```python
import numpy as np
import matplotlib.pyplot as plt

# the formula for the p-norm is:
# ||x||_p = (|x_1|^p + |x_2|^p + ... + |x_n|^p)^(1/p)
# where x is a vector in R^n
# here we have x = (x, y) in R^2
# so the formula becomes:
# ||x||_p = (|x|^p + |y|^p)^(1/p)
def is_inside_unit_ball(x, y, p):
    return (np.abs(x)**p + np.abs(y)**p)**(1/p) <= 1

def plot_random_points_in_unit_ball(p, num_points=1000):
    inside_points_x = []
    inside_points_y = []
    outside_points_x = []
    outside_points_y = []

    for _ in range(num_points):
        x = np.random.uniform(-1, 1)
        y = np.random.uniform(-1, 1)

        if is_inside_unit_ball(x, y, p):
            inside_points_x.append(x)
            inside_points_y.append(y)
        else:
            outside_points_x.append(x)
            outside_points_y.append(y)

    plt.scatter(inside_points_x, inside_points_y, label=f'p = {p}', color='red')
    plt.scatter(outside_points_x, outside_points_y, label=f'p = {p}', color='grey')

p_values = [1.25, 1.5, 3, 8]

# Plot random points inside the unit ball for each value of p
for p in p_values:
    plt.figure(figsize=(8, 8))
    plot_random_points_in_unit_ball(p, 2500)
    plt.title('Random Points in Unit Ball in R^2 for p-norm = ' + str(p))
    plt.xlabel('x')
    plt.ylabel('y')
    plt.legend()
    plt.grid(True)
    plt.axis('equal')
    plt.show()
```

