$\boxed{6}$

(7) $x^2 y'' - xy' - 3y = 5x^4 \qquad y_1 = 1/x$

$$y'' - \frac{y'}{x} - \frac{3y}{x^2} = 5x^2$$

$$y_2 = \frac{1}{x} \int x^2 e^{\int \frac{dx}{x}} dx = \frac{1}{x} \int x^2 e^{\ln x} dx = \frac{1}{x} \int x^3 dx = \frac{x^4}{x \cdot 4} = \frac{x^3}{4}$$

$$y_2 = \frac{x^3}{4} \Rightarrow y_{oo} = \tilde{C}_1 \frac{1}{x} + \tilde{C}_2 \frac{x^3}{4}$$

$\exists\ \tilde{C}_1 = C_1(x) \quad \tilde{C}_2 = C_2(x)$

$$y_{on} = C_1(x) \frac{1}{x} + C_2(x) \frac{x^3}{4}$$

$$\left(\frac{1}{x}\right)' = (x^{-1})' = -1\, x^{-2}$$

$$\frac{3x^2}{4}$$

$$\begin{cases} C_1'(x) \frac{1}{x} + C_2'(x) \frac{x^3}{4} = 0 \\ C_1'(x) \left(-\frac{1}{x^2}\right) + C_2'(x) \frac{3x^2}{4} = 5x^2 \end{cases}$$

$$\Delta = \begin{vmatrix} \frac{1}{x} & \frac{x^3}{4} \\ -\frac{1}{x^2} & \frac{3x^2}{4} \end{vmatrix} = \frac{3x}{4} + \frac{x}{4} = x$$

$$\Delta C_1 = \begin{vmatrix} 0 & x^3/4 \\ 5x^2 & \frac{3x^2}{4} \end{vmatrix} = -\frac{5}{4} x^5 \qquad \Delta C_2 = \begin{vmatrix} 1/x & 0 \\ -1/x^2 & 5x^2 \end{vmatrix} = 5x$$

$$C_1'(x) = \frac{\Delta C_1}{\Delta} = -\frac{5x^5}{4x} = -\frac{5x^4}{4} \qquad C_2'(x) = \frac{\Delta C_2}{\Delta} = 5$$

$$C_1(x) = -\frac{5}{4} \int x^4 dx = -\frac{5 \cdot x^5}{5 \cdot 4} = -\frac{x^5}{4} + C_1 \qquad C_2(x) = 5\int dx = 5x + C_2$$

$$y_{on} = \frac{1}{x}\left(-\frac{x^5}{4} + C_1\right) + \frac{x^3}{4}(5x + C_2)$$

$\boxed{6}$ 5

$$y'' - 2y' + y = \frac{e^x}{\sqrt{4-x^2}}$$

1) ЛОДУ: $y'' - 2y' + y = 0$

хар. ур: $\lambda^2 - 2\lambda + 1 = 0$

$(\lambda - 1)^2 = 0$

$\lambda_{1,2} = 1$

$y_1 = e^x \quad y_2 = xe^x$

$y_{oo} = C_1 e^x + C_2 x e^x$

2) Пусть $C_1 = C_1(x) \quad C_2 = C_2(x)$

$y_{on} = C_1(x) e^x + C_2(x) x e^x$

$$\begin{cases} C_1'(x) e^x + C_2'(x) x e^x = 0 \\ C_1'(x) e^x + C_2'(x)(e^x + xe^x) = \dfrac{e^x}{\sqrt{4-x^2}} \end{cases}$$

$$\Delta = \begin{vmatrix} e^x & xe^x \\ e^x & e^x + xe^x \end{vmatrix} = e^{2x} + \cancel{xe^{2x}} - \cancel{xe^{2x}} = e^{2x}$$

$$\Delta C_1 = \begin{vmatrix} 0 & xe^x \\ \dfrac{e^x}{\sqrt{4-x^2}} & (e^x + xe^x) \end{vmatrix} = \frac{x e^{2x}}{\sqrt{4-x^2}}$$

$$\Delta C_2 = \begin{vmatrix} e^x & 0 \\ e^x & \dfrac{e^x}{\sqrt{4-x^2}} \end{vmatrix} = \frac{e^{2x}}{\sqrt{4-x^2}}$$

3) $C_1'(x) = \dfrac{x \cancel{e^{2x}}}{\sqrt{4-x^2}\,\cancel{e^{2x}}} = \dfrac{x}{\sqrt{4-x^2}} \qquad C_2'(x) = \dfrac{1}{\sqrt{4-x^2}}$

$$C_1(x) = \int \frac{x\,dx}{\sqrt{4-x^2}} = \left| dx^2 = 2x\,dx \right| = -\frac{1}{2}\int \frac{d(-x^2+4)}{\sqrt{-x^2+4}} = -\frac{1}{2}\,\frac{\sqrt{4-x^2}}{\frac{1}{2}} = -\sqrt{4-x^2} + C_1$$

$$C_2(x) = \int \frac{dx}{\sqrt{4-x^2}} = \arcsin\frac{x}{2} + C_2$$

$$y_{on} = e^x\left[C_1 - \sqrt{4-x^2}\right] + xe^x\left[\arcsin\frac{x}{2} + C_2\right]$$

6

8) $y'' - \frac{xy'}{x-1} + \frac{y}{x-1} = (x-1)e^x$ $\quad \boxed{y_1 = x}$

$$y_2 = x\int \frac{1}{x^2} e^{+\int \frac{x}{x-1}dx} dx = x\int \frac{1}{x^2} e^x (x-1)\,dx \ominus$$

$$\int \frac{(x-1)}{x^2} e^x dx = \int \frac{e^x \cdot x}{x^2}dx - \int \frac{e^x}{x^2}dx = \int \frac{e^x}{x}dx - \int \frac{e^x}{x^2}dx =$$

$$= \left[\begin{array}{cc|cc} u = \frac{1}{x} & du = -\frac{1}{x^2} & u = \frac{1}{x^2} & du = -2\frac{1}{x^3} \\ v = e^x & & v = e^x & \end{array}\right] = \frac{e^x}{x} + \cancel{\int \frac{e^x}{x^2}dx} - \cancel{\int \frac{e^x}{x^2}dx} = \frac{e^x}{x}$$

$$\ominus\ x \cdot \frac{e^x}{x} = \underline{\underline{e^x}}$$

$$y_{oo} = \tilde{C}_1 x + \tilde{C}_2 e^x$$

Пусть $\tilde{C}_1 = C_1(x)$ $\quad \tilde{C}_2 = C_2(x)$

$$\begin{cases} C_1'(x)\,x + C_2'(x)\,e^x = 0 \\ C_1'(x) + C_2'(x)\,e^x = (x-1)e^x \end{cases}$$

$$\Delta = \begin{vmatrix} x & e^x \\ 1 & e^x \end{vmatrix} = xe^x - e^x = e^x(x-1)$$

$$\Delta C_1 = \begin{vmatrix} 0 & e^x \\ (x-1)e^x & e^x \end{vmatrix} = -(x-1)e^{2x} \qquad \Delta C_2 = \begin{vmatrix} x & 0 \\ 1 & e^x(x-1) \end{vmatrix} = xe^x(x-1)$$

$$C_1'(x) = \frac{-\cancel{(x-1)}\cancel{e^x}\cdot e^x}{\cancel{e^x(x-1)}} = -e^x \qquad C_2'(x) = \frac{xe^x(x-1)}{e^x(x-1)} = x$$

