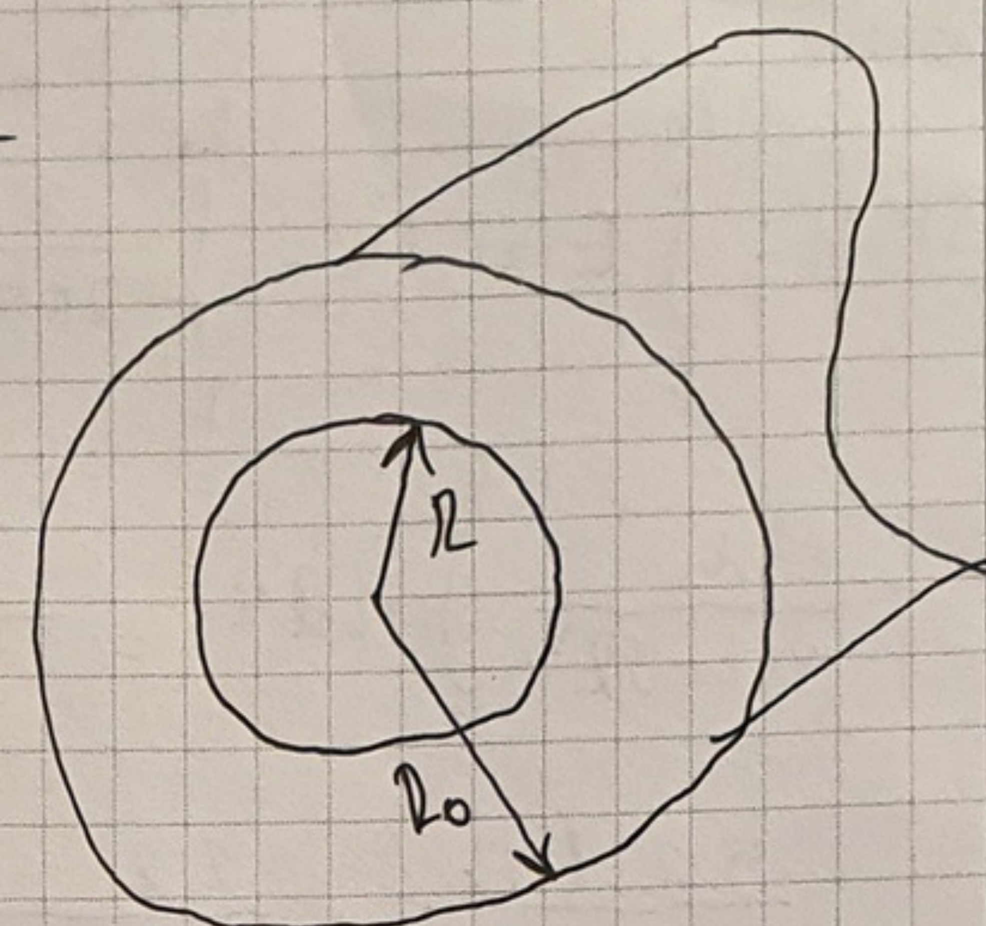
Перминов Дмитрий ИУ5-33Б ДЗ 1 вар. 11.

$R_0 = 3R \qquad \varepsilon = \dfrac{R_0^n}{R_0^n + R^n - r^n} \qquad n = 2$

Защита



Реш.

1) $\varepsilon = \dfrac{9R^2}{9R^2 + R^2 - r^2} = \dfrac{9R^2}{10R^2 - r^2}$

2) т. Гаусса $\quad R < r < R_0$

$\oint\limits_S D_n\, dS = \sum\limits_i q_i = q \;\rightarrow\; \oint D_n dS = D \cdot 2\pi r H = q \;\rightarrow\; \left| \lambda = \dfrac{dq}{dx} \right| \rightarrow$

$D(r) = \dfrac{\lambda}{2\pi r}$ (1)

3) $\bar{D} = \varepsilon\varepsilon_0 \bar{E} \;\rightarrow\; E = \dfrac{D}{\varepsilon\varepsilon_0} =$

