and-matching market. Matches are drawn at random from the distribution of firm vacancy postings with probability density function $f_\nu(z,l,t)$. The indicator function $\mathbf{1}_{agree}$ denotes matches for which the worker will agree to move firms. Moving to a new firm yields a capital gain (or loss) of $V(s,y,l_x,t,) - V(s,z,l,t)$, and a one-off premium payment $(m(\cdot))$.

In the third line, the term $i(s,z)$ denotes the probability that the worker improves their skill through learning on the job, yielding a capital gain of $\partial V(s,z,l,t)/\partial s$. At each point in time, the worker faces an exogenous probability δ that they separate into unemployment yielding a capital gain of $V^u(s,t) - V(s,z,L,t)$, where $V^u(s,t)$ is the value of being an unemployed worker of skill s.

The fourth line of equation 11 relates to the expected capital gain as a result of the firm improving its productivity through the spillover of knowledge via labor mobility (the firm hires new labor).

The final term in equation 11 relates to the capital gain/loss resulting from changes in the number of other incumbent skill level $\hat{s}$ workers employed by the firm. From the point of view of an individual worker, this is taken as exogenous. $\Psi(\hat{s}, z, l, t)$ denotes the probability that the number of employed workers of skill $\hat{s}$ increases at the firm, and is given by

$$\begin{aligned}\Psi(\hat{s}, z, l, t) \equiv\ & -[\lambda + \delta] l(\hat{s}) \\ & -l(\hat{s})\varepsilon(\hat{s})\theta q(\theta) \int_{x=0}^{\infty} \int_{l_x} \mathbf{1}_{accept} f_\nu(x, l_x, t)\, dl_x\, dx \\ & +\nu q(\theta) \int_{x=z}^{\infty} \mathbf{1}_{accept} h_\varepsilon(x, t, \hat{s})\, dx \\ & +i(\hat{s}, z) l(\hat{s})\end{aligned}$$

where each of the terms have a similar interpretation to those in the firm's Hamiltonian-Jacobi-Bellman equation.

Given the value function defined in equation 11, the employed worker's choice of search effort ε satisfies the FOC give by

$$\frac{\partial c_\varepsilon(\varepsilon)}{\partial \varepsilon} = \theta q(\theta) \int_{x=0}^{\infty} \int_{l_x} \left(\int_y \mathbf{1}_{agree} \left[V(s, y, l_x, t,) + m(\cdot) - V(s, z, l, t) \right] T(y; z, x, s)\, dy \right) f_\nu(x, l, t)\, dl_x\, dx \tag{12}$$

where the left hand side is the marginal cost of additional search effort, and the right hand side is the expected capital gain from finding a new job at another firm.

B.3.2 Unemployed Workers

Unemployed workers of skill s are assumed to undertake home production. The real flow value of their home production is $b(t)s$ each period. Unemployed workers can leave the unemployment state by successfully searching for a job. From the perspective of the firms, the main distinction between employed and unemployed workers is that unemployed workers do not have a stock of productive knowledge that could spillover into the firm. The productive knowledge from an unemployed worker's last employer prior to becoming unemployed is assumed to fully depreciate upon entering unemployment. The unemployed worker still retains their innate skill level s while unemployed, and this skill level does not depreciate. However, the worker's skill cannot improve while the worker is unemployed.

Let $V^u(s, t)$ denote the value of being an unemployed worker will skill s at time t. The value function satisfies the following Hamilton-Jacobi-Bellman equation

$$[r+\lambda]V^u(s,t) = b(t)s + \frac{\partial V^u(s,t)}{\partial t} + \max_{\varepsilon\in[0,\varepsilon_{max}]}\Big\{-c_\varepsilon(\varepsilon)$$
$$+\,\varepsilon\theta q(\theta)\int_{z,l}\max\{V(s,z,l,t)+m(s,z,U,t)-V^u(s,t)\ 0\}f_\nu(z,l,t)\,dz\,dl\Big\} \tag{13}$$

where $r+\lambda$ is the effective discount rate, $b(t)s$ is the flow payment of benefits (home production), $\partial V^u(s,t)/\partial t$ is the capital gain/loss due to changing labor market conditions over time, $c_\varepsilon(\varepsilon)$ is the cost of searching with effort ε, and the final term is the expected capital gain from matching with a searching firm and leaving unemployment.

The unemployed worker's choice of search effort satisfies the following first order condition

$$\frac{\partial c_\varepsilon(\varepsilon)}{\partial \varepsilon} = \theta q(\theta)\int_{z,l}\max\{V(s,z,l,t)+m(s,z,U,t)-V^u(s,t)\ 0\}f_\nu(z,l,t)\,dz\,dl \tag{14}$$

which balances the marginal cost (left hand side) against the marginal benefit (the right hand side) of searching.

B.4 Worker's compensation

A worker's total compensation is modeled in two parts. First there is the wage the worker receives for the total number of effective labor (their skill) they supply to the firm's production process. The second part is a premium they earn above, or below, the baseline wage. This premium nets out a number of factors that relates to the firms and workers incentivising the other to agree to the new match.

