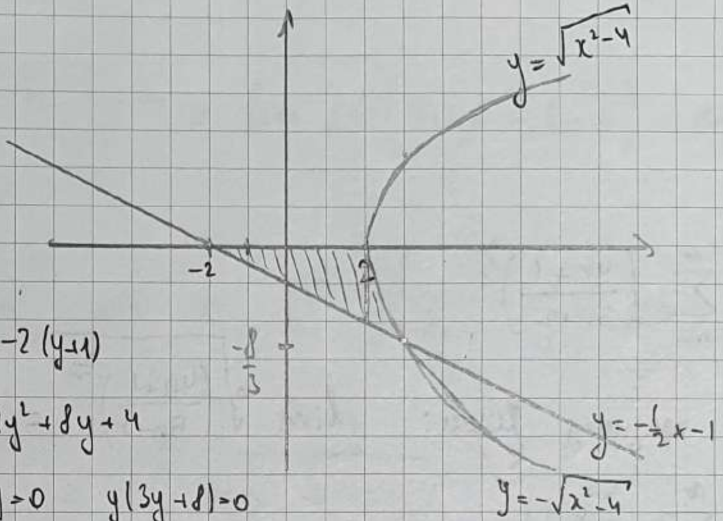
№1. $\boxed{\text{Перемяков Дмитрий ИУ5-33Б}}$ ТВиМС ДЗ №1 Вар: 11.

$$\int_{-\frac{8}{3}}^{0} dy \int_{-2(y+1)}^{\sqrt{4+y^2}} f(x,y)\,dx = \int_{-2}^{2} dx \int_{-\frac{1}{2}x-1}^{0} f(x,y)\,dy + \int_{2}^{\frac{10}{3}} dx \int_{-\frac{1}{2}x-1}^{-\sqrt{x^2-4}} f(x,y)\,dy$$



$$\sqrt{4+y^2} = -2(y+1)$$

$$4+y^2 = 4y^2+8y+4$$

$$3y^2+8y=0 \qquad y(3y+8)=0$$

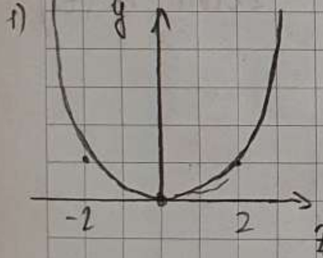
$$\underline{y=-\frac{8}{3}}$$

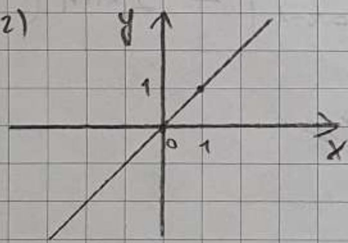
№2.