$$C_1(x) = -\int e^x dx = -e^x + C_1 \qquad C_2(x) = \frac{x^2}{2} + C_2$$

$$y_{on} = xC_1 + e^x C_2 - xe^x + \frac{e^x \cdot x^2}{2}$$

$$-\frac{1}{2}\int \frac{d(2-y^2)}{\sqrt{2-y^2}} = \int dx \qquad x = -\sqrt{2-y^2} + C_2$$

$$Н.у: \ 1 = -\sqrt{2} + C_2 \qquad C_2 = 1+\sqrt{2}$$

$$x = -\sqrt{2-y^2} + 1 + \sqrt{2}$$

5) $y^{IV} + y'' = \begin{array}{c|c|c|c} f_1 & f_2 & f_3 & f_4 \\ x e^{-x} & + 2 - x & + x \sin x & - e^{x} \sin x \\ \lambda=-1,\ \beta=0,\ z=-1 & \lambda=0,\ \beta=0,\ z=0 & \lambda=0,\ \beta=1,\ z=\pm i & \lambda=1,\ \beta=1,\ z=1\pm i \end{array}$

ЛОДУ: $y^{IV} + y'' = 0$

хар.ур: $\lambda^4 + \lambda^2 = 0$

$\lambda^2(\lambda^2+1) = 0$

$\lambda_{12} = 0 \qquad \lambda^2 = -1 = i^2$

$\lambda = \pm i$

$y_1 = e^{0x} = 1 \qquad y_3 = \sin x$

$y_2 = x \qquad y_4 = \cos x$

$$y_{oo} = C_1 + C_2 x + C_3 \sin x + C_4 \cos x$$

ЛНОУ:

$f_1 = \boxed{x} e^{-x}$

$f_{r.1} = (A_1 x + B_1) e^{-x}$

$f_2 = 2\boxed{-x}$

$f_{r.2} = (A_2 x + B_2) x^2$

$f_3 = \boxed{x} \sin x$

$f_{r.3} = [(A_3 x + B_3)\sin x + (D_3 x + F_3)\cos x] x$

$f_4 = -e^{x} \sin x$

$f_{r.4} = [A_4 \sin x + B_4 \cos x] e^{x}$

$$y_{on} = y_{oo} + y_{r1} + y_{r2} + y_{r3} + y_{r4}$$

4.1

$xy'' + y' + x = 0$ явно не присут. y

$y' = p(x)$ $y'' = p'(x)$

$p' + \frac{p}{x} + 1 = 0$ – лин. $p = u(x)v(x) \Rightarrow u'v + v'u + \frac{uv}{x} = -1$

$u'v + u\left(v' + \frac{v}{x}\right) = -1$

$\begin{cases} v' + \frac{v}{x} = 0 & (1) \\ u'v = -1 & (2) \end{cases}$

(1): $\frac{dv}{dx} = -\frac{v}{x}$ $\int\frac{dv}{v} = -\int\frac{dx}{x}$

$\ln v = -\ln x$ $v_{част} = \frac{1}{x}$

(2) $\frac{du}{dx} = -x$ $\int du = -\int x dx$ $u = -\frac{x^2}{2} + C_1$

$p = \frac{1}{x}\left(-\frac{x^2}{2} + C_1\right) = y'$

$y' = -\frac{x}{2} + \frac{C_1}{x} \Rightarrow \frac{dy}{dx} = -\frac{x}{2} + \frac{C_1}{x} \Rightarrow \int dy = -\frac{1}{2}\int x dx + C_1\int\frac{dx}{x}$

$y = -\frac{x^2}{4} + C_1 \ln|x| + C_2$

4.2

$1 + yy'' + (y')^2 = 0$ $|:y$ $\boxed{y(1) = 1 \quad y'(1) = 1}$

$y'' + \frac{(y')^2}{y} + \frac{1}{y} = 0$ явно не прис x.

$y' = p(y)$ $y'' = p'p$ $p'p + \frac{p^2}{y} + \frac{1}{y} = 0$ $|:p$ $p' + \frac{p}{y} + \frac{1}{py} = 0$ ~~Берн~~ Берн.

$u(y)v(y) = p$ $u'v + v'u + \frac{uv}{y} = -\frac{1}{uvy}$

$u'v + u\left(v' + \frac{v}{y}\right) = \frac{-1}{uvy}$

$\begin{cases} v' + \frac{v}{y} = 0 & (1) \\ u'v = \frac{-1}{uvy} & (2) \end{cases}$

(1): $\frac{dv}{dy} = -\frac{v}{y}$

$\frac{dv}{v} = -\frac{dy}{y}$ $\ln v = -\ln y$ $\underline{v_{част} = \frac{1}{y}}$

(2): $\frac{du}{dy} \cdot \frac{1}{y} = \frac{-1}{u}$

$\int u\, du = -\int y\, dy$ $\frac{u^2}{2} = -\frac{y^2}{2} + \frac{C_1}{2}$

$u = \sqrt{-y^2 + C_1}$

$p = \frac{1}{y}\sqrt{C_1 - y^2} = y'$

$1 = \sqrt{C_1 - 1}$ $\underline{C_1 = 2}$

$p = \frac{1}{y}\sqrt{2 - y^2} = y'$ $\frac{dy}{dx} = \frac{1}{y}\sqrt{2-y^2}$ $\int\frac{y\,dy}{\sqrt{2-y^2}} = \int dx$ $[dy^2 = 2y dy]$ $\frac{1}{2}\int\frac{dy^2}{\sqrt{2-y^2}}$

$\boxed{1.7}$ $\int \frac{x^2-1}{x^3+x}\,dx \;\ominus$

$$\frac{x^2-1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1} = \frac{Ax^2+A+Bx^2+Cx}{x(x^2+1)}$$

$$x^2-1 = Ax^2 + Bx^2 + Cx + A$$

x^0: $-1 = A$ $\qquad A=-1$

x^1: $0 = C$ $\qquad C=0$

x^2: $1 = A+B$ $\qquad B=2$

$$\ominus\; -\int \frac{dx}{x} + 2\int \frac{x\,dx}{x^2+1} = -\ln|x| + \ln|x^2+1| + C$$

$\boxed{1.8}$

$$\int \frac{e^{\operatorname{arctg} x} + x\ln(1+x^2)+1}{1+x^2}\,dx = \int e^{\operatorname{arctg} x}\,d(\operatorname{arctg} x) + \int \frac{x\ln(1+x^2)}{1+x^2}\,dx + \int \frac{dx}{1+x^2} =$$

$$= e^{\operatorname{arctg} x} + \frac{1}{2}\int \frac{\ln(1+x^2)\,d(1+x^2)}{1+x^2} + \operatorname{arctg} x = e^{\operatorname{arctg} x} + \operatorname{arctg} x + \frac{\ln^2(1+x^2)}{4} + C$$

$$\left\{ \int \frac{\ln t\,dt}{t} = \int \ln t\; d(\ln t) = \frac{\ln^2 t}{2} \qquad \frac{\ln^2(1+x^2)}{2} \right\}$$

