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mihaicrisan@Mihais-MacBook-Air:~/Developer/UBB/UBB-School-work/Analisy
```

```
python homework4.py
n = 10 -> sum = 0.6456349206349207 (~ln2)
n = 50 -> sum = 0.6832471605759183 (~ln2)
n = 100 -> sum = 0.688172179310195 (~ln2)
n = 1000 -> sum = 0.6926474305598223 (~ln2)
n = 10000 -> sum = 0.6930971830599583 (~ln2)
n = 20000 -> sum = 0.6931221811849583 (~ln2)
n = 50000 -> sum = 0.6931371806599672 (~ln2)
n = 100000 -> sum = 0.6931421805849816 (~ln2)
n = 1000000 -> sum = 0.6931466805602525 (~ln2)
Number of positive terms on the sum: 1
Number of negative terms on the sum: 2
p = 1, q = 2 -> sum = 0.34657309028043704
```

```
✓ < base ✘ < 20:44:32 ⏺
```

```
8s ✘ < base ✘ < 20:44:45 ⏺
```

The screenshot shows a dark-themed code editor interface with the following details:

- Top Bar:** Includes standard window controls (red, yellow, green buttons), a back/forward navigation bar, a search bar containing "Analisy", and system status icons.
- Left Sidebar (EXPLORER):** Shows a tree view of open files:
 - OPEN EDITORS:** main.py (closed), homework4.py (selected).
 - ANALYSIS:** homework4.py (selected).
- Central Editor Area:** Displays the content of the selected file, `homework4.py`. The code defines two functions: `calculate_sum` and `calculate_sum_p_q`, and a `main` function.

