Assignment-12

1. Show that $f_n(x) = \dfrac{x^2}{x^2 + (1-nx)^2}$ is not equicontinuous on $[0,1]$.

2. Show that $f_n(x) = \dfrac{e^{-nx}}{n^2}$ is not equicontinuous on $\mathbb{R}$.

3. (f_n) ptwise bdd. seq. on a countable set E. Show that $\exists\ (f_{n_k})$ s.t. (f_{n_k}) cvgs. to $f(x)$ for each $x \in E$.

4. Let K be a compact metric space with f_n's cts. on K. Show that if $f_n \to f$ unif. then (f_n) is equicts. on K.

evaluation

5. For $a, b \in \mathbb{R}$ with $a < b$, let (f_n) be a seq. of differentiable functions on $[a,b]$. Suppose (f_n) and (f_n') are unif. bounded. Show that (f_n) is equicts. and has a uniformly cvgt. subseq.

6. The following example shows that the Arzela-Ascoli thm. does not hold if the metric space X is not totally bdd.

Consider $f_n : \mathbb{R} \to \mathbb{R}$ given by $f_n(x) = \begin{cases} \dfrac{|x|}{n}, & \text{if } |x| \le n \\ 1, & \text{if } |x| > n. \end{cases}$

Show that (f_n) is unif. bounded, but does not have any unif. cvgt. subsequence.

7. Let $f : [0,1] \to \mathbb{R}$ cts. and $\displaystyle\int_0^1 f(x) \cdot x^n \, dx = 0$ for $n = 0, 1, 2, \ldots$

Then, show that $f(x) = 0$ on $[0,1]$.

8. Suppose (f_n) equicts. on a compact set K.

$f_n \to f$ ptwise on K.

Show that $f_n \to f$ uniformly on K.