



```
1 import numpy as np
2 import matplotlib.pyplot as plt
3 from mpl_toolkits.mplot3d import Axes3D
4
5 def quadratic_function(x, A):
6     return 0.5 * x.T @ A @ x # 0.5 * x^T * A * x where @ is the matrix multiplication operator used in Python 3.5+
7
8 def plot_surface_contour_gradient(A, title):
9     x = np.linspace(-5, 5, 100)
10    y = np.linspace(-5, 5, 100)
11    X, Y = np.meshgrid(x, y)
12    Z = np.zeros_like(X)
13
14    for i in range(X.shape[0]):
15        for j in range(X.shape[1]):
16            Z[i, j] = quadratic_function(np.array([X[i, j], Y[i, j]]), A)
17
18    fig = plt.figure(figsize=(8, 8))
19
20    # Plot 3D Surface
21    ax = fig.add_subplot(111, projection='3d')
22    ax.plot_surface(X, Y, Z, cmap='viridis', alpha=0.5, label='Surface')
23
24    # Plot The level sets
25    contour = ax.contour(X, Y, Z, levels=20, cmap='viridis')
26
27    # Plot Gradient at Three Different Points
28    for point in [(1, 2), (-2, -3), (3, -2)]:
29        gradient = A @ np.array(point)
30        ax.quiver(point[0], point[1], quadratic_function(np.array(point), A),
31                  gradient[0], gradient[1], quadratic_function(gradient, A),
32                  color='red', length=50, normalize=True, arrow_length_ratio=0.05,
33                  label=f'Gradient at ({point[0]}, {point[1]})')
34
35    ax.set_title(f'{title}')
36    ax.legend()
37    plt.show()
38
39 # Matrices for each scenario
40 A_min = np.array([[2, 0], [0, 3]])
41 A_max = np.array([[-2, 0], [0, -3]])
42 A_saddle = np.array([[2, 0], [0, -3]])
43
44 # Plot for each scenario
45 plot_surface_contour_gradient(A_min, "Unique Minimum")
46 plot_surface_contour_gradient(A_max, "Unique Maximum")
47 plot_surface_contour_gradient(A_saddle, "Unique Saddle Point")
```