$\boxed{1.9}$

$$\int \sin\sqrt{x}\,dx = \left| \begin{matrix} \sqrt{x}=t \\ x = t^2 \\ dx = 2t\,dt \end{matrix} \right| = 2\int \sin t \cdot t\,dt = \left\{ \begin{matrix} u = t & du = dt \\ v = -\cos t & \end{matrix} \right\} = -t\cos t + \sin t + C =$$

$$= -2\sqrt{x}\cos\sqrt{x} + 2\sin\sqrt{x} + C$$

1.10

$$\int \frac{x\,dx}{x^3-3x+2} = \left\{ \begin{array}{cccc|c} 1 & 0 & -3 & 2 & \\ & 1 & 1 & -2 & 1 \\ \hline 1 & 1 & -2 & 0 & \end{array} \quad \begin{array}{c} -2 \\ 1 \end{array} \quad (x^2+x-2)(x-1) = (x+2)(x-1)^2 \right\} =$$

$$= \int \frac{x\,dx}{(x+2)(x-1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+2}$$

1.11

$$\int \frac{dx}{e^x\sqrt{4e^{2x}-1}} = \left\{ \begin{array}{l} e^x = t \\ x = \ln t \\ dx = \frac{1}{t}\,dt \end{array} \right\} = \int \frac{dt}{t^2\sqrt{4t^2-1}} = \left| \begin{array}{l} t = \frac{1}{y} \\ dt = -\frac{1}{y^2} \end{array} \right| = \Big\{ de^{-x} =$$

$$= -\int \frac{\cancel{y^2}\,dy}{\cancel{y^2}\sqrt{\frac{4}{y^2}-1}} = -\int \frac{dy}{\sqrt{\frac{4}{y^2}-1}} = \left| \frac{4-y^2}{y^2} \right| = -\int \frac{y\,dy}{\sqrt{4-y^2}} = +\frac{1}{2}\int \frac{d(y^2+4)}{\sqrt{-y^2+4}} =$$

$$= \sqrt{4-y^2} + C = \sqrt{4-\frac{1}{t^2}} + C = \sqrt{4-\frac{1}{e^{x2}}} + C$$

1.13

$$\int t^2\sqrt[4]{1+t^3}\,dt = \left[dt^3 = 3t^2 \right] = \frac{1}{3}\int (1+t^3)^{1/4}\,d(t^3+1) = \frac{(t^3+1)^{\frac{5}{4}}}{5}\cdot\frac{4}{3} =$$

$$= (\ln^3 x+1)^{\frac{5}{4}}\cdot\frac{4}{15} + C$$

6.1 $y'' + y = tg x \frac{1}{\cos x}$

1ООУ: $y'' + y = 0$

хар.ур: $\lambda^2 + 1 = 0$

$\lambda^2 = i^2$

$\lambda = \pm i$

$y_1 = \sin x \quad y_2 = \cos x$

$$\underline{y_{oo} = C_1 \sin x + C_2 \cos x}$$

Пусть $C_1 = C_1(x) \quad C_2 = C_2(x)$

$y_{on} = C_1(x) \sin x + C_2(x) \cos x$

$$\begin{cases} C_1'(x) \sin x + C_2'(x) \cos x = 0 \\ C_1'(x) \cos x + C_2'(x)(-\sin x) = tg x \frac{1}{\cos x} \end{cases}$$

$$\Delta = \begin{vmatrix} \sin x & \cos x \\ \cos x & -\sin x \end{vmatrix} = -\sin^2 x - \cos^2 x = -1$$

$$\Delta C_1 = \begin{vmatrix} 0 & \cos x \\ tg x \frac{1}{\cos x} & -\sin x \end{vmatrix} = -tg x$$

$$\Delta C_2 = \begin{vmatrix} \sin x & 0 \\ \cos x & \frac{tg x}{\cos x} \end{vmatrix} = \frac{\sin x}{\cos x} tg x = tg^2 x$$

$C_1'(x) = tg x$

$C_2'(x) = -tg^2 x$

$$C_1(x) = \int tg x \, dx = \int \frac{\sin x \, dx}{\cos x} = -\ln|\cos x| + \tilde{C}_1$$

$$C_2(x) = -\int \frac{\sin^2 x}{\cos^2 x} dx = -\int \frac{1 - \cos^2 x}{\cos^2 x} dx = -\int \frac{dx}{\cos^2 x} + \int \frac{\cos^2 x}{\cos^2 x} dx =$$

$$= -tg x + x + \tilde{C}_2$$

$$y_{on} = \underbrace{\left(\tilde{C}_1 \sin x + \tilde{C}_2 \cos x\right)}_{y_{oo}} \underbrace{- \sin x \ln|\cos x| - \sin x + \cos x \cdot x}_{y_{чн}}$$

$\boxed{1.4}$

$$\int \frac{dx}{3-2\sin x+\cos x} = \left\{ \begin{array}{ll} \boxed{\text{УПП}}\ \ tg\frac{x}{2}=t & \frac{x}{2}=2\,arctg\,t \\ dx=\frac{2\,dt}{1+t^2} & \\ \sin x=\frac{2t}{1+t^2} & \cos x=\frac{1-t^2}{1+t^2} \end{array} \right\} = 2\int \frac{dt}{(1+t^2)\left(3-\frac{4t}{1+t^2}+\frac{1-t^2}{1+t^2}\right)} =$$

$$= 2\int \frac{dt}{3(1+t^2)-4t+1-t^2} = 2\int \frac{dt}{3+3t^2-4t+1-t^2} = 2\int \frac{dt}{2t^2-4t+4} =$$

$$= \int \frac{dt}{t^2-2t+2} = \int \frac{dt}{(t-1)^2+1} = arctg\,\frac{t-1}{1}+C = arctg\left(tg\frac{x}{2}-1\right)+C$$

$\boxed{1.5}$

$$\int \frac{dx}{x\sqrt{x^2-2x-1}} = \left| \begin{array}{ll} x=\frac{1}{t} & t=\frac{1}{x} \\ dx=-\frac{dt}{t^2} & \end{array} \right| = -\int \frac{\cancel{t}\,dt}{t^2\sqrt{\frac{1}{t^2}-\frac{2}{t}-1}} = -\int \frac{dt}{\sqrt{1-2t-t^2}} =$$

$$= -\int \frac{dt}{\sqrt{-(t^2+2t-1)}} = \left[\, t^2+2t+1-2 \,\right] = -\int \frac{dt}{\sqrt{2-(t+1)^2}} = -\int \frac{d(t+1)}{\sqrt{\ldots}} =$$

$$= -\arcsin\frac{t+1}{\sqrt{2}}+C = -\arcsin\frac{\frac{1}{x}+1}{\sqrt{2}}+C$$

$$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right|$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a}$$

$\boxed{1.6}$

$$\int \sqrt{x^2-2x+5}\,dx = \int \sqrt{(x-1)^2+4}\,dx \;\ominus$$

$$\int \sqrt{x^2+a^2}\,dx = \left\{ \begin{array}{ll} u=\sqrt{x^2+a^2} & \\ & du=\frac{2x\,dx}{2\sqrt{x^2+a^2}} \\ V=x & \end{array} \right\} = x\sqrt{x^2+a^2} - \int \frac{x^2\,dx}{\sqrt{x^2+a^2}} =$$

$$= x\sqrt{x^2+a^2} - \int \frac{x^2+a^2}{\sqrt{x^2+a^2}}\,dx + \int \frac{a^2\,dx}{\sqrt{x^2+a^2}} = x\sqrt{x^2+a^2} - \int\sqrt{x^2+a^2}\,dx + a^2\ln\left(x+\sqrt{x^2+a^2}\right)$$

$$\int \sqrt{x^2+a^2}\,dx = \frac{x\sqrt{x^2+a^2}+a^2\ln\left(x+\sqrt{x^2+a^2}\right)}{2}+C$$

$$\ominus\; \frac{(x-1)\sqrt{(x-1)^2+4}+4\ln\left|x-1+\sqrt{(x-1)^2+4}\right|}{2}+C$$

