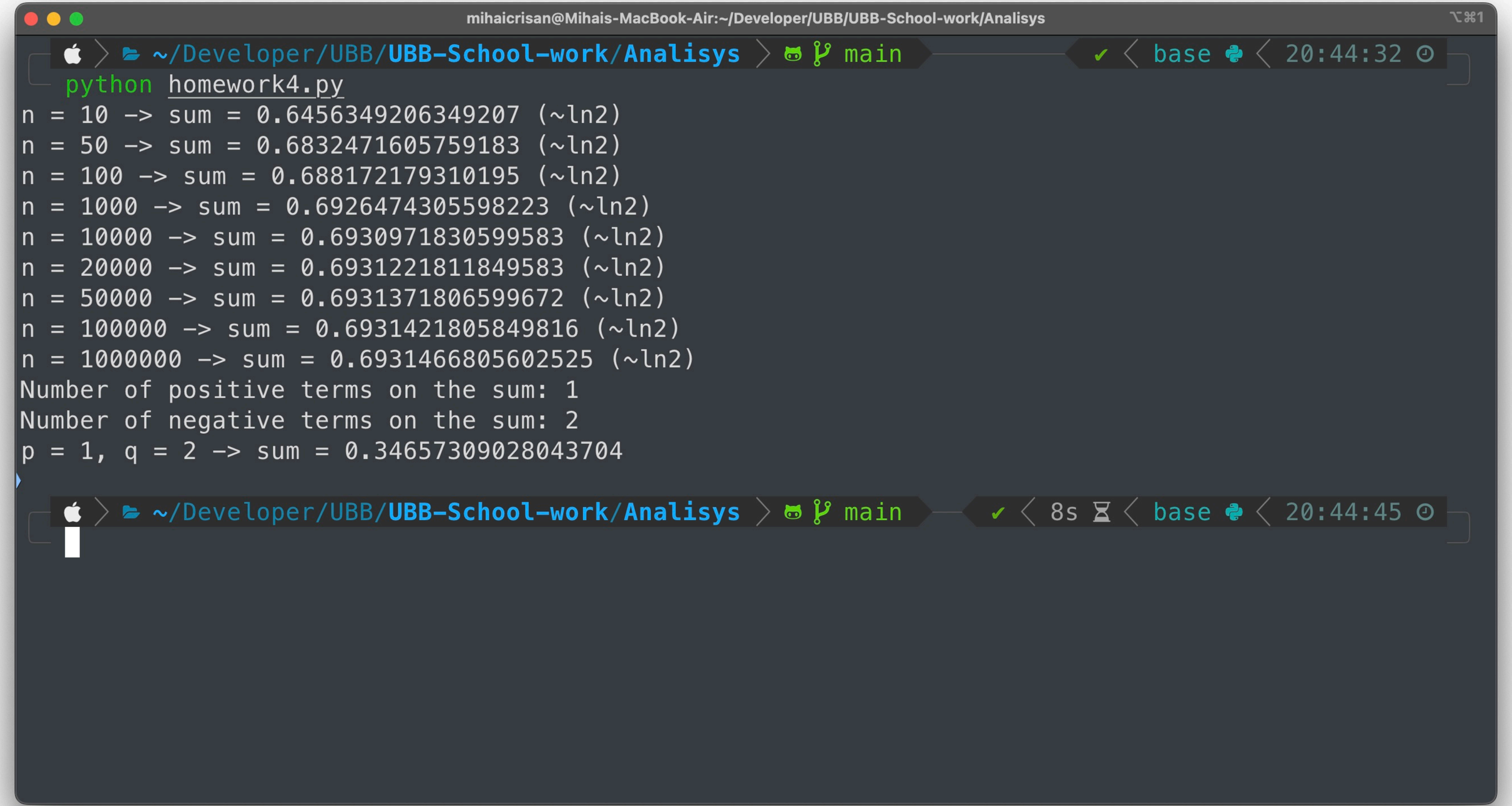
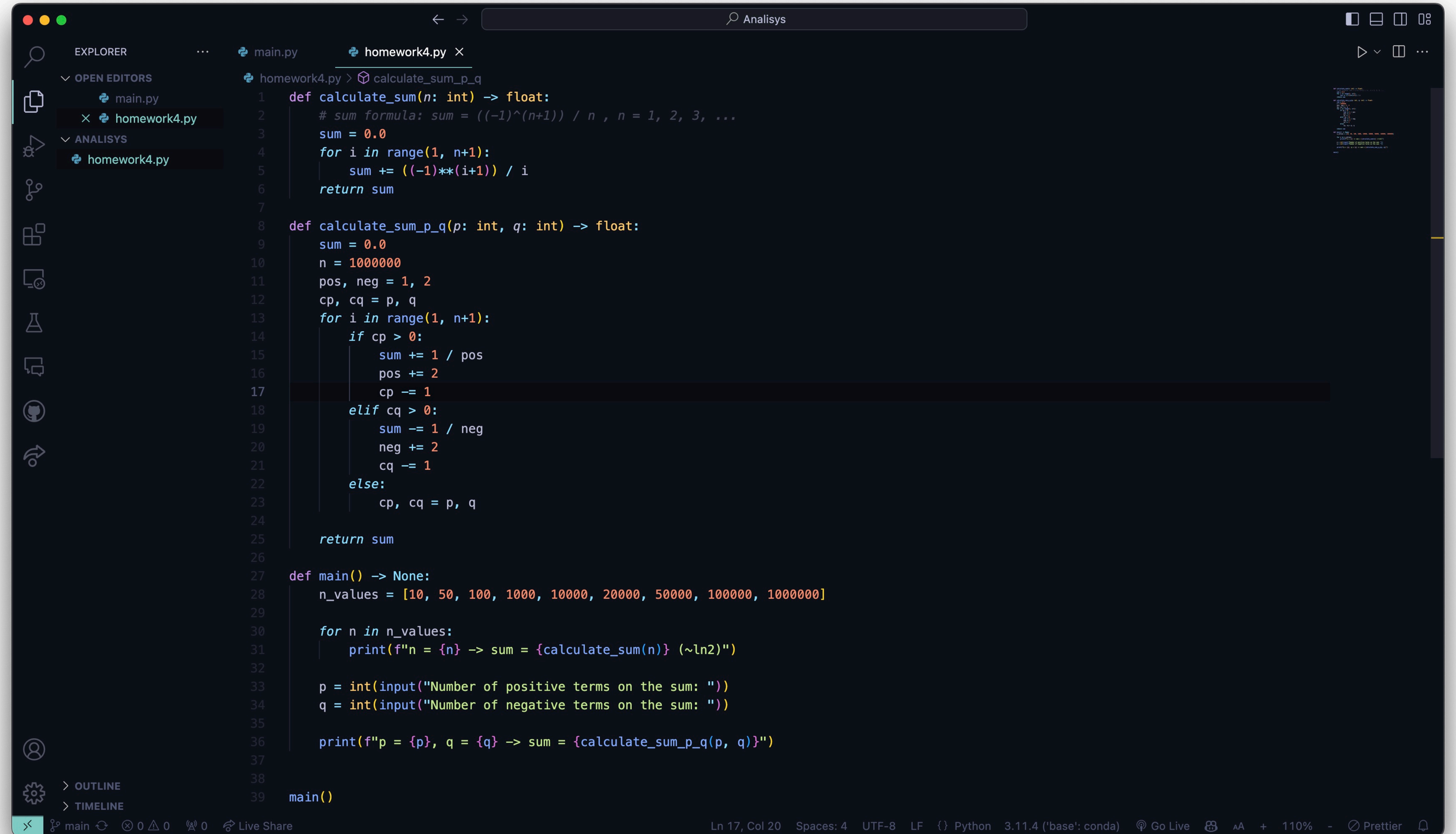

