4 (a) Let $A, B \subset \mathbb{R}$. Show that $A^\circ \cup B^\circ \subseteq (A \cup B)^\circ$. Can the inclusion be strict? Justify! [3]

(b) Let A be a compact subset and B be a closed subset of a metric space (X, d). Show that $A \cap B$ is also compact. [3]

(c) Let $U_n = (-1 + \frac{1}{n}, 1 - \frac{1}{n})$. Is $\{U_n\}$ an open cover for $(-1, 1)$? Show that finitely many U_n's cannot cover $(-1, 1)$? Does $[-\frac{1}{2}, \frac{5}{6}]$ can be covered by finitely many U_n's. Justify! [4]

(d) By assuming $A = \{m + \pi n : m, n \in \mathbb{Z}\}$ is dense in $\mathbb{R}$ (also, for $y \in [-1, 1]$ we know that there exists a $x \in \mathbb{R}$ such that $\sin x = y$). Show that for every $y \in [-1, 1]$ there exists a sequence $\{n_k\} \subset \mathbb{N}$ such that $\lim_{k\to\infty} \sin n_k = y$. [5]