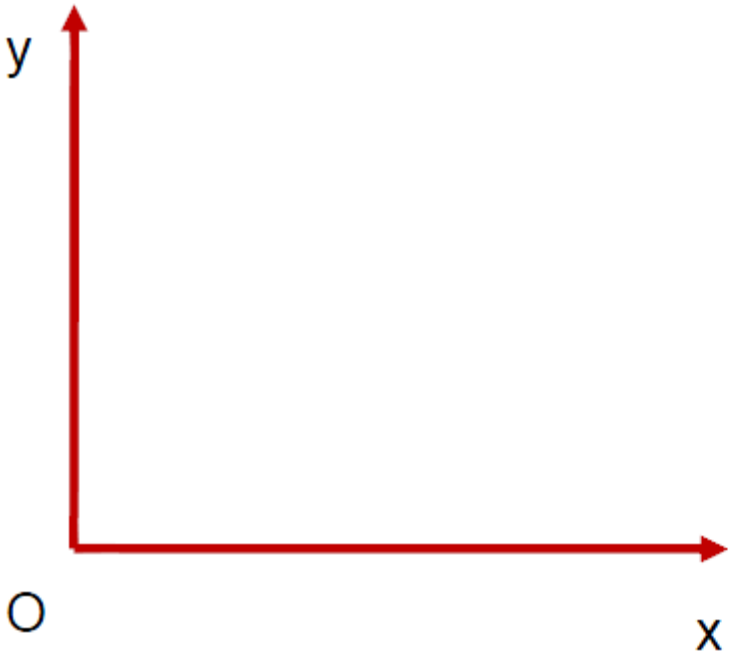
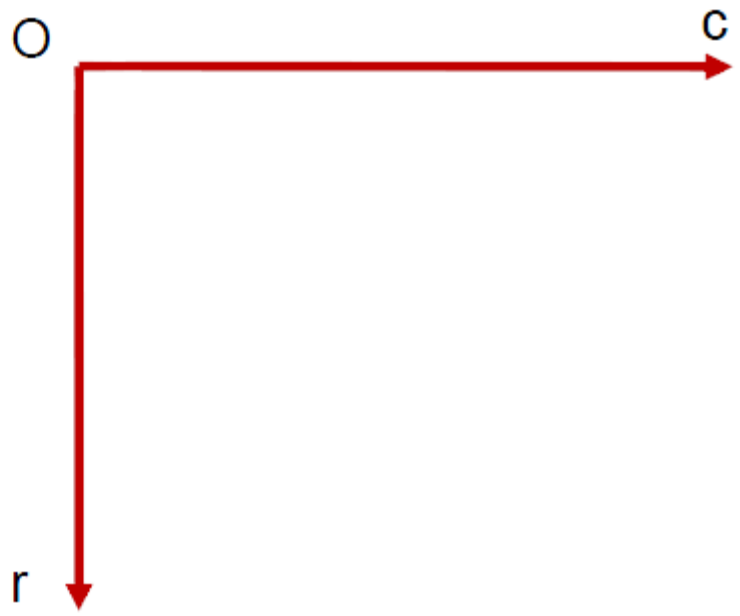
# 2D Line Drawing

