

Indian Institute of Technology Kanpur
Department of Mathematics and Statistics

Surfaces: Assignment 1

Subject: Several variables calculus & differential geometry
(MTH305A)

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- (1) Let S be a regular surface and $p \in S$. Show that there exists a local coordinate chart (\mathbb{R}^2, ϕ) such that $p \in \phi(\mathbb{R}^2)$.
- (2) Let S be a regular surface in \mathbb{R}^3 . A subset \tilde{S} of S is called open in S , if for every $p \in \tilde{S}$, there exists $p \in V \subset_{\text{open}} \mathbb{R}^3$ such that $V \cap S \subset \tilde{S}$.

Show that an open subset \tilde{S} of a regular surface is a regular surface.

- (3) Show that the cylinder

$$C = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 = 1\}$$

is a regular surface and find parameterisations whose coordinate neighbourhoods cover the cylinder.

- (4) Does there exist a smooth function $f : \mathbb{R}^3 \rightarrow \mathbb{R}$ and $r \in f(\mathbb{R}^3)$ which is not a regular value of f but $f^{-1}(r)$ is a regular surface.
- (5) Let us consider the plane $P = \{(x, y, z) \in \mathbb{R}^3 \mid x = y\}$ in \mathbb{R}^3 and $\phi : U \rightarrow \mathbb{R}^3$ be given by $\phi(x, y) = (x + y, x + y, xy)$, where $U = \{(x, y) \in \mathbb{R}^2 \mid x > y\}$.
- (a) Show that $\phi(U) \subset P$.
- (b) Is ϕ a parameterisation of P ?