B.4.1 Wages

Wages are set independently independently at each firm. The wage setting mechanism used here follows the approach used by Mortensen (2010), which is in the spirit of Stole and Zwiebel (1996). The firm bargains individually with each worker in their labor force. Long-term wage contracts are not possible, and the worker and firm bargain over the marginal surplus from the worker's labor each period. In the bargaining between the firm and an individual worker, the firm's outside position is to not use that particular worker this period, and instead produce with the other workers who have agreed to contracts. The worker's outside option is to undertake home production, with payoff $b(t)$.

Let β denote the worker's relative bargaining strength. The wage rate, $\omega(z,l,t)$, for each effective labor unit at a firm with productivity z and labor distribution l satisfies the Mortensen (2010) bargaining equation:

$$\beta\frac{\partial \pi(z,l,t)}{\partial l} = (1-\beta)[\omega(z,l,t)-b(t)] \tag{15}$$

B.4.2 Knowledge premium

When a worker arrives at a new firm, they negotiate the knowledge premium component of their compensation with the firm. It is through this premium workers are paid for their diffusion of productive knowledge, or workers pay firms for the learning opportunities the firm provides. The knowledge premium negotiation is assumed to occur after the firm and worker observe the amount of knowledge spillover that will occur if the match was to go ahead.

To drastically simplify the numerical computation of the model, the knowledge premium is assumed to be a fixed, one-off payment when the worker starts. [2] In this sense, the knowledge premium can either be thought of in terms of a signing bonus, or as the net present value of a series of premium payments that would have occurred over the worker's employment spell.

Let z denote the firm's productivity, and x denote the productivity of the worker's previous firm. In addition, let z' denote the productivity of the hiring firm after the occurrence of any knowledge transfer. The expected capital gain for the worker of accepting the new job offer from firm (z, l_z) is given by

$$V(s, z', l_z, t) - V(s, x, l_x, t) + m(z'; z, l_z, x, l_x, s, t)$$

where $m(\cdot)$ denotes the knowledge premium payment.

From the perspective of the firm, the gains from hiring the new worker comprise of the expected capital gain due to the knowledge spillover (if any), and the capital gain from having another worker of skill s employed

$$\Pi(z', l_z, t) - \Pi(z, l_z, t) - m(z', z, l_z, x, l_x, s, t) + \frac{\partial \Pi(z, l_z, t)}{\partial l(s)}$$

Assuming the workers and firms have same relative bargaining strengths as used for the wage equation above, the premium that an employed worker and firm negotiate will be the value of m that satisfies the following Nash-bargaining equation

$$\begin{aligned} \beta \left(\Pi(z', l_z, t) - \Pi(z, l_z, t) - m + \frac{\partial \Pi(z, l_z, t)}{\partial l(s)} \right) = \\ (1 - \beta) \left(V(s, z', l_z, t) + m - V(s, x, l_x, t) \right) \end{aligned} \tag{16}$$

Similarly, for an unemployed worker (who has no knowledge spillover), the knowledge premium payment is the value of m that satisfies

$$\beta \left(-m + \frac{\partial \Pi(z, l_z, t)}{\partial l(s)} \right) = (1 - \beta) \left(V(s, z, l_z, t) + m - V^u(s, t) \right) \tag{17}$$

[2] If the knowledge premium was modeled as an ongoing payment, we would have to not only keep track of the worker's premium at each point in time, but also the entire joint distribution of premiums and worker skill within each firm's pool of labor.

Because workers will be willing to offer a negative knowledge premium to work at the most productive firms (pay the firms for the learning opportunities), for some matches, the total worker compensation (wages plus knowledge premium) will be negative. In effect, the most productive firms will be paid by the factor inputs. To avoid this situation, the premium payment is bounded to be a non-negative value, so that workers must receive some positive compensation from each job.[3]

B.5 Labor markets

As is standard for search-and-matching models, the labor market tightness, θ, is characteristics by the ratio of vacancy postings to worker search effort. The total number of vacancies posted in the economy is given by $\mathcal{F}^{act}(t)\int_{z=0}^{\infty}\int_{l}\nu(z,l,t)f(z,l,t)\,dl\,dz$. The total amount of worker search effort comes from two sources, unemployed and employed workers. Therefore, the labor market tightness, θ, is defined as

$$\theta \quad = \quad \frac{\mathcal{F}^{act}(t)\int_{z=0}^{\infty}\int_{l}v(z,l,t)f(z,l,t)\,dl\,dz}{\int_{s}\left[\mathcal{N}^{unemp}(s,t)\varepsilon(s,U,t)+\mathcal{N}^{emp}(s,t)\int_{z}\int_{l}\varepsilon(s,z,l,t)h^{emp}(z,l,t,s)\,dl\,dz\right]\,ds} \tag{18}$$