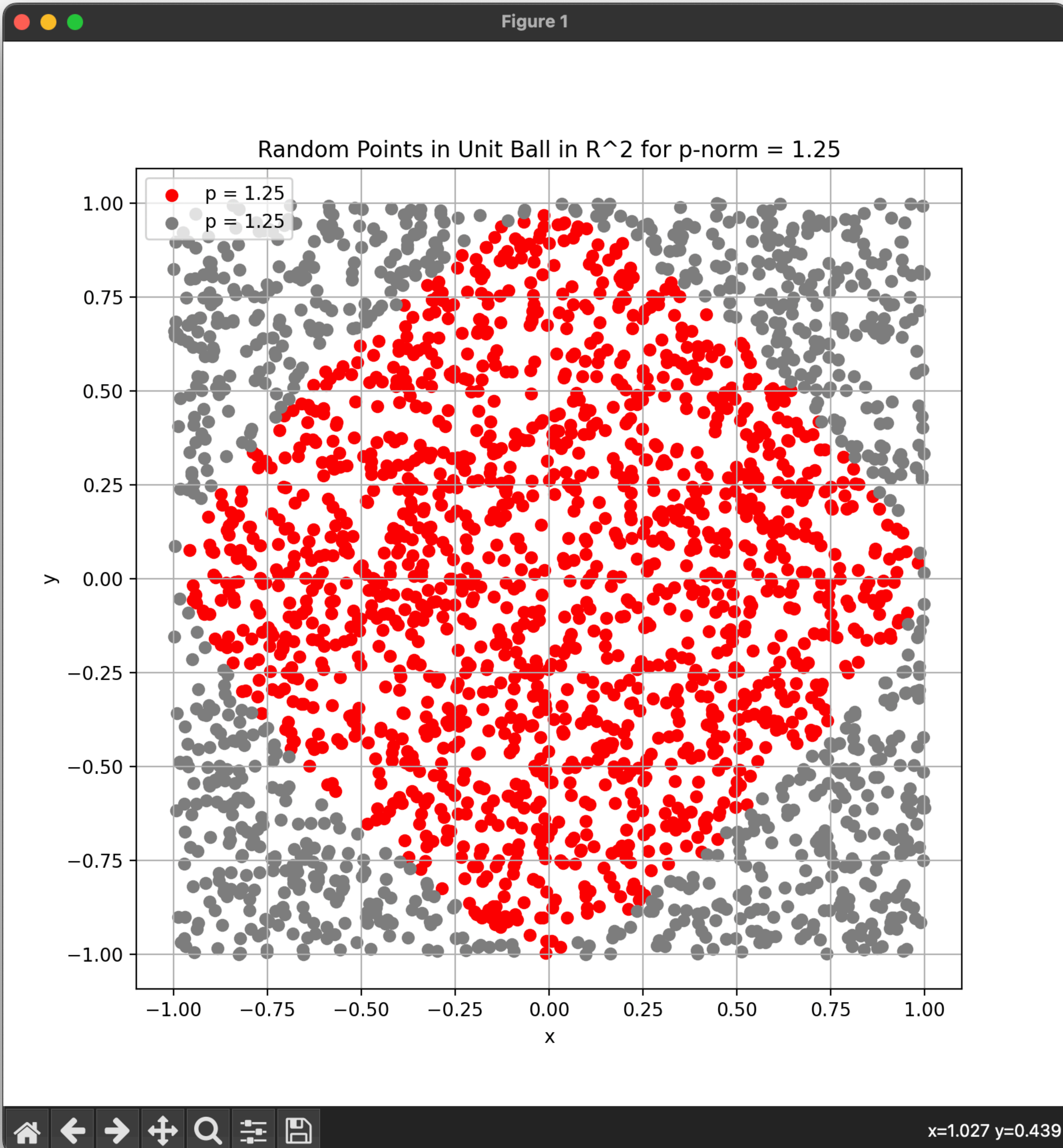

Figure 1
Random Points in Unit Ball in R^2 for p-norm = 1.25
p = 1.25
p = 1.25
y
x
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00
x=1.027 y=0.439