$= \dfrac{\lambda}{2\pi r \varepsilon_0} \cdot \dfrac{(10R^2 - r^2)}{9R^2}$ (2)

4) $\bar{P} = \bar{E}(\varepsilon - 1)\varepsilon_0 \;\rightarrow\; P(r) = \dfrac{\lambda(10R^2 - r^2)}{2\pi r \varepsilon_0 \cdot 9R^2}\left(\dfrac{9R^2}{10R^2 - r^2} - 1\right)\varepsilon_0 =$

$= \left(\dfrac{\lambda}{2\pi r \varepsilon_0} - \dfrac{\lambda(10R^2 - r^2)}{2\pi r \varepsilon_0\, 9R^2}\right)\varepsilon_0 = \dfrac{\lambda(r^2 - R^2)}{2\pi r\, 9R^2}$ (3)

5) $\sigma'(r) = P_n = \dfrac{\lambda(r^2 - R^2)}{2\pi r \cdot 9R^2} \cos\alpha$, α – угол между нормалью к поверх и поляризации $\Rightarrow$ σ'_1 – знак „+"; σ'_2 – знак „-"

$\sigma'_1 = -\sigma(R) = \dfrac{-\lambda(R^2 - R^2)}{2\pi \cdot 9R^3} = 0$ – внут. поверх. (4)

$\sigma'_2 = \sigma(R_0) = \dfrac{\lambda(9R^2 - R^2)}{2\pi \cdot 9R^3 \cdot 3} = \dfrac{\lambda \cdot \cancel{8}^{4} \cancel{R^2}}{\cancel{2} \cdot 27\pi R^3} = \dfrac{4\lambda}{27\pi R}$ – внеш. поверх. (5)

6) $\rho' = -\operatorname{div}\bar{P} = -\dfrac{1}{r}\dfrac{\partial}{\partial r}\left(\dfrac{\lambda(r^2 - R^2)\cancel{r}}{2\pi \cancel{r}\, 9R^2}\right) = \dfrac{-\lambda \cdot 2r}{2\pi \cdot 9R^2 r} = \dfrac{-\lambda}{9\pi R^2}$ (6)

7) Проверка: $q' = \sigma'_1 S_1 + \sigma'_2 S_2 + \iiint\limits_{R<r<R_0} \rho'\, dV = 0 + \dfrac{4\lambda}{27\pi R} 2\pi H (3R) -$

$- \dfrac{\lambda}{9\pi R^2}\pi H (R_0^2 - R^2) = \dfrac{8\lambda H \cancel{R}}{9\cancel{R}} - \dfrac{\lambda H 8\cancel{R^2}}{9\cancel{R^2}} = 0$ //

8) $E_{max} = E(R) = \frac{\lambda \cdot 9R^2}{2\pi\varepsilon_0 \cdot 9R^3} = \frac{\lambda}{2\pi\varepsilon_0 R}$ (7)