Searching firms and workers are matched at random according to a matching function. Let $Q(\cdot)$ denote the constant returns to scale, homogeneous of degree one, matching function. Then the probability that a firm's vacancy is matched with a worker is given by

$$q(\theta)=Q(\theta,1)$$

Given the distribution of firms, $f(z,l,t)$, and their vacancy posting choices, $\nu(z,l,t)$, workers sample from the distribution of vacancy postings has the probability density function given by

$$f_{\nu}(z,l,t)=\frac{\nu(z,l,t)f(z,l,t)}{\int_{x}\int_{l}\nu(x,l,t)f(x,l,t)\,dl\,dx} \tag{19}$$

On the other side of the market, firms sample from the distribution of search effort with a probability density function given by

$$h_{\varepsilon}(z,l,t,s) \quad = \quad \frac{\varepsilon(s,z,l,t)h(z,l,t,s)}{\int_{s}\left[\int_{\tilde{z}=0}^{\infty}\int_{l}\varepsilon(s,\tilde{z},l,t)h(\tilde{z},l,t,s)\,dl\,d\tilde{z}+\frac{\mathcal{N}^{unemp}(s,t)}{\mathcal{N}^{emp}(t)}\varepsilon^{unemp}(s,t)\right]\,ds} \tag{20}$$

for all pairs of (z,l) corresponding to productivity-labor points in which firms are active,

[3] In the cases where the premium payment is at the boundary, a match only occurs if both the firm and worker receive some positive surplus from the match.

and

$$h_\varepsilon(0,0,t,s) = \frac{\frac{\mathcal{N}^{unemp}(s,t)}{\mathcal{N}^{emp}(t)}\varepsilon^{unemp}(s,t)h_{unemp}(s,t)}{\int_s \left[\int_{\tilde{z}=0}^{\infty}\int_l \varepsilon(s,\tilde{z},l,t)h(\tilde{z},l,t,s)\,dl\,d\tilde{z} + \frac{\mathcal{N}^{unemp}(s,t)}{\mathcal{N}^{emp}(t)}\varepsilon^{unemp}(s,t)\right] ds} \tag{21}$$

corresponding to unemployed workers search effort.

B.6 Kolmogorov Forward Equation

The search-and-matching labor model described above determines the search effort (policy choices) for both worker and firms. Given the policy choices of the workers and firms in the economy, the distribution of firms (and hence also workers) across the productivity-labor space evolves over time according to a Kolmogorov Forward Equation (KFE).

Let $f(z,l,t)$ denote the PDF for the distribution of firms who have productivity z, and labor distribution l.[4] The change in the mass of firms at (z,l,t) over time is given by

$$\begin{aligned}
\frac{\partial f(z,l,t)}{\partial t} &= -\gamma_I \frac{\partial f(z,l,t;s)}{\partial z} + \frac{1}{2}\sigma^2 \frac{\partial^2 f(z,l,t;s)}{\partial z \partial z} \\
&+ \int_{\tilde{z}=0}^{z} f(\tilde{z},l,t)\,\nu(\tilde{z},l,t)q(\theta) \int_s \left(\int_{\hat{z}=z}^{\infty}\int_{\hat{l}} \mathbf{1}_{accept} T(z,\tilde{z},\hat{z},s) h_\varepsilon(\hat{z},\hat{l},t,s)\,d\hat{l}\,d\hat{z}\right) d\tilde{z}\,ds \\
&- f(z,l,t)\nu(z,l,t)q(\theta) \int_s \left(\int_{\hat{z}=z}^{\infty}\int_{\hat{l}} \mathbf{1}_{accept} h_\varepsilon(\hat{z},\hat{l},t,s)\,d\hat{l}\,d\hat{z}\right) ds \\
&+ \mathbf{1}_{l=0} \frac{\mathcal{F}^{inact}(t)}{\mathcal{F}(t)} f_{Ent}(z,t) \mathbf{1}_{\Pi(z,0,t)>E(t)} \\
&+ \int_s \frac{\partial}{\partial l(s)} \Psi(s,z,l,t)\,ds
\end{aligned} \tag{22}$$

The first line on the right hand side contains terms related to the Brownian motion process for productivity shock innovations that hit the firms, with drift γ_I and variance σ^2.

The second line represents the inflow of firms with productivity $\tilde{z} < z$, who hire a new worker who previously worked at a firm with productivity $\hat{z} > z$, and the amount of knowledge that spills over is the exact amount needed to boost the hiring firm's productivity to z (which occurs with probability $T(z,\hat{z},\tilde{z},s)$).

The third line represents firms who currently have productivity z and firm size l, who hire a new worker from a move productive firm and receive any amount of knowledge spillover which will improve their productivity above z.

The fourth line denotes the entry of inactive firms. $\mathcal{R}(t)/\mathcal{F}_{act}(t)$ is the relative size of inactive to active firms. The probability that the inactive firms entering the market with productivity

[4] Inactive firms can be considered a point mass below the lower bound of productivity level for active firms.

z is given by the probability density function for the distribution of initial ideas $f_{new}(z,t)$. Finally, when an entrepreneur becomes an active firm, it starts with an initial firm size of zero. Hence the indicator function $\mathbf{1}_{l=0}$ so this term only applies in cases when the firm size is zero.

