Introduction to 3D Vision

: A Tutorial for Everyone

Sunglok Choi, Senior Researcher

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Electronics and Telecommunication Research Institute (ETRI)

An Invitation

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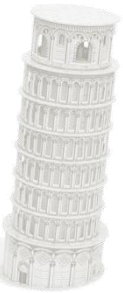
What is 3D Vision?

Computer Vision

What is it?

- Label (e.g. Tower of Pisa)
- Shape (e.g. )

Where is it?

- Place (e.g. Piazza del Duomo, Pisa, Italy)
- Location (e.g. )

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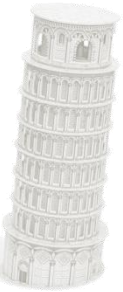
- Location (e.g. )

(84, 10, 18) [m]

Recognition Problems **v.s.** **Reconstruction Problems**

What is 3D Vision?

Visual Geometry (Multiple View Geometry)

Geometric Vision

Computer Vision

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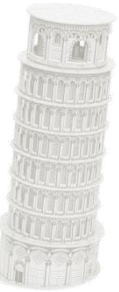
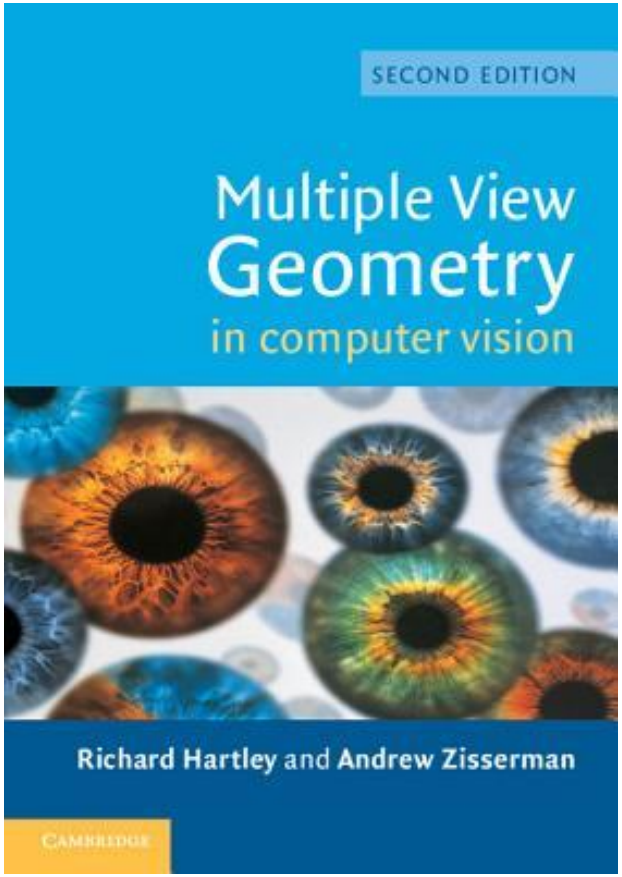
- Location (e.g. )

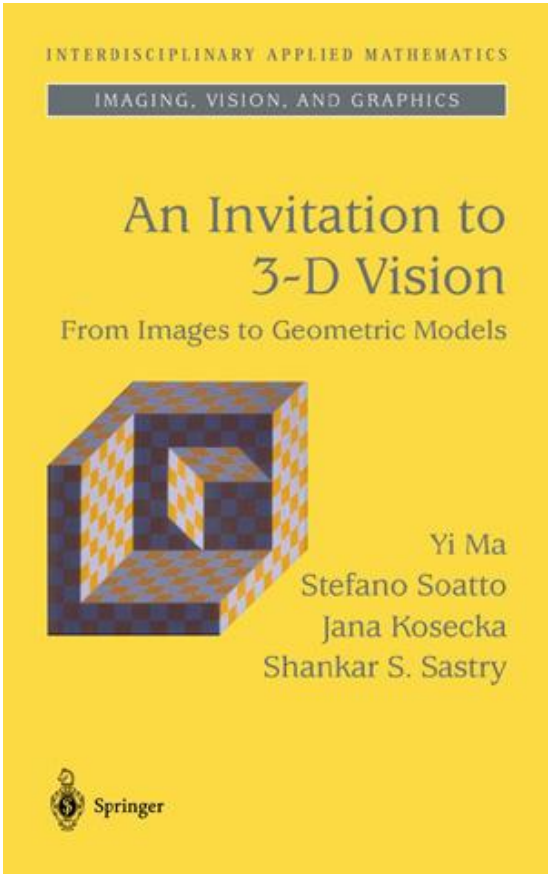
(84, 10, 18) [m]

Recognition Problems **v.s.** **Reconstruction Problems**

What is 3D Vision?

- Reference Books





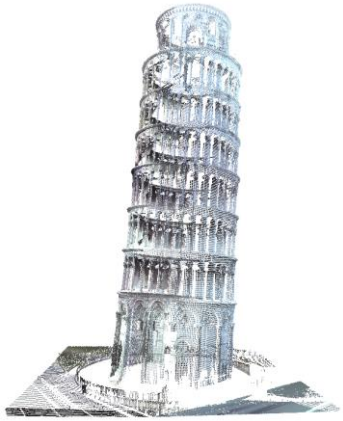
What is 3D Vision?