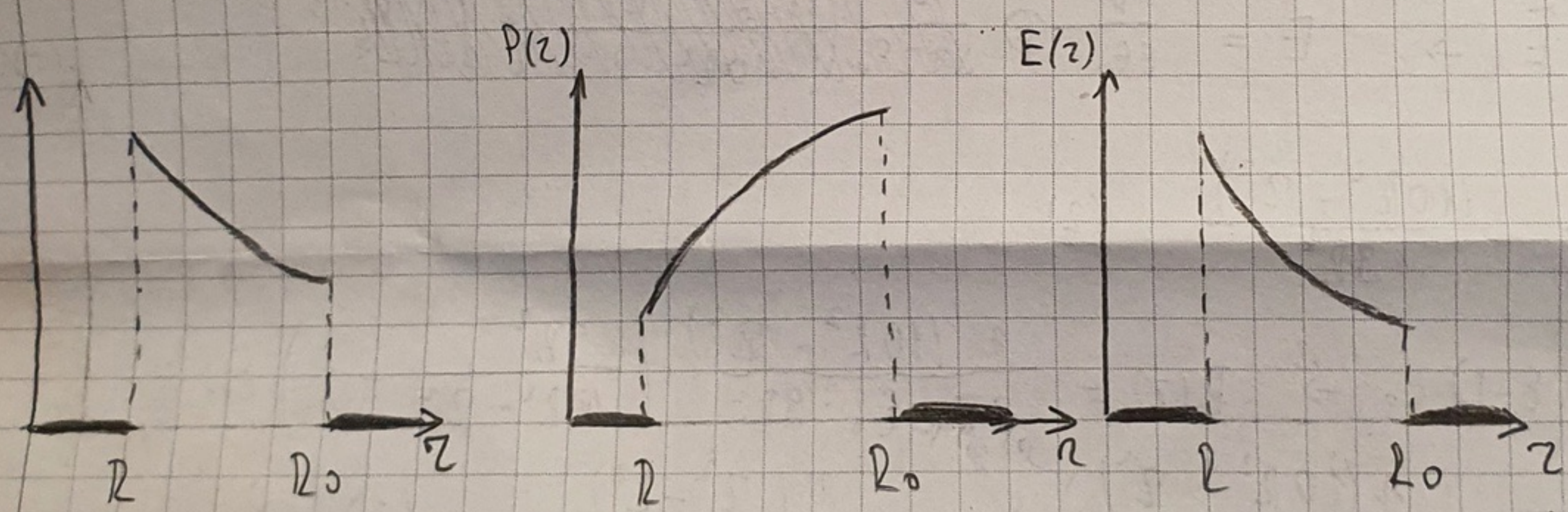
9) $U = \int\limits_R^{R_0} E(r)\,dr = \int\limits_R^{R_0} \frac{\lambda}{2\pi r\varepsilon_0}\left(\frac{10R^2 - r^2}{9R^2}\right)dr = \frac{\lambda \cdot 10\cancel{R^2}}{2\pi\varepsilon_0\, 9\cancel{R^2}}\int\limits_R^{R_0}\frac{dr}{r} -$

$-\frac{\lambda}{2\pi\varepsilon_0\, 9R^2}\int\limits_R^{R_0} r\,dr = \frac{5\lambda}{9\pi\varepsilon_0}\ln\left(\frac{R_0}{R}\right) - \frac{\lambda}{2\pi\varepsilon_0\, 9R^2}\,\frac{1}{2}\left(R_0^2 - R^2\right) =$

$= \frac{5\lambda\ln 3}{9\pi\varepsilon_0} - \frac{2\lambda}{9\pi\varepsilon_0} = \frac{\lambda(5\ln 3 - 2)}{9\pi\varepsilon_0}$

$C = \frac{q}{UH} = \frac{\lambda}{U} = \cancel{\frac{5\ln 3 - 2}{9\pi\varepsilon_0}} = \frac{9\pi\varepsilon_0}{5\ln 3 - 2}$ (8)

Ответы: (1) ... (8).



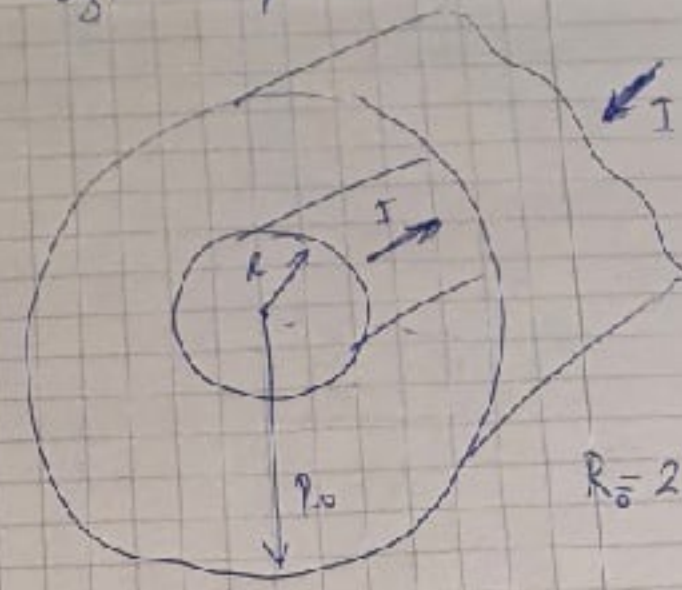
при $r < R$:

$D \cdot 2\pi r H = \sum_i q_i = 0 \rightarrow D = 0 \rightarrow E = 0 \rightarrow P = 0$

при $r > R_0$:

$D \cdot 2\pi r H = \sum_i q_i = +q - q = 0 \rightarrow D = 0 \rightarrow E = 0 \rightarrow P = 0$

Пермяков Дмитрий ИУ5-33Б Дз 2 Вар. 11



$R_0 = 2R$

1) $\mu = \frac{R^2 + r^2}{2R^2}$

2) По т.о циркуляции: $\oint_L H_\varphi \, dl = \sum_i I_i = \iint_S j \, dS$

$R < r < R_0$

$H \cdot 2\pi r = I$

Защита

$H = \frac{I}{2\pi r}$

3) $\bar{B} = \mu\mu_0 \bar{H}$

$\Rightarrow B = \frac{R^2 + r^2}{2R^2} \mu_0 \frac{I}{2\pi r} = \frac{I\mu_0 (r^2 + R^2)}{4\pi r R^2}$

4) $\bar{J} = (\mu - 1)\bar{H}$

при $R < r < R_0$: $J = \frac{R^2 + r^2 - 2R^2}{2R^2} \frac{I}{2\pi r} = \frac{(r^2 - R^2) I}{4\pi r R^2}$

5) $\bar{j}' = \operatorname{rot} \bar{J}$

$\operatorname{rot} \bar{J} = \frac{1}{r}\left(\frac{\partial J_z}{\partial \varphi} - \frac{\partial (r J_\varphi)}{\partial \varphi}\right)\bar{e}_r + \left(\frac{\partial J_z}{\partial z} - \frac{\partial J_r}{\partial r}\right)\bar{e}_\varphi + \frac{1}{r}\left(\frac{\partial (r J_\varphi)}{\partial r} - \frac{\partial J_z}{\partial \varphi}\right)\bar{e}_z$

$J_z = J_r = 0, \Rightarrow (\operatorname{rot} \bar{J})_z = (j')_z = \frac{1}{r}\frac{\partial (r J_\varphi)}{\partial r} = \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{(r^2 - R^2) I}{4\pi R^2}\right) =$

$= \frac{1}{r} \frac{I}{4\pi R^2} 2r = \frac{I}{2\pi R^2}$

6) Т.о циркуляции

$\oint_L (\bar{J}, d\bar{l}) = I' \to (J_{2z} - J_{1z}) l = I'$, I' - пов ток намагнич.

$\partial I'_{пов} = (\bar{i}'_{пов}, \bar{\tau}) dl = (\bar{i}'_{пов})_\tau dl \to I'_{пов} = \int_0^l (\bar{i}'_{пов})_\tau \, dl$

$\to (J_{2z} - J_{1z}) l = (\bar{i}'_{пов})_\tau l \to J_{2z} - J_{1z} = (\bar{i}'_{пов})_\tau = (\bar{i}'_{пов})_z$

Р-м цирк. поверх. рад. R_0:

$i'_{пов} = J_1 - J_2$ $\quad i'_{П внут} = J_1 - J_2 = 0$

в вакуме $J_1 = 0$, т.к. $\mu - 1 = 0$. В среде $J_2 = \frac{I(r^2 - R^2)}{4\pi R^2 r}$

