Slides and example codes are available:
https://github.com/mint-lab/cv_tutorial

Image Geometry

Sunglok Choi, Assistant Professor, Ph.D.
Computer Science and Engineering Department, SEOULTECH
sunglok@seoultech.ac.kr | https://mint-lab.github.io/

Review) Absolute Camera Pose Estimation

- Example) **Pose estimation (book) + camera calibration – initially given** K `[pose_estimation_book3.py]`

(due to wrong initial K)

- Load the cover image
- Extract ORB features
- Build **3D object points** $\mathbf{X}_1, \mathbf{X}_2, \ldots$

Pass tentative matches

Pass correct matches

- Grab an input image
- Extract **ORB features** $\mathbf{x}_1, \mathbf{x}_2, \ldots$
- Match them to the object features

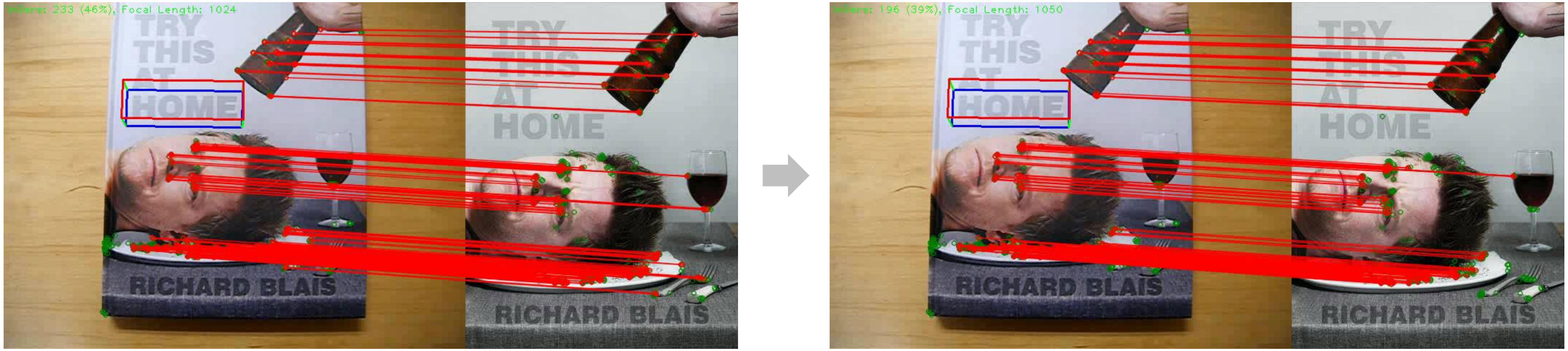
[Outlier Rejection]
- Remove wrong matches using `cv::findHomography()`

(free from K)

[Camera Pose Estimation]
- Estimate camera parameters using `cv::calibrateCamera()`

[Camera Pose Update]
- Intrinsic: K (initially known)
- Extrinsic: R, $\mathbf{t}$

