Assignment-11.

1. Let $f_n : \mathbb{R} \to \mathbb{R}$ and suppose $f_n \to 0$ uniformly on every closed and bounded interval $[a,b]$. Does it follow that $f_n \to 0$ uniformly on $\mathbb{R}$? Explain.

2. For each of the sequences below, determine whether (f_n) cvg. ptwise. If so, then does it converge uniformly?

(a) $f_n(x) = x^n$ on $(-1,1]$.

(b) $f_n(x) = n^2 x (1-x^2)^n$ on $[0,1]$.

(c) $f_n(x) = nx/(1+nx)$ on $[0,\infty)$

(d) $f_n(x) = nx/(1+n^2x^2)$ on $[0,\infty)$

(e) $f_n(x) = x e^{-nx}$ on $[0,\infty)$

(f) $f_n(x) = nx\, e^{-nx}$ on $[0,\infty)$.

3. Let $f_n : \mathbb{R} \to \mathbb{R}$ cts. for each n, and suppose that $f_n \to f$ uniformly on each closed bounded interval $[a,b]$. Show that f is cts. on $\mathbb{R}$.

Evaluation

4. Suppose $(f_n) \in C[0,1]$ and that $f_n \to f$ unif. on $[0,1]$.

5pts.

Is it true that
$$\int_0^{1-1/n} f_n \to \int_0^1 f \quad ?$$
Explain.

5. Show that $\sum_{n=1}^{\infty} x^2/(1+x^2)^n$ cvgs. for all $|x| \le 1$, but the convergence is not uniform.

6. For which values of x does $\sum_{n=1}^{\infty} n e^{-nx}$ converge?

On which intervals is the convergence uniform? Explain.