# Assignment 5:

1. If $E$ is a connected subset of $M$, and if $A$ and $B$ are disjoint open sets in $M$ with $E \subset A \cup B$, prove that either $E \subset A$ or $E \subset B$.

2. If $E$ and $F$ are connected subsets of $M$ with $E \cap F \neq \phi$, then show that $E \cup F$ is connected.

3. If every pair of points in $M$ is contained in some connected set, show that $M$ is itself connected.

4. If $E$ and $F$ are nonempty subsets of $M$, and if $E \cup F$ is connected, show that $\overline{E} \cap \overline{F} \neq \phi$.

5. If $M$ is connected and has at least two points, show that $M$ is uncountable.

6. If $f: \mathbb{R} \to \mathbb{R}$ is cts. and open, show that $f$ is strictly monotone.

7. Prove that there does not exist a cts. function $f: \mathbb{R} \to \mathbb{R}$ satisfying
$$f(\mathbb{Q}) \subset \mathbb{R} \setminus \mathbb{Q} \quad \text{and} \quad f(\mathbb{R} \setminus \mathbb{Q}) \subset \mathbb{Q}.$$

8. Let $A$ and $B$ be closed subsets of $M$, and suppose that both $A \cup B$ and $A \cap B$ are connected. Prove that $A$ and $B$ are connected.

evaluation!

9. Let $I = (\mathbb{R} \setminus \mathbb{Q}) \cap [0,1]$ and $Q = \mathbb{Q} \cap [0,1]$, with their usual metrics. Prove that there is a cts. map from $I$ onto $Q$, but that there does not exist a cts. map from $[0,1]$ onto $Q$.

10. If $f: \mathbb{R} \to \mathbb{R}$ is differentiable, prove that $f'$ has the intermediate value property.

11. Let $V$ be a normed vector space, and let $x \neq y \in V$. Show that the map $f(t) = x + t(y-x)$ is a homeomorphism from $[0,1]$ into $V$.
Also show that any normed space $V$ is connected.