Indian Institute of Technology Kanpur

Department of Mathematics and Statistics

Surfaces: Assignment 1

Subject: Several variables calculus & differential geometry (MTH305A)

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DISCLAIMER

(1) Let S be a regular surface and $p \in S$. Show that there exists a local coordinate chart $(\mathbb{R}^2, \phi)$ such that $p \in \phi(\mathbb{R}^2)$.
(2) Let S be a regular surface in $\mathbb{R}^3$. A subset $\tilde{S}$ of S is called open in S, if for every $p \in \tilde{S}$, there exists $p \in V \subset_{\text{open}} \mathbb{R}^3$ such that $V \cap S \subset \tilde{S}$.
Show that an open subset $\tilde{S}$ of a regular surface is a regular surface.
(3) Show that the cylinder
$$C = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 = 1\}$$
is a regular surface and find parameterisations whose coordinate neighbourhoods cover the cylinder.
(4) Does there exist a smooth function $f : \mathbb{R}^3 \to \mathbb{R}$ and $r \in f(\mathbb{R}^3)$ which is not a regular value of f but $f^{-1}(r)$ is a regular surface.
(5) Let us consider the plane $P = \{(x, y, z) \in \mathbb{R}^3 \mid x = y\}$ in $\mathbb{R}^3$ and $\phi : U \to \mathbb{R}^3$ be given by $\phi(x, y) = (x + y, x + y, xy)$, where $U = \{(x, y) \in \mathbb{R}^2 \mid x > y\}$.
(a) Show that $\phi(U) \subset P$.
(b) Is ϕ a parameterisation of P?