Department of Mathematics and Statistics, IIT Kanpur

First Course Handout (FCH)

Course. MTH305A: Several Variable Calculus & Differential Geometry

Semester: 2022-23 (I)

Instructor. Dr. Bidyut Sanki

1. Course Description.

Differential geometry is the study of geometry using the techniques of differential calculus, integral calculus, linear algebra and multi-linear algebra (though mostly differential calculus). The course MTH305A: Several variables calculus & differential Geometry is a basic course that introduces the fundamentals of differential geometry primarily by focusing on the theory of curves and surfaces in the Euclidean space of dimension three. For our purpose, we begin the course with studying several variable calculus. We study a few topics in several variable calculus, e.g., chain rule, inverse and implicit function theorem, Taylor's theorem and applications etc, those are essential to study differential geometry of curves and surfaces. Next, we study geometric properties of curves both local (e.g., tangent, normal, binormal, regularity, curvature, torsion etc.) and global (e.g., isoperimetric inequality). Finally, we study geometric properties of surfaces and their higher dimensional analogue to built a solid mathematical understanding of the fundamental notions of differential geometry and sufficient visual intuition of the subject.

2. Course Content

Several variables calculus. Differentiation: Definition and examples, Mean value inequality, Tangent planes to level sets of functions; Implicit mapping theorem, Inverse mapping theorem and applications; Taylor's theorem and applications.

Differential geometry. Curves: Definition and examples, Regular curves, Plane curves, Curvature of plane curves, Isoperimetric inequality for plane curves; Space curves, Frenet-Serret formula for space curves; Local existence theorem curves.

Surfaces: Definition and examples; Tangent planes, Maps between surfaces; First fundamental and second fundamental forms; Curvature of surface; Hilbert's theorem for compact surfaces; Gauss theorem a Egregium.

3. Prerequisite.

Analysis – I (MTH301A)

4. References.

- (i) Spivak: Calculus on manifolds, Springer.
- (ii) Tom M. Apostol: Mathematical Analysis, Narosa Publishing House, India.
- (iii) W. Rudin: Principles of Mathematical Analysis.
- (iv) A Pressley: Elementary differential geometry, Springer India.
- (v) M P do Carmo: Differential geometry of curves and surfaces, Prentice Hall.

5. Contact instructor.

Instructor name. Dr. Bidyut Sanki Email. bidyut@iitk.ac.in Office. FB587 Home page. click here

6. Course conduct.

Lecture. Schedule: MWF 11:00-12:00; Venue: L3

First lecture: July 29, 2022.

Tutorial. We will have Discussion/Tutorial on every Thursday 11:00 AM - 11:50 AM, Venue: L3, and will be started from August 4, 2022.

Assignment. We will have one assignment per week that will be sent by email.

7. Evaluation

• Quiz: 30%

Mid-sem (announced): 30%End-sem (announced): 40%