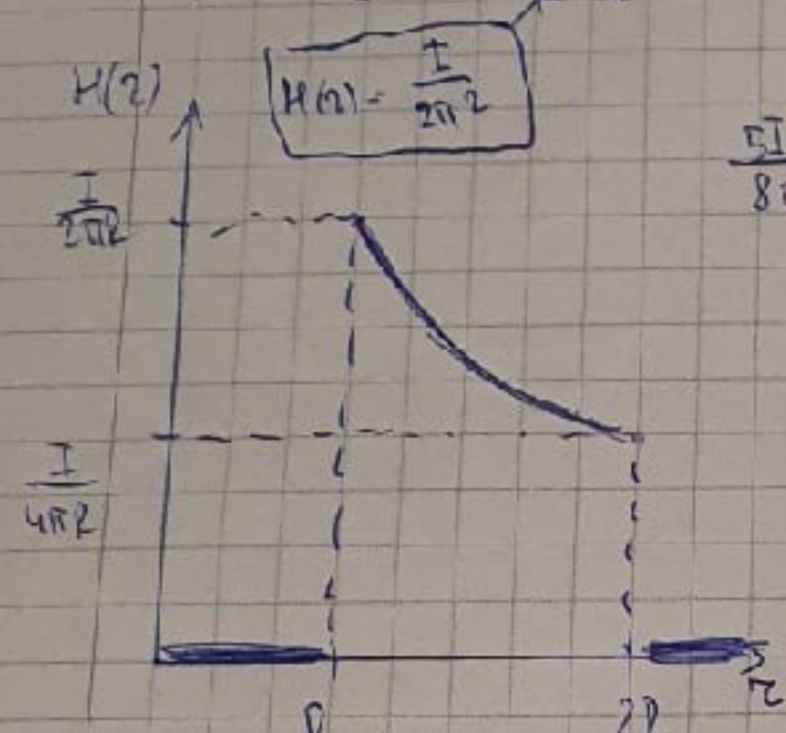
$i'_{П внеш} = J_1 - J_2 = 0 - \frac{(r^2 - R^2) I}{4\pi r R^2} = \frac{-I(r^2 - R^2)}{4\pi r R^2} = \frac{I(R^2 - r^2)}{4\pi r R^2}$

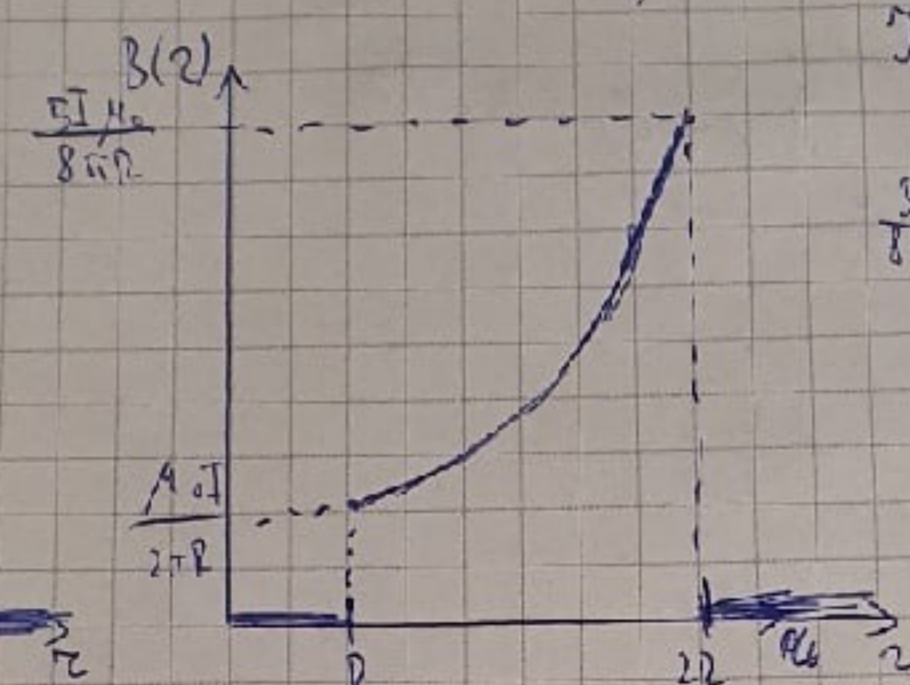
А при $r = R_0$: $(i'_{пов})_z = \frac{I(R^2 - (2R)^2)}{4\pi (2R) R^2} = -\frac{3I}{8\pi R}$

