

Homework

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Group 313

1. c) $x^2 - y^2$ subject to $x^2 + y^2 = 1$

$$f(x, y) = x^2 - y^2 \quad g(x, y) = x^2 + y^2 - 1 = 0$$

$$\begin{aligned} L(x, y, \lambda) &= f(x, y) + \lambda \cdot g(x, y) \\ &= x^2 - y^2 + \lambda (x^2 + y^2 - 1) \end{aligned}$$

$$\begin{aligned} \frac{\partial L}{\partial x} &= 2x + 2\lambda x = 0 \Rightarrow 2x(1 + \lambda) = 0 \\ &\Rightarrow \lambda = -1 \end{aligned}$$

$$\begin{aligned} \frac{\partial L}{\partial y} &= -2y + 2\lambda y = 0 \Rightarrow 2y(-1 + \lambda) = 0 \\ &\Rightarrow \lambda = 1 \end{aligned}$$

$$\frac{\partial L}{\partial \lambda} = x^2 + y^2 - 1 = 0$$

$$\begin{aligned} \text{i.) } \lambda = -1 &\Rightarrow -2y = 0 \Rightarrow y = 0 \Rightarrow x = 1 \Rightarrow \\ &\Rightarrow f(1, 0) = 1 \end{aligned}$$

$$\begin{aligned} \text{ii.) } \lambda = 1 &\Rightarrow 2x = 0 \Rightarrow x = 0 \Rightarrow y = 1 \Rightarrow \\ &\Rightarrow f(0, 1) = -1 \end{aligned}$$

2.) $x^3 + y^3 + z^3$ subject to $x^2 + y^2 + z^2 = 1$

$$f(x, y, z) = x^3 + y^3 + z^3 \quad g(x, y, z) = x^2 + y^2 + z^2 - 1 = 0$$

$$\begin{aligned} L(x, y, z, \lambda) &= f(x, y, z) + \lambda \cdot g(x, y, z) \\ &= x^3 + y^3 + z^3 + \lambda (x^2 + y^2 + z^2 - 1) \end{aligned}$$

$$(1) \frac{\partial L}{\partial x} = 3x^2 + 2\lambda x = 0 \Rightarrow x(3x + 2\lambda) = 0$$

$$x=0 \quad x = -\frac{2}{3}\lambda$$

$$(2) \frac{\partial L}{\partial y} = 3y^2 + 2\lambda y = 0 \Rightarrow y(3y + 2\lambda) = 0$$

$$y=0 \quad y = -\frac{2}{3}\lambda$$

$$(3) \frac{\partial L}{\partial z} = 3z^2 + 2\lambda z = 0 \Rightarrow z(3z + 2\lambda) = 0$$

$$z=0 \quad z = -\frac{2}{3}\lambda$$

$$\frac{\partial L}{\partial \lambda} = x^2 + y^2 + z^2 - 1 = 0$$

$$(1), (2), (3) \rightarrow 3(\underbrace{x^2 + y^2 + z^2}_{=1}) + 2\lambda(x + y + z) = 0$$

$$\Rightarrow 3 + 2\lambda(x + y + z) = 0$$

$$\Rightarrow x + y + z = -\frac{3}{2\lambda}$$

$$\text{So } \lambda = -\frac{3}{2}x \Rightarrow y = z = x$$

$$\text{So } \lambda = -\frac{3}{2}y \Rightarrow x = z = y$$

$$\text{So } \lambda = -\frac{3}{2}z \Rightarrow x = y = z$$

$$\left. \begin{array}{l} \lambda = -\frac{3}{2}x \\ \lambda = -\frac{3}{2}y \\ \lambda = -\frac{3}{2}z \end{array} \right\} \Rightarrow \partial(x, y, z) = 3x^3$$