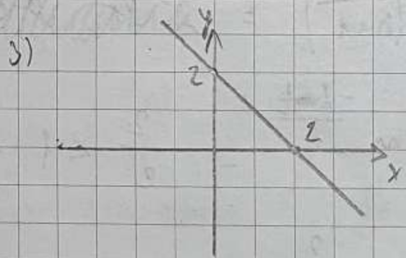
1) $z^2 = 4y$

2) $x = y$

3) $x+y=2$ $\qquad V-?$

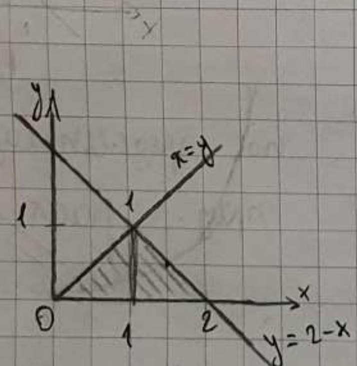


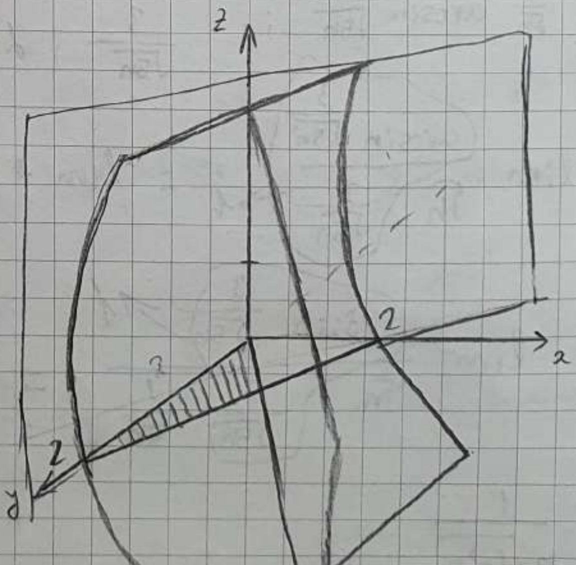




$$z = \pm 2\sqrt{y}$$

$$z = 2\sqrt{y} \qquad z = -2\sqrt{y}$$





симметрия

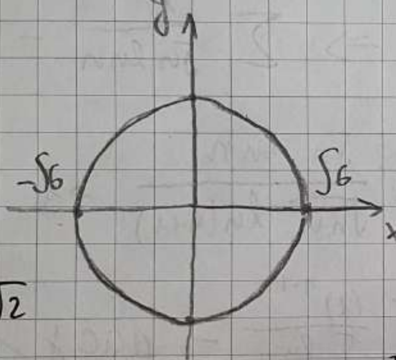
$$V = \left[\int_0^1 dx \int_0^x 2\sqrt{y}\,dy + \int_1^2 dx \int_0^{2-x} 2\sqrt{y}\,dy\right]\cdot 2 = 2\left[\int_0^1 dx\; 2\cdot\frac{y^{3/2}}{3/2}\Bigg|_0^x + \int_1^2 dx\; 2\cdot\frac{y^{3/2}}{3/2}\Bigg|_0^{2-x}\right] =$$

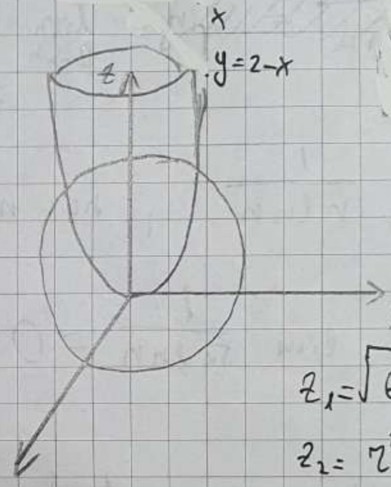
$$= 2\left[\frac{4}{3}\int_0^1 x^{3/2}dx + \frac{4}{3}\int_1^2 (2-x)^{3/2}dx\right] = 2\left[\frac{4}{3}\cdot\frac{x^{\frac{5}{2}}}{\frac{5}{2}}\Bigg|_0^1 - \frac{4}{3}\,\frac{(2-x)^{5/2}}{\frac{5}{2}}\Bigg|_1^2\right] = 2\left[\frac{8}{15}+\frac{8}{15}\right] = \frac{32}{15}$$

$\int 3.$

$$x^2+y^2+z^2 = 6$$

$$z = x^2+y^2$$





$$\begin{cases} z=\sqrt{6-x^2-y^2} \\ z = x^2+y^2 \end{cases}$$

$$\begin{cases} z=\sqrt{6-r^2} \\ z = r^2 \end{cases} \rightarrow r = \sqrt{2}$$

$$z_1=\sqrt{6-x^2-y^2}=\sqrt{6-r^2}$$

$$z_2 = r^2$$

$$V=\int_0^{2\pi} d\varphi \int_0^{\sqrt{2}} \left(\sqrt{6-r^2}-r^2\right) r\,dr = \int_0^{2\pi} d\varphi \left[-\frac{1}{2}\int_0^{\sqrt{2}} \sqrt{6-r^2}\, d(6-r^2) - \frac{r^4}{4}\Big|_0^{\sqrt{2}} \right] =$$

$$= \int_0^{2\pi} d\varphi \left[\frac{6\sqrt{6}}{3} - \frac{4\sqrt{4}}{3} - 1 \right] = \left(2\sqrt{6} - \frac{11}{3}\right) 2\pi$$

№4.

а) $\sum_{n=1}^{\infty}\left(\frac{4n+1}{5n+2}\right)^{\frac{n}{2}}$

по признаку Коши: $\lim \sqrt[n]{\left(\frac{4n+1}{5n+2}\right)^{\frac{n}{2}}} = \lim \sqrt{\frac{4n+1}{5n+2}} = \sqrt{\frac{4}{5}} < 1$ – сх.