Figure 1

Unique Minimum

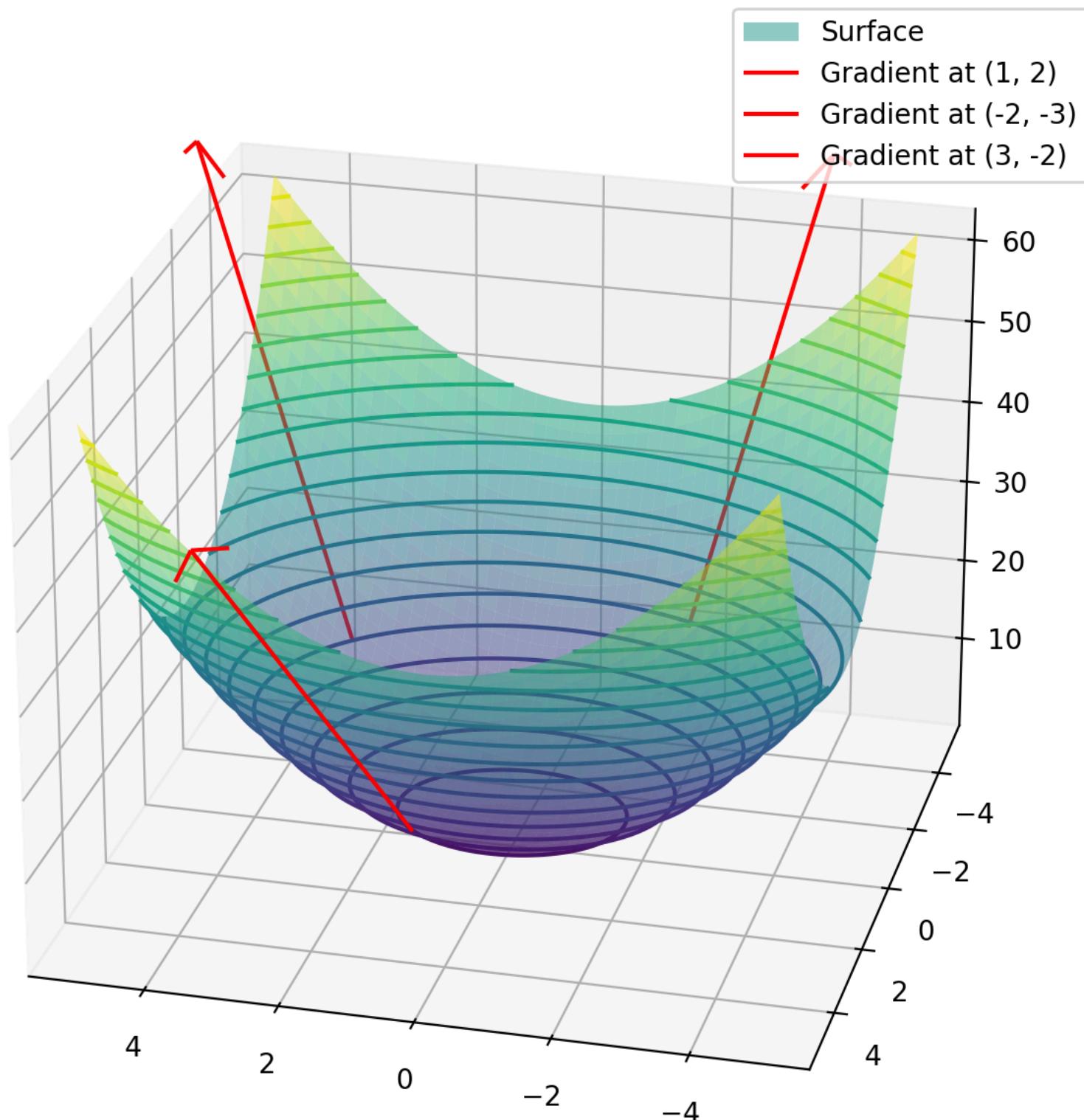


Figure 1

Unique Minimum

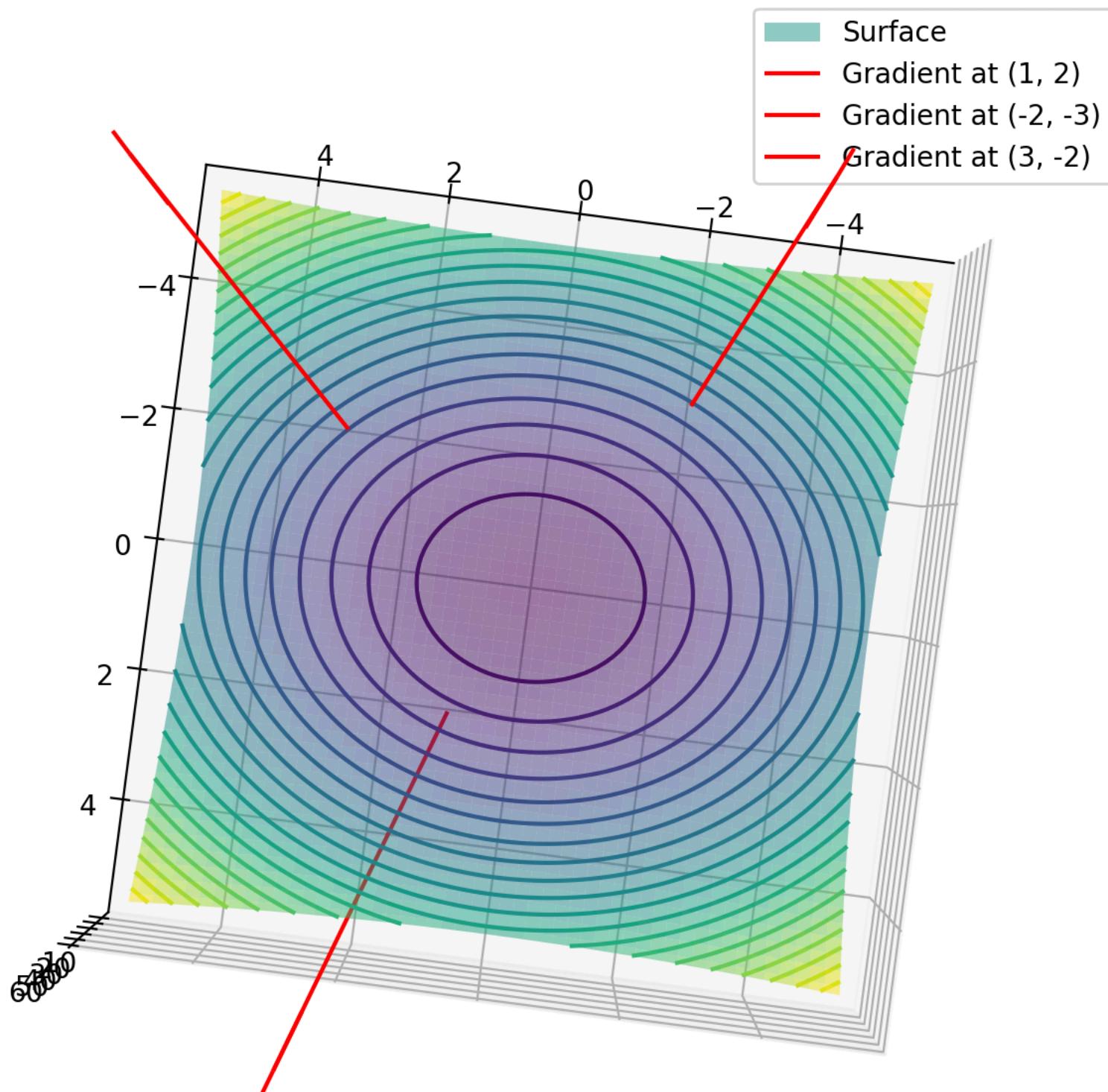


Figure 1

Unique Maximum

- Surface
- Gradient at (1, 2)
- Gradient at (-2, -3)
- Gradient at (3, -2)

