Indian Institute of Technology Kanpur

Department of Mathematics and Statistics

Curves: Assignment 1

Subject: Several variables calculus & differential geometry (MTH305A)

Semester: 2020-2021, I

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(1) Let $\alpha : (a, b) \to \mathbb{R}^3$ be a parameterized curve does not pass through the origin. If $\alpha(t_0)$ is the point on the trace of α closest to the origin and $\alpha'(t_0) \neq 0$, then the position vector $\alpha(t_0)$ is orthogonal to $\alpha'(t_0)$.

(2) Let $\alpha : I \to \mathbb{R}^3$ be a parametrized curve and let $v \in \mathbb{R}^3$ be a fixed vector. Assume that $\alpha'(t) \perp v$ for all $t \in I$ and that $\alpha(0) \perp v$. Prove that

$$\alpha(t) \perp v, \text{ for all } t \in I.$$

(3) Let $\alpha : I \to \mathbb{R}^3$ be a parameterized curve, with $\alpha'(t) \neq 0$, for all $t \in I$. Show that $\|\alpha(t)\|$ is a non-zero constant if and only if $\alpha(t)$ if orthogonal to $\alpha'(t)$ for all $t \in I$.

(4) Is $\alpha(t) = (t^2, t^4)$ is a parameterisation of $y = x^2$?

(5) Find the parametric equation of the level curves:
   (a) $y^2 - x^2 = 1$
   (b) $\frac{x^2}{4} + \frac{y^2}{9} = 1$.