- OpenCV (Open Source Computer Vision)
  - `calib3d` Module: Camera Calibration and 3D Reconstruction

- OpenCV API Reference
  - Introduction
  - core. The Core Functionality
  - imgproc. Image Processing
  - imgcodecs. Image file reading and writing
  - videoio. Media I/O
  - highgui. High-level GUI and Media I/O
  - video. Video Analysis
  - calib3d. Camera Calibration and 3D Reconstruction
  - features2d. 2D Features Framework
  - objdetect. Object Detection
  - ml. Machine Learning
  - flann. Clustering and Search in Multi-Dimensional Spaces
  - photo. Computational Photography
  - stitching. Images stitching
  - cuda. CUDA-accelerated Computer Vision
  - cudaarithm. CUDA-accelerated Operations on Matrices
  - cudabgsegm. CUDA-accelerated Background Segmentation
  - cudacodec. CUDA-accelerated Video Encoding/Decoding
  - cudafeatures2d. CUDA-accelerated Feature Detection and Description
  - cudafilters. CUDA-accelerated Image Filtering
  - cudaimgproc. CUDA-accelerated Image Processing
  - cudaoptflow. CUDA-accelerated Optical Flow
  - cudastereo. CUDA-accelerated Stereo Correspondence
  - cudawarping. CUDA-accelerated Image Warping
  - shape. Shape Distance and Matching
  - superres. Super Resolution
  - videostab. Video Stabilization
  - viz. 3D Visualizer

What is 3D Vision?

image



point cloud, depth/range image, polygon mesh, …

3D Vision **v.s.** **3D Data Processing**

Perspective Camera

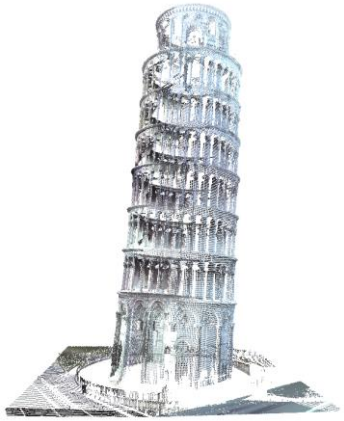
Omni-directional Camera

RGB-D Camera
(Stereo, Structured Light, ToF, Light Field)

Range Sensor
(LiDAR, RADAR)

What is 3D Vision?

image



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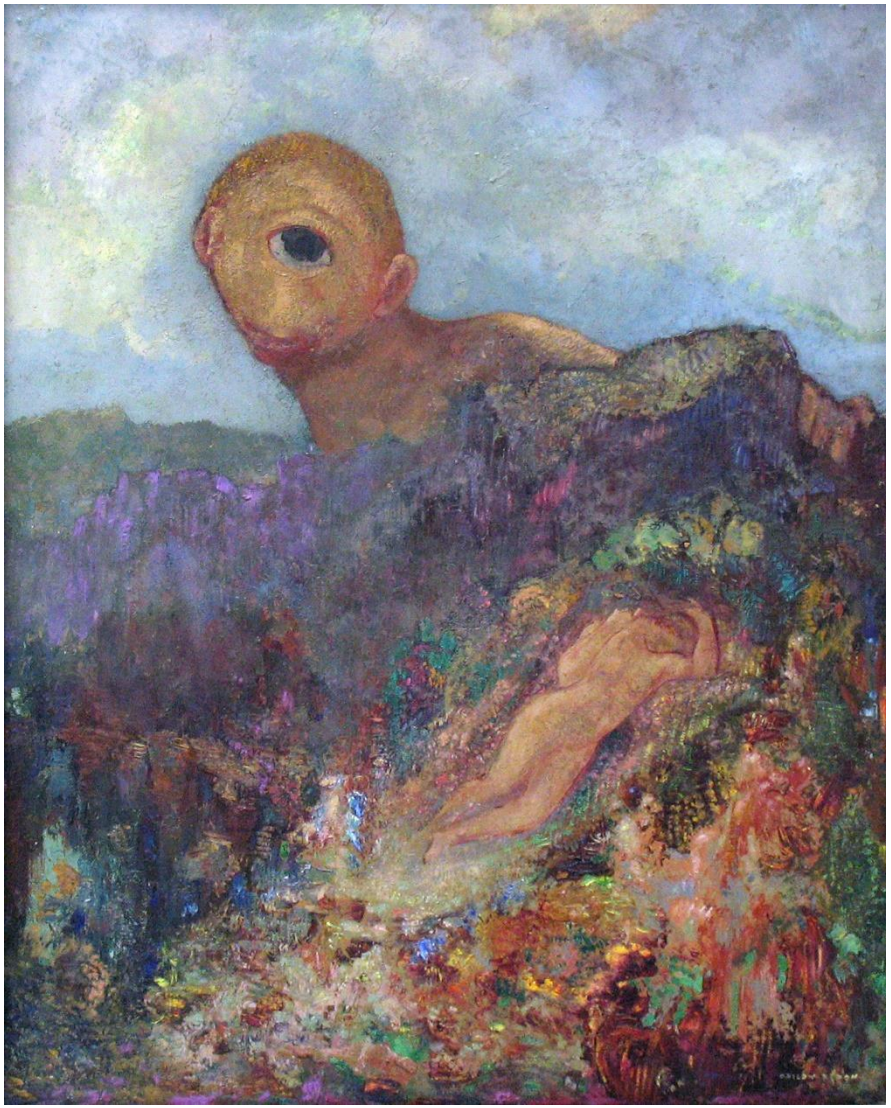
(LiDAR, RADAR)