# Computer coordinate

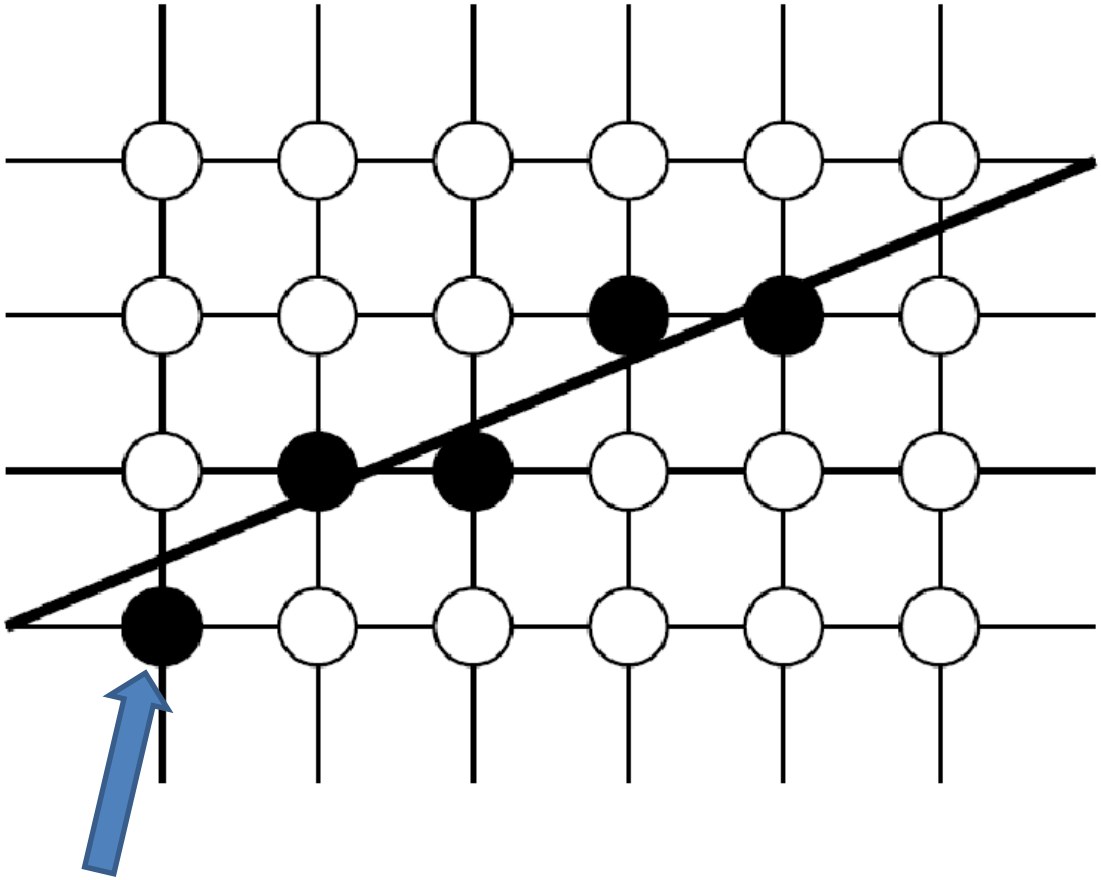



Right-hand rule




Left-hand rule

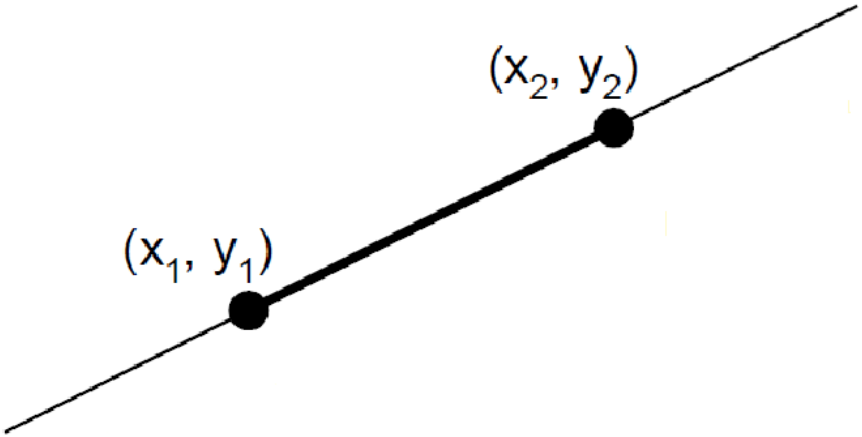
# Continuous and Discrete Line



Multi small pixels => Scan-converted line

# Line segment

- Starting point p(x1,y1) and end point q(x2,y2)