1.1 изи

1.2 $\int x^2 \sin x \, dx = \left| \begin{matrix} u = x^2 & du = 2x\,dx \\ V = \int \sin x \, dx = -\cos x \end{matrix} \right| = -x^2\cos x + 2\int \cos x \cdot x \, dx \ominus$

$\cancel{\int x^2 \sin x}$ $\int x \cos x \, dx = \left| \begin{matrix} u = x & du = dx \\ V = \int \cos x \, dx = \sin x \end{matrix} \right| = x\sin x - \int \sin x \, dx = x \cdot \sin x + \cos x$

$\ominus -x^2 \cos x + 2\sin x \cdot x - 2\cos x + C$

1.3

$\int \sin^2 x \cos^4 x \, dx = \left\{ \begin{matrix} \cos 2x = \cos^2 x - \sin^2 x = \\ = 1 - 2\sin^2 x & \sin^2 x = \frac{1-\cos 2x}{2} \\ = 2\cos^2 x - 1 & \Rightarrow \cos^2 x = \frac{\cos 2x + 1}{2} \end{matrix} \right\} = \int \frac{1-\cos 2x}{2} \left(\frac{1+\cos 2x}{2}\right)^2 =$

$= \frac{1}{8}\int (1-\cos 2x)(1+\cos 2x)^2 dx = \frac{1}{8}\int (1-\cos 2x)(1+2\cos 2x + \cos^2 2x)\,dx =$

$= \frac{1}{8}\int (1 + \cancel{2}\cos 2x + \cancel{\cos^2 2x} - \cancel{\cos 2x} - \cancel{2}\cos^2 2x - \cos^3 2x)\, dx =$

$= \frac{1}{8}\left[x + 1\int \cos 2x \, dx - \int \cos^2 2x \, dx - \int \cancel{\cos 2x} \, dx \int \cos^3 2x \, dx \right] =$

$= \frac{1}{8}x + \cancel{\frac{1}{16}\sin 2x} - \frac{1}{16}x - \frac{1}{8 \cdot 8}\sin 4x - \cancel{\frac{1}{16}\sin 2x} - \frac{1}{48}\sin^3 2x + C$

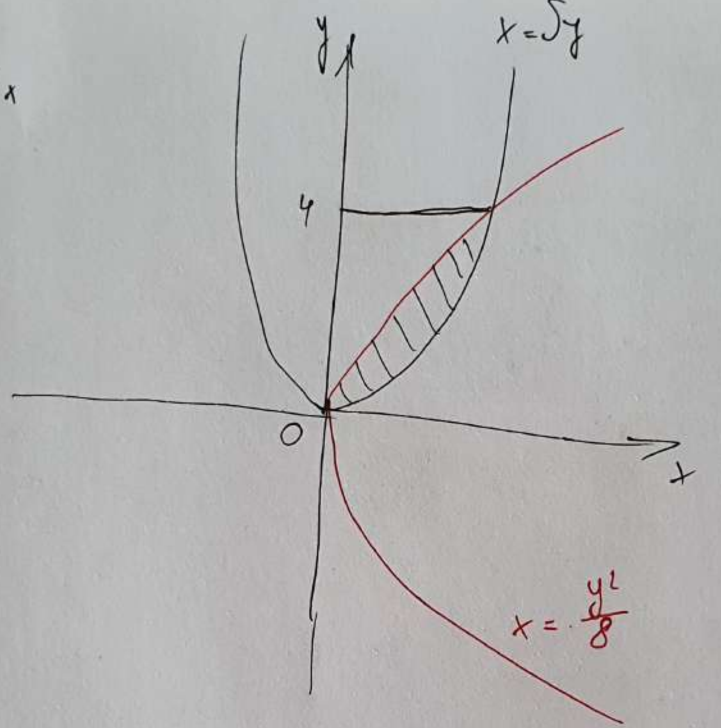
$\int \cos^2 2x \, dx = \left[\frac{1+\cos 4x}{2}\right] = \frac{1}{2}\int dx + \frac{1}{2 \cdot 4}\int \cos 4x \, d4x = \frac{1}{2}x + \frac{1}{8}\sin 4x$

$\int \cos^2 2x \cdot \cos 2x \, dx = \frac{1}{2}\int \cos^2 2x \, d\sin 2x = \frac{1}{2}\int d\sin 2x - \frac{1}{2}\int \sin^2 2x \, d\sin 2x \ominus$

$d\sin 2x = \cos 2x \cdot 2 \cdot dx \mid 1 - \sin^2 2x \mid$ $\ominus \frac{1}{2}\sin 2x - \frac{1}{2} \frac{\sin^3 2x}{3}$

$\boxed{2.2}$

$y = x^2 \quad y^2 = 8x$



$$V_y = \pi\int_c^d x^2(y)\,dy$$

$\sqrt{y} = \frac{y^2}{8} \quad \cancel{y} = \frac{y^{\cancel{4}3}}{64}$

$y^3 = 64 \quad \underline{y = 4}$

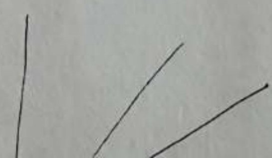
рабочее поле

$$V_y = \pi\int_0^4 \left(y - \frac{y^4}{64}\right)dy = \pi\frac{y^2}{2}\Big|_0^4 - \pi\frac{y^5}{5\cdot 64}\Big|_0^4 = \pi\left(8 - \frac{1024}{5\cdot 64}\right) =$$

$$= \pi\left(8 - \frac{\cancel{2^{10}}\,2^4}{5\cdot \cancel{2^6}}\right) = \pi\left(8 - \frac{16}{5}\right) = \pi\left(\frac{40-16}{5}\right) = \pi\left(\frac{24}{5}\right) = \frac{24\pi}{5}$$

$\boxed{2.3}$

$r = a\sin^3\frac{\varphi}{3}$



$\boxed{2.3}$

$r = a \sin^3 \frac{\varphi}{3}$ $\qquad l = \int\limits_{L}^{\beta} \sqrt{r_\varphi^2 + r_\varphi'^2}\, d\varphi$

$r'_\varphi = a \frac{3}{3} \sin^2 \frac{\varphi}{3} \cos \frac{\varphi}{3} = a \sin^2 \frac{\varphi}{3} \cos \frac{\varphi}{3}$

