(1) (a) Let $\{r_n\}$ be an enumeration of rationals in $(0,1)$. Does $\lim\limits_{n\to\infty} r_n$ exists? Justify! [3]

(b) We say that l is a limit point of the sequence $\{x_n\}$ if there exists a sub-sequence $\{x_{n_k}\}$ such that $x_{n_k} \to l$ as $k \to \infty$.
If $\{x_n\}$ is a bounded sequence with 1 being the only limit point show that the sequence $\{x_n\}$ converges. [4]

(c) Let $\{a_j\}$ and $\{b_j\}$ be two sequences of real numbers. Define $c_n = \sum\limits_{j=0}^{n} a_{n-j} b_j$.

Show that if $\sum\limits_{j=0}^{\infty} |a_j| < \infty$ and $\sum\limits_{j=0}^{\infty} |b_j| < \infty$ then the series $\sum\limits_{n=0}^{\infty} c_n$ converges. [4]

Let $a_j = \begin{cases} 2 & j = 0 \\ 2^j & j \geq 1 \end{cases}$ and $b_j = \begin{cases} -1 & j = 0 \\ 1 & j \geq 1 \end{cases}$. What can you say about the series $\sum\limits_{n=0}^{\infty} c_n$?
Justify your answer. [4]