As in the case of the Hamilton-Jacobi-Bellman equations for the firm and workers, the final term in the terms in the KFE relates to changes in the measure of firms due to changes in distribution of incumbent labor being employed by the firm. Where $\Psi(s,z,l,t)$ is given by:

$$\begin{aligned}\Psi(s,z,l,t) \equiv\ & (\delta+\lambda)l(s,z)f(z,l,t) \\ & + \left[l(s)\varepsilon(s,z,l,t)\theta q(\theta)\int_{\tilde{z}}\int_{\tilde{l}} 1_{accept} f_{\nu}(\tilde{z},\tilde{l},t)\,d\tilde{l}\,d\tilde{z}\right] f(z,l,t) \\ & - \left[\nu(z,l,t)q(\theta)\int_{\tilde{z}=0}^{z}\int_{\tilde{l}} h_{\varepsilon}(\tilde{z},\tilde{l},t,s)\,d\tilde{l}\,d\tilde{z}\right] f(z,l,t) \\ & + i(s,z)\left(f(z,l,t)l(s)\right)\end{aligned}$$

The interpretations of these terms is the same as in the case of the Hamilton-Jacobi-Bellman equations.

B.6.1 Number of active and inactive firms

The value of a firm is increasing in the firm's productivity level. Within the economy there will exist some low bound on productivity z_{lb} such that inactive firms will choose not to enter the market. I.e. $\Pi(z,0,t) \leq \Pi^{I}(t)$ for all $z \leq z_{lb}$. In addition, active firms who happen to have a series of bad productivity shocks such that their productivity falls below this bound will also choose to shut down and become inactive firms (since the value of being inactive is greater than being active).

Following the approach of Luttmer (2012) the measure of active firms (or equivalently the measure of inactive firms) varies according to the flow of firms over the lower bound z_{lb} according to the expression:[5]

$$\frac{\partial(1-F(z_{lb},t))}{\partial t} = \frac{1}{2}\sigma^2\frac{\partial^2(1-F(z_{lb},t))}{\partial z\partial z} + \frac{\mathcal{R}(t)}{\mathcal{F}_{act}(t)}[1-F_{Ent}(z_{lb},t)] \tag{23}$$

where $F(z,t) = \int_l \int_{x=z_{lb}}^{z} f(x,l)\,dx\,dl$ is the CDF of distribution of firms in log-productivity space. The left hand side of equation 23 denotes the change in mass of firms above the lower bound, i.e. the measure of active firms. The final term on the right hand side denotes the inflow of inactive firms who draw an initial productivity above the level z_{lb}. Therefore, the first term on the right hand side denotes the flow of active firms who cross the lower bound and choose to become inactive as a result of the Brownian motion shocks.

[5]For more details, see appendix A.1

B.6.2 Number of unemployed workers

The change in the measure of unemployed for skill level s, denoted by $\mathcal{N}^{unemp}(s,t)$, is found using the PDE:

$$\begin{aligned}\frac{\partial \mathcal{N}^{unemp}(s,t)}{\partial t} &= \delta \mathcal{N}^{emp}(s,t) \\ &+ \mathbf{1}_{s=s_{min}} \lambda \left[\mathcal{N}^{emp}(t) + \int_s \mathcal{N}^{unemp}(s,t)\, ds \right] - \lambda \mathcal{N}^{unemp}(s,t) \\ &- \mathcal{N}^{unemp}(s,t) \varepsilon(s,U,t) \theta q(\theta) \int_z \int_l \mathbf{1}_{accept} f_\nu(z,l,t)\, dl\, dz \end{aligned} \tag{24}$$

The first term on the right hand side in the inflow of employed workers who are separated from their employers at rate δ. The second term relates to worker deaths. When $s = s_{min}$, there is an inflow in new-born workers who replace all those agents who die at the exogenous rate λ. The final term in the right hand side is the outflow of unemployed workers who successfully find a job at one of the firms posting vacancies.

The measure of employed workers of skill level s is found by integrating over the distribution of labor across active firms:

$$\mathcal{N}^{emp}(s,t) = \int_z \int_l l(s) \mathcal{F}_{active}(t) f(z,l,t)\, dl\, dz \tag{25}$$

Finally, the probability density function for the distribution of employed labor is given by

$$h(z,l,t,s) = \frac{l(s) f(z,l,t)}{\int_z \int_l \int_s l(s) f(z,l,t)\, ds\, dl\, dz} \tag{26}$$

C Additional empirical results

The following appendix provides further empirical results from the analysis of the firm-level data. For a more detailed discussion, the reader is referred to Kirker and Sanderson (2017).