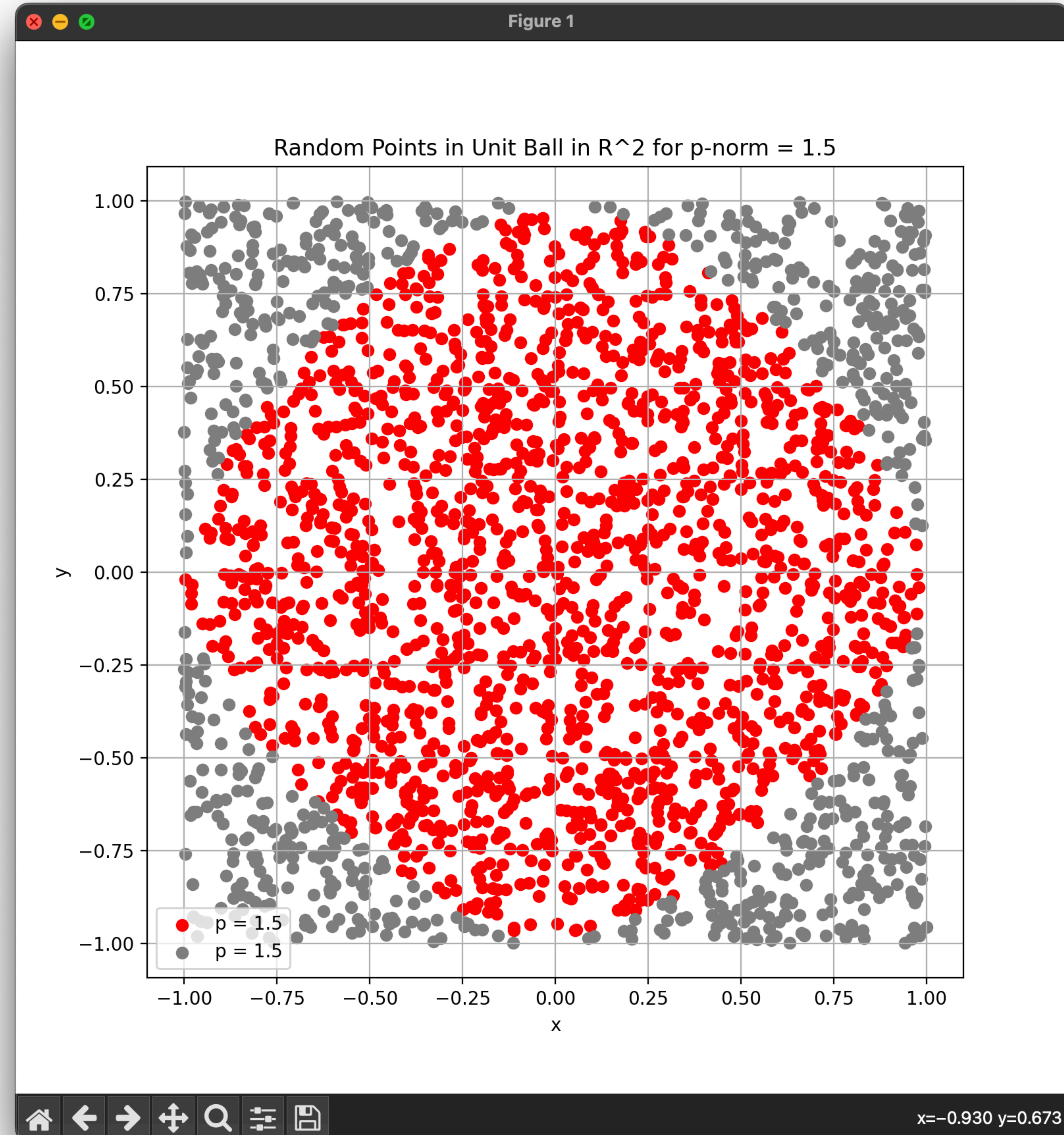

Figure 1
Random Points in Unit Ball in R^2 for p-norm = 1.5
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
y
x
p = 1.5
p = 1.5
x=−0.930 y=0.673