mihaicrisan@Mihais-MacBook-Air:~/Developer/UBB/UBB-School-work/Analisys
~/Developer/UBB/UBB-School-work/Analisys
main
base
20:44:32
python homework4.py
n = 10 -> sum = 0.645634920634920 7 (~ln2)
n = 50 -> sum = 0.6832471605759183 (~ln2)
n = 100 -> sum = 0.688172179310195 (~ln2)
n = 1000 -> sum = 0.6926474305598223 (~ln2)
n = 10000 -> sum = 0.6930971830599583 (~ln2)
n = 20000 -> sum = 0.6931221811849583 (~ln2)
n = 50000 -> sum = 0.6931371806599672 (~ln2)
n = 100000 -> sum = 0.6931421805849816 (~ln2)
n = 1000000 -> sum = 0.6931466805602525 (~ln2)
Number of positive terms on the sum: 1
Number of negative terms on the sum: 2
p = 1, q = 2 -> sum = 0.34657309028043704
~/Developer/UBB/UBB-School-work/Analisys
main
8s
base
20:44:45


Analisys
EXPLORER
OPEN EDITORS
main.py
homework4.py
ANALISYS
homework4.py
main.py
homework4.py
homework4.py > calculate_sum_p_q
def calculate_sum(n: int) -> float:
# sum formula: sum = ((-1)^(n+1)) / n , n = 1, 2, 3, ...
sum = 0.0
for i in range(1, n+1):
sum += ((-1)**(i+1)) / i
return sum
def calculate_sum_p_q(p: int, q: int) -> float:
sum = 0.0
n = 1000000
pos, neg = 1, 2
cp, cq = p, q
for i in range(1, n+1):
if cp > 0:
sum += 1 / pos
pos += 2
cp -= 1
elif cq > 0:
sum -= 1 / neg
neg += 2
cq -= 1
else:
cp, cq = p, q
return sum
def main() -> None:
n_values = [10, 50, 100, 1000, 10000, 20000, 50000, 100000, 1000000]
for n in n_values:
print(f"n = {n} -> sum = {calculate_sum(n)} (~ln2)")
p = int(input("Number of positive terms on the sum: "))
q = int(input("Number of negative terms on the sum: "))
print(f"p = {p}, q = {q} -> sum = {calculate_sum_p_q(p, q)}")
main()
OUTLINE
TIMELINE
main
0 0
0
Live Share
Ln 17, Col 20
Spaces: 4
UTF-8
LF
Python
3.11.4 ('base': conda)
Go Live
110%
Prettier