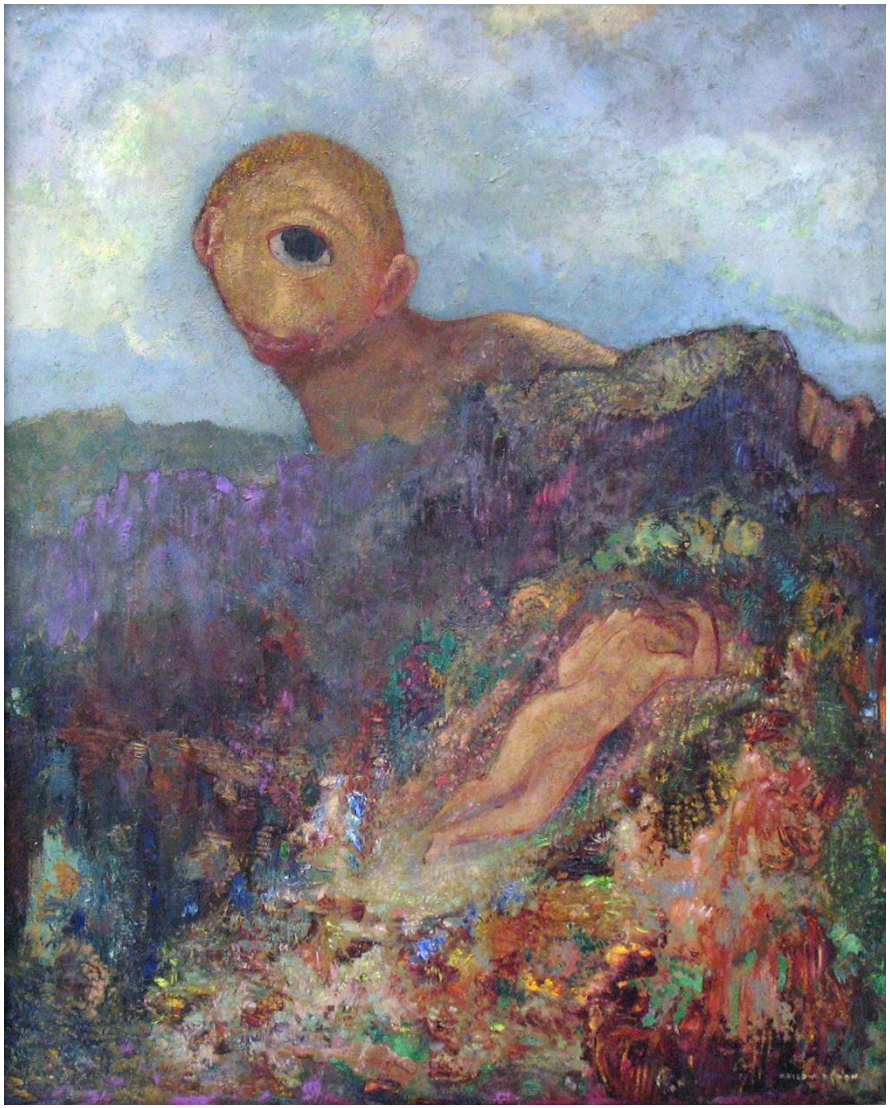
Table of Contents

Single-view Geometry



The Cyclops, gouache and oil by Odilon Redon

Single-view Geometry



Camera Projection Model

General 2D-3D Geometry

The Cyclops, gouache and oil by Odilon Redon

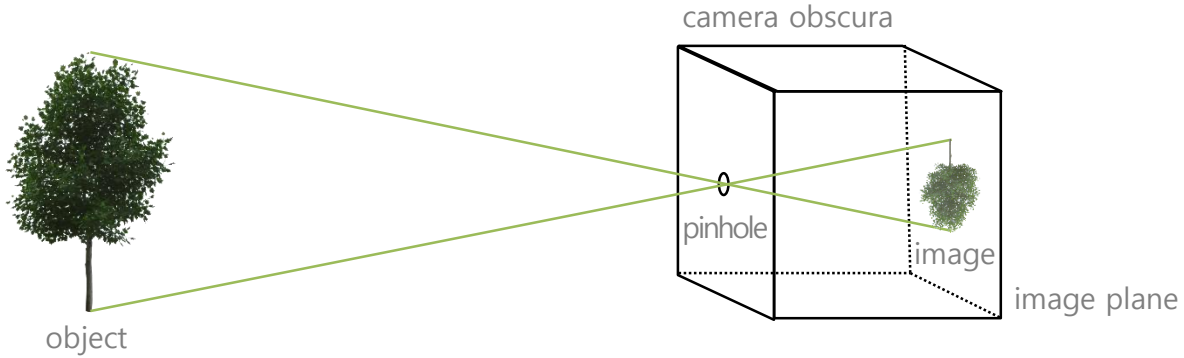
Camera Projection Model

- **The Pinhole Camera Model**

A large-scale camera obscura at San Francisco, California