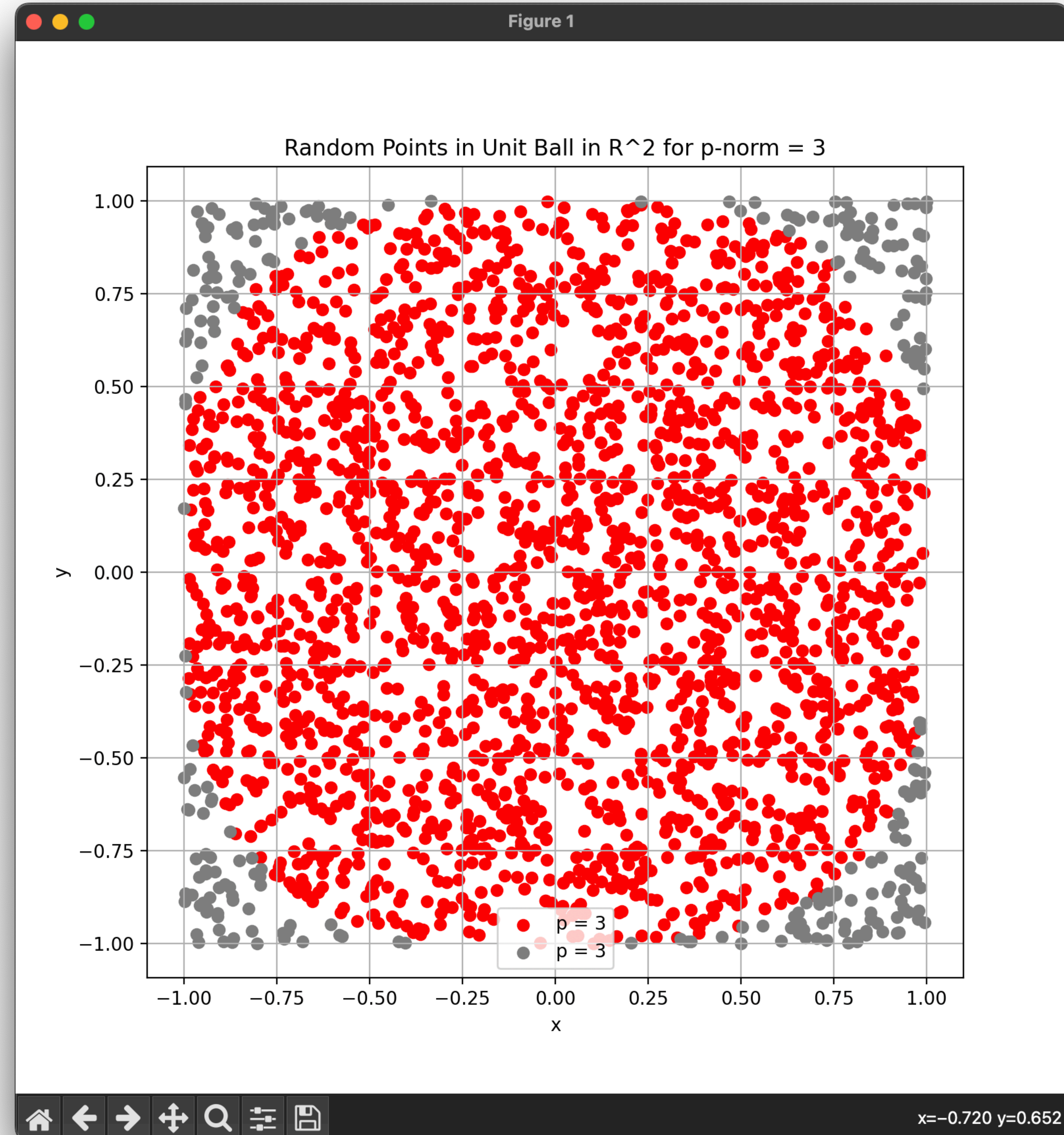

Figure 1
Random Points in Unit Ball in R^2 for p-norm = 3
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
y
p = 3
p = 3
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00
x
x=−0.720 y=0.652

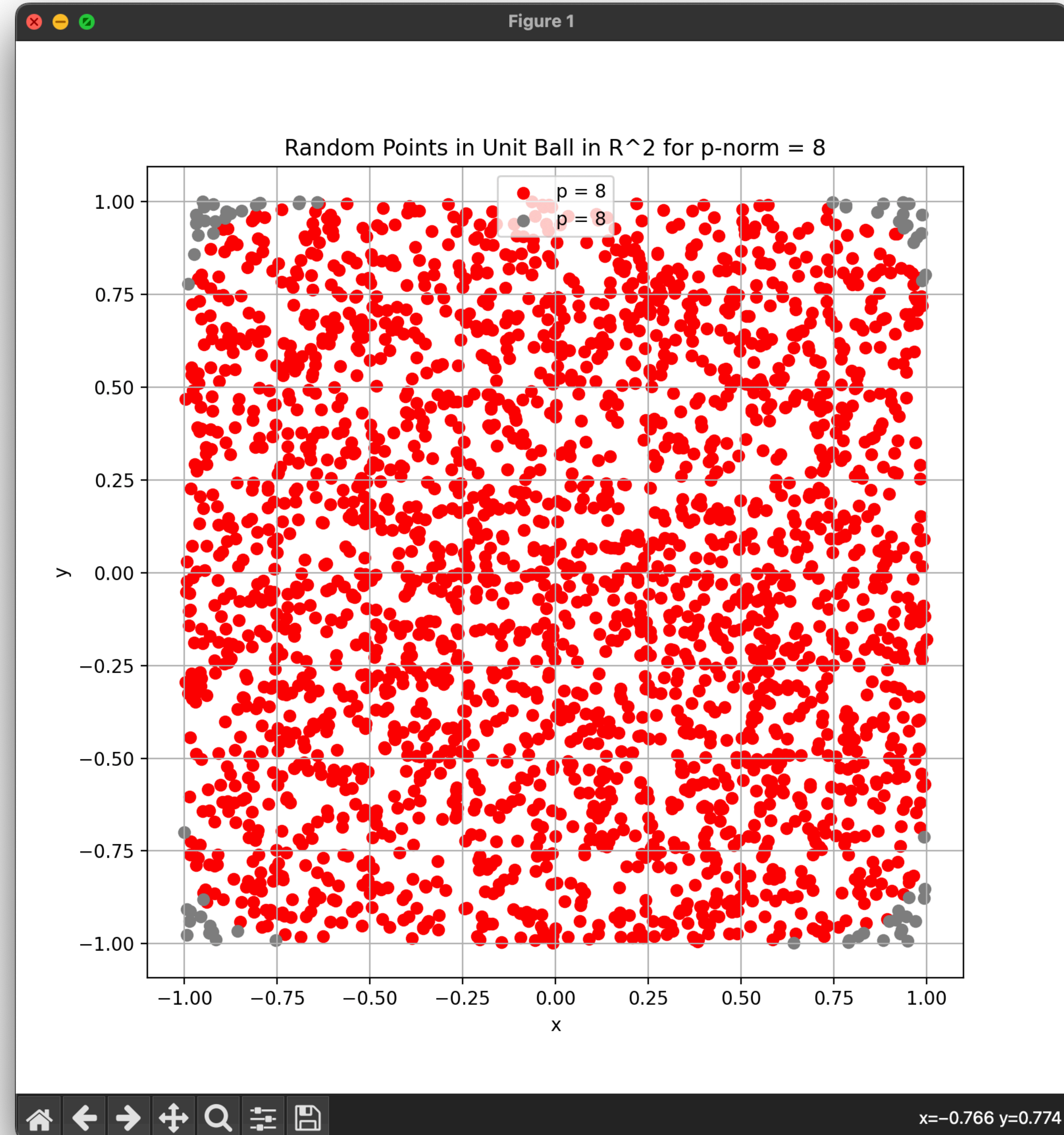
Figure 1
Random Points in Unit Ball in R^2 for p-norm = 8
p = 8
p = 8
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
y
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00
x
x=−0.766 y=0.774

