(b) Parametric test for existence of trend

This is a parametric test for existence of trend using significance of regression approach.

underlying assumption : normality

Testing for existence of linear trend

$$Y_t = \alpha + \beta t + \epsilon_t; \quad t = 1(1)n$$

$\{\epsilon_t\}$ seq of i.i.d. $N(0, \sigma^2)$

We set $H_0: \beta = 0$ ag $H_A: \beta \neq 0$

no linear trend   linear trend

usual t/F test is used for the testing (Ref: Linear regression - Montgomery).

Testing for existence of quadratic ~~for~~ trend

$$Y_t = \alpha + \beta t + \gamma t^2 + \epsilon_t; \quad \epsilon_t \overset{iid}{\sim} N(0, \sigma^2)$$

We can set the following type of hypotheses testing

$H_{01}: \gamma = 0$ ag $H_{A1}: \gamma \neq 0$ (testing for 2nd order polynomial time trend given that 1st order time trend is accepted)

OR

$H_{02}: \beta = 0, \gamma = 0$ ag H_{A2}: not H_{02}

(joint testing for β and γ)

Joint testing is done using the standard F testing with restricted and unrestricted sum of squares

(Ref: Linear regression - Montgomery)

Remark: If test for existence of trend indicates significant trend component is present, then the next step would be to estimate the trend, $\hat{m}_t$ (say)

Remark: After trend estimation, detrend the data as $y_t - \hat{m}_t$ (for additive model) and check for existence of trend in the detrended data to ensure trend is captured properly through trend estimation.

II Testing for existence of seasonality

Friedman's test (Friedman, Journal of American Statistical Association; 1937)

This is once again a non-parametric test procedure.

Null hypothesis of "no seasonality" is tested against the alternate hypothesis of "presence of seasonality".

For testing seasonality the underlying data is either monthly or quarterly.

Steps for Friedman's test (for a monthly data)

Step 1: Remove trend, if necessary, from the time series (this will involve estimation of trend component, if it is present - we will shortly consider the topic of estimation of trend)

Step 2 Rank the values obtained from step 1 within each year from smallest (1) to largest (12).

Let M_{ij} denote the rank corresponding to the i^{th} month for the j^{th} year

Let c denote the number of years of data

Let M_i denote the total rank for the month i (total over the c years)

$i = 1(1)12$

i.e. $M_i = \sum_j M_{ij}$

we thus obtain following rank table from the date

month \ yr	1	2	- - - -	c	M_i
1	$M_{1,1}$	$M_{1,2}$		$M_{1,c}$	M_1
2	$M_{2,1}$	$M_{2,2}$		$M_{2,c}$	M_2
⋮	⋮	⋮		⋮	⋮
$r=12$	$M_{12,1}$	$M_{12,2}$		$M_{12,c}$	M_{12}

Note that each column $(M_{1j}, M_{2j}, \ldots, M_{12,j})'$ for a particular j (year) is a permutation of $\{1, 2, \ldots 12\}$.

Under the null hypothesis of no seasonality, each permutation of $\{1, 2, \ldots, 12\}$ is equally likely.

In such a situation

M_{ij} can take any of $1, 2, \ldots r$ with equal probability $\frac{1}{r}$ and $E(M_{ij}) = \frac{1}{r}(1+2+\ldots+r)$

$$= \frac{r+1}{2}$$

$$\& \; E(M_i) = E\left(\sum_{j=1}^{c} M_{ij}\right) = \frac{c(r+1)}{2}$$

Step 3 : Compute the asymptotic test statistic

$$X = 12 \sum_{i=1}^{r} \left(M_i - \frac{c(r+1)}{2}\right)^2 \Big/ cr(r+1) \quad ; \; r = 12$$

Using asymptotic theory (I would avoid deriving that), it can be shown that

$X \overset{asym}{\sim} \chi^2_{r-1}$ (a central χ^2 on $r-1$ d.f)

under the null hypothesis of no seasonality

The ~~asy~~ asymptotic test would reject null hypothesis of no seasonality at level of significance α if

$$obsd\ (X) > \chi^2_{r-1}(\alpha)$$

where $\chi^2_{r-1}(\alpha)$ is the upper α cutoff point of a central χ^2 dist^n on $r-1$ degrees of freedom.

i.e $\quad P\left(\chi^2 > \chi^2_{r-1}(\alpha)\right) = \alpha$

$\chi^2 \sim$ central chi-square on $r-1$ d.f.