Problem Set-6
MTH-204, 204A
Abstract Algebra

1. The additive group $\mathbb{R}$ acts on $\mathbb{C}$ by $a.z = e^{ia}z$, for $a \in \mathbb{R}$ and $z \in \mathbb{C}$. Find all orbits and stabilizers for this action.

2. Let H and K be two subgroups of a group G. Compute the order of HK using the orbit stabilizer theorem.

3. If a nontrivial finite group G acts on a finite set of more than one elements and the action has only one orbit then show that some $g \in G$ has no fixed points.

4. Prove that A_4 has no subgroup of order 6.

5. Prove that the number of conjugacy classes in S_n is the number of partitions of n.

6. Find all conjugacy classes and verify the class equation for the following groups.

a. D_n b. Q_8 c. A_4

7. Prove that a group G is not cyclic if and only if G is a union of proper subgroups.

8. Prove that in a finite group the union of the subgroups conjugate to a proper subgroup do not fill up the whole group.

9. Let p be a prime and let G be a group of order p^a. Prove that G has a subgroup of order p^b for all $0 \leq b \leq a$.