C.1 Firm-level summary statistics

Table 1 describes the characteristics of the private-for-profit firms in the sample based on firm-year data. Across all the firms in the sample, the average size of the productivity gaps associated with hiring from more and less productive firms are similar (0.064 vs 0.060), leading to an aggregate productivity gap close to zero (0.005). If we interpret the aggregate productivity gap through the lens of exposure to new knowledge from other firms, the productivity gap suggests that due to labor mobility, the average worker in a hiring firm gains exposure to productive knowledge that is around half a percent higher than their employer's current level. However, there is significant variation in the knowledge exposure measures for different firms as represented by the large standard deviation of the productivity gaps. Primarily this is due to the lumpy nature of the number of new hires each year, especially for smaller firms.

Table 1: Summary statistics at the firm-year level (Value-added per worker)

	Firms in sample ($FTE \geq 10$)			**Firms that hire new workers**			**Firms that hire from more productive firms**			**Firms that do not hire**		
Variable	**Mean**	**Median**	**S.D.**	**Mean**	**Median**	**S.D.**	**Mean**	**Median**	**S.D.**	**Mean**	**Median**	**S.D.**
Labor productivity												
log V.A. per worker	11.102	11.094	N.A.	11.101	11.093	N.A.	10.962	10.983	N.A.	11.176	11.165	N.A.
Growth rate V.A. per worker (%)	-0.004	0.000	0.432	-0.003	0.001	0.432	-0.003	0.001	0.457	-0.040	-0.019	0.388
Productivity gap												
Aggregate gap	0.005	0	0.208	0.005	0	0.209	0.041	0.015	0.228	0	0	0
More prod. firms gap	0.064	0.015	0.164	0.065	0.016	0.165	0.100	0.046	0.196	0	0	0
Less prod. firms gap	-0.060	-0.022	0.123	-0.061	-0.023	0.124	-0.059	-0.027	0.108	0	0	0
Labor force												
Total FTE units of labor	56.230	17.961	255.994	56.953	18.166	258.128	75.169	21.743	317.416	14.248	12.181	8.657
Share of FTE from new hires	0.194	0.155	0.169	0.198	0.157	0.169	0.218	0.180	0.162	0	0	0
Share of FTE from exiting workers	0.172	0.136	0.150	0.174	0.138	0.150	0.192	0.157	0.148	0.086	0.042	0.165
Excess (annual) turnover	0.514	0.457	0.329	0.522	0.462	0.325	0.594	0.538	0.330	0.019	0	0.054
New Hires												
No. of new employees	22.070	7	101.667	22.448	7	102.498	31.686	11	125.734	0	0	0
Share of hires from brand new workers	0.001	0	0.018	0.001	0	0.018	0.001	0	0.010	0	0	0
Share of hires from non-market	0.116	0.062	0.166	0.116	0.062	0.165	0.105	0.079	0.120	0	0	0
Share of hires from small firms (L<5)	0.288	0.250	0.232	0.288	0.250	0.231	0.260	0.250	0.171	0	0	0
Share of hires from missing prod. data	0.102	0.051	0.154	0.102	0.053	0.154	0.091	0.069	0.107	0	0	0
Share of hires from PFP	0.489	0.500	0.257	0.489	0.500	0.257	0.540	0.519	0.198	0	0	0
within same industry	0.131	0.061	0.180	0.131	0.062	0.180	0.148	0.105	0.170	0	0	0
More productive sources	0.205	0.167	0.219	0.205	0.167	0.219	0.305	0.250	0.202	0	0	0
Obs.	126048			124146			80700			1902		

Notes: Summary statistics based on the sample of firm-year observations in the data set. FTE refers to Full Time Equivalent units of labor (1 FTE = 1 worker per year). Shares of hires are computed as the number of hires from the subgroup relative to the total number of new hires for that firm-year. N.A. denotes values that have been censored in accordance with Statistics New Zealand's confidentiality guidelines. PFP denotes Private For Profit firms (those for which we have productivity data). 'Firms that hire from more productive firms' denotes any firm that hires at least one worker from a more productive firm during that year

The distribution of labor across firms is highly skewed. In the sample of firms (with an average of more than 10 FTE workers), the average firm uses the equivalent of around 56 full time employees, while the median firm employees the equivalent of around 18 full time employees on average across the year.[6] The average firm also features a large amount of labor churn. On average, new workers, who were not employed by the firm during the previous financial year, supply just under 20 percent of the FTE labor units used by a firm in the current year. And workers who will leave the firm sometime during the year supply on average around 17 percent of the firm's labor input for that year. The large contributions by new and exiting workers contribute to an excess turnover rate of around 50 percent.[7]

The average firm hires around 22 new employees each year, and the overwhelming majority of firm-years feature the firm employing at least one new worker. These new workers are sourced from a wide variety of sources. Around 11 percent come from non-market firms, around 29 percent come from small firms (less than 5 FTE workers). More importantly, around 49 percent of observed hires come from other PFP firms that we observe productivity data from in the data set. In addition, 13 percent of all new hires by the average firm are from PFP firms within the same industry, and around 20 percent of new hires are from more productive PFP firms.

Table 1 also describes the characteristics of the subsets of firms that hire new workers, firms that hire at least one new worker from a more productive firm, and firms that do not hire new workers. Most firms within the sample hire new workers each year. Firms that hire at least one worker tend to have slightly lower productivity than firms that do not, but are also significantly larger in terms of labor force size, and have higher rates of labor market churn.