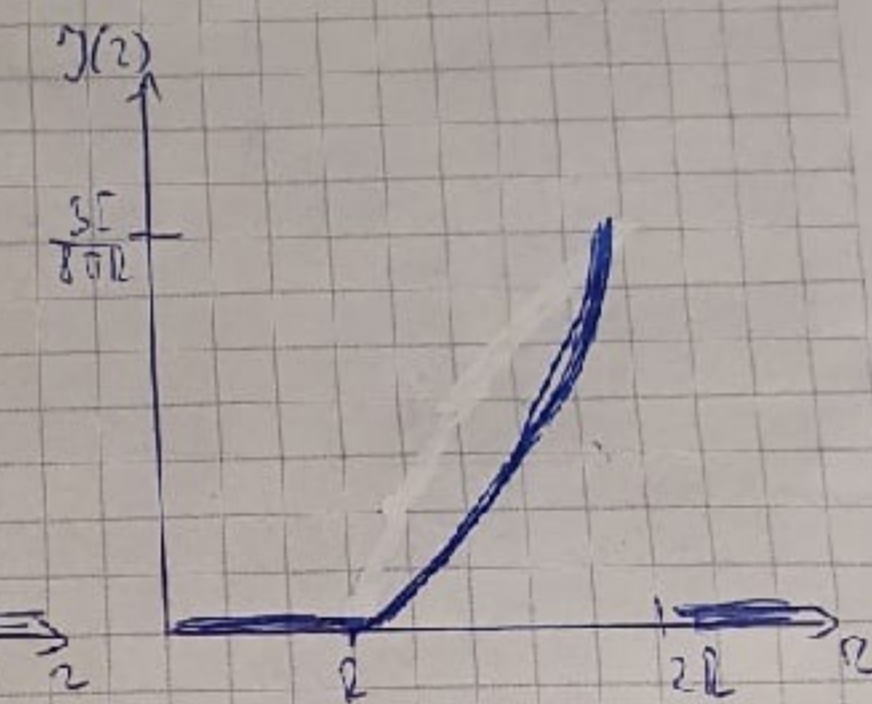
Проверка:

$$I' = \int_0^{2\pi R_0} i_{пов}\, dl + \int_R^{R_0} \frac{I}{2\pi R^2}\, 2\pi r\, dr = \frac{-3I}{2\cancel{\pi R}}\cancel{4\pi R} + \frac{I}{R^2}\,\frac{(R_0^2 - R^2)}{2} = |R_0 = 2R| =$$

$$= -\frac{3I}{2} + \frac{I\,3\cancel{R^2}}{\cancel{R^2}\,2} = -\frac{3I}{2} + \frac{3I}{2} = 0$$







Т. Стокса: $\oint_L H_l\, dl = \sum_i I_i$

при $r<R$ и при $r>2R$ $\sum_i I_i = 0 \Rightarrow$
$\Rightarrow H = 0$ $(\sum I = I - I = 0)$