- Line equation

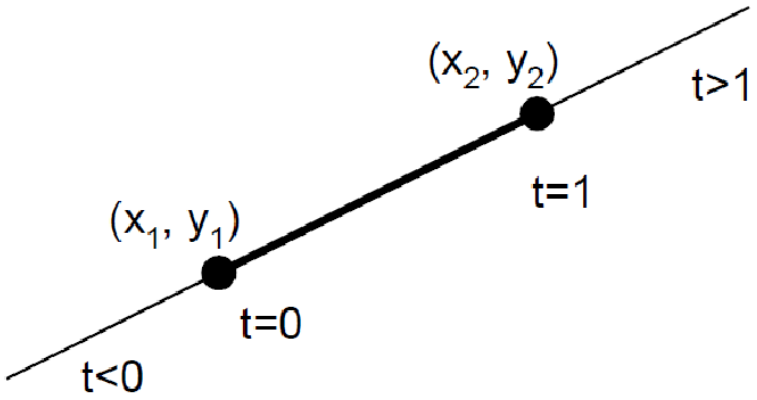
$$y = mx + b$$

$$m = \frac{Dy}{Dx}, Dy = y_2 - y_1, Dx = x_2 - x_1; b = y_1 - mx_1$$

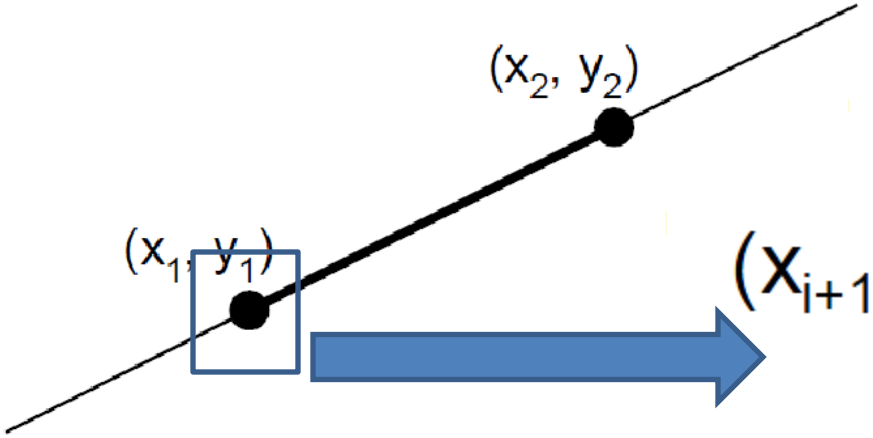
# Line segment

- Parametric equation

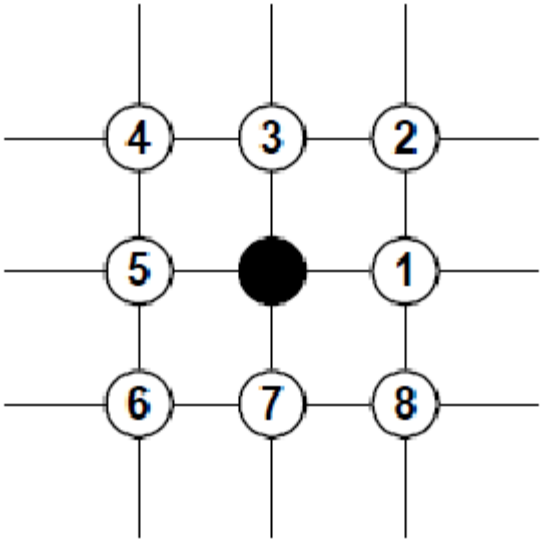
$$\begin{cases} x = (1-t)x_1 + tx_2 \\ y = (1-t)y_1 + ty_2 \end{cases}$$



