Quiz-I
MTH-204, MTH-204A
ABSTRACT ALGEBRA
Spring-2023
Date: 16th February 2023

Time Allowed: 30 mins (6.15-6.45 PM) Max. Marks: 15

Write your answer in the space provided and explain all the major steps

1. Show that the additive group $\mathbb{Q}$ does not have a proper subgroup of finite index. [4]

Ans: Let H be a subgroup of finite index, say n. Then $n(q+H) = H$ for every $q \in \mathbb{Q}$. So in particular $n(\frac{q}{n} + H) = H$. So $q \in H$ and hence $\mathbb{Q} \subseteq H$, a contradiction.

2. For any two elements x and y in a group G prove that xy and yx have the same order. [4]

Ans: $x^{-1}(xy)x = yx$. So xy and yx are conjugate and hence have the same order.

3. Let Q_8 be the quaternion group. Is the map $f : Q_8 \to Q_8$ given by $f(x) = x^2$ a homomorphism ? Justify your answer. [4]

Ans: No, as $f(ij) = (ij)^2 = k^2 = -1$ whereas $f(i).f(j) = i^2.j^2 = -1.-1 = 1$

4. Write $\sigma = (456)(23)(12)(678)$ as a product of disjoint cycles and find the order of σ. [3]

Ans: Note that $\sigma = (132)(45678)$ and hence order of $\sigma = lcm\{3, 5\} = 15$.

———————————ROUGH———————————