Assignment 4: Several variables calculus & differential geometry (MTH305A)

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(1) Let $S \subset \mathbb{R}^n$ be open and $f : S \to \mathbb{R}$ is differentiable at each point in S. Let $x, y \in S$ such that $L(x, y) \subset S$. Then show that there exists a point z in the interior of $L(x, y)$ such that $f(y) - f(x) = f'(z)(y - x)$.

(2) Find all first and second partial derivatives of z with respect to x and y if $xy + yz + xz = 1$.

(3) Prove that $f(x, y) = \sin(y - ax) + \ln(y + ax)$ is a solution to the wave equation $D_{1,1}f = a^2 D_{2,2}f$.

(4) Let α and k be constants. Prove that the function $f(x, t) = e^{-\alpha^2 k^2 t} sin(kt)$ is a solution to the heat equation $D_2 f = \alpha^2 D_{1,1} f$.

(5) Find all local maxima and minima for the following functions:
(i) $f_1(x, y) = x^2 + y^2$,
(ii) $f_2(x, y) = x^2 - y^2$,
(iii) $f_3(x, y) = x^4 + y^4$
(iv) $f_4(x, y) = x^3 + y^3$

(6) The plane $x + y - z = 1$ intersects the cylinder $x^2 + y^2 = 1$ in an ellipse. Find the points on the ellipse closest to and farthest from the origin applying Lagrange's method.

(7) Find all points on the surface $xy - z^2 + 1 = 0$ that are closest to the origin.