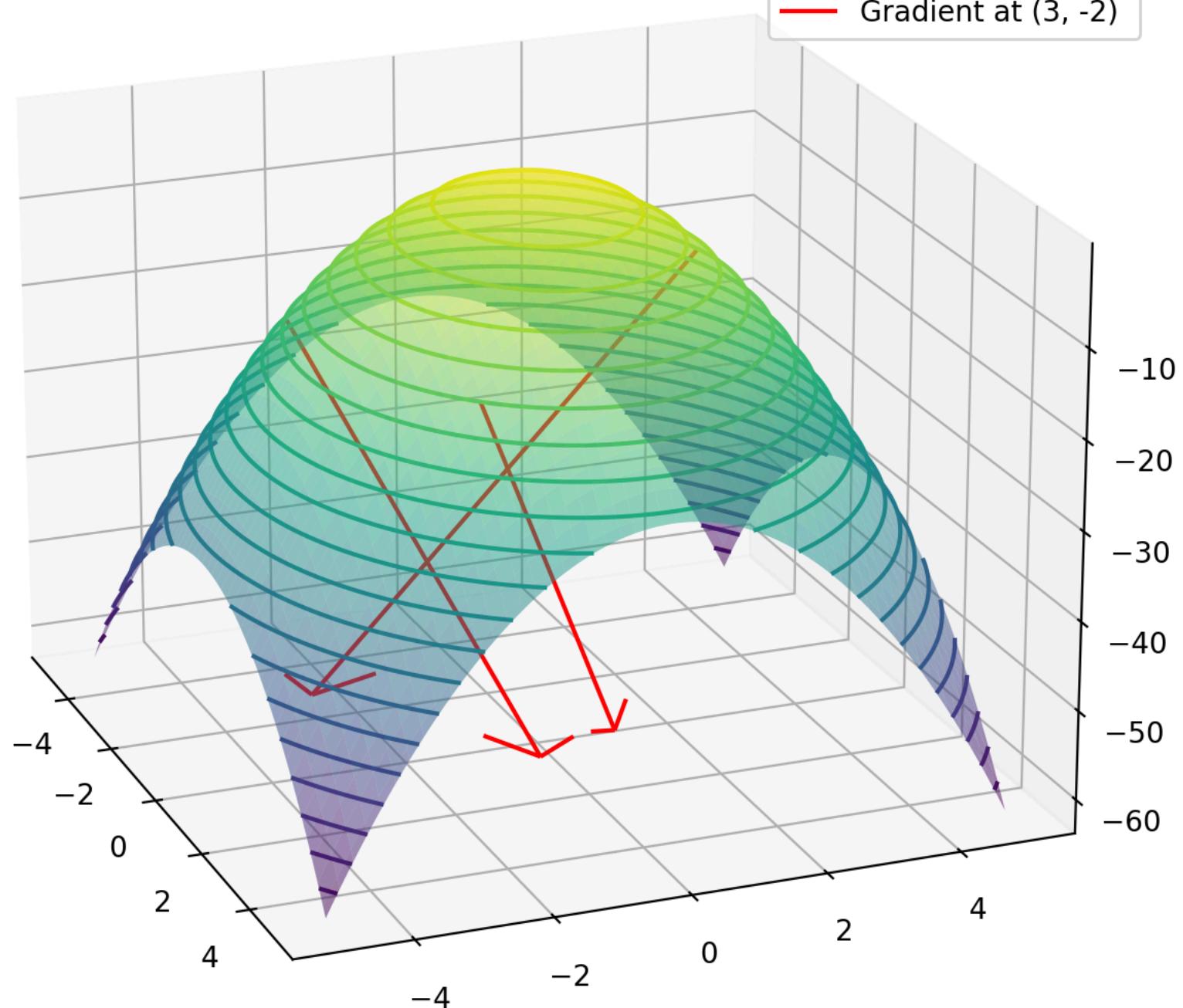


Figure 1

Unique Maximum

- Surface
- Gradient at (1, 2)
- Gradient at (-2, -3)
- Gradient at (3, -2)

