Solution: Suppose A is open.

Assume that $\text{int}(\partial A) \neq \phi$. Let $x \in \text{int}(\partial A)$.

Also $\text{int}(\partial A) = \text{int}(\overline{A} \cap \overline{M \setminus A})$

$= \text{int}(\overline{A} \cap M \setminus A)$ (Since A is open implies that $M \setminus A$ is closed.)

Since for any sets S and T, $\text{int}(S \cap T) = \text{int}(S) \cap \text{int}(T)$,

$$\text{int}(\overline{A} \cap M \setminus A) = \text{int}(\overline{A}) \cap \text{int}(M \setminus A)$$

Since $x \in \text{int}(\partial A)$, $x \in \text{int}(\overline{A})$ and $x \in \text{int}(M \setminus A)$

$\Downarrow$ $\Downarrow$

$\exists\, r_1 > 0$ s.t. $B(x, r_1) \subset \overline{A} = A \cup A'$ $\exists\, r_2 > 0$, s.t. $B(x, r_2) \subset \text{int}(M \setminus A)$

Take $r := \min\{r_1, r_2\}$. Consider $B(x, r)$.

$$B(x,r) \subset A \cup A' \text{ and } B(x,r) \subset \text{int}(M \setminus A) \subset M \setminus A. \quad \text{—— } (1)$$

$\Rightarrow B(x,r) \subset (A \cup A') \cap (M \setminus A)$

If $x \in A$, then $x \notin M \setminus A$. But from (1), we have $B(x,r) \subset M \setminus A$ implying $x \in M \setminus A$, contradicting $x \notin M \setminus A$.

If $x \in A'$, then $B(x,r) \cap A \neq \phi$, in particular.
But $B(x,r) \subset M \setminus A$ (from (1)) implying $B(x,r) \cap A = \phi$ contradicting $B(x,r) \cap A \neq \phi$.

Therefore, $\text{int}(\partial A) = \phi$.

Suppose A is closed. Then $M \setminus A$ is open. Hence $\text{int}(\partial(M \setminus A)) = \phi$.

Note that $\partial(M \setminus A) = \overline{M \setminus A} \cap \overline{A} = \partial A$

$\Rightarrow \text{int}\, \partial(M \setminus A) = \text{int}(\partial A)$

$\Rightarrow \text{int}(\partial A) = \phi$ as $\text{int}(\partial(M \setminus A)) = \phi$.

Example: Consider $M = \mathbb{R}$ and $A = \mathbb{Q}$.

Then $\partial A = \overline{A} \cap \overline{\mathbb{R} \setminus \mathbb{Q}} = \mathbb{R}$.

So, $\operatorname{int}(\partial A) = \mathbb{R}$.