Draw the box using `cv::projectPoints()`



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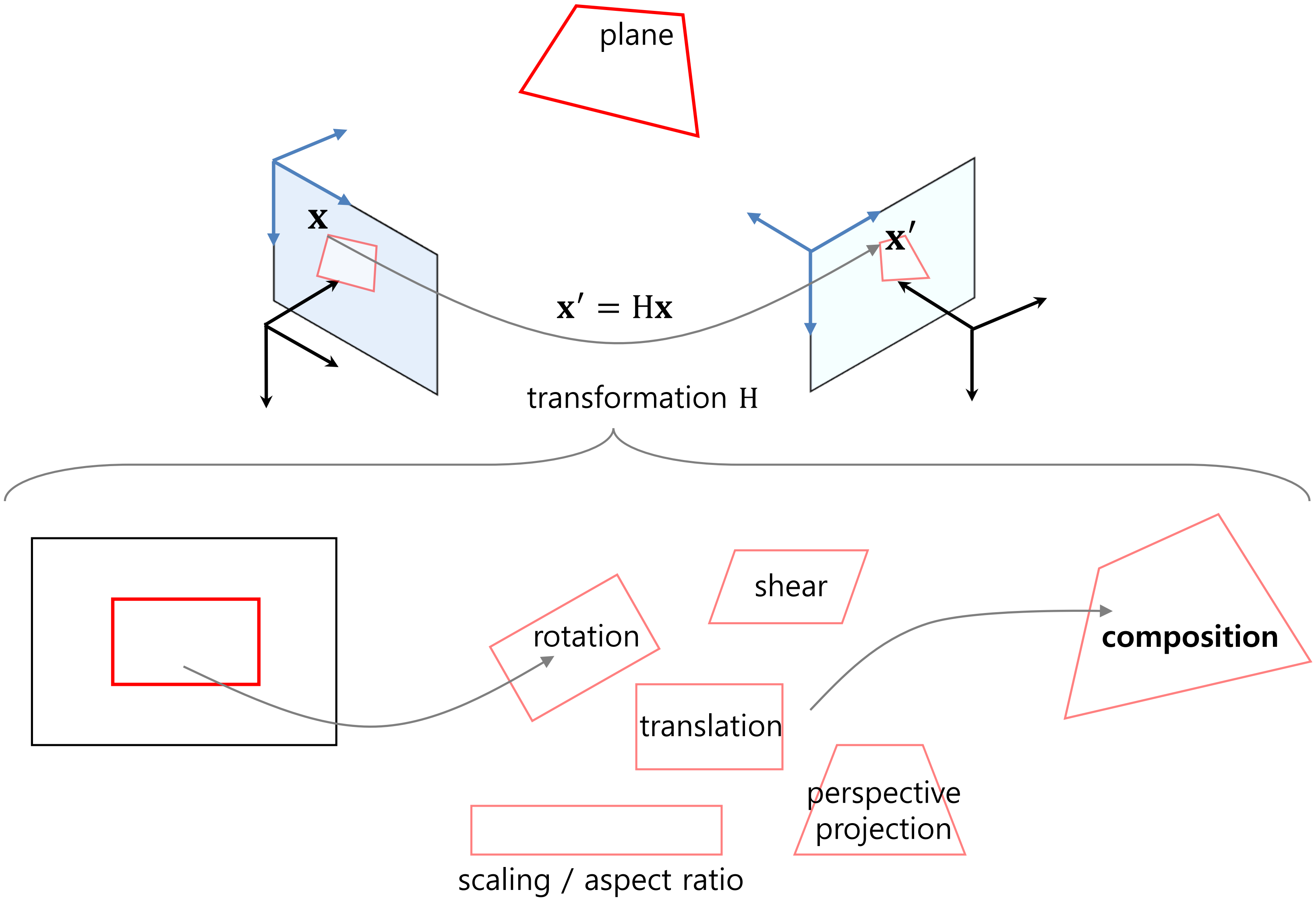
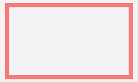
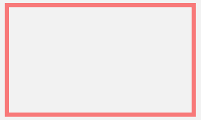


Planar Homography

	**Euclidean Transformation** (a.k.a. Rigid Transform) 	**Similarity Transformation** 	**Affine Transformation** 	**Projective Transformation** (a.k.a. Planar Homography) 
Matrix Forms H	$\begin{bmatrix} \cos\theta & -\sin\theta & t_x \\ \sin\theta & \cos\theta & t_y \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} s\cos\theta & -s\sin\theta & t_x \\ s\sin\theta & s\cos\theta & t_y \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} a_{11} & a_{12} & t_x \\ a_{21} & a_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} a_{11} & a_{12} & t_x \\ a_{21} & a_{22} & t_y \\ v_1 & v_2 & 1 \end{bmatrix}$
DOF	3	4	6	8
Transformations - rotation - translation - scaling - aspect ratio - shear - perspective projection	 O O X X X X	 O O O X X X	 O O O O O X	 O O O O O O
Invariants - length - angle - ratio of lengths - parallelism - incidence - cross ratio	 O O O O O O	 X O O O O O	 X X X O O O	 X X X X O O

OpenCV APIs			`cv.getAffineTransform()`
			`cv.estimateRigidTransform()`
			-
			`cv.warpAffine()`
				`cv.getPerspectiveTransform()`
				-
				`cv.findHomography()`
				`cv.warpPerspective()`

Note) Similarly **3D transformations** (3D-3D geometry) are represented as **4x4 matrices**.