A modern-day camera obscura

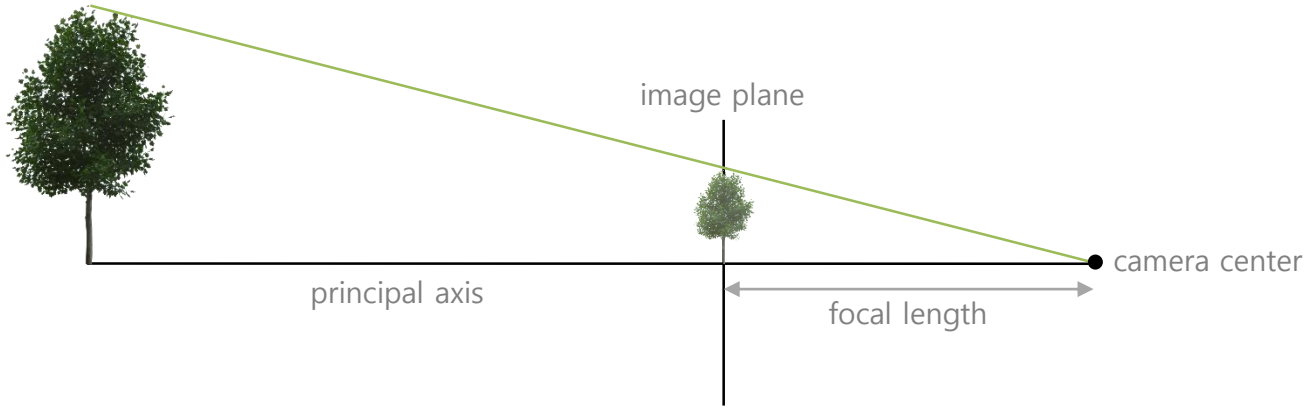
An Image in camera obscura at Portslade, England

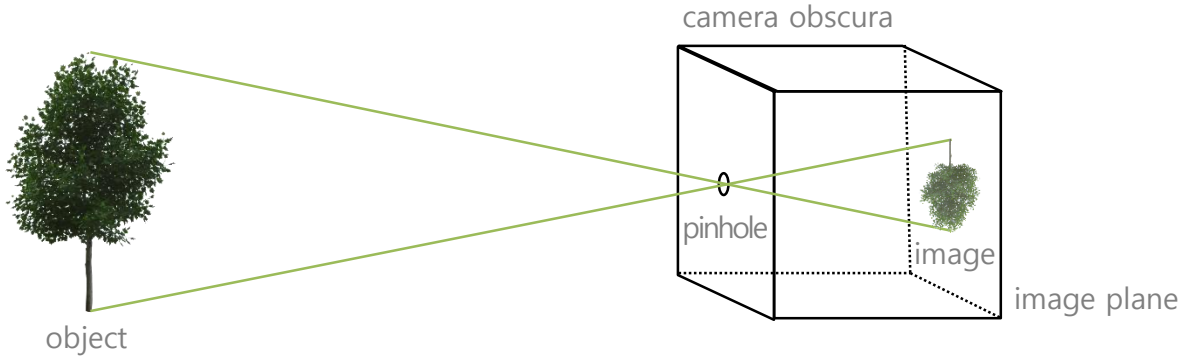


Images are from Wikipedia (https://en.wikipedia.org/wiki/Camera_obscura).

Camera Projection Model

- **The Pinhole Camera Model**





Camera Projection Model

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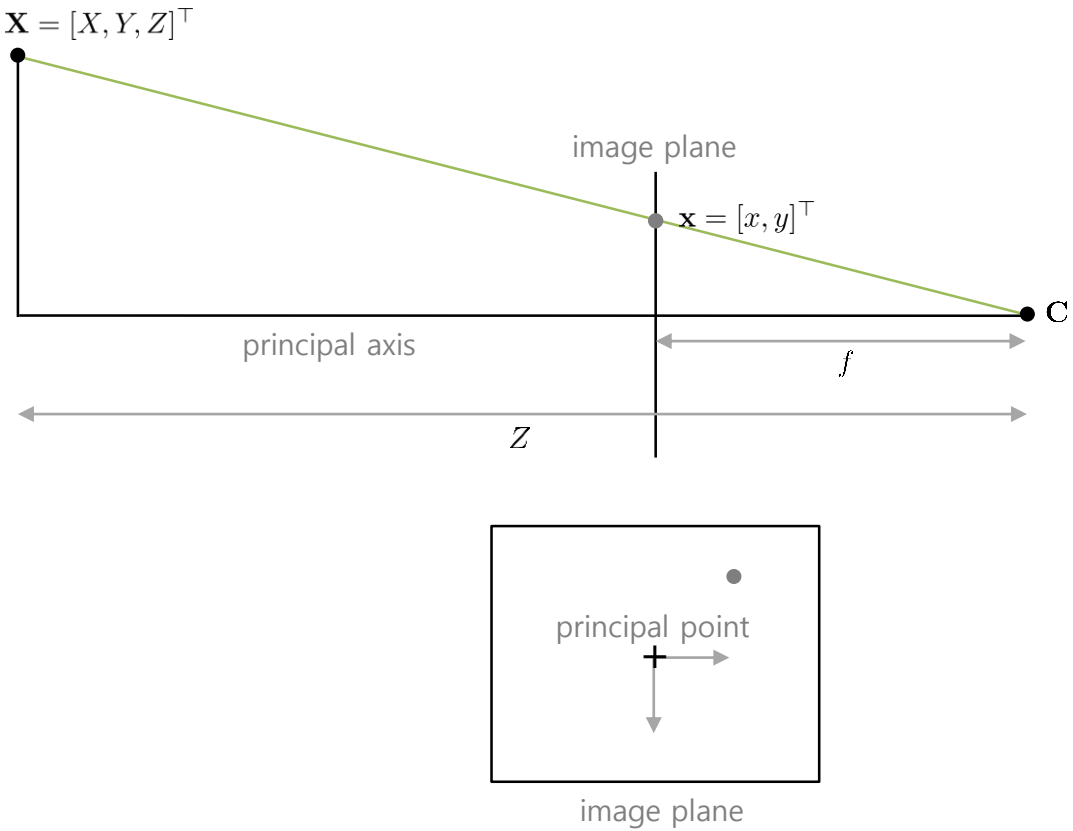