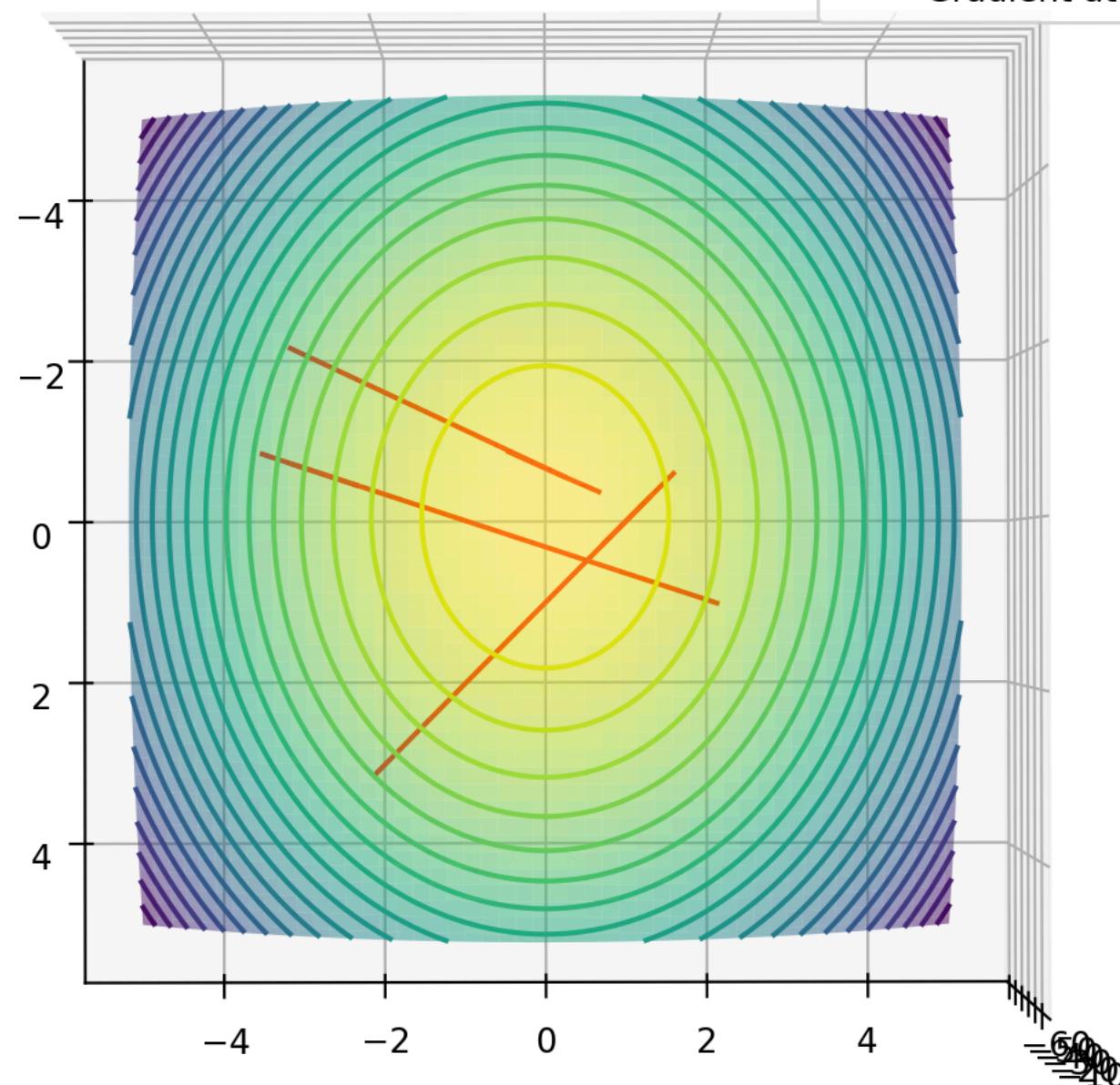


Figure 1

Unique Saddle Point

- Surface
- Gradient at (1, 2)
- Gradient at (-2, -3)
- Gradient at (3, -2)