Note) Affine Transformation

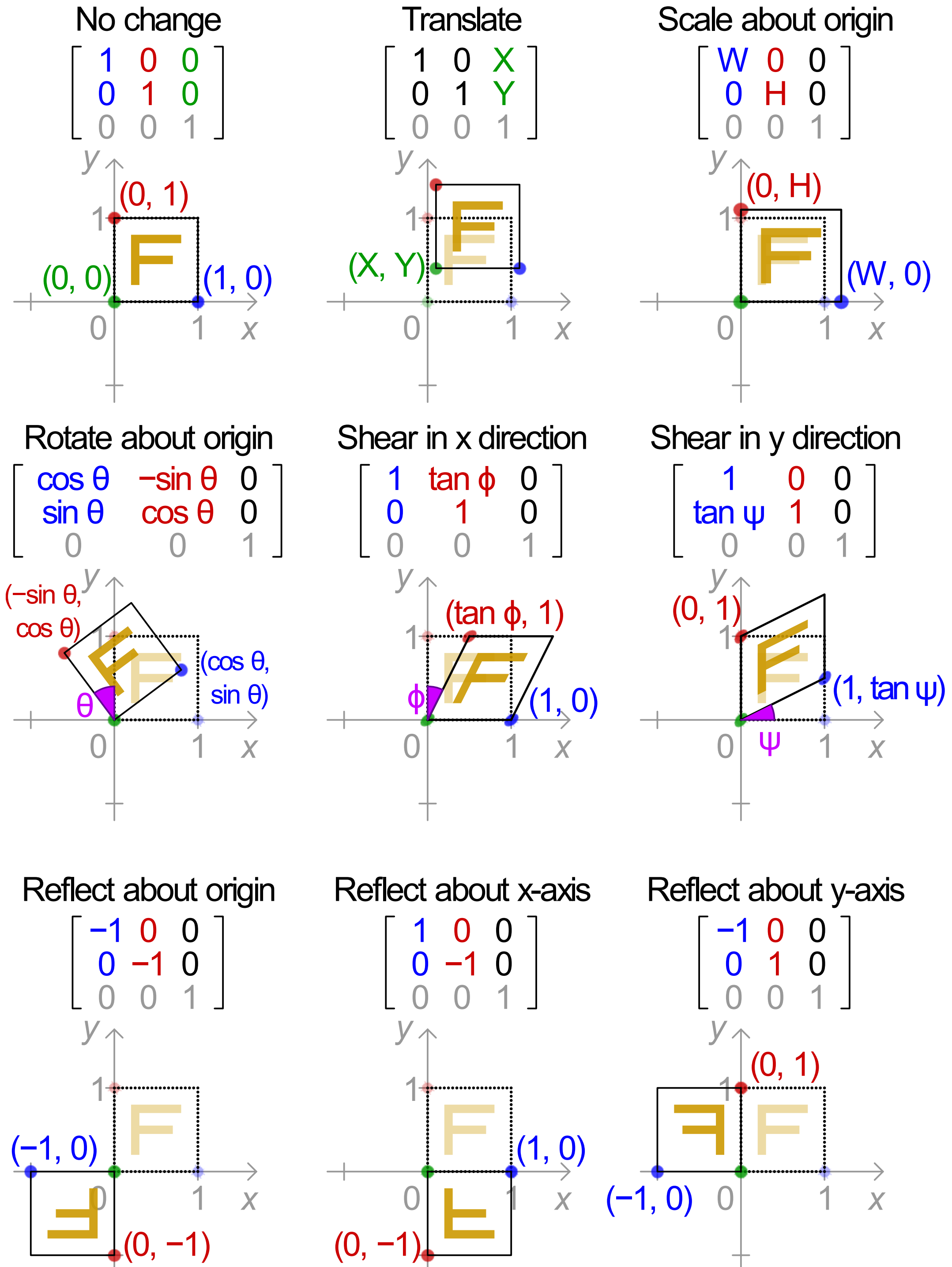
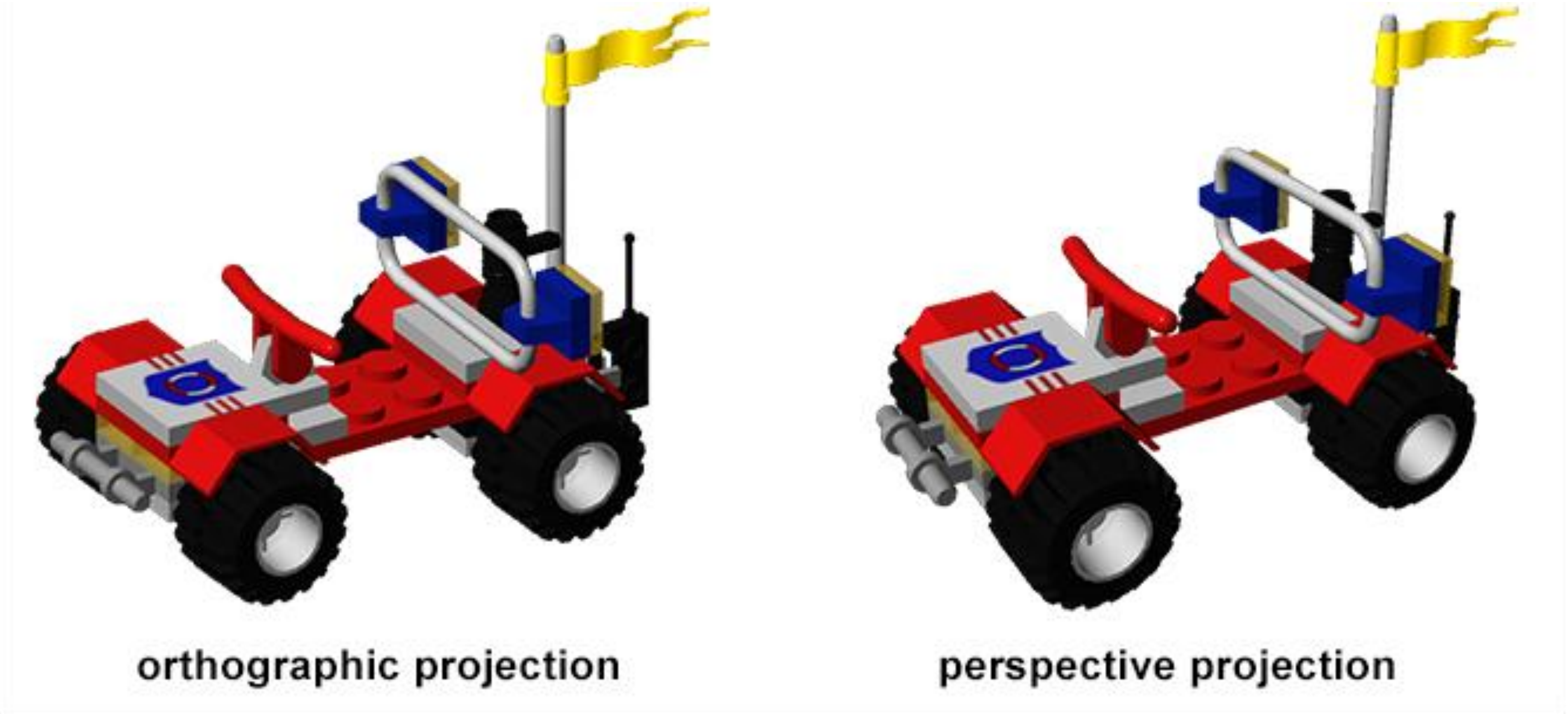
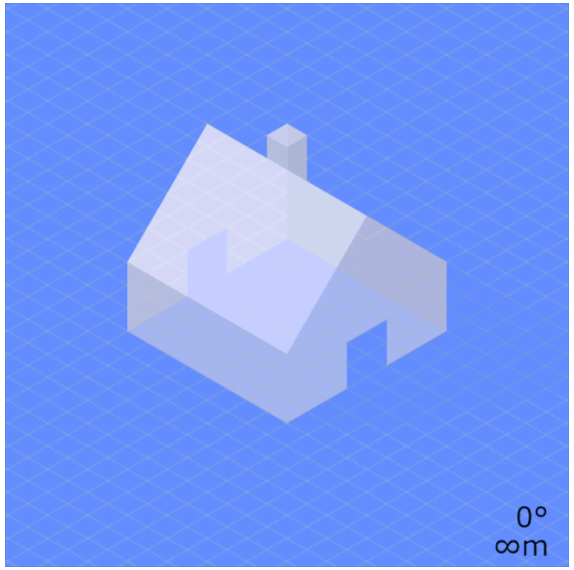