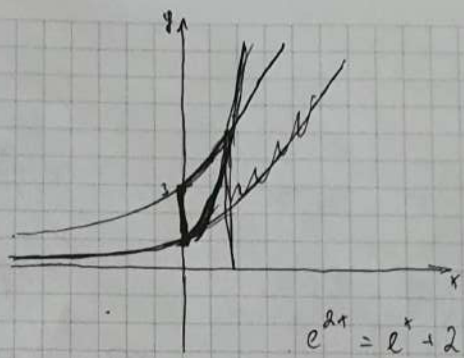
$$l = \int_0^{3\pi} \sqrt{a^2 \sin^6 \frac{\varphi}{3} + a^2 \sin^4 \frac{\varphi}{3} \cos^2 \frac{\varphi}{3}}\, d\varphi = a \int_0^{3\pi} \sqrt{\sin^6 \frac{\varphi}{3} + \sin^4 \frac{\varphi}{3} \cos^2 \frac{\varphi}{3}}\, d\varphi =$$

$$= a \int_0^{3\pi} \sin^2 \frac{\varphi}{3}\, d\varphi = \left| \begin{array}{l} \cos 2x = \cos^2 x - \sin^2 x = \\ = \cancel{\cos^2 x}\ 1 - 2\sin^2 x \\ \sin^2 x = \frac{1 - \cos 2x}{2} \end{array} \right| = a \int \frac{1 - \cos 2\frac{\varphi}{3}}{2}\, d\varphi =$$

$$= \frac{a}{2} \int d\varphi - \frac{a}{2} \frac{3}{2} \int \cos \frac{2\varphi}{3}\, d\frac{2\varphi}{3} = \frac{a}{2} \varphi \Big|_0^{3\pi} - \frac{3a}{4} \sin \frac{2\varphi}{3} \Big|_0^{3\pi} = \frac{3\pi \cdot a}{2} -$$

$$- \frac{3a}{4} \left[\underbrace{\sin 2\pi}_{0} - 0 \right] = \frac{3\pi}{2} a$$

① $y = e^{2x}$ $\quad y = e^x + 2$ $\quad x = 0$



x	0	1
y	1	2..

x	0	1
y	3	4

$e^{2x} = e^x + 2$

$\ln e^{2x} = \ln e^x \cdot \ln 2$

$2x = x \ln 2$

$e^{2x} = e^x + 2$

$\cancel{\ln e^{2x} = \ln e^x \cdot \ln 2}$

$t^2 - t - 2 = 0$

$\left[\begin{array}{l} t = 2 \\ t = -1 \end{array}\right.$ $\quad e^x = 2$, $x = \ln 2$

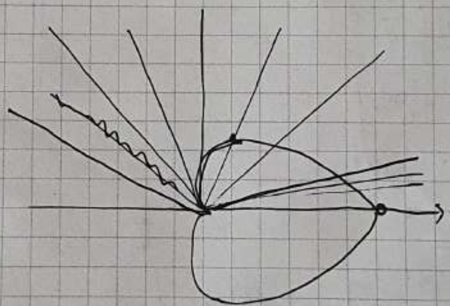
$$S = \int_0^{\ln 2} (e^x + 2 - e^{2x})\,dx =$$

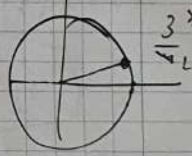
$$= \int_0^{\ln 2} e^x\,dx + 2\int_0^{\ln 2} dx - \int_0^{\ln 2} e^{2x}\,dx =$$

$$= e^x \Big|_0^{\ln 2} + 2 \cdot x \Big|_0^{\ln 2} - \frac{1}{2} e^{2x} \Big|_0^{\ln 2} = e^{\ln 2} + 2\ln 2 - \frac{1}{2} e^{2\ln 2} = \cancel{2} + 2\ln 2 - \cancel{2} =$$

$$= 2\ln 2$$

② $\rho = 2\cos^2\frac{\varphi}{2} = 1 + \cos 2\varphi$





$$V = \frac{2}{3}\pi \int_L^\beta \rho^3(a) \sin\varphi\, d\varphi$$

$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1$

$\cos^2 x = \dfrac{\cos 2x + 1}{2}$

$$V = \frac{2}{3}\pi \int_0^\pi (1 + \cos 2\varphi)^3 \sin\varphi\, d\varphi =$$

$$= \frac{4}{3}\pi \int_0^\pi (1 + 3\cos 2\varphi + 3\cos^2 2\varphi + \cos^3 2\varphi) \sin\varphi\, d\varphi =$$

$$= \frac{4}{3}\pi \left[\int_0^\pi \sin\varphi\, d\varphi + 3\int_0^\pi \cos 2\varphi \sin\varphi\, d\varphi + 3\int_0^\pi \cos^2 2\varphi \sin\varphi + \int_0^\pi \cos^3 2\varphi \sin\varphi\, d\varphi \right] =$$

$$= \frac{4}{3}\pi \left[-\cos\varphi \Big|_0^\pi - \frac{3}{2}\, \frac{\cos^2 2\varphi}{2} \Big|_0^\pi - \frac{3}{2}\, \frac{\cos^3 2\varphi}{3} \Big|_0^\pi - \frac{1}{2}\, \frac{\cos^4 2\varphi}{4} \Big|_0^\pi \right] =$$

$$= \frac{4}{3}\pi \left[\cancel{1} - \cancel{1} - \frac{3}{2}\frac{1}{2} + \frac{3}{4} - \frac{3}{2}\frac{1}{3} + \frac{3}{2 \cdot 3} - \frac{1}{2}\frac{1}{4} + \frac{1}{2 \cdot 4} \right] = -\frac{4\pi}{3 \cdot 2} + \frac{2}{3}\pi - \frac{4\pi}{8 \cdot 3} +$$

$$+ \frac{4}{8 \cdot 3}\pi =$$

$$= -\frac{4\pi}{6} + \frac{2}{3}\pi - \frac{\pi}{6} + \frac{\pi}{6} = 0$$

$\int \cos^2 x\, dx =$ $\quad \int \cos 2\varphi\, d(\cos 2\varphi)$

$\boxed{К4}$

① $\sqrt{y+1}\,(yy''-(y')^2) = yy'\sqrt{yy'}$ $\quad y(0)=1 \quad y'(0)=2$

явно отсут. $x \to$ $\quad y'=p(y) \quad y''=p'p$

$\sqrt{y+1}\,(y\cdot p'p - p^2) = y\cdot p\sqrt{yp}$

$p'py - p^2 = \dfrac{yp\sqrt{py}}{\sqrt{y+1}}$; $\quad p' - \dfrac{p}{y} = \dfrac{\sqrt{py}}{\sqrt{y+1}}$

$p = u(y)\,v(y) \Rightarrow \quad u'v + v'u - \dfrac{uv}{y} = \dfrac{\sqrt{uvy}}{\sqrt{y+1}}$

$u'v + u\left(v' - \dfrac{v}{y}\right) = \dfrac{\sqrt{uvy}}{\sqrt{y+1}}$

$$\begin{cases} v' - \dfrac{v}{y} = 0 & :(1) \\ u'v = \dfrac{\sqrt{uvy}}{\sqrt{y+1}} & (2): \end{cases}$$

(1): $\dfrac{dv}{dy} = \dfrac{v}{y}$

$\dfrac{dv}{v} = \dfrac{dy}{y}$ $\quad \boxed{v_{част} = y}$

(2):

$\dfrac{du}{dy}\,y = \dfrac{\sqrt{u}\sqrt{y}}{\sqrt{y+1}} \quad \Big| :y$

$\dfrac{du}{\sqrt{u}} = \dfrac{d(y+1)}{\sqrt{y+1}}$

$2\sqrt{u} = \ln|y+1| + \ln|c|$

$\sqrt{u} = \dfrac{\ln|c(y+1)|}{2}$ $\quad u = \dfrac{\ln^2(c(y+1))}{4}$

$p = \dfrac{\ln^2[c(y+1)]\,y}{4}$

н. у.: $p = \dfrac{\ln^2(2c)\,\cancel{1}}{\cancel{4}\,2} = \dfrac{\ln^2(2c)}{2} = 2$