C.2 Worker-level summary statistics

Table 2 summarizes the key worker-level characteristics of new hires relative to different groups of workers. Panel A of the table shows the characteristics of new workers relative to the average incumbent worker in the hiring firm, one month after hiring. The average new worker earns an FTE adjusted income that is roughly 85 percent of the average incumbent worker at the hiring firm. Workers sourced from more productive firms tend to earn marginally more than workers from less productive firms (86.2 vs 84.5 percent of the average incumbent's earnings). New workers tend to be younger (about 88 percent of the average age) than the average incumbent, and less skilled (around 87 percent of the worker quality of the average incumbent worker). New workers are also more likely on average to be multiple job-holders, working an average of 10 percent more jobs in the same month.

[6] According to Mills and Timmins (2004), the average firm size of all employing firms in New Zealand is around 13.7 workers.

[7] Excess turnover is computed as

$$\text{Excess turnover} = \frac{\text{starts} + \text{exits} - |\text{net change}|}{(FTE_t + FTE_{t-1})/2}$$

where FTE is the number of full time equivalent units of labor in the final month of the firm's financial year.

Table 2: Summary statistics for new workers

	All new hires			New hires from more productive firms			New hires from less productive firms		
Variable	**Mean**	**Median**	**S.D.**	**Mean**	**Median**	**S.D.**	**Mean**	**Median**	**S.D.**
A) New worker's characteristics (at the hiring firm) relative to incumbent workers									
Real earnings percentile	0.450	0.407	0.309	0.422	0.369	0.303	0.464	0.438	0.305
FTE supplied relative to avg. incument	0.903	1.003	0.690	0.904	1.002	0.616	0.918	1.008	0.715
Age relative to avg. incumbent	0.889	0.825	0.343	0.899	0.835	0.343	0.860	0.793	0.332
Worker quality percentile	0.490	0.478	0.299	0.460	0.429	0.293	0.511	0.500	0.295
Number of jobs relative to avg. incumbent	1.141	0.975	0.533	1.135	0.971	0.535	1.130	0.975	0.486
Obs.	4094400			1154500			1335200		
B) New worker's characteristics (at last main job) relative to the workers who stays									
Real earnings percentile	0.450	0.407	0.309	0.422	0.369	0.303	0.805	0.713	0.612
FTE supplied relative to avg. stayer	0.920	1.001	1.456	0.980	1.015	0.939	0.883	1	0.858
Age relative to avg. stayer	0.879	0.813	0.345	0.889	0.825	0.345	0.850	0.783	0.334
Worker quality percentile	0.490	0.478	0.299	0.460	0.429	0.293	0.511	0.500	0.295
Number of jobs relative to avg. stayer	1.145	0.972	0.546	1.131	0.965	0.543	1.139	0.974	0.504
Obs.	4005200			1131200			1314900		
C) New worker's characteristics at their new job relative to their own characteristics at the last main job									
Real earning per FTE	1.119	1.025	0.494	1.064	1.002	0.459	0.464	0.438	0.305
FTE supplied: new job relative to old job	2.346	1	228.217	2.178	1	205.699	2.532	1	330.921
No. of months between jobs	5.484	0	13.167	4.823	0	11.572	4.630	0	11.539
Prob. working in same industry	0.226	0	0.418	0.284	0	0.451	0.275	0	0.447
Obs.	4202000			1180200			1367800		

Notes: Summary statistics are computed at the worker-month level. Summary statistics reported as percentiles are the percentile within the firm (e.g. 0.45 implies the new worker is above 45 percent of workers in the firm). Summary statistics that are reported as relative to the average are computed as a fraction relative to the average member of the control group (e.g. 0.5 implies that the new worker's characteristic is half that of the average control group member). The human capital measure consists of the worker's fixed effect and contribution from observable characteristics (see section for more details). FTE denoted Full Time Equivalent measure of labor. Real earnings are computed controlling for FTEs supplied.

Panel B of Table 2 shows the characteristics of new workers relative to the average worker at their previous main job (in the month prior to them leaving). On average, workers who change jobs tend to earn below average pay (around 86 percent of the average pay), supply less FTE units of labor (88 percent of the average), and also be younger and less skilled than the average worker.[8] These results do not differ dramatically whether we consider workers coming from more or less productive firms. It is likely that the general negative selection in workers who leave firms may reflect the proportionally higher job mobility by younger/junior employees.

The final panel, Panel C, shows the worker's characteristics at their new job, relative to their own characteristics at their last main job. Taking a new job is profitable on average for workers. Their FTE-adjusted monthly earnings are around 13 percent higher than in their previous job, and they work supply around twice the FTE units of labor. This large increase in labor supply is primarily driven by part-time workers and those with multiple jobs transitioning to full-time jobs.[9] The median worker supplies the same number of hours at their new job as they did at their previous job.