Image: Wikipedia

Note) Affine Transformation



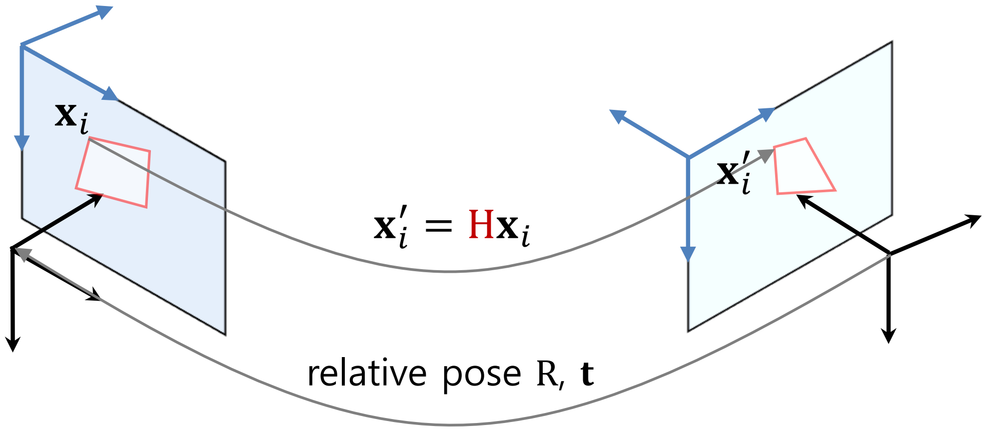


- **Q) What is difference between two images?**
  - Parallel projection vs. Perspective projection
    - Affine camera (a.k.a. weak perspective camera; $f = \infty$)
    - Less natural but (sometimes) useful in technical visualization