$$\boxed{B = \frac{I\mu_0 (R^2 + r^2)}{4\pi R^2}}$$

$$\boxed{J = \frac{I(r^2 - R^2)}{4\pi R^2 r}}$$

при $r < R$ и при $r > R_0$, т.е. $r > 2R$ $\mu = 1 \Rightarrow$

$$J = (\mu - 1)\bar{H} = 0$$

Перминов Дмитрий ИУ5-33Б

Вар. 2

Дано: $C = C_0$

$R = R_0$

m

l

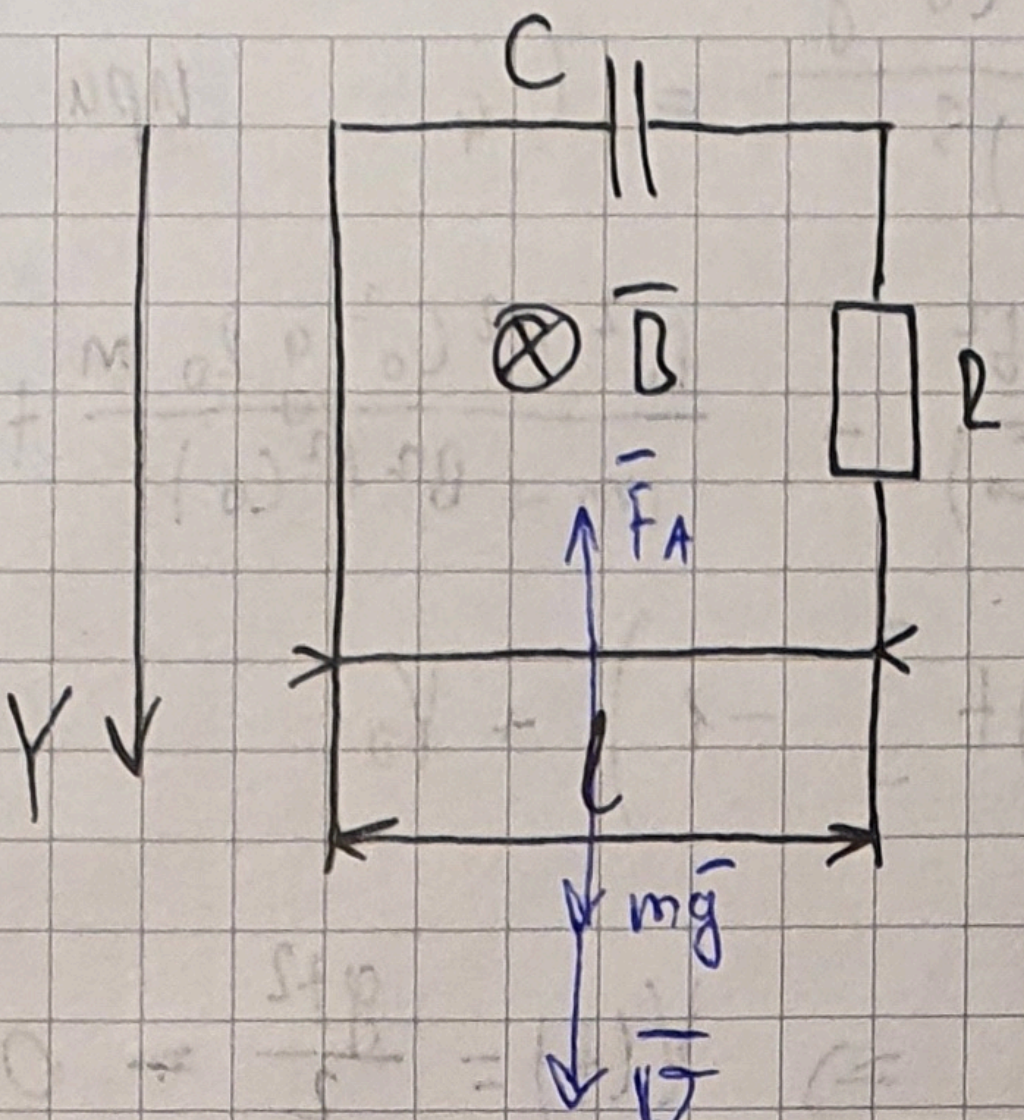
$\dot{y}(0)=0$

$q(0)=0$

B

$I(0)=0$

$Y(0)=Y_0$



Реш:

ПЗН: $\sum_{k=1}^{n} F_k = m\bar{a}$, $m\ddot{y} = mg - IBl$; $\ddot{y} = g - \frac{IBl}{m}$

ЭДС индукции: $|\varepsilon_i| = \left|-\frac{d\Phi}{dt}\right|$, где $\Phi = \int_S \bar{B}\,d\bar{S}$ $\quad d\Phi = Bl\,dy$

$$|\varepsilon_i| = \left|\frac{-Bl\,dy}{dt}\right| = Bl\dot{y}$$

ЭДС всей цепи: $\varepsilon_i = IR_0 + U_{конд}$, $U_{конд} = \frac{q}{C_0}$

$$Bl\dot{y} = IR_0 + \frac{q}{C_0} \Rightarrow Bl\ddot{y} = \dot{I}R_0 + \frac{\dot{q}}{C_0} , \quad \dot{q} = I \quad и \quad \ddot{y} = g - \frac{IBl}{m}$$