The average new employee has an average of just over five months break between jobs, with a median of zero months break. As stated previously, we do not explicitly model depreciation of productive knowledge in the model. The results from these summary statistics tend to suggest that for most workers in the data set, the depreciation of skill/knowledge between jobs is not a significant concern. Around 20 percent of the worker moves observed in the data set are within the same 3-digit industry as their previous employer.

Overall, workers who move between firms tend to come from the lower half of the sending firm's pool of labor (in terms of earnings, age, and labor supplied), and they also tend to have a similar ranking in the firms that thy join. Caution should be applied when interpreting these worker-level summary statistics from the point of view of the productivity spillover story. The aggregate labor flows in the economy are dominated by workers who have a high turnover rate in their careers, and those that tend to work multiple jobs. These types of employees are less likely to be the main vector of productivity knowledge spillover between firms. For the majority of the analysis below, the spillover effects from new hires are examined based on a subset of new workers defined by various characteristics of the new hires and their previous employer. This approach attempts to control for the fact that not all new workers will be equal in terms of either the knowledge they have or their ability to transfer productive knowledge between firms.

[8] Because FTE is a derived measure of labor supply, there may be some measurement error in the amount of labor supplied by individual workers.

[9] The large variance seen the the value of FTE supplied is drive by a relatively small number of outlier workers who work multiple jobs at the time of their previous employment (giving them a low FTE in their previous main job) and move to full time work at their new job. Some of these large changes could be due to the the way FTEs are calculated. If a part-time worker moves to another part-time job it is possible for them to appear to become a full-time worker (or increase their labor supply) if their hourly pay increases by a sufficient amount.

C.3 Industry-specific knowledge

If workers facilitate the spillover of knowledge between firms, not all knowledge that workers bring into the firm will be of equal value. The structure of the baseline model already allows for different effects from knowledge coming from more and less productive firms through the disaggregated productivity gaps. However, this is not the only dimension along which the value of knowledge will differ for the hiring firm. The knowledge spillover channel predicts that workers hired away from other firms within the same industry (whose knowledge should be more valuable to the hiring firm) should have a larger positive benefit for the hiring firm's productivity than workers hired away from firms in other industries.

To examine if this is the case, the productivity gaps (and hire intensities) related to hires from more and less productive PFP firms are further subdivided into two groups: hires from within the same industry, and hires from different industries, i.e.:

$$\begin{aligned}
\text{Exposure} \;=\; & \sum_{\text{ind}\in\{\text{same},\text{diff}\}} \beta_{M,ind} \frac{\sum_{n\in\mathcal{N}_j,t-1} \mathbb{D}_{\text{ind}}(n)\mathbb{D}_n[\ln(A_{n,\tau-\delta}) - \ln(A_{i,t-1-\delta})]}{L_{i,t-1}} \\
& + \sum_{\text{ind}\in\{\text{same},\text{diff}\}} \beta_{L,\text{ind}} \frac{\sum_{n\in\mathcal{N}_j,t-1} \mathbb{D}_{\text{ind}}(n)(1-\mathbb{D}_n)[\ln(A_{n,\tau-\delta}) - \ln(A_{i,t-1-\delta})]}{L_{i,t-1}} \\
& + \sum_{s\in\mathcal{S}_{i,t-1}} \lambda_s \frac{H_{i,s,t-1}}{L_{i,t-1}} \qquad (27)
\end{aligned}$$

where 'ind = same' denotes the hire is from the same industry, 'ind = diff' denotes the hire is from a different industry, $\mathbb{D}_{\text{ind}}(n)$ is a dummy variable based on the 'ind' classification. So when $\mathbb{D}_{\text{ind}}(n) = \mathbb{D}_{\text{same}}(n)$, $\mathbb{D}_{\text{same}}(n)$ takes on the value of 1 if worker n's previous main job was in the same industry (and a similar definition for the case when 'ind = diff'). Therefore, $\beta_{M,\text{same}}$ denotes the effect of the productivity gap for hires from more productive firms within the same industry. In addition, the set of sources, $\mathcal{S}_{i,t-1}$, is expanded to include hires from the same and different industries who worked at more or less productive firms.

Table 3 presents the key results for estimating this extended version of the model for the three main productivity measures. With results to the definition of 'same industry', results are presented using all 39 different industries within the data set (at roughly a 3-digit level of classification), as well as industry classifications based on the 1-digit level (e.g. all of manufacturing industries are grouped together).

