MSO 201A : Quiz 1 solution

(1) $F(.)$ has 2 pts of jump discontinuities at $x=1$ and $x=2$

$$P(X=1) = F(1) - F(1-) = \frac{1}{6}$$

$$P(X=2) = F(2) - F(2-) = \frac{1}{2}$$

$F(.)$ is continuous elsewhere

(a) $$P(X=2 \mid X>0.5) = \frac{P(X=2)}{1-P(X\le 0.5)} = \frac{1/2}{1-F(0.5)}$$

$$= \frac{1/2}{1-\frac{1}{3}(1-e^{-1/6})} \quad - 2$$

(b) $$P\left(X=\tfrac{11}{6} \,\middle|\, 1<X\le 3\right) = \frac{P(X=11/6)}{P(1<X\le 3)} = 0 \quad - 2$$

(c) $$P\left(1\le X<3 \,\middle|\, \tfrac{1}{4}\le X<2\right) = \frac{P(1\le X<2)}{P(\frac{1}{4}\le X<2)}$$

$$= \frac{F(2-)-F(1-)}{F(2-)-F(\frac{1}{4}-)} = \frac{F(2-)-F(1-)}{F(2-)-F(\frac{1}{4})} \quad - 1$$

$$= \frac{\left(\frac{1}{2}-\frac{1}{3}e^{-2/3}\right)-\left(\frac{1}{3}-\frac{1}{3}e^{-1/3}\right)}{\left(\frac{1}{2}-\frac{1}{3}e^{-2/3}\right)-\left(\frac{1}{3}-\frac{1}{3}e^{-\frac{1}{12}}\right)} \quad - 2$$

$$= \frac{\frac{1}{6}-\frac{1}{3}\left(e^{-2/3}-e^{-1/3}\right)}{\frac{1}{6}-\frac{1}{3}\left(e^{-2/3}-e^{-1/12}\right)}$$

(d) Discrete part of $F(x)$:

say, $$F_1(x) = \begin{cases} 0, & x<1 \\ \frac{1}{6}, & 1\le x<2 \\ \frac{2}{3}, & x\ge 2 \end{cases}$$

$\alpha = \frac{2}{3}$ & $$F_d(x) = \begin{cases} 0, & x<1 \\ \frac{1}{4}, & 1\le x<2 \\ 1, & x\ge 2 \end{cases} \quad - 2\tfrac{1}{2}$$

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$1-\alpha = \frac{1}{3}$ &

$$F_c(x) = \begin{cases} 0, & x < 0 \\ 1-e^{-x/3}, & x \geq 0 \end{cases} \quad \text{— ① } \; 2\tfrac{1}{2}$$

$$F(x) = \frac{2}{3} F_d(x) + \frac{1}{3} F_c(x)$$

(e) p.d.f. for $F_c(x)$:

$$f_c(x) = \begin{cases} \frac{1}{3} e^{-x/3}, & x \geq 0 \\ 0 & \text{o/w} \end{cases}$$

m.g.f. $M_X(t) = E(e^{tX}) = \frac{1}{3} \int_0^{\infty} e^{tx} e^{-x/3} \, dx$

$$= \frac{1}{3} \frac{1}{\left(\frac{1}{3} - t\right)} \qquad \boxed{t < \frac{1}{3}} \text{ — ②}$$

Deduct ½ mark If $t < \frac{1}{3}$ is not written

i.e. $M_X(t) = \frac{1}{1-3t}$ for $t < \frac{1}{3}$

(2) Z : r.v. denoting lifetime

For vendor X: $P(Z > 2) = \int_2^{\infty} \frac{1}{2} e^{-x/2} dx = e^{-1}$ — ①

For vendor Y: $P(Z > 2) = \frac{1}{4} \int_2^4 dx = \frac{1}{2}$ — ①

Let A be the event that bulb is supplied by vendor X &
B - - - - - - - - - - Y;
R be the event that lifetime is > 2, i.e. $Z > 2$

reqd prob $= P(A|R) = \dfrac{P(A)\,P(R|A)}{P(A)\,P(R|A) + P(B)\,P(R|B)}$

$$= \frac{\frac{1}{2} e^{-1}}{\frac{1}{2} e^{-1} + \frac{1}{2} \cdot \frac{1}{2}} \text{ — ③}$$

$$= \frac{e^{-1}}{e^{-1} + \frac{1}{2}}$$