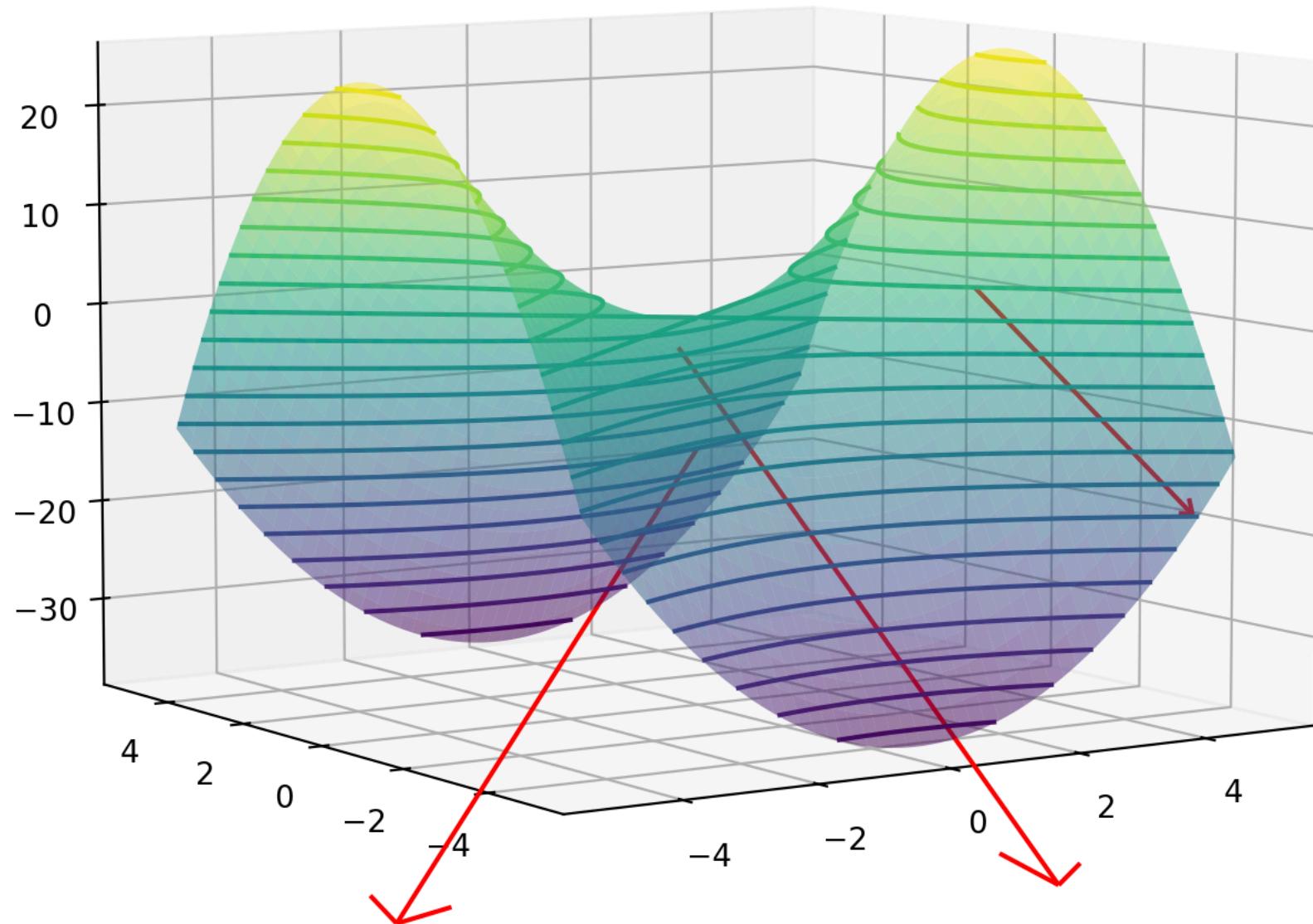


Figure 1

Unique Saddle Point