б) $\sum_{n=1}^{\infty} \frac{2^n n!}{n^n}$

по Даламберу: $\lim \frac{\cancel{2^{n+1}}\,\cancel{(n+1)!}}{(n+1)^{n+1}} \cdot \frac{n^n}{\cancel{2^n}\,\cancel{n!}} = \lim \frac{2n^n}{(n+1)^n} = 2\lim \frac{n^n}{(n+1)^n} =$

$= 2\lim \left(\frac{n}{n+1}\right)^n = \left| \begin{matrix} n+1=t \\ n=t-1 \end{matrix} \right| = 2\lim \left(\frac{t-1}{t}\right)^{t-1} = 2\lim\left(1+\left(\frac{-1}{t}\right)\right)^{\frac{-t(t-1)}{-t}} =$

$= 2 \cdot e^{\lim \frac{-t}{t}} = \frac{2}{e} < 1 \to$ сх

в) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \arcsin \frac{3}{\sqrt{5n}}$; $\frac{3}{\sqrt{5n}}$, $\alpha = \frac{1}{2} < 1$ – расх по Дирехле

$\lim \frac{\arcsin \frac{3}{\sqrt{5n}}}{\sqrt{n} \cdot \frac{3}{\sqrt{5n}}} \to 1 = \lim \frac{1}{\sqrt{n}}$, р-м. $\frac{3}{\sqrt{n}\sqrt{5n}}$, $\lambda = 1 \le 1$ – расх по Дир.

$\to \lim \frac{\arcsin \frac{3}{\sqrt{5n}} \nearrow 1}{\cancel{\sqrt{n}} \cdot \frac{3}{\sqrt{5n}} \cdot \frac{1}{\cancel{\sqrt{n}}}} = 1 = \text{const} \neq 0 \neq \infty \to$ по предельному приз. расх

г) $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{\ln^3 n}}$

$\lim\limits_{R\to\infty} \int_2^R \frac{1\,dx}{n\sqrt{\ln^3 n}} = \lim \int_2^R \frac{d\ln n}{\ln^{+\frac{3}{2}} n} = \lim \frac{1}{\sqrt{\ln n}}\Big|_2^R = -\frac{1}{2}\lim\left(\underbrace{\frac{1}{\sqrt{\ln R}}}_{\to 0} - \frac{1}{\ln 2}\right) =$

$= \frac{1}{2\ln 2} \to$ сх $\to \sum \frac{1}{n\sqrt{\ln^3 n}}$ – сх

(55) а) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}\ln n}$; $\left\{\sum \frac{1}{\sqrt{n}\ln n}\right\}$

1) ~~lim $\frac{n \ln n}{\sqrt{n}\ln n}$ = lim~~ $\lim_{R\to\infty} \int_2^R \frac{dn}{n\ln n} = \lim \ln(\ln R - \ln 2) = \infty$ - расх

$\frac{1}{\sqrt{n}\ln n} > \frac{1}{n\ln n}$, по приз. срав $\Rightarrow \sum \frac{1}{\sqrt{n}\ln n}$ - расх

2) $\lim \frac{1}{\sqrt{n}\ln n} = 0$ 3) $\frac{\sqrt{n}\ln n}{\sqrt{n+1}\ln(n+1)} > 1 \Rightarrow$

не выполнилось усл. Лейбница $\Rightarrow \sum \frac{(1)^{n-1}}{\sqrt{n}\ln n}$ - расх

б) $\sum_{n=1}^{\infty} (-1)^n \frac{(n^2+4)}{3n^2}$; $\left\{\sum \frac{n^2+4}{3n^2}\right\}$

1) $\lim \frac{n^2+4}{2n^2} = \frac{1}{2} \neq 0 \to$ расх.

2) $\lim \frac{n^2+4}{2n^2} \neq 0 \to$ усл. Лейбница не выполнено $\Rightarrow \sum (-1)^n \frac{(n^2+4)}{3n^2}$ расх

в) $\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{2n^4+9n^2+1}$; $\left[\frac{n^2}{2n^4+9n^2+1}\right]$ $\alpha = +2 > 1$ - сх по дирехле

$\lim \frac{n^2}{(2n^4+9n^2+1)n^{-2}} = \frac{1}{2} \Rightarrow$ т.к. $\frac{1}{n^2}$ - сх по дирехле $\Rightarrow$ по пред. приз.
$\sum \frac{n^2}{2n^4+9n^2+1}$ - сх

$\Rightarrow$ сх. абсолютно