Homework 8

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2\. a) $\langle x, y \rangle = 0$

$|\langle x, y \rangle| \leq \|x\| \cdot \|y\|$ (Cauchy Schwarz Theorem)

$\Rightarrow |0| \leq \|x\| \cdot \|y\|$

$0 \leq \|x\| \cdot \|y\|$

Take $\|x\| \cdot \|y\| = 0 \Rightarrow x$ or y is $0 \Rightarrow x \perp y$

$x \perp y \iff \langle x, y \rangle = \|x\| \cdot \|y\| \cdot \cos(\alpha)$

$\Rightarrow \langle x, y \rangle = \|x\| \cdot \|y\| \cdot \cos 90^\circ$

$\langle x, y \rangle = \|x\| \cdot \|y\| \cdot 0$

$\langle x, y \rangle = 0 \quad , \forall x, y \in \mathbb{R}^n$

b) $\|x+y\| = \|x-y\| \quad |^2$

$\|x+y\|^2 = \|x-y\|^2$

$\|x+y\|^2 = \langle x+y, x+y \rangle = \langle x, x \rangle + \langle y, y \rangle + 2\langle x, y \rangle$

$\|x-y\|^2 = \langle x-y, x-y \rangle = \langle x, x \rangle + \langle y, y \rangle - 2\langle x, y \rangle$

~~$\langle x, x \rangle$~~

$\|x+y\|^2 - \|x-y\|^2 = 4\langle x, y \rangle$

$\|x+y\|^2 - \|x-y\|^2 = 4 \cdot 0$ (from point a)

$\|x+y\|^2 - \|x-y\|^2 = 0$

$\|x+y\|^2 = \|x-y\|^2$

$\|x+y\| = \|x-y\|$

e.) $\|x+y\|^2 = \|x\|^2 + \|y\|^2$

$\|x+y\|^2 = \langle x+y, x+y\rangle = \langle x, x\rangle + \langle y, y\rangle + 2\langle x, y\rangle$

(point a)

$\Rightarrow \|x+y\|^2 = \langle x, x\rangle + \langle y, y\rangle \Rightarrow$

$\|x+y\|^2 = \|x\|^2 + \|y\|^2$