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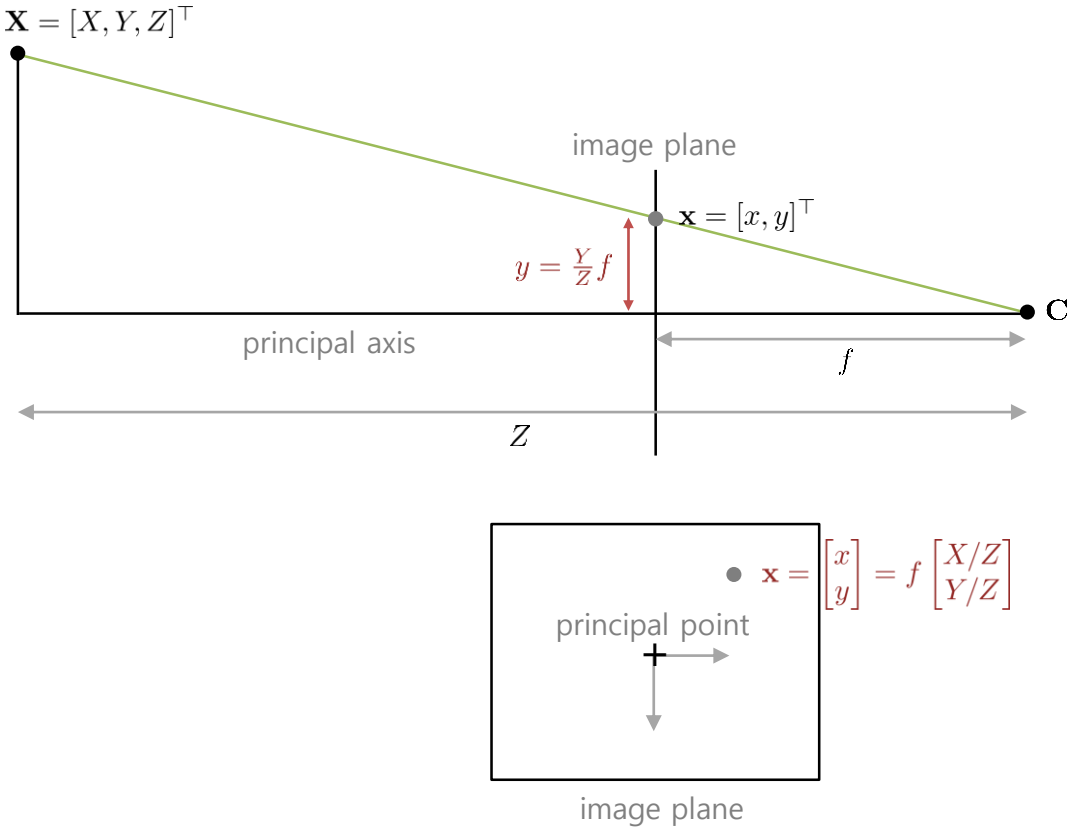

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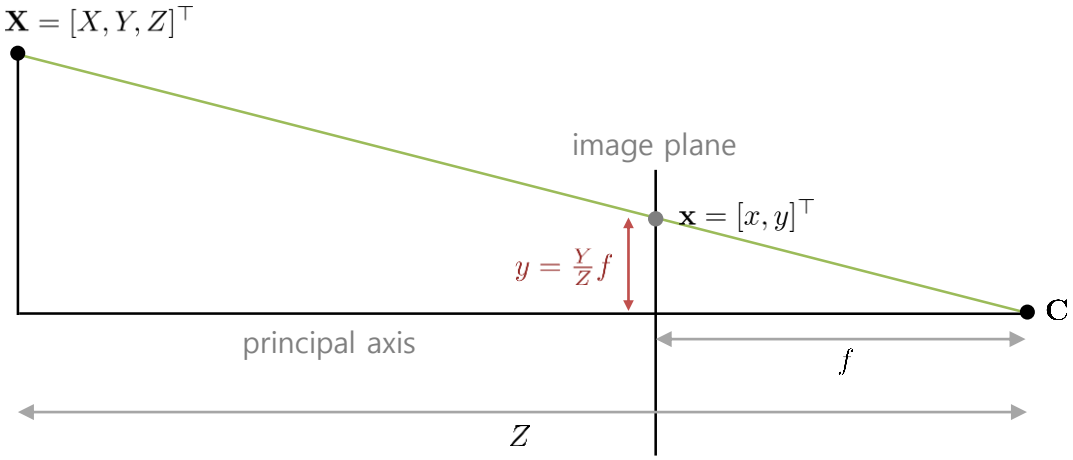
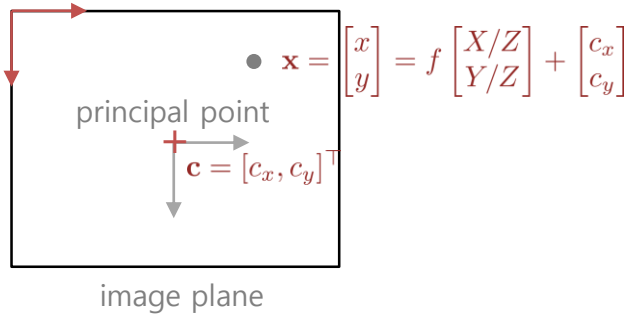
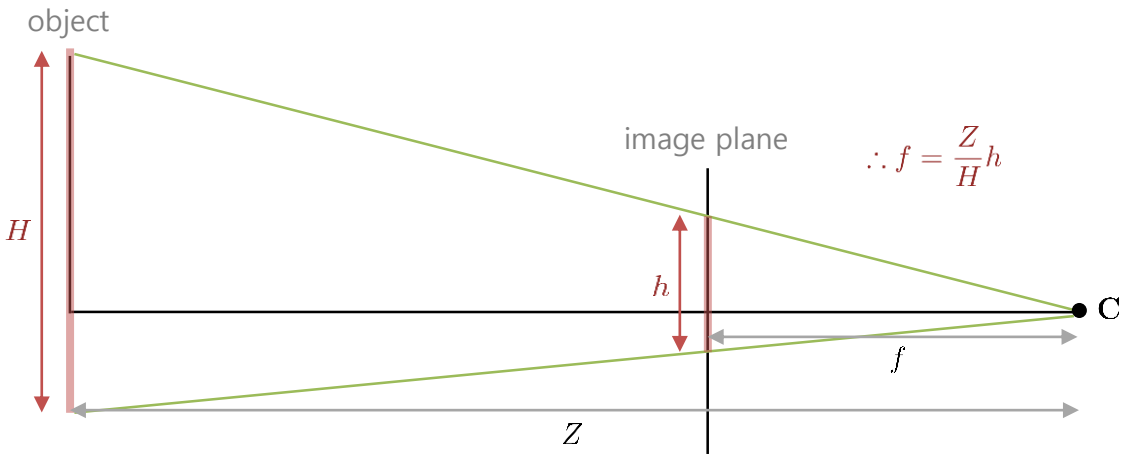




