Albedo iteration in 3D setups

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1 Introduction

In this document the albedo iterations are explained that make sure that also with a varying albedo between day and nightside of the planet, the total energy is conserved. The problem is here that the 3D scheme in principle uses a β-map that is not aware of if the local PT point is on the day side or on the night side. This creates a problem when the night side has a different reflectivity as the day side as the night side cannot reflect stellar radiation.

1.1 Equations

As explained in the WASP-43b paper (Chubb & Min), the β-map represents a way of redistributing heat from the day to the night side. We have first the static β-map:

$$\beta_\star = \cos\Lambda\cos\Phi, \tag{1}$$

on the dayside and

$$\beta_\star = 0, \tag{2}$$

on the nightside. Here Λ and Φ are the longitude and latitude respectively. The β map is computed from a diffusion equation using $\beta_\star$ as source term. Note that β and $\beta_\star$ have the property that:

$$\int_{\text{planet}} \beta = \int_{\text{planet}} \beta_\star = \int_{\text{day}} \beta_\star = 1 \tag{3}$$

The total amount of reflected light from the planet is:

$$f_{\text{ref}} = \int_{\text{day}} \beta_\star \omega \tag{4}$$

If the albedo, ω, is a constant over the day and the night side, the fraction of starlight reflected taken into the computation of the P-T structure is:

$$f'_{\text{ref}} = \int_{\text{planet}} \beta\omega = \omega \int_{\text{planet}} \beta = \int_{\text{day}} \beta_\star \omega = \omega \tag{5}$$

This only works if the albedo is constant. If the albedo on the day and the night side is different $f'_{\text{ref}} \neq f_{\text{ref}}$, which makes the energy balance incorrect.

To solve this issue we can add a scaling to the β-map. The total emission needs to be $1 - f_{\text{ref}}$. If we take the new β-map to be:

$$\beta' = \gamma\beta \tag{6}$$

with γ a constant. We now have that the emission we use for the computation of the PT structure is equal to

$$\gamma - \int_{\text{planet}} \gamma\beta(\omega) \tag{7}$$

To have the right energy balance we need to make sure that

$$\gamma - \int_{\text{planet}} \gamma\beta\omega = 1 - \int_{\text{day}} \beta_\star\omega \tag{8}$$

which gives for the scaling factor

$$\gamma = \frac{1 - \int_{\text{day}} \beta_\star\omega}{1 - \int_{\text{planet}} \beta\omega} \tag{9}$$

Note that for constant ω we can use Eq. 3 to show that $\gamma = 1$ (as expected).

To compute γ we need to know the albedo at each location of the planet. For this we start with $\gamma = 1$ and compute the albedo everywhere. We use this to compute a new estimate of γ. Usually this is already enough and the value of γ does not change after one iteration. In the case of self-consistent cloud formation the albedo might depend more heavily on the PT structure and more than 1 iteration is required for γ to converge.