Image: Scatchpixel, Wikipedia

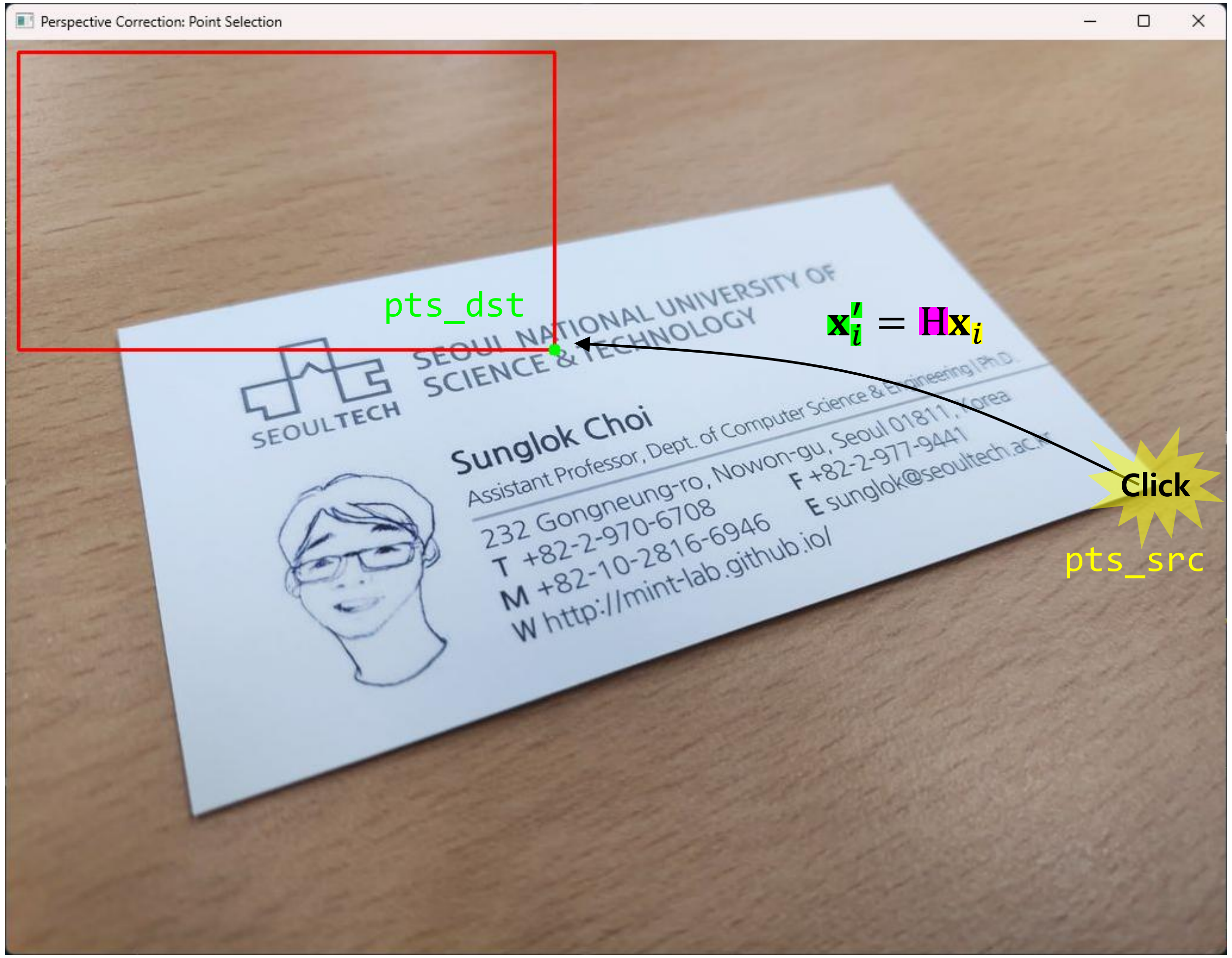
Planar Homography

- **Planar homography estimation**
  - Unknown: Planar homography H (8 DOF)
  - Given: Point correspondence $(\mathbf{x}_1, \mathbf{x}'_1)$, …, $(\mathbf{x}_n, \mathbf{x}'_n)$
  - Constraints: n x projective transformation $\mathbf{x}'_i = \mathrm{H}\mathbf{x}_i$
  - Solutions ($n \geq 4$) → 4-point algorithm
    - OpenCV: `cv.getPerspectiveTransform()` and `cv.findHomography()`
    - Note) More simplified transformations need less number of minimal correspondence.
      - Affine ($n \geq 3$), similarity ($n \geq 2$), Euclidean ($n \geq 2$)
  - Note) Planar homography can be decomposed as relative camera pose.
    - OpenCV: `cv.decomposeHomographyMat()`
    - The decomposition needs to know camera matrices.



Planar Homography

- Example) **Perspective distortion correction** `[perspective_correction.py]`



Planar Homography

- Example) **Perspective distortion correction** [perspective_correction.py]

```python
def mouse_event_handler(event, x, y, flags, param):
    if event == cv.EVENT_LBUTTONDOWN:
        param.append((x, y))

if __name__ == '__main__':
    img_file = '../data/sunglok_card.jpg'
    card_size = (450, 250)
    offset = 10

    # Prepare the rectified points
    pts_dst = np.array([[0, 0], [card_size[0], 0], [0, card_size[1]], [card_size[0], card_size[1]]])

    # Load an image
    img = cv.imread(img_file)

    # Get the matched points from mouse clicks
    pts_src = []
    wnd_name = 'Perspective Correction: Point Selection'
    cv.namedWindow(wnd_name)
    cv.setMouseCallback(wnd_name, mouse_event_handler, pts_src)
    while len(pts_src) < 4:
        img_display = img.copy()
        cv.rectangle(img_display, (offset, offset), (offset + card_size[0], offset + card_size[1]), (0, 0, 255), 2)
        idx = min(len(pts_src), len(pts_dst))
        cv.circle(img_display, offset + pts_dst[idx], 5, (0, 255, 0), -1)
        cv.imshow(wnd_name, img_display)
```

