# Assignment 7

1. $(M,d)$ : metric space. $A \subset B \subset M$ and if $B$ is totally bdd, show that $A$ is totally bdd.

2. Show that a subset $A$ of $\mathbb{R}$ is totally bdd. iff $A$ is bdd.

3. Is total boundedness preserved by homeomorphism? Explain.

4. Show that $A$ is totally bdd iff $A$ can be covered by finitely many closed sets of diameter at most $\varepsilon$ for every $\varepsilon > 0$.

5. Prove that $A$ is totally bdd iff $\bar{A}$ is totally bdd.

6. Prove that $A$ is totally bdd iff every seq. $(x_n)$ in $A$ has a subsequence $(x_{n_k})$ for which $d(x_{n_k}, x_{n_{k+1}}) < \frac{1}{2^k}$.

evaluation 7. If $A$ is not totally bdd, show that $A$ has an infinite subset $B$ that is homeomorphic to a discrete space.

8. Prove that a totally bdd metric space is separable.

9. Prove that the Hilbert cube $H^\infty$ is totally bdd.