Name: **Roll No:**

MSO201A/ESO 209: PROBABILITY & STATISTICS

Quiz # 1: Full Marks 20

[1] Let $F(x)=\begin{cases} 0, & x<0 \\ 3\alpha x, & 0\le x\le 1 \\ (\alpha+\beta)(x^2+1), & 1<x\le 2 \\ 3x/8, & 2<x<5/2 \\ \gamma & x\ge 5/2. \end{cases}$

(a) Find the values of α, β and γ such that $F(.)$ is a distribution function.

(b) Let X be the random variable with distribution function $F(.)$, with the values of α, β and γ as obtained in (a). Find $P(1<X<5/2 \mid X\ge 3/2)$, $P(X=5/2 \mid X>1)$ and $P(X=3/2 \mid X>1/2)$. [4+3+2+2]

[2] Let $(\Omega,\mathcal{F},P)$ be a probability space and $A,B,C\in\mathcal{F}$ be such that $P(A)=0.5$, $P(B)=0.4, P(C)=0.3, P(AB)=P(AC)=P(BC)=0.2$ and $P(ABC)=0.1$. Find $P\left(A^c\cap B^c\cap C^c\right), P\left(\left(A^c\cup B^c\right)\cap C^c\right)$ and $P\left(\left(A^c\cap B^c\right)\cup C^c\right)$. [3+3+3]

Quiz #1 : Solution & Marking Scheme

(1)(a) For $F(.)$ to be dist^n f^n, $F(.)$ have to be right cont.

Right continuity of $F(.)$ at $0 \Rightarrow F(0) = F(0+)$ . . .(i)

. . . . $1 \Rightarrow F(1) = F(1+)$ . . (ii)

. . . . . . $2 \Rightarrow F(2) = F(2+)$ . . .(iii)

. . . . . . $5/2 \Rightarrow F(5/2) = F(5/2+)$ . . (iv)

$F(1) = F(1+) \Rightarrow 3\alpha = (\alpha+\beta)\,2$

i.e. $\alpha = 2\beta$

$F(2) = F(2+) \Rightarrow (\alpha+\beta) \times 5 = \frac{6}{8}$ (2)

i.e. $\alpha+\beta = \frac{6}{40}$

$\Rightarrow 3\beta = \frac{6}{40}$ i.e. $\beta = \frac{1}{20}$ & $\alpha = \frac{1}{10}$ (1.5)

Further for $F(.)$ to be a d.f., we need $F(\infty) = 1$ (0.5)

$\Rightarrow \gamma = 1$.

$$\Rightarrow \quad F(x) = \begin{cases} 0, & x < 0 \\ 3x/10, & 0 \le x \le 1 \\ \frac{3}{20}(x^2+1), & 1 < x \le 2 \\ 3x/8, & 2 < x < 5/2 \\ 1, & x \ge 5/2 \end{cases}$$

pt(s) of jump(s): $5/2$

magnitude of jump : $\frac{1}{16} = F(5/2) - F(5/2-) = P(X = 5/2)$.

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(b) $$P\left(1 < X < 5/2 \mid X \geq 3/2\right) = \frac{P\left((1 < X < 5/2) \cap (X \geq 3/2)\right)}{P(X \geq 3/2)}$$

$$= \frac{P(3/2 \leq X < 5/2)}{P(X \geq 3/2)} \quad \text{— (1)}$$

$$= \frac{F(5/2-) - F(3/2-)}{1 - F(3/2-)}.$$

$$= \underbrace{\frac{15/16 - 39/80}{1 - 39/80}}_{(2)} = \frac{36/80}{41/80} = \frac{36}{41}$$

$$P(X = 5/2 \mid X > 1) = \frac{P(X = 5/2 \cap X > 1)}{P(X > 1)} = \frac{P(X = 5/2)}{1 - F(1)}$$

$$= \frac{1/16}{\underbrace{1 - 3/10}_{(2)}} = \frac{1/16}{7/10} = \frac{1}{16} \times \frac{10}{7}$$

$$P(X = 3/2 \mid X > 1/2) = \frac{P(X = 3/2 \cap X > 1/2)}{P(X > 1/2)}$$

$$= \frac{P(X = 3/2)}{1 - F(1/2)} = 0 \quad \text{— (2)}$$

(2) (a) $P(A^c \cap B^c \cap c^c) = P(A \cup B \cup c)^c$

$= 1 - P(A \cup B \cup c)$ — (1)

$P(A \cup B \cup c) = P(A) + P(B) + P(c) - P(AB) - P(Ac) - P(B$

$+ P(ABC)$ — (1)

$= .5 + .4 + .3 - 3 \times 0.2 + .1 = 0.7$

$\Rightarrow P(A^c \cap B^c \cap c^c) = 1 - 0.7 = 0.3$. — (1)

(b) $P((A^c \cup B^c) \cap c^c) = P((A \cap B)^c \cap c^c)$.

$= P((A \cap B) \cup c)^c$ — (1)

$= 1 - P[(A \cap B) \cup c]$

$= 1 - [P(AB) + P(c) - P(ABC)]$ — (1)

$= 1 - [.2 + .3 - .1] = 0.6$ — 1(

(c) $P((A^c \cap B^c) \cup c^c) = P((A \cup B)^c \cup c^c)$

$= P((A \cup B) \cap c)^c$ — (1)

$= 1 - P((A \cup B) \cap c)$

$= 1 - [P(AC \cup BC)]$

$= 1 - [P(AC) + P(BC) - P(ABC)]$ — (1)

* $= 1 - [.2 + .2 - .1] = 0.7$ — (1)