3 (a) (i) Find all the continuous function $f : \mathbb{R} \to \mathbb{R}$ such that $f(x) + f(2x) = 0$. [2]

(ii) Find all the continuous function $f : (0,1) \to \mathbb{R}$ such that $f(x)^2 - f(x) = 6,\ \forall x \in (0,1)$ and $f(\frac{1}{2}) > 0$. [2]

(b) Let $f, g : A \to \mathbb{R}$ be two continuous functions such that $f(x) < g(x),\ \forall x \in A$. Answer the following, by proving it in case the answer is in affirmative or produce a counter example.

(i) Suppose $A = (0,1)$. Can we say that there exists $\lambda > 0$ such that $f(x) < \lambda g(x),\ \forall x \in (0,1)$. [2]

(ii) Suppose $A = [0,1]$. Can we say that there exists $\lambda > 0$ such that $f(x) < \lambda g(x),\ \forall x \in (0,1)$. [4]

(c) Let S be a bounded set in $\mathbb{R}$ and $f : S \to \mathbb{R}$ be continuous on S. Will f always be bounded? If f is uniformly continuous will f be bounded? Justify your answers. [5]