Assignment 1.

1. If A is a nonempty set bounded below, show that A has a greatest lower bound.

2. Let A be a nonempty subset of $\mathbb{R}$ bounded above. Show that there is a sequence (x_n) of elements in A that converges to $\sup A$.

3. Prove that every convergent seq. of real nos. is bounded. Moreover, if (a_n) is convergent, show that $\inf_n a_n \leq \lim_{n\to\infty} a_n \leq \sup_n a_n$.

4. Show that the least upper bound property holds in $\mathbb{Z}$.

5. Let $a_n \geq 0$ for all $n \geq 1$, and let $s_n := \sum_{i=1}^{n} a_i$. Show that (s_n) converges iff (s_n) is bdd.
   (if and only if)

6. Prove that a convergent seq. is Cauchy, and any Cauchy seq. is bdd.

7. Show that a Cauchy seq. with a convergent subsequence actually converges.

8. Show that (x_n) converges to $x \in \mathbb{R}$ iff every subseq. $(x_{n_k})_{k=1}^{\infty}$ of (x_n) has a further subseq. $(x_{n_{k_\ell}})_{\ell=1}^{\infty}$ that converges to x.

9. Suppose that $a_n \geq 0$ and $\sum_{n=1}^{\infty} a_n < \infty$.

   (i) Show that $\liminf_n \{n a_n\} = 0$.

   (ii) Give an example showing that $\limsup_n n a_n > 0$ is possible.