$\ln^2(2c) = 4 \quad \ln(2c) = 2$

$2c = e^2$

$\boxed{c = \dfrac{e^2}{2}} \longrightarrow y' = \dfrac{\ln^2\left|\frac{e^2}{2}(y+1)\right|}{2\!\!\!/4}$

$y' = \dfrac{\ln^2\left(\frac{e^2}{2}\right) + \ln^2(y+1)}{4}$

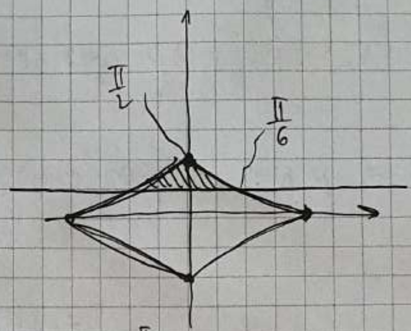
Рупинтов

K2

(3) $S_{\varphi} = ?$ $\begin{cases} x = 4\cos t \\ y = 2\sin t \end{cases}$ $y = 1$ $M(0;2)$

t	0	$\frac{\pi}{2}$	$\frac{\pi}{4}$
x	4	0	
y	0	2	

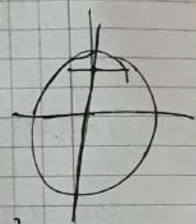
$$S = \int_{t_1}^{t_2} y(t)\, x'_t \, dt$$



$2\sin t = 1$

$\sin t = \frac{1}{2}$

$t = \frac{\pi}{6}$



сим.

$$S = 4\cdot 2\cdot 2\int_{\frac{\pi}{2}}^{\frac{\pi}{6}} \sin t\cdot(-\sin t)\,dt = -16\int_{\frac{\pi}{2}}^{\frac{\pi}{6}} \sin^2 t\,dt = [(*)] = +\frac{\cancel{16}^{8}\pi}{\cancel{6}_{3}} + \frac{\cancel{16}^{2}\sqrt{3}}{\cancel{8}} =$$

$$= \boxed{\frac{8\pi}{3} + 2\sqrt{3}}$$

(*)

$$\int \sin^2 t\,dt = \cancel{\int(1-\cos 2t)\,dt} = \frac{1}{2}\int(1-\cos 2t)\,dt =$$

$\cos 2t = \cos^2 t - \sin^2 t = 1 - 2\sin^2 t$

$\sin^2 t = \frac{1-\cos 2t}{2}$

$$= \frac{1}{2}t\Big| - \frac{1}{2\cdot 2}\int \cos 2t\, d2t =$$

$$= \frac{1}{2}t\Big|_{\frac{\pi}{2}}^{\frac{\pi}{6}} - \frac{1}{4}\sin 2t\Big|_{\frac{\pi}{2}}^{\frac{\pi}{6}} =$$

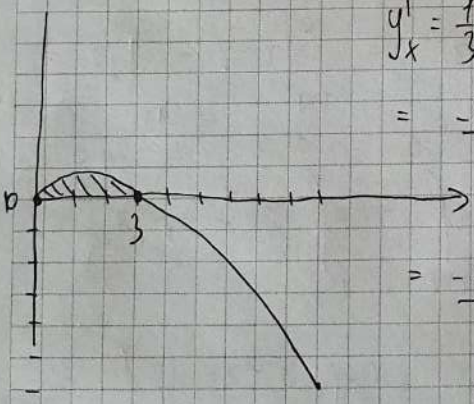
$$= \frac{1}{2}\Big(\frac{\pi}{6} - \frac{3\pi}{6}\Big) - \frac{1}{4}\Big(\frac{\sqrt{3}}{2}\Big) =$$

$-\frac{\pi}{3}$

$\sin\frac{\pi}{3}$ $\sin\pi$ $= -\frac{\pi}{6} - \frac{\sqrt{3}}{8}$

(4) $l = ?$ $y = \frac{1}{3}(3-x)\sqrt{x}$

x	0	9
y	0	-6



$$y'_x = \frac{1}{3}\Big[-\sqrt{x} + (3-x)\frac{1}{2\sqrt{x}}\Big] =$$

$$= \frac{-\sqrt{x}}{3} + \frac{3-x}{2\sqrt{x}\cdot 3} =$$

$$= \frac{-2x + 3 - x}{6\sqrt{x}} =$$

$$= \frac{3-3x}{6\sqrt{x}} = \frac{1-x}{2\sqrt{x}}$$

$$l = \int_0^3 \sqrt{1 + \Big(\frac{1-x}{2\sqrt{x}}\Big)^2}\,dx = \int\sqrt{1 + \frac{(1-x)^2}{4x}}\,dx =$$

$$= \frac{1}{4}\int\sqrt{\frac{x^2+2x+1}{x}}\,dx = \frac{1}{4}\int\frac{x+1}{\sqrt{x}}\,dx =$$

$$= \frac{1}{4}\int\frac{x}{\sqrt{x}}\,dx + \frac{1}{4}\int\frac{dx}{\sqrt{x}} = \dots$$

③ $y''(y^2+2) = 2y(y'+1)y' \quad \Big| \; y(0)=0 \quad y'(0)=1$

K4

$y' = p(y) \qquad y'' = p'p$

$p'\cancel{p}(y^2+2) = 2y(p+1)\cancel{p} \qquad p' = 2y\,\frac{p+1}{y^2+2} \qquad \int \frac{dp}{p+1} = \int \frac{2y\,dy}{y^2+2}$

$\ln|p+1| = \frac{2}{2}\int \frac{d(y^2+2)}{y^2+2} \qquad \ln|p+1| = \ln|y^2+2| + \ln|\tilde{c}|$

$|p+1| = |c(y^2+2)|$

$p+1 = c(y^2+2)$

$p = c(y^2+2)-1 \quad \Rightarrow \text{Hy:} \quad 1 = c(2)-1$

$2c = 2 \quad \underline{c=1}$

$y' = y^2+1$

$\frac{dy}{dx} = y^2+1$

$\frac{dy}{y^2+1} = dx \qquad x = \operatorname{arctg}\frac{y}{1} + C \qquad \Big| \int \frac{dx}{x^2+a^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} \Big|$

$C=0$

$x = \operatorname{arctg} y \;\rightarrow\; y = \operatorname{tg} x$ //

K1

(1) $\int e^{\operatorname{arctg} 3x} \frac{dx}{1+9x^2} = \frac{1}{3}\int e^{\operatorname{arctg} 3x}\, d(\operatorname{arctg} 3x) = \frac{1}{3} e^{\operatorname{arctg} 3x} + C$

(2) $\int \frac{dx}{x\sin^2(\ln x)} = \frac{d\ln x}{\sin^2(\ln x)} = \frac{dt}{\sin^2 t} = -\operatorname{ctg} t = -\operatorname{ctg}(\ln x) + C$

(3) $\int \frac{x\,dx}{x^4+a^2} = \frac{1}{2}\int \frac{dx^2}{(x^2)^2+a^2} = \left\{ \int \frac{dx}{x^2+a^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \right\} = \frac{1}{2}\cdot\frac{1}{a}\operatorname{arctg}\frac{x^2}{a} + C$

(4) $\int \frac{x^2dx}{x^6+4} = \left\{ dx^3 = 3x^2dx \right\} = \frac{1}{3}\int \frac{dx^3}{(x^3)^2+4} = \frac{1}{3}\,\frac{1}{2}\operatorname{arctg}\frac{x^3}{2} + C$