Planar Homography

- Example) **Perspective distortion correction** [perspective_correction.py]

```
if __name__ == '__main__':
    img_file = '../data/sunglok_card.jpg'
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    # Prepare the rectified points
    pts_dst = np.array([[0, 0], [card_size[0], 0], [0, card_size[1]], [card_size[0], card_size[1]]])

    # Load an image
    img = cv.imread(img_file)

    # Get the matched points from mouse clicks
    pts_src = []
    ...

    if len(pts_src) == 4:
        # Calculate planar homography and rectify perspective distortion
        H, _ = cv.findHomography(np.array(pts_src), pts_dst)
        img_rectify = cv.warpPerspective(img, H, card_size)

        # Show the rectified image
        cv.imshow('Perspective Correction: Rectified Image', img_rectify)
        cv.waitKey(0)

    cv.destroyAllWindows()
```
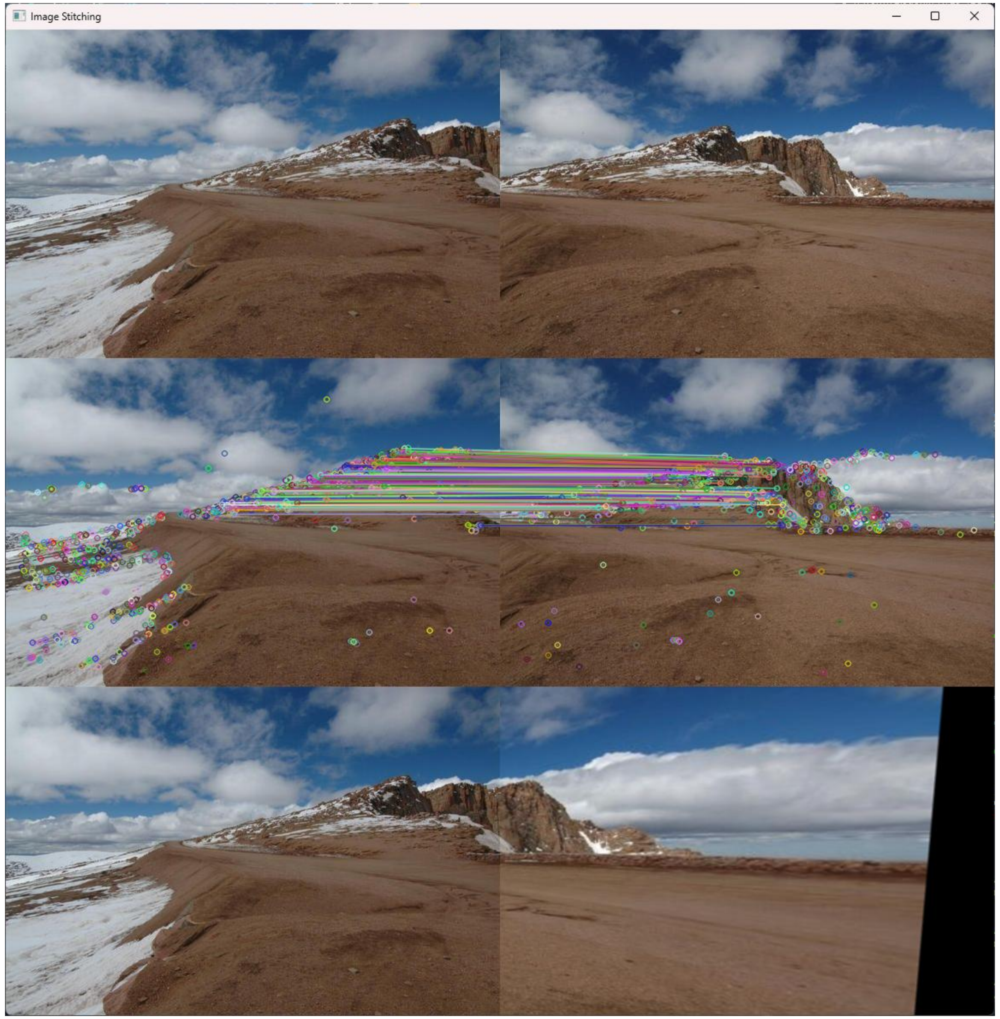

Planar Homography

- Example) **Planar image stitching** [image_stitching.py]

```
# Load two images
img1 = cv.imread('../data/hill01.jpg')
img2 = cv.imread('../data/hill02.jpg')

# Retrieve matching points
brisk = cv.BRISK_create()
keypoints1, descriptors1 = brisk.detectAndCompute(img1, None)
keypoints2, descriptors2 = brisk.detectAndCompute(img2, None)

fmatcher = cv.DescriptorMatcher_create('BruteForce-Hamming')
match = fmatcher.match(descriptors1, descriptors2)

# Calculate planar homography and merge them
pts1, pts2 = [], []
for i in range(len(match)):
    pts1.append(keypoints1[match[i].queryIdx].pt)
    pts2.append(keypoints2[match[i].trainIdx].pt)
pts1 = np.array(pts1, dtype=np.float32)
pts2 = np.array(pts2, dtype=np.float32)

H, inlier_mask = cv.findHomography(pts2, pts1, cv.RANSAC)
img_merged = cv.warpPerspective(img2, H, (img1.shape[1]*2, img1.shape[0]))
img_merged[:,:img1.shape[1]] = img1 # Copy

# Show the merged image
img_matched = cv.drawMatches(img1, keypoints1, img2, keypoints2, match, None, None, None,
```