# Concept




$$(x_{i+1}, y_{i+1}) = (x_i \pm 1, y_i \pm 1)$$



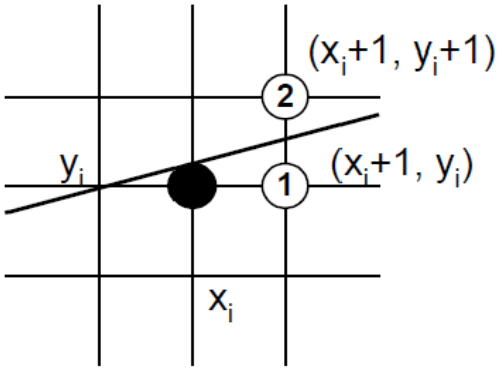
# Concept

- Let *m* denote the slope of segment line, and it satisfies

$$0 < m < 1.$$

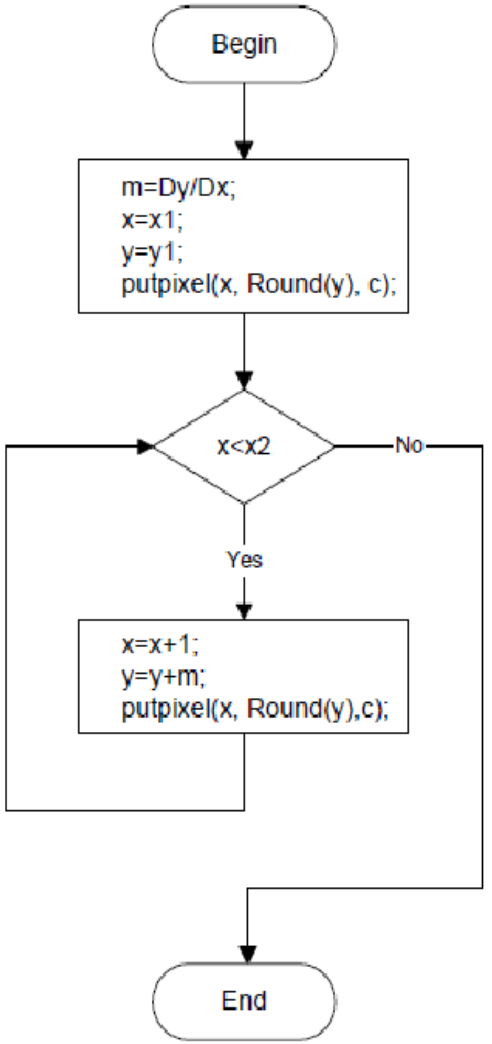
- If given point $(x_i, y_i)$, the next point can be defined by