(5) $\int \frac{x^2dx}{\sqrt{x^6+a^2}} = \frac{1}{3}\int \frac{dx^3}{\sqrt{x^6+a^2}} = \left\{ \int \frac{dx}{\sqrt{x^2+a^2}} = \ln\left(x+\sqrt{x^2+a^2}\right) \right\} = \frac{1}{3}\ln\left(x^3+\sqrt{x^6+a^2}\right) + C$

(6) $\int \frac{\ln x - 3}{x\sqrt{\ln x}}dx = \int \frac{\ln x - 3}{\sqrt{\ln x}}\, d\ln x = \int \frac{\ln x}{\sqrt{\ln x}}\, d\ln x - 3\int \frac{d\ln x}{\sqrt{\ln x}} = \frac{2}{3}\ln^{\frac{3}{2}} x - 3\,\frac{2}{3}\ln^{\frac{3}{2}} x = \frac{2}{3}\ln^{\frac{3}{2}} x - 2\ln^{\frac{3}{2}} x + C$

(7) $\int \frac{2^x dx}{\sqrt{1-4^x}} = \left\{ \begin{array}{l} 2^x = t \\ dt = 2^x \ln 2\, dx \end{array} \; dx = \frac{dt}{t\ln 2} \right\} = \frac{1}{\ln 2}\int \frac{dt}{\sqrt{1-t^2}} = \frac{1}{\ln 2}\arcsin\frac{2^x}{1} + C$

(8) $\int e^{2x^2+\ln x}dx = \int x\, e^{2x^2}dx = \left\{ \begin{array}{l} x^2 = t \\ dt = 2x\,dx \\ \frac{1}{2}dt = x\,dx \end{array} \right\} = \frac{1}{2}\int e^{2t}\,dt = \frac{1}{4}e^{2x^2} + C$

(9) $\int \sin\frac{1}{x}\cdot\frac{dx}{x^2} = \left\{ \begin{array}{l} \frac{1}{x} = t \\ dx = -\frac{1}{t^2}dt \end{array} \right\} = -\int \sin t\, \frac{dt\, t^2}{t^2} = -\int \sin t\, dt = \cos\frac{1}{x} + C$

(10) $\int \frac{dx}{x\ln x\ln(\ln x)} = \int \frac{dt}{t\ln t} = \int \frac{da}{a} = \ln|a| = \ln|\ln(\ln x)| + C$

(11) $\int \frac{x^2}{\sqrt{2-x}}dx = \left\{ \begin{array}{l} 2-x = t \\ x = t-2 \end{array} \right\} = \int \frac{t^2-4t+4}{\sqrt{t}}dt = \int \frac{t^2dt}{\sqrt{t}} - 4\int \frac{t\,dt}{\sqrt{t}} + 4\int \frac{dt}{\sqrt{t}} = \frac{2t^{\frac{5}{2}}}{5} - \frac{8t^{3/2}}{3} + 8\sqrt{t} =$

$= \frac{2}{5}(2-x)^{5/2} - \frac{8}{3}(2-x)^{3/2} + 8\sqrt{2-x} + C$

(12) $\int \frac{\sin 2x\, dx}{\sqrt{2+\cos^2 x}} = -\frac{1}{2}\int \frac{d\cos 2x}{\sqrt{2+\frac{\cos 2x+1}{2}}} = -\frac{\sqrt{2}}{2}\int \frac{d(\cos 2x+5)}{\sqrt{5+\cos 2x}} = -\sqrt{2}\sqrt{\cos 2x+5} + C$

(13) $\int x^2\arcsin 2x\, dx = \left\{ \begin{array}{ll} u = \arcsin 2x & du = \frac{2}{\sqrt{1-4x^2}} \\ v = \frac{x^3}{3} & \end{array} \right\} = \arcsin 2x\,\frac{x^3}{3} - \frac{2}{3}\int \frac{x^3}{\sqrt{1-4x^2}}dx = \left\{ \begin{array}{l} t = 1-4x^2 \\ dt = -8x\,dx \\ -\frac{1}{8}dt = x\,dx \end{array} \right\} =$

$= \underbrace{\arcsin 2x}\,\frac{x^3}{3} + \frac{2}{3}\cdot\frac{1}{8}\cdot\frac{1}{4}\int \frac{1-t}{\sqrt{t}}dt = \underline{\quad} + \frac{1}{48}\int \frac{dt}{\sqrt{t}} - \frac{1}{48}\int \frac{t\,dt}{\sqrt{t}} = \underline{\quad} + \frac{2}{48}\sqrt{t} - \frac{2}{48\cdot 3}t^{3/2} =$

$= \arcsin 2x \cdot \frac{x^3}{3} + \frac{1}{24}\sqrt{1-4x^2} - \frac{1}{24\cdot 3}(1-4x^2)^{3/2} + C$

(14) $\int \frac{x\arcsin x}{\sqrt{1-x^2}}dx = \left\{ \begin{array}{ll} u = \arcsin x & du = \frac{dx}{\sqrt{1-x^2}} \\ v = -\frac{1}{2}\int \frac{d(-x^2)}{\sqrt{1-x^2}} & = -\sqrt{1-x^2} \end{array} \right\} = -\arcsin x\sqrt{1-x^2} + x + C$

(15) $\int \ln(x+\sqrt{4+x^2})dx = \begin{cases} u=\ln(x+\sqrt{4+x^2}) & du = \dfrac{1+\frac{2x}{2\sqrt{4+x^2}}}{x+\sqrt{4+x^2}} = \dfrac{1+\frac{x}{\sqrt{x^2+4}}}{x+\sqrt{4+x^2}} \\ v=x \end{cases} = \ln(\dots) - \int \dfrac{x+\frac{x^2}{\sqrt{x^2+4}}}{x+\sqrt{4+x^2}}dx =$

$= \ln(\dots)x - \int \dfrac{x\sqrt{4+x^2}+x^2}{\sqrt{4+x^2}(x+\sqrt{4+x^2})} = \ln(x)\,x - \int \dfrac{x(\cancel{\sqrt{4+x^2}+x})}{\sqrt{4+x^2}(\cancel{x+\sqrt{4+x^2}})}dx = \ln(\dots)\cancel{x} - \frac{1}{2}\int \dfrac{dt^2}{\sqrt{x^2+4}} =$

$= x\ln(\dots) - \cancel{\frac{1}{2}}\dfrac{\sqrt{x^2+4}}{\cancel{\frac{1}{2}}} + C$ //

(16) изи

(17) $\int \dfrac{x\cos x\,dx}{\sin^3 x} = \begin{cases} u=x & du=dx \\ v=\frac{\sin^{-2}x}{-2} \end{cases} = \dfrac{-x\sin^{-2}x}{2} + \frac{1}{2}\int\dfrac{dx}{\sin^2 x} = \dfrac{-x\sin^{-2}x}{2} - \dfrac{ctg\,x}{2} + C$ //

(18) $\int \dfrac{\ln x+2}{x\sqrt{1-\ln x-\ln^2 x}}dx = \begin{cases} \ln x = t \\ dt = \frac{1}{x}dx \end{cases} = \int \dfrac{t+2}{\sqrt{t^2-t+1}} = \{(t^2-t+1)' = 2t-1\} = \int \dfrac{(2t-1)\frac{1}{2}+\frac{5}{2}}{\sqrt{t^2-t+1}}dt =$