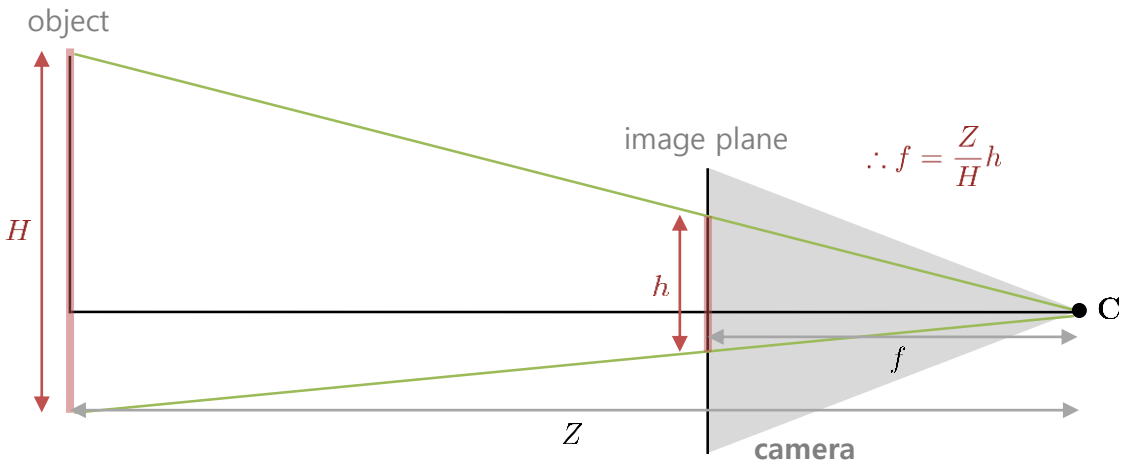
Camera Projection Model

- **The Pinhole Camera Model**
  - Example: **Simple Camera Calibration #1**
    - Unknown: **Focal length** (unit: [pixel])
    - Assumptions
      - The object length and distance from the camera are known.
      - The object is aligned with the image plane.

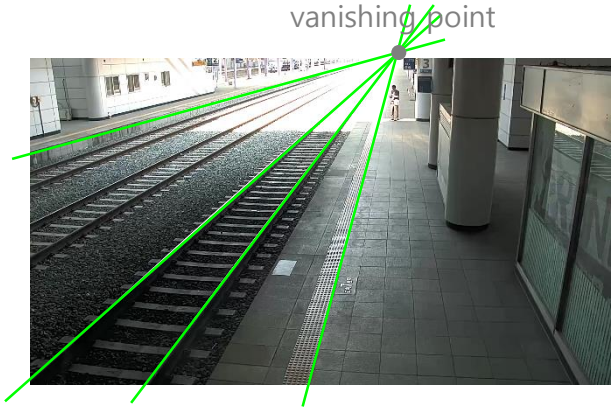


Camera Projection Model

- **The Pinhole Camera Model**
  - Example: **Simple Camera Calibration #1**
    - Unknown: **Focal length** (unit: [pixel])
    - Assumptions — Simple Object Measurement
      - The object length and distance from the camera are known.
      - The object is aligned with the image plane.

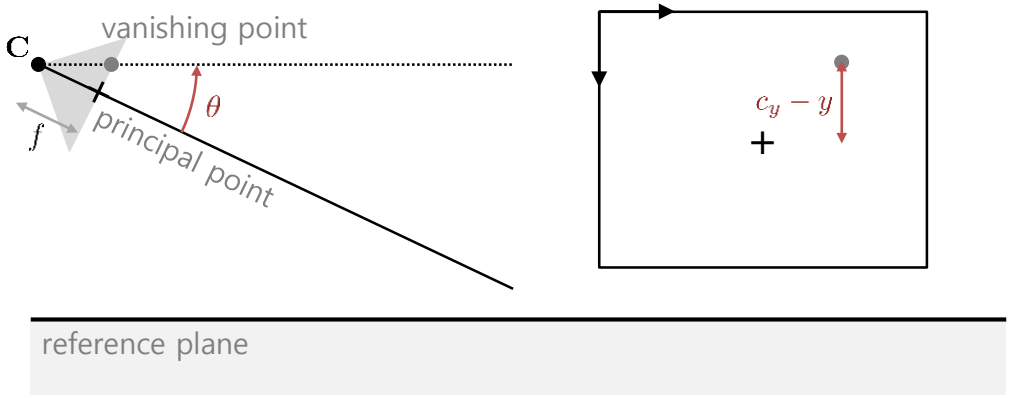


Camera Projection Model



- **The Pinhole Camera Model**
  - Example: **Simple Camera Calibration #2**
    - Unknown: **Tilt angle** (unit: [rad])
    - Assumptions
      - The focal length is known.
      - The principal point is known or selected as the center of images.
      - A vanishing point from the reference plane is known.