Table 3 shows that when value-added per worker is used to measure firm productivity, the coefficient on the productivity gap from workers from more productive firms within the same industry is nearly three times as large as the coefficient on the productivity gap from workers from more productive firms in other industries (the p-value is 0.03). The coefficients on the productivity gap from less productive firms are (i) significantly lower then the coefficients on the productivity gap from more productive firms and (ii) not significantly different between hires from the same and hires from different industries. Relative to the baseline results, this pattern in productivity gap coefficients supports the predictions of a

Table 3: Regression results featuring between and within industry productivity gaps

	Value-added			**Trans-log**		
	Baseline	**Productivity gaps by ind.**		**Baseline**	**Productivity gaps by ind.**	
		3-digit	**1-digit**		**3-digit**	**1-digit**
Prod. gap, hires from (β):						
More prod. firms	0.480***			0.354***		
	(0.098)			(0.068)		
Within same ind.		1.016***	0.926***		0.311**	0.287***
		(0.230)	(0.164)		(0.123)	(0.105)
From diff. ind.		0.367***	0.326***		0.373***	0.390***
		(0.123)	(0.121)		(0.090)	(0.097)
Less prod. firms	0.153***			0.374***		
	(0.030)			(0.056)		
Within same ind.		0.188***	0.156***		0.278***	0.314***
		(0.068)	(0.052)		(0.103)	(0.079)
From diff. ind.		0.139***	0.152***		0.415***	0.418***
		(0.035)	(0.039)		(0.071)	(0.086)
Hire intensity (λ):						
More prod. firms	-0.200***			-0.037*		
	(0.057)			(0.021)		
Within same ind.		-0.310***	-0.312***		0.008	0.002
		(0.075)	(0.064)		(0.028)	(0.028)
From diff. ind.		-0.168*	-0.154*		-0.059**	-0.066**
		(0.078)	(0.081)		(0.028)	(0.031)
Less prod. firms	-0.117***			0.004		
	(0.027)			(0.019)		
Within same ind.		-0.069	-0.099***		-0.001	-0.005
		(0.040)	(0.035)		(0.028)	(0.023)
From diff. ind.		-0.139***	-0.120***		0.004	0.010
		(0.033)	(0.037)		(0.025)	(0.029)
Parameter tests:						
$\Pr(\beta_M = \beta_L)$	0.001			0.808		
$\Pr(\beta_{M,\text{same}} = \beta_{L,\text{same}})$		0.001	0.000		0.825	0.832
$\Pr(\beta_{M,\text{diff}} = \beta_{L,\text{diff}})$		0.064	0.159		0.712	0.824
$\Pr(\beta_{M,\text{same}} = \beta_{M,\text{diff}})$		0.029	0.005		0.724	0.517
$\Pr(\beta_{L,\text{same}} = \beta_{L,\text{diff}})$		0.531	0.955		0.302	0.406
$\Pr(\lambda_M = \lambda_L)$	0.237			0.174		
$\Pr(\lambda_{M,\text{same}} = \lambda_{L,\text{same}})$		0.008	0.006		0.846	0.851
$\Pr(\lambda_{M,\text{diff}} = \lambda_{L,\text{diff}})$		0.748	0.733		0.111	0.095
$\Pr(\lambda_{M,\text{same}} = \lambda_{M,\text{diff}})$		0.228	0.117		0.124	0.117
$\Pr(\lambda_{L,\text{same}} = \lambda_{L,\text{diff}})$		0.169	0.659		0.895	0.694
Obs.	37269	37269	37269	38037	38037	38037

Notes: The dependent variable in the regressions is the change in log productivity ($\Delta \ln A_{i,j,t}$), where the measure of productivity differs by column. The 3-digit classification refers to the level of industry classification used by Fabling and Maré (2015) which is very similar to the level 3 ANZSIC06 categories. The 1-digit classification refers to the level 1 ANZSIC06 categories. Standard errors are reported in parentheses. Each regression includes industry-year fixed effects, lagged productivity changes, hiring intensities from other sources, and excess turnover as additional regressors. The regressor $\Delta \ln A_{i,j,t-1}$ is instrumented for using $\ln A_{i,j,t-2}$ in response to the presence of Nickell bias. Productivity lag length is chosen to eliminate autocorrelation in the residual term. Productivity gaps are constructed using the subset of new hires from other private for profit firms for which productivity can be observed. The regression standard errors are clustered at the firm level. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

productive knowledge spillover channel that productive knowledge from within a firm's own industry is more applicable to the hiring firm and provides a larger boost to firm productivity than productive knowledge from outside the industry. Less productive knowledge, whether from inside or outside the firm's industry, is less useful to the hiring firm, and will likely be discarded, and hence there is not a significant difference in the effect of less productive knowledge from within or between industries.

For both of the MFP measures considered, the coefficients related to the productivity gaps from hires in the same industry are not significantly different from those related to hires from different industries (or the baseline results that do not distinguish between industries). This broadly lines up with the prediction from the unmeasured worker quality channel that the benefit to the hiring firm is unlikely be affected by the industry the worker previously worked in, given that it is hard to motivate how there will be a systemic difference in the ability of firms to screen or train workers within and between industries.[10]

The remaining parameters in the model are generally no significantly affected by the distinction between hires from within or between industries.Most notably, the coefficients related to the hire intensities (shown in table 3), which capture the effect of hiring intensity from the various sources, do not differ significantly with hiring from the same or different industries.

[10]An exception to this would be if unmeasured worker quality was related to on-the-job training, and workers received training that was industry specific. In such a case, we would expect to see some differences in the coefficients related to the same and different industries.

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