6. c.) $\sum_{n \geq 1} \frac{n \cdot x^n}{2^n}$

Homework 4 Rihai Cipan

1. b) $\sum \frac{1 \cdot 3 \cdot \ldots \cdot (2n-1)}{2 \cdot 4 \cdot \ldots \cdot 2n} \cdot \frac{1}{n^2}$

Root test:

$$\frac{x_{n+1}}{x_n} = \frac{1 \cdot 3 \cdot \ldots \cdot (2n+1)}{2 \cdot 4 \cdot \ldots \cdot 2n \cdot (2n+2)} \cdot \frac{1}{(n+1)^2} \cdot \frac{2 \cdot 4 \cdot \ldots \cdot 2n}{1 \cdot 3 \cdot \ldots \cdot (2n-1)} \cdot \frac{n^2}{1}$$

$$= \frac{2n+1}{2n+2} \cdot \frac{n^2}{(n+1)^2} \longrightarrow 1 \quad \text{inconclusive}$$

R-D test:

$$\lim_{n \to \infty} n \cdot \left(\frac{x_n}{x_{n+1}} - 1 \right) = \lim_{n \to \infty} n \cdot \left(\frac{(2n+2) \cdot (n+1)^2}{(2n+1) \cdot n^2} - 1 \right)$$

$$\lim_{n \to \infty} n \cdot \left(\frac{(2n+2) \cdot (n^2+2n+1)}{(2n+1) \cdot n^2} - 1 \right)$$

$$\lim_{n \to \infty} n \cdot \left(\frac{2n^3+4n^2+2n+2n^2+4n+2}{2n^3+n^2} - 1 \right)$$

$$\lim_{n \to \infty} n \cdot \left(\frac{2n^3+6n^2+6n+2}{2n^3+n^2} - 1 \right)$$

$$\lim_{n \to \infty} n \cdot \left(\frac{2n^3+6n^2+6n+2-2n^3-n^2}{2n^3+n^2} \right)$$

$$\lim_{n \to \infty} n \cdot \left(\frac{5n^2+6n+2}{2n^3+n^2} \right)$$

$$\lim_{n \to \infty} \frac{5n^3+6n^2+2}{2n^3+n^2} = \frac{5}{2} > 1 \Rightarrow \text{convergent}$$

6. c) $\sum_{n\geq 1} \frac{n \cdot x^n}{2^n}$

Power series $\sum_{n\geq 1} a_n (x-c)^n$

$c = 0$

$a_n = \frac{n}{2^n}$

$\sum_{n\geq 1} \frac{n}{2^n} \cdot x^n$

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{n+1}{2^{n+1}} \cdot \frac{2^n}{n} =$

$= \lim_{n\to\infty} \frac{n+1}{n} \cdot \frac{2^n}{2^n \cdot 2} = \lim_{n\to\infty} \frac{n+1}{n} \cdot \frac{1}{2} = \frac{1}{2} \in [0,\infty]$

$\Rightarrow$ L exists and $L = \frac{1}{2}$

$\Rightarrow$ $R = \frac{1}{L}$, $L \in (0,\infty)$

$R = \frac{1}{\frac{1}{2}} = 2$

The convergence set:

$C := \{x \in \mathbb{R} \mid \sum_{n=0}^{\infty} a_n (x-c)^n \text{ converges}\}$

$C = (0-2, 0+2) = (-2, 2)$

Check endpoints:

$x = 2 \rightarrow \sum_{n\geq 1} \frac{n \cdot 2^n}{2^n} = \sum_{n\geq 1} n \longrightarrow$ ~~con~~ diverges

$$x := -2 \qquad \sum_{n \geq 1} n \cdot \frac{(-2)^n}{2^n} = \sum n \cdot (-1)^n \cdot \frac{2^n}{2^n} = \sum n \cdot (-1)^n$$

$$\sum n(-1)^n$$

R.T : $\frac{x_{n+1}}{x_n} = \frac{(n+1)(-1)^n \cdot (-1)}{n \cdot (-1)^n} \longrightarrow -1 < 1$

$\Rightarrow$ diverges (alternating $-1+2-3+4-\dots$)

$$\Rightarrow \quad C = (-2, 2)$$