$$\begin{cases} x_{i+1} = x_i + 1 \\ y_{i+1} \in \{y_i, y_i + 1\} \end{cases}$$

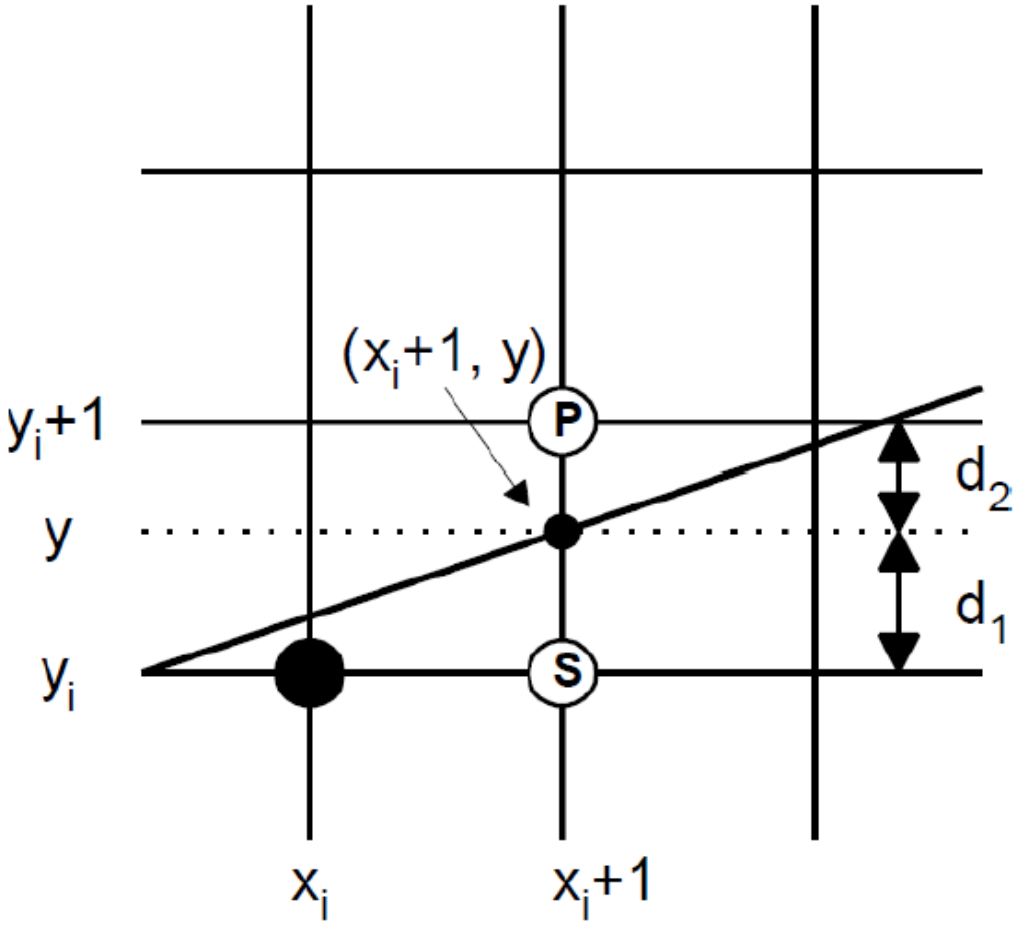


# Basic Incremental Algorithm (Digital Differential Analyzer)

$$
\begin{aligned}
y_{i+1} &= mx_{i+1} + B \\
&= m(x_i + \Delta x) + B \\
&= y_i + m\Delta x
\end{aligned}
$$



# Bresenham Algorithm



# Bresenham Algorithm

$d_1 = y - y_i$; $d_2 = (y_i + 1) - y$

$$y_{i+1} = \begin{cases} y_i, & \text{if } d_1 \le d_2 \\ y_i + 1, & \text{otherwises} \end{cases}$$

- Compare values of $d_1$ and $d_2$ by

$d_1-d_2=2(Dy/Dx)x_i+2(Dy/Dx)-2y_i+2b-1$

with $y=(Dy/Dx)(x_i+1)+b$

- Eliminate division by Dx

$p_i=Dx(d_1-d_2)=$**$2(Dy)x_i-2(Dx)y_i+c$** **(*)**

**with** $c = 2Dy + (2b - 1)Dx$

# Bresenham Algorithm

- Reduce calculation of $p_i(x_i,y_i)$ in (*) by

$$p_{i+1} - p_i = 2(Dy)(x_{i+1} - x_i) - 2(Dx)(y_{i+1} - y_i)$$

$$\Rightarrow p_{i+1} - p_i = 2(Dy) - 2(Dx)(y_{i+1} - y_i), \text{ with } x_{i+1} = x_i + 1$$

and define a constant increment for each step

- $p_i < 0, p_{i+1} = p_i + 2Dy$ by $y_{i+1} = y_i$
- $p_i \geq 0, p_{i+1} = p_i + 2Dy - 2Dx$ by $y_{i+1} = y_i + 1$

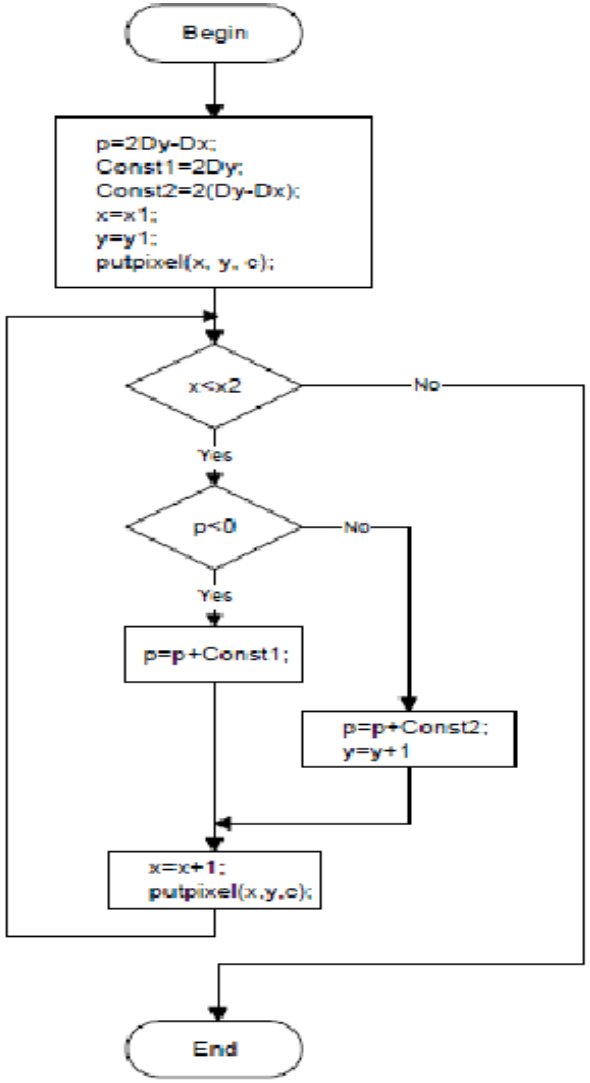
where the initial value of $p_i$ is defined by