Тогда $Bl\left(g - \frac{IBl}{m}\right) = \dot{I}R_0 + \frac{I}{C_0} \Rightarrow \dot{I}R_0 + I\underbrace{\left(\frac{1}{C_0} + \frac{B^2l^2}{m}\right)}_{C_1} = \underbrace{Blg}_{C_2}$

$$\dot{I}R_0 + IC_1 = C_2 \quad (1)$$

реш-им ДУ при усл. $t_0 = 0$ $I(t_0) = 0$ $\Rightarrow I = \frac{C_2}{C_1} - \frac{C_2}{C_1} e^{-\frac{C_1 t}{R_0}}$

$$I = \frac{Blg}{\frac{1}{C_0} + \frac{(Bl)^2}{m}}\left(1 - e^{-\left(\frac{1}{C_0R_0} + \frac{B^2l^2}{mR_0}\right)t}\right) \Rightarrow I = \frac{BlgC_0m}{m + B^2l^2C_0}\left(1 - e^{\frac{-m+B^2l^2C_0}{C_0R_0m}t}\right)$$

$\Rightarrow$ подставим в исх. ур-е. $\Rightarrow \ddot{y} = g - \frac{Bl}{m}\cdot\frac{BlC_0gm}{m+B^2l^2C_0}\left(1 - e^{\frac{-m+B^2l^2C_0}{C_0R_0m}t}\right)$

$\Rightarrow$ интегр $\Rightarrow \dot{y} = gt - \frac{B^2l^2gC_0}{m+B^2l^2C_0}t - \frac{B^2l^2C_0^2R_0gm}{(m+B^2l^2C_0)^2}\exp\left[\frac{-m+B^2l^2C_0}{C_0R_0m}t\right] + C_3$

где $\dot{y} = \upsilon(t)$, $C_3 = \frac{B^2l^2C_0^2R_0gm}{(m+B^2l^2C_0^2)^2}$ при $t_0 = 0$ $\upsilon(t_0) = 0$

$\rightarrow$ инт $\rightarrow y = \frac{gt^2}{2} - \frac{B^2l^2gC_0}{2(m+B^2l^2C_0)}t^2 + \frac{B^2l^2C_0^2gR_0m}{(m+B^2l^2C_0)^2}t + \frac{B^2l^2C_0^3gm^2R_0^2}{(m+B^2l^2C_0)^3}\cdot\exp\left[-\left(\frac{m+B^2l^2C_0}{C_0R_0m}\right)\cdot t\right] + Y_0 - \frac{B^2l^2gR_0^2C_0^3m^2}{(m+B^2l^2C_0)^3}$

где $Y_0 - \frac{B^2 l^2 R_0^2 m^2 C_0^3 g}{(m+B^2 l^2 C_0)^3} = C_4$ при $\begin{cases} t_0 = 0 \\ y(t_0) = Y_0 \end{cases}$

$$\boxed{Y(t) = \frac{gt^2}{2} - \frac{B^2 l^2 g C_0 t^2}{2(m+B^2 l^2 C_0)} + \frac{B^2 l^2 C_0^2 g R_0 m}{(m+B^2 l^2 C_0)^2} t + \frac{B^2 l^2 C_0^3 m^2 R_0^2 g}{(m+B^2 l^2 C_0)^3} \cdot \left(\exp\left[-\left(\frac{m+B^2 l^2 C_0}{C_0 R_0 m}\right)t\right] - 1\right) + Y_0}$$

Проверка: при $B \to 0 \Rightarrow Y(t) = \frac{gt^2}{2} - 0 + 0 + 0 + Y_0$

$$Y(t) = \frac{gt^2}{2} + Y_0$$

при отсутств. ~~перемычки~~ маг. поле, перемычка двит. только под действ. силы тяжести $\Rightarrow$ всё верно.