$$\therefore \theta = \tan^{-1} \frac{c_y - y}{f}$$

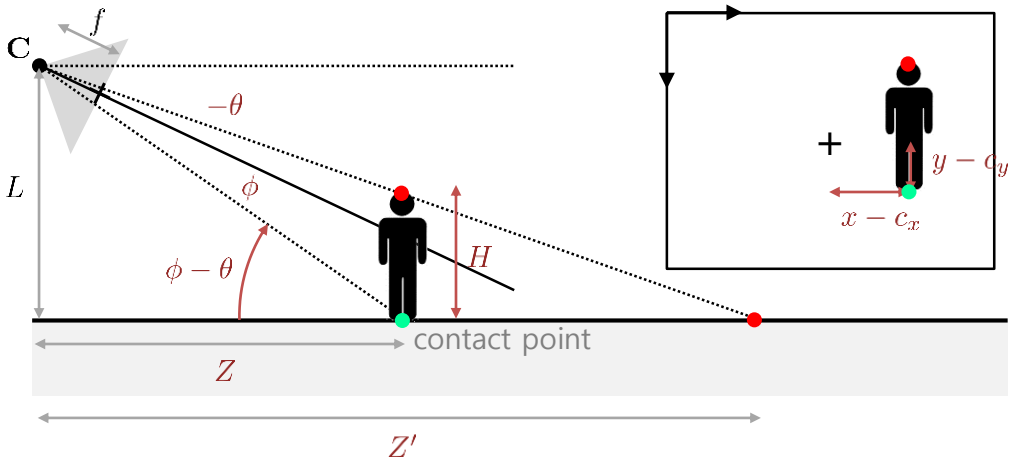


Camera Projection Model

- **The Pinhole Camera Model**
  - Example: **Simple Object Localization**
    - Unknown: **Position** (and also **height**)
    - Assumptions
      - The focal length, principal points, camera height, and tilt angle are known.
      - The object is on the reference plane.

$$\therefore Z = \frac{H}{\tan(\phi - \theta)} \quad \left(\phi = \tan^{-1}\frac{y - c_y}{f}\right)$$

$$X = \frac{x - c_x}{f} Z \qquad H = \frac{Z' - Z}{Z'} L$$



[참고] Hoiem et al., Putting Objects in Perspective, CVPR, 2006

Camera Projection Model

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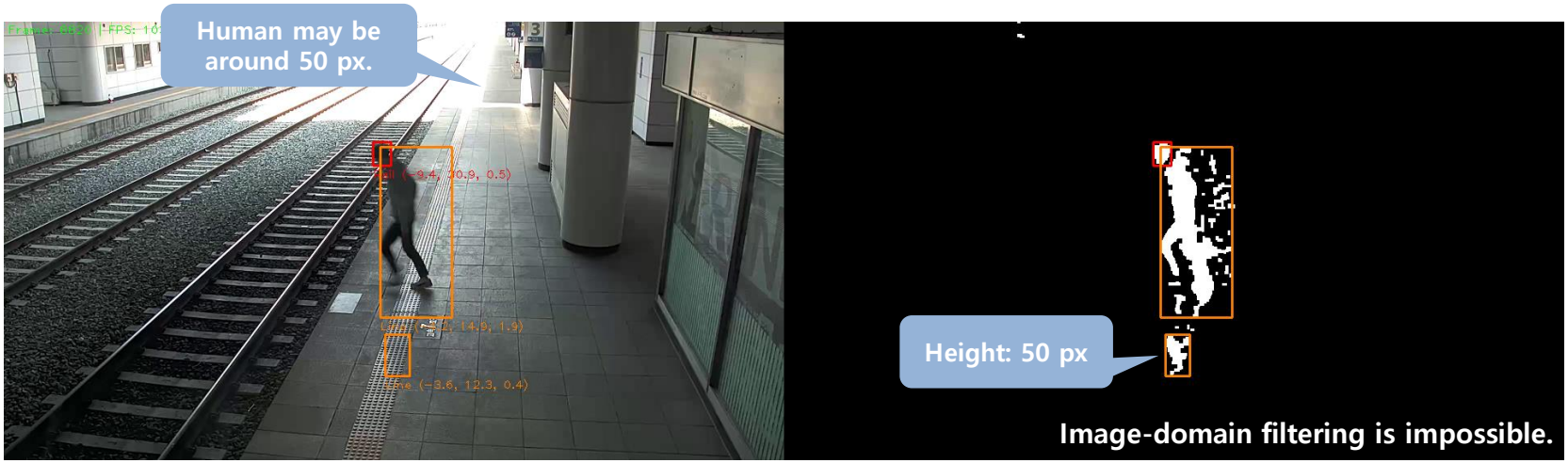
  - Example: **Simple Object Localization**