$$p_0 = 2(Dy)x_0 - 2(Dx)y_0 + c$$

$$= 2(Dy)x_0 - 2(Dx)y_0 + 2Dy + (2b-1)Dx$$

**$p_0$= 2Dy – Dx** with $b = y_0 - (Dy / Dx)x_0$

# Bresenham Algorithm



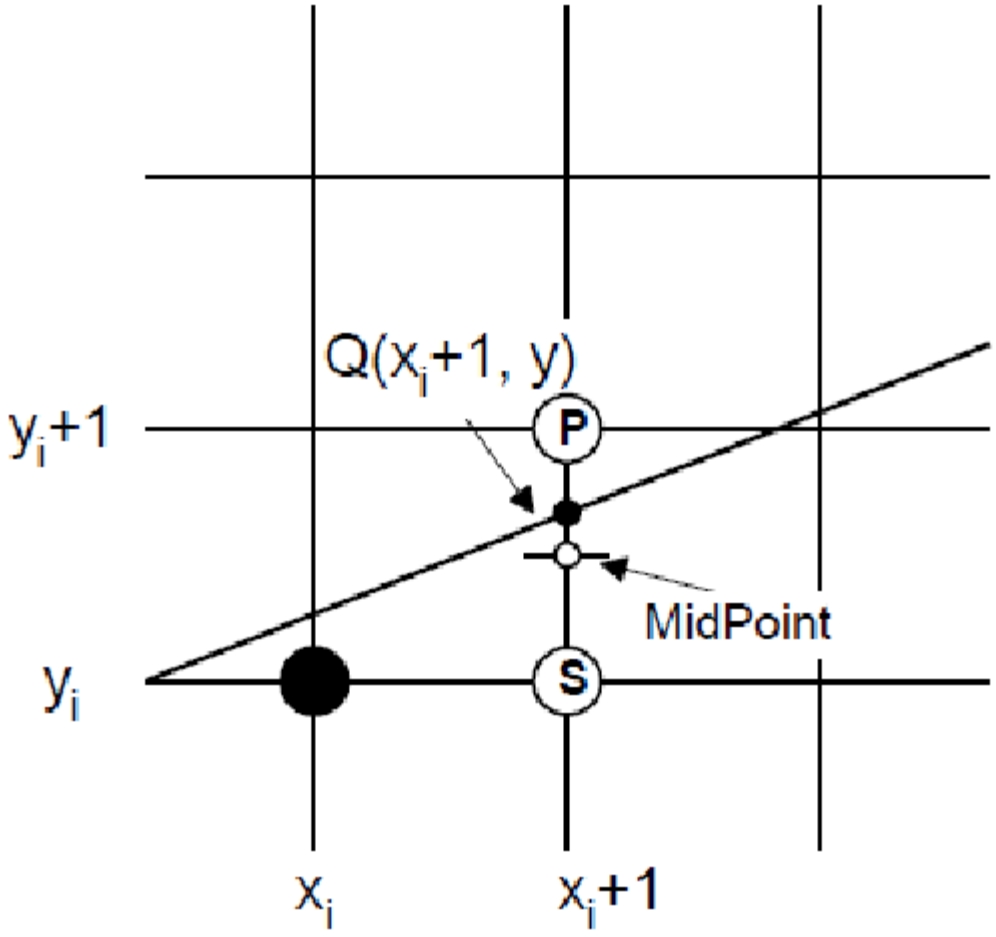
# Examples

Given two points A(12,20) and B(22,27)

