Name:
Roll No:

MTH442A: Time Series Analysis
Quiz 1: Full Marks 20

Let $Y_t = \alpha + \beta t^2 + S_t + \epsilon_t$; α and β are fixed constants, S_t is seasonal component with periodicity 6 and $\{\epsilon_t\}$ is a sequence of i.i.d. $N(0,1)$ random variables.
Prove or disprove the following statements:

(a) $\{\nabla_6 Y_t\}$ is free from seasonal factor and trend
(b) $\{\nabla_6 Y_t\}$ is a Gaussian process
(c) $\{\nabla^2 \nabla_6 Y_t\}$ is covariance stationary $MA(7)$ process
(d) $Cov\,(\nabla^8 \epsilon_t, \nabla_8\, \epsilon_t) = -1$; for all t
(e) $Cov\,(\nabla^2\, Y_t, \nabla^2\, Y_{t+h}) = 0$; for all $|h| \geq 2$ and for all t.
(f) $X_t = |\epsilon_t - \epsilon_{t-1}|$ is a Gaussian process
(g) $Z_t = \epsilon_{6t} + \epsilon_{2(t+6)}$ is strict stationary

[2 + 2 + 3 + 3 + 3 + 3 + 4]

(a) $\nabla_6 Y_t = Y_t - Y_{t-6}$

$= (\alpha + \beta t^2 + S_t + \epsilon_t)$
$- (\alpha + \beta (t-6)^2 + S_{t-6} + \epsilon_{t-6}$

$= 12\beta t - 36\beta + \epsilon_t - \epsilon_{t-6}$

Seasonal factor eliminated but trend is present

(b) $P_t = \nabla_6 Y_t$ say

For any n and any admissible $(t_1, \dots, t_n)$, consider

$\underset{\sim}{P} = \begin{pmatrix} P_{t_1} \\ \vdots \\ P_{t_n} \end{pmatrix}$

No random variable part

$\forall \underline{\alpha} \in \mathbb{R}^n$, $\underline{\alpha} \neq \underline{0}$; $\underline{\alpha}' \underset{\sim}{P}$ = fixed const + lin comb of indep $N(0,1)$ r.v.s.

$\Rightarrow \underline{\alpha}' \underset{\sim}{P} \sim N_1 \quad \forall\, \underline{\alpha} \in \mathbb{R}^n$

$\Rightarrow \underset{\sim}{P} \sim N_n$

$\Rightarrow \{\nabla_6 Y_t\}$ is Gaussian

(c) $\nabla \nabla_6 Y_t = (12\beta t - 36\beta + \epsilon_t - \epsilon_{t-6})$
$- (12\beta(t-1) - 36\beta + \epsilon_{t-1} - \epsilon_{t-7})$

$= 12\beta + \epsilon_t - \epsilon_{t-1} - \epsilon_{t-6} + \epsilon_{t-7}$

$$\nabla^2 \nabla_6 Y_t = (\epsilon_t - \epsilon_{t-1} - \epsilon_{t-6} + \epsilon_{t-7})$$
$$- (\epsilon_{t-1} - \epsilon_{t-2} - \epsilon_{t-7} + \epsilon_{t-8})$$

$\nearrow$ Stationary MA(8)

(d) $\nabla^8 \epsilon_t = (1-B)^8 \epsilon_t = f^n$ of $\epsilon_t, \epsilon_{t-1}, \ldots, \epsilon_{t-8}$

coeff of ϵ_t & ϵ_{t-8} : 1

$\nabla_8 \epsilon_t = \epsilon_t - \epsilon_{t-8}$

$\Rightarrow \mathrm{Cov}(\nabla^8 \epsilon_t, \nabla_8 \epsilon_t) = 1 - 1 = 0 \ \forall t$

(e) $\nabla Y_t = Y_t - Y_{t-1} = (\alpha + \beta t^2 + S_t + \epsilon_t) - (\alpha + \beta(t-1)^2 + S_{t-1} + \epsilon_{t-1})$

$= 2\beta t - \beta + S_t - S_{t-1} + \epsilon_t - \epsilon_{t-1}$

$\nabla^2 Y_t = 2\beta + (S_t - 2S_{t-1} + S_{t-2}) + (\epsilon_t - 2\epsilon_{t-1} + \epsilon_{t-2})$

$\Rightarrow \mathrm{Cov}(\nabla^2 Y_t, \nabla^2 Y_{t+h}) = 0 \quad \forall |h| \geq 3$

(f) $X_t = |\epsilon_t - \epsilon_{t-1}|$

$\epsilon_1 - \epsilon_0 \sim N(0, 2) \Rightarrow X_1 = |\epsilon_1 - \epsilon_0| \nsim N,$

$\Rightarrow \{X_t\}$ is not Gaussian process.

(g) $Z_t = \epsilon_{6t} + \epsilon_{2(t+6)}$

$Z_1 = \epsilon_6 + \epsilon_{14}$

$Z_3 = 2\epsilon_{18}$

$V(Z_1) \neq V(Z_3)$

$\Rightarrow \{Z_t\}$ is not even covariance stationary