```
1 def calculate_sum(n: int) -> float:
2     # sum formula: sum = ((-1)^(n+1)) / n , n = 1, 2, 3, ...
3     sum = 0.0
4     for i in range(1, n+1):
5         sum += ((-1)**(i+1)) / i
6     return sum
7
8 def calculate_sum_p_q(p: int, q: int) -> float:
9     sum = 0.0
10    n = 1000000
11    pos, neg = 1, 2
12    cp, cq = p, q
13    for i in range(1, n+1):
14        if cp > 0:
15            sum += 1 / pos
16            pos += 2
17            cp -= 1
18        elif cq > 0:
19            sum -= 1 / neg
20            neg += 2
21            cq -= 1
22        else:
23            cp, cq = p, q
24
25    return sum
26
27 def main() -> None:
28     n_values = [10, 50, 100, 1000, 10000, 20000, 50000, 100000, 1000000]
29
30     for n in n_values:
31         print(f"n = {n} -> sum = {calculate_sum(n)} (~ln2)")
32
33     p = int(input("Number of positive terms on the sum: "))
34     q = int(input("Number of negative terms on the sum: "))
35
36     print(f"p = {p}, q = {q} -> sum = {calculate_sum_p_q(p, q)}")
37
38
39 main()
```
- Bottom Status Bar:** Shows file information (main.py), line/col counts (Ln 17, Col 20), encoding (UTF-8), and other details (Spaces: 4, LF, Python 3.11.4 ('base': conda)). It also includes zoom controls (110%), a Prettier button, and a Go Live button.

6. c.) $\sum_{m \geq 1} \frac{m \cdot x^m}{2^m}$

Homework 4

Rihai Cipam

1. b) $\sum \frac{1 \cdot 3 \cdots (2m-1)}{2 \cdot 4 \cdots 2m} \cdot \frac{1}{m^2}$

Root test:

$$\frac{x_{m+1}}{x_m} = \frac{1 \cdot 3 \cdots (2m-1) \cdot (2m+1)}{2 \cdot 4 \cdots 2m \cdot (2m+2)} \cdot \frac{1}{(m+1)^2} \cdot \frac{2 \cdot 4 \cdots 2m}{1 \cdot 3 \cdots (2m-1)} \cdot \frac{m^2}{1}$$

$$= \frac{2m+1}{2m+2} \cdot \frac{m^2}{(m+1)^2} \rightarrow 1 \text{ inconclusive}$$

R-D test:

$$\lim_{m \rightarrow \infty} m \cdot \left(\frac{x_m}{x_{m+1}} - 1 \right) = \lim_{m \rightarrow \infty} m \cdot \left(\frac{(2m+2) \cdot (m+1)^2}{(2m+1) \cdot m^2} - 1 \right)$$

$$\lim_{m \rightarrow \infty} m \cdot \left(\frac{(2m+2) \cdot (m^2 + 2m + 1)}{(2m+1) \cdot m^2} - 1 \right)$$

$$\lim_{m \rightarrow \infty} m \cdot \left(\frac{2m^3 + 4m^2 + 2m + 2m + 1}{2m^3 + 5m^2} - 1 \right)$$

$$\lim_{m \rightarrow \infty} m \cdot \left(\frac{2m^3 + 6m^2 + 6m + 2m + 2}{2m^3 + m^2} - 1 \right)$$

$$\lim_{m \rightarrow \infty} m \cdot \left(\frac{2m^3 + 6m^2 + 6m + 2}{2m^3 + m^2} - 1 \right)$$

$$\lim_{m \rightarrow \infty} m \cdot \left(\frac{5m^2 + 6m + 2}{2m^3 + m^2} \right)$$

$$\lim_{m \rightarrow \infty} \frac{5m^3 + 6m^2 + 2}{2m^3 + m^2} = \frac{5}{2} > 1 \Rightarrow \text{convergent}$$

$$6. \text{ c) } \sum_{m \geq 1} \frac{m \cdot x^m}{2^m}$$

Power series $\sum_{m \geq 1} a_m (x-c)^m$

$$c = 0$$

$$a_m = \frac{m}{2^m}$$

$$\sum_{m \geq 1} \frac{m}{2^m} \cdot x^m$$

$$\lim_{m \rightarrow \infty} \frac{|a_{m+1}|}{|a_m|} = \lim_{m \rightarrow \infty} \frac{m+1}{2^{m+1}} \cdot \frac{2^m}{m} = \\ = \lim_{m \rightarrow \infty} \frac{m+1}{m} \cdot \frac{2^m}{2^m \cdot 2} = \lim_{m \rightarrow \infty} \frac{m+1}{m} \cdot \frac{1}{2} = \frac{1}{2} \in [0, \infty]$$

$$\Rightarrow L \text{ exists and } L = \frac{1}{2}$$

$$\Rightarrow R = \frac{1}{L}, \quad L \in (0, \infty)$$

$$R = \frac{1}{\frac{1}{2}} = 2$$

The convergence set:

$$C := \{x \in \mathbb{R} \mid \sum_{m=0}^{\infty} a_m (x-c)^m \text{ converges}\}$$

$$c = (-2, 2) = (-2, 2)$$

Check endpoints:

$$x = 2 \rightarrow \sum_{m \geq 1} \frac{m \cdot 2^m}{2^m} = \sum_{m \geq 1} m \rightarrow \text{converges}$$

$$x = -2 \quad \sum_{n=1}^{\infty} n \cdot \frac{(-2)^n}{2^n} = \sum_{n=1}^{\infty} n \cdot (-1)^n \cdot \cancel{2^n} = \sum_{n=1}^{\infty} n \cdot (-1)^n$$

$$\sum_{n=1}^{\infty} n (-1)^n$$

$$\text{R.T.: } \frac{x_{m+1}}{x_m} = \frac{(m+1)(-1)^{m+1} \cdot (-1)}{m \cdot (-1)^m} \rightarrow -1 < 1$$

\Rightarrow diverges

Alternating

$$-1 + 2 - 3 + 4 - \dots$$

$$\Rightarrow c = (-2, 2)$$

$$\frac{1}{\sqrt{n}} \rightarrow 0 \quad (\text{known step}) \Leftrightarrow$$

$$(\infty, 0) \ni \frac{1}{\sqrt{n}} = k \Leftrightarrow$$

$$k = \frac{1}{\sqrt{n}} = \frac{1}{\sqrt{0}} = \infty$$

Stetig unregelmäßig - mit

$$\lim_{n \rightarrow \infty} \left(\frac{1}{\sqrt{n}} \right) = 0 \quad \text{und} \quad \lim_{n \rightarrow \infty} \left(\frac{1}{\sqrt{n}} \right)' = 0$$

$$(\sum, \cdot, +) \ni (\sum_{n=0}^{\infty} 0, 0 + 0, 0 \cdot 0) = 0$$