$= \frac{1}{2}\int \dfrac{d(t^2-t+1)}{\sqrt{t^2-t+1}} + \frac{5}{2}\int\dfrac{dt}{\sqrt{t^2-t+1}} = \sqrt{t^2-t+1} + \frac{5}{2}\int\dfrac{dt}{\sqrt{(t-\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}} = \sqrt{\dots} + \frac{5}{2}\ln\left((\cdot)+\sqrt{\dots}\right) + C$ //

(19) $\int \dfrac{\cos x\,dx}{\sqrt{1-4\sin x+\cos^2 x}} = [d\sin x = \cos x\,dx] = \int \dfrac{d\sin x}{\sqrt{-\sin^2 x-4\sin x+2}} = [\sin x = t] = \int\dfrac{dt}{\sqrt{-t^2-4t+2}} =$

$= \left\{ -(t^2+4t-2) = -(t+2)^2+6 \right\} = \int \dfrac{dt}{\sqrt{6-(t+2)^2}} = \arcsin\dfrac{\sin x+2}{\sqrt{6}} + C$ //

(20) $\int\dfrac{4x-11}{\sqrt{1+x-x^2}}dx = \{(1+x-x^2)' = -2x+1\} = \int \dfrac{(-2x+1)(-2)-9}{\sqrt{1+x-x^2}}dx = -2\int\dfrac{d(-x^2+x+1)}{\sqrt{\dots}} - 9\int\dfrac{dx}{\sqrt{\dots}} =$

$= -4\sqrt{1+x-x^2} - 9\int\dfrac{dx}{\sqrt{\frac{5}{4}-(x-\frac{1}{2})^2}} = -4\sqrt{\dots} - 9\arcsin\dfrac{2(x-\frac{1}{2})}{\sqrt{5}} + C$ //

(21) $\int\sqrt{\cos 2x}\cos x\,dx = \int\sqrt{1-2\sin^2 x}\,d\sin x \ominus (*) \ominus \frac{1}{2}\left(\sqrt{2}t\sqrt{1-t^2} + \arcsin\frac{t\sqrt{2}}{1}\right) = \dots$ // $|t=\sin x|$

(22) $\int\dfrac{2x^4-x^2+1}{x^3-x}dx = \left| \begin{array}{l|l} 2x^4-x^2+1 & x^3-x \\ -\,2x^4-2x^2 & 2x \\ \quad x^2+1 & \end{array} \Rightarrow 2x + \dfrac{x^2+1}{x^3-x} \right| = 2\int x\,dx + \int\dfrac{x^2+1}{x^3-x}dx \ominus$

$\dfrac{x^2+1}{x^3-x} = \dfrac{x^2+1}{x(x^2-1)} = \dfrac{x^2+1}{x(x-1)(x+1)} = \dfrac{A}{x} + \dfrac{B}{x-1} + \dfrac{C}{x+1} = \dfrac{Ax^2-A+Bx^2+Bx+Cx^2-Cx}{x(x-1)(x+1)}$

$x^2+1 = Ax^2-A+Bx^2+Bx+Cx^2-Cx$

$x^0:\ 1=-A \quad A=-1$

$x^1:\ 1=B-C$

$x^2:\ 1=A+B+C$

$\begin{cases} B+C=2 & B=2-C \\ B-C=1 & 2-2C=1 \\ B=\frac{3}{2} & 2C=1 \quad C=\frac{1}{2} \end{cases}$

$\ominus -\int\dfrac{dx}{x} + \frac{3}{2}\int\dfrac{d(x-1)}{x-1} + \frac{1}{2}\int\dfrac{d(x+1)}{x+1} = -\ln|x| + \frac{3}{2}\ln|x-1| + \frac{1}{2}\ln|x+1| + C$ //

$\boxed{K5}$

① $y^{IV} - 16y = 1 - (x^2+1)e^{-2x} + e^{2x}\sin 2x + x\cos 2x - 5x^3$

1	$(x^2+1)e^{-2x}$	$e^{2x}\sin 2x$	$x\cos 2x$	$-5x^3$
$\lambda=0$, $\beta=0$, $z=0$	$\lambda=-2$, $\beta=0$, $z=-2$	$\lambda=2$, $\beta=2$, $z=2\pm 2i$	$\lambda=0$, $\beta=2$, $z=\pm 2i$	$\lambda=0$, $\beta=0$, $z\neq 0$

ЛОДУ: $y^{IV} - 16y = 0$

хар. ур.: $\lambda^4 - 16 = 0$

$(\lambda^2-4)(\lambda^2+4)=0$

$(\lambda-2)(\lambda+2)(\lambda^2+4)=0$

$\lambda_1 = 2$ $\quad \lambda^2 = -4 = 4i^2$

$\lambda_2 = -2$ $\quad \lambda = \pm 2i$

$\lambda_{34} = \pm 2i$

$y_1 = e^{2x}$ $\quad y_2 = e^{-2x}$ $\quad y_3 = \cos 2x$ $\quad y_4 = \sin 2x$

$y_{oo} = C_1 e^{2x} + C_2 e^{-2x} + C_3 \cdot \cos 2x + C_4 \sin 2x$

ЛНОУ:

$f_1 = 1 - \boxed{5x^3}$ $\quad \lambda = \beta = z = 0$

$f_{2.н.1} = A_1x^3 + B_1x^2 + C_1x + D_1$

$f_2 = -e^{-2x}(x^2+1) \Rightarrow \lambda=-2 \quad \beta=0 \quad z=-2$

$f_{r.н.2} = [A_2x^2 + B_2x + D_2]e^{-2x} \cdot x$

$f_3 = e^{2x}\sin 2x$ $\quad \lambda=2 \quad \beta=2 \quad z=2\pm 2i$

$f_{ч.н.3} = e^{2x}[A_3\sin 2x + B_3\cos 2x]$

$f_4 = \boxed{x}\cos 2x$ $\quad \lambda=0 \quad \beta=2 \quad z=\pm 2i$

$f_{rн4} = ([A_4x + B_4]\cos 2x + [D_4x + F_4]\sin 2x) \cdot x$

$y_{он} = y_{oo} + y_{rн1} + y_{rн2} + y_{rн3} + y_{rн4}$

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$\lambda_1 = 0 \quad \lambda_2 = 0 \quad \lambda_{34} = 2\pm 3i$ $\qquad f(x) = x^2 + e^{2x}\cos 3x + x\sin 3x$

$y_1 = 1 \quad y_2 = x \quad y_{34} = \cancel{(\cos 3x + \sin 3x)e^{2x}}$ $\quad y_3 = e^{2x}\cos 3x \quad y_4 = e^{2x}\sin 3x$

$y_{oo} = C_1 + C_2x + C_3e^{2x}\cos 3x + C_4e^{2x}\sin 3x$

ЛНОУ:

$f_1 = \boxed{x^2}$ $\quad \lambda=0 \quad \beta=0 \quad z=0$

$f_{r1} = [A_1x^2 + B_1x + D_1]x^2$

$f_2 = e^{2x}\cos 3x$ $\quad \lambda=2 \quad \beta=3 \quad z=2\pm 3i$

$f_{r.2} = [A_2e^{2x}\cos 3x + B_2e^{2x}\sin 3x]\,x$

$f_3 = x\sin 3x$ $\quad \lambda=0 \quad \beta=3 \quad z=\pm 3i$

$f_{r.3} = (A_3x + B_3)\sin 3x + (D_3x + F_3)\cos 3x$

$y_{он} = y_{oo} + y_{r1} + y_{r2} + y_{r3}$