Fundamental components of a time series

A time series may contain deterministic component(s) and stochastic component.

Deterministic components are non-random in nature and are of following types:

m_t : trend or long term movement / tendency characterizing a time series

s_t : seasonal components are distinguishable patterns of regular annual variations in a time series.

c_t : cyclical components are more or less regular long range swings above and below some equilibrium level or trend line

stages of cyclical component : upswing, peak downswing, trough

Stochastic random component of a time series is referred to as the irregular component. This component accounts for the random nature of any time series sequence.

Models of time series

- Additive model

$$Y_t = m_t + s_t + c_t + e_t$$

- Multiplicative model

$$Y_t = m_t \, s_t \, c_t \, e_t$$

Note: e_t is the irregular random component

Remark: A particular time may have one or more deterministic components present in it; e.g. suppose a time series has trend and seasonal components, we call it is trend-seasonal model. Such a trend-seasonal model will also contain the irregular random component.

Preliminary tests of a time series

(I) Testing for existence of trend

(a) Relative ordering test

This is a non-parametric test procedure used for testing existence of trend component

Null hypothesis ag Alternate hypothesis

H_0 : no trend ag H_A: trend is present

Let the time series be denoted by $\{Y_1, \ldots, Y_n\}$ (at n time points)

Define

$$q_{ij} = \begin{cases} 1, & \text{If } Y_i > Y_j \text{ when } i<j \\ 0, & \text{o/w} \end{cases}$$

$$Q = \sum_{\substack{i \\ i<j}} \sum_{j} q_{ij}$$

Note that Q counts the # of decreasing points in the time series and is also the # discordances.

If there is no trend (increasing or decreasing) in the time series,

$$P(q_{ij}=0) = P(q_{ij}=1) = \frac{1}{2}$$

(i.e equally likely to be concordant or discordant)

$\Rightarrow$ under no trend (i.e under H_0),

$$E(Q) = \sum_{i<j}\sum E(q_{ij}) = \frac{n(n-1)}{4}$$

If observed $Q \ll E(Q)$ then it would be an indication of rising trend and if observed $Q \gg E(Q)$ then it would be an indication of a falling trend.

If obsd Q does not differ "significantly" from $E(Q)$ (under H_0) then it would indicate no trend.

Q is related with Kendall's τ, the rank correlation coefficient, through the relationship

$$\tau = 1 - \frac{4Q}{n(n-1)}$$

Using the standard results of Kendall's τ, we have that, under the null hypothesis of no trend

$$E(\tau) = 0 \quad \& \quad V(\tau) = \frac{2(2n+5)}{9n(n-1)}.$$

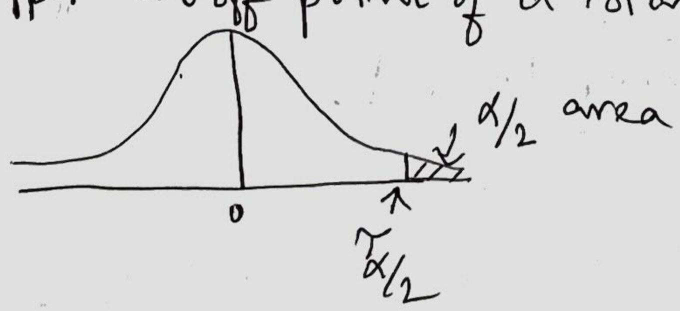
Asymptotic test for H_0: no trend is based on the statistic

$$Z = \frac{\tau - E(\tau)}{\sqrt{V(\tau)}} \overset{asym}{\sim} N(0,1) \text{ under } H_0.$$

We would reject the null hypothesis of no trend at level of significance α if

observed $|Z| > \tau_{\alpha/2}$

($\tau_{\alpha/2}$ is the $\alpha/2$ th upper cut off point of a standard normal dist^n, i.e



$P(Z > \tau_{\alpha/2}) = \alpha/2$)

$Z \sim N(0,1)$