Planar Homography

- Example) **2D video stabilization** [video_stabilization.py]

```
# Open a video and get the reference image and feature points
video = cv.VideoCapture('../data/traffic.avi')

_, gray_ref = video.read()
if gray_ref.ndim >= 3:
    gray_ref = cv.cvtColor(gray_ref, cv.COLOR_BGR2GRAY)
pts_ref = cv.goodFeaturesToTrack(gray_ref, 2000, 0.01, 10)

# Run and show video stabilization
while True:
    # Read an image from `video`
    valid, img = video.read()
    if not valid:
        break
    if img.ndim >= 3:
        gray = cv.cvtColor(img, cv.COLOR_BGR2GRAY)
    else:
        gray = img.copy()

    # Extract optical flow and calculate planar homography
    pts, status, err = cv.calcOpticalFlowPyrLK(gray_ref, gray, pts_ref, None)
    H, inlier_mask = cv.findHomography(pts, pts_ref, cv.RANSAC)

    # Synthesize a stabilized image
    warp = cv.warpPerspective(img, H, (img.shape[1], img.shape[0]))
```

A shaking CCTV video

Planar Homography

- Assumption) **A <u>plane</u> is observed by two views.**
  - Perspective distortion correction: A complete plane


Planar Homography

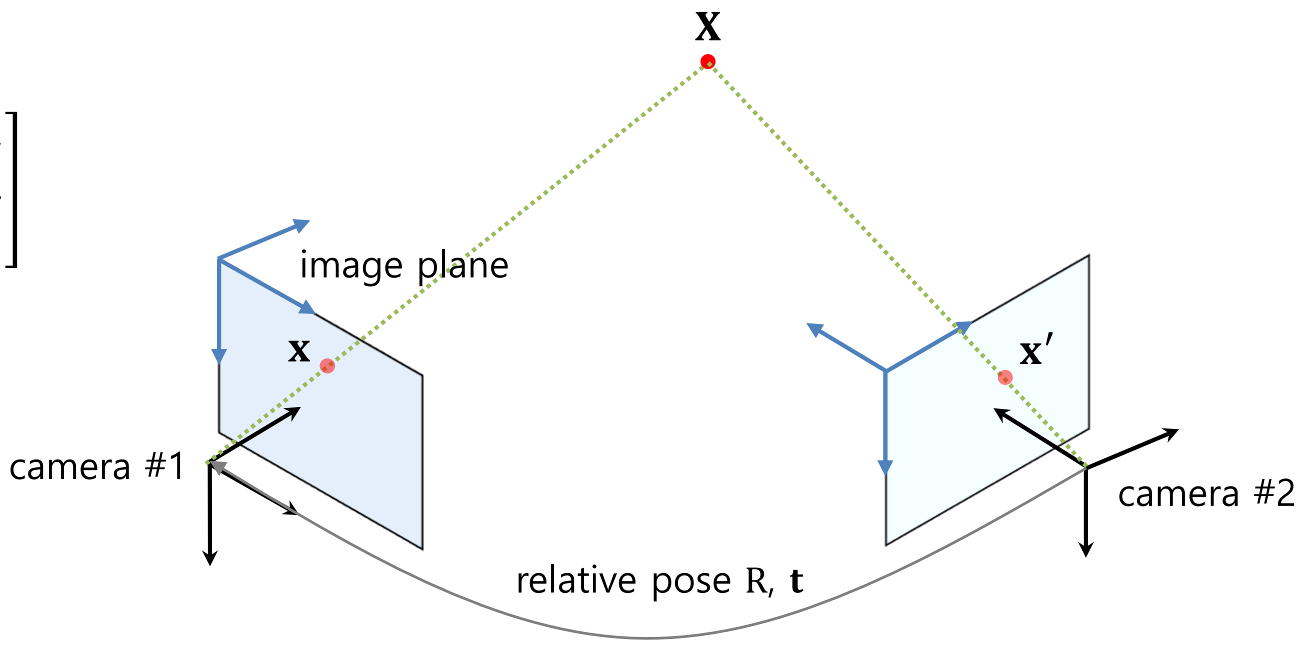
- Assumption) **A <u>plane</u> is observed by two views.**
  - Perspective distortion correction: A complete plane
  - Planar image stitching: An approximated plane (← distance ≫ depth variation)
  - 2D video stabilization: An approximated plane (← small motion)