| | | DDA (m=0.7) | | Bresenham (Dx=10,Dy=7,$p_0$ =4, $Const_1$=2Dy=14, $Const_2$= 2(Dy-Dx)=-6 ) | |
|---|---|---|---|---|---|
| i | $x_i$ | y | $y_i$ | $p_i$ | $y_i$ |
| 0 | 12 | 20 | 20 | 4 | 20 |
| 1 | 13 | 20.7 | 21 | -2 | 21 |
| 2 | 14 | 21.4 | 21 | 12 | 21 |
| 3 | 15 | 22.1 | 22 | 6 | 22 |
| 4 | 16 | 22.8 | 23 | 0 | 23 |
| 5 | 17 | 23.5 | 24 | -6 | 24 |
| 6 | 18 | 24.2 | 24 | 8 | 24 |
| 7 | 19 | 24.9 | 25 | 2 | 25 |
| 8 | 20 | 25.6 | 26 | -4 | 26 |
| 9 | 21 | 26.3 | 26 | 10 | 26 |
| 10 | 22 | 27 | 27 | 4 | 27 |

(Provided by Dang Nguyen Duc Tien-Vu Quoc Hoang-Le Phong)

# Midpoint (Pitteway and Van Aken)




- Compare position of Q and Midpoint, where Q is the intersection point between drawing line and vertical line defined by two points P and S, and the Midpoint is the middle point of two points P and S.

# Midpoint

- Drawing line equation: $Ax + By + C = 0$

$A = y_2 - y_1,\ B = -(x_2 - x_1),\ C = x_2y_1 - x_1y_2$

- Let F(x,y) be an implicit function

$$F(x, y) = Ax + By + C, \text{ where } A > 0$$

$$F(x, y) = \begin{cases} < 0, (x, y) \text{ positions above the line} \\ = 0, \ (\text{x,y}) \text{ positions on the line} \\ \geq 0, \ (\text{x,y}) \text{ positions below the line} \end{cases}$$

- Eliminate division by 2 in *F*(MidPoint) by

$$p_i = 2F(MidPoint) = 2F(x_i + 1, y_i + \tfrac{1}{2}) \ (**)$$

$$p_i < 0 \Rightarrow y_{i+1} = y_i$$
$$p_i \geq 0 \Rightarrow y_{i+1} = y_i + 1$$

# Midpoint

- Reduce calculation of $p_i(x_i,y_i)$ in **(**)** by

$$p_{i+1} - p_i = 2F\left(x_{i+1}+1, y_{i+1}+\frac{1}{2}\right) - 2F\left(x_i+1, y_i+\frac{1}{2}\right)$$

$$= \ldots = 2A + 2B(y_{i+1} - y_i)$$

$$\Rightarrow p_{i+1} = p_i + 2Dy - 2Dx(y_{i+1} - y_i)$$

and define a constant increment for each step

- $p_i < 0, p_{i+1} = p_i + 2Dy$ by $y_{i+1} = y_i$
- $p_i \geq 0, p_{i+1} = p_i + 2Dy - 2Dx$ by $y_{i+1} = y_i + 1$

where the initial value of $p_i$ is defined by

$$p_0 = 2F\left(x_0+1, y_0+\frac{1}{2}\right) = 2A + B = 2Dy - Dx$$

# Midpoint

Bresenham Algorithm = ? Midpoint Algorithm

What is the difference between Bresenham and Midpoint algorithms ?  Or which one is more general ?

# References

[1] H. Kiem, D.A. Duc, L.D. Duy, V.H. Quan, Cơ Sở Đồ Họa Máy Tính, NXB. Giáo Dục, 2005.

[2] D.N.D.Tien, V.Q. Hoang, L. Phong, CG-Course Slide, HCM-University of Science.