Triangulation

- **Triangulation** (point localization)
  - Unknown: Position of a 3D point $\mathbf{X}$ (3 DOF)
  - Given: Point correspondence $(\mathbf{x}, \mathbf{x}')$, camera matrices $(\mathrm{K}, \mathrm{K}')$, and relative pose $(\mathrm{R}, \mathbf{t})$
  - Constraints: $\mathbf{x} = \mathrm{K}\,[\,\mathrm{I} \mid \mathbf{0}\,]\,\mathbf{X} = \mathrm{P}\,\mathbf{X}$, $\mathbf{x}' = \mathrm{K}'\,[\,\mathrm{R} \mid \mathbf{t}\,]\,\mathbf{X} = \mathrm{P}'\,\mathbf{X}$
  - Solution
    - OpenCV `cv.triangulatePoints()`
    - Special case) Stereo cameras

$$\mathrm{R} = \mathrm{I}_{3\times 3},\ \mathbf{t} = \begin{bmatrix} -b \\ 0 \\ 0 \end{bmatrix},\ \text{and}\ \mathrm{K} = \mathrm{K}' = \begin{bmatrix} f & 0 & c_x \\ 0 & f & c_y \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore Z = \frac{f}{x - x'} b,\ X = \frac{x - c_x}{f} Z,\ \text{and}\ Y = \frac{y - c_y}{f} Z$$



Triangulation

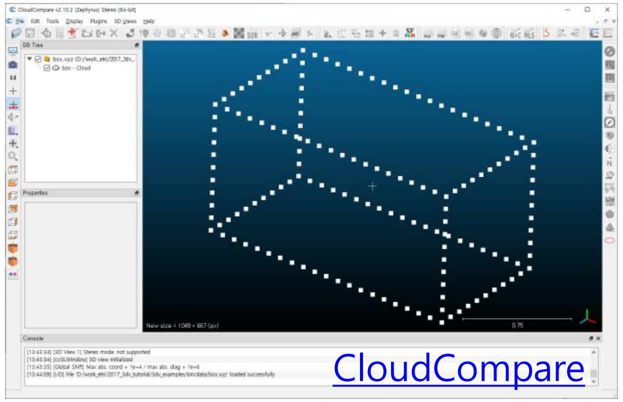
- Example) Triangulation

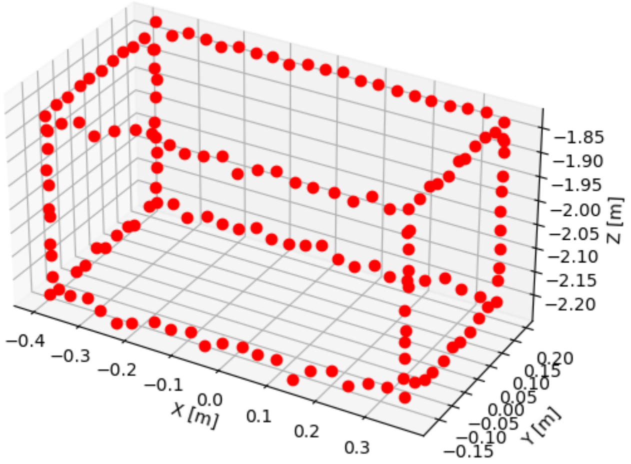
```python
f, cx, cy = 1000., 320., 240.
pts0 = np.loadtxt('../data/image_formation0.xyz')[:,:2]
pts1 = np.loadtxt('../data/image_formation1.xyz')[:,:2]
output_file = '../data/triangulation.xyz'

# Estimate relative pose of two view
F, _ = cv.findFundamentalMat(pts0, pts1, cv.FM_8POINT)
K = np.array([[f, 0, cx], [0, f, cy], [0, 0, 1]])
E = K.T @ F @ K
_, R, t, _ = cv.recoverPose(E, pts0, pts1)

# Reconstruct 3D points (triangulation)
P0 = K @ np.eye(3, 4, dtype=np.float32)
Rt = np.hstack((R, t))
P1 = K @ Rt
X = cv.triangulatePoints(P0, P1, pts0.T, pts1.T)
X /= X[3]
X = X.T

# Write the reconstructed 3D points
np.savetxt(output_file, X)
```

$$\mathbf{x} = \mathbf{K}\,[\,\mathbf{I}\,|\,\mathbf{0}\,]\,\mathbf{X} = \mathbf{P}\,\mathbf{X}$$

$$\mathbf{x}' = \mathbf{K}'\,[\,\mathbf{R}\,|\,\mathbf{t}\,]\,\mathbf{X} = \mathbf{P}'\,\mathbf{X}$$

A point cloud: data/box.xyz





output_file: data/triangulation.xyz

Summary

- **Planar Homography**: $\mathbf{x}_i' = \mathrm{H}\mathbf{x}_i$
  - Example) Perspective distortion correction
  - Example) Planar image stitching
  - Example) 2D video stabilization
- **Triangulation**: Finding $\mathbf{X}$ (3 DOF)
  - Example) Triangulation