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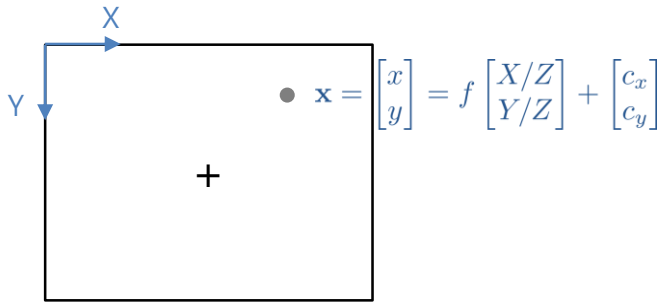
Camera Projection Model

- **Camera Matrix** (Intrinsic Parameters)



$$\mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix} = f \begin{bmatrix} X/Z \\ Y/Z \end{bmatrix} + \begin{bmatrix} c_x \\ c_y \end{bmatrix} \longrightarrow \mathbf{x} = \mathrm{K}\hat{\mathbf{x}} \quad \text{where} \quad \mathrm{K} = \begin{bmatrix} f & 0 & c_x \\ 0 & f & c_y \\ 0 & 0 & 1 \end{bmatrix}, \ \mathbf{x} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}, \ \text{and} \ \hat{\mathbf{x}} = \begin{bmatrix} X/Z \\ Y/Z \\ 1 \end{bmatrix}$$

generalized

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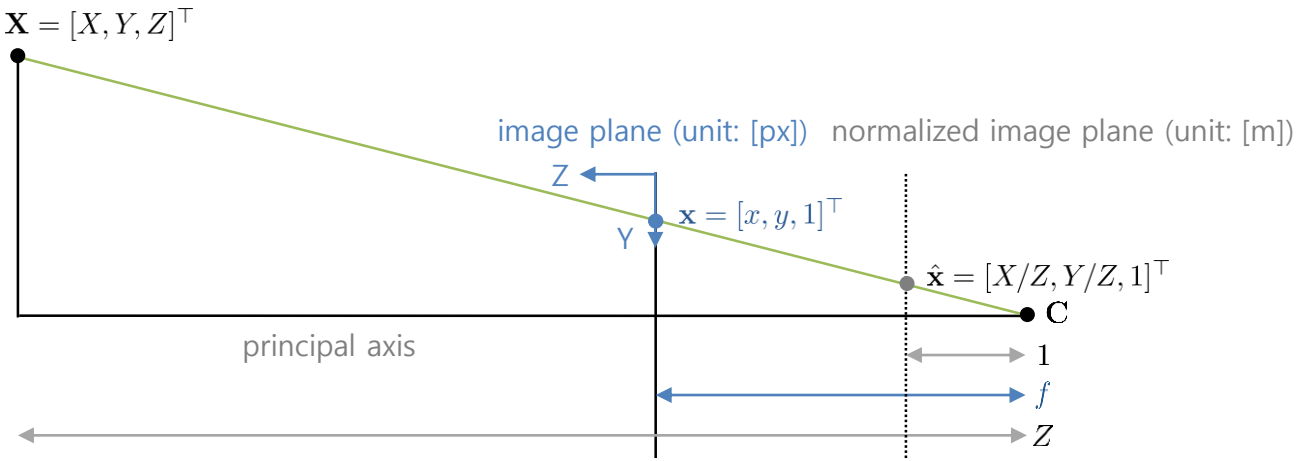

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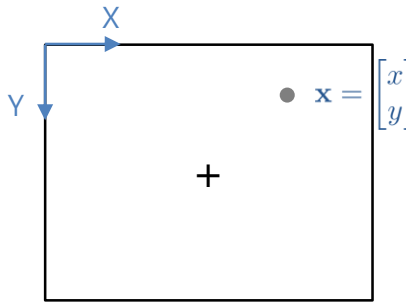

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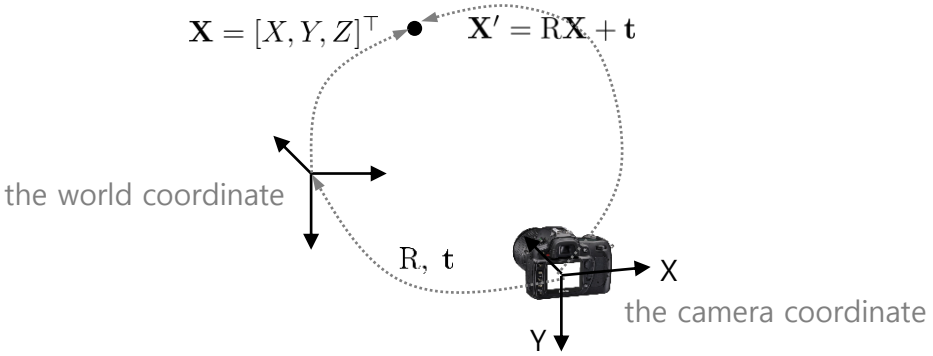
- **Projection Matrix** (Intrinsic/Extrinsic Parameters)
  - If a 3D point is not based on the camera coordinate, the point should be transformed.

$$\mathbf{X}' = \mathrm{R}\mathbf{X} + \mathbf{t} \longrightarrow \mathbf{X}' = \underbrace{\left[\mathrm{R} \,|\, \mathbf{t}\right]}_{\text{3x4 matrix}} \mathbf{X} \quad \text{where} \quad \mathbf{X} = \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\mathbf{x} = \mathrm{P}\mathbf{X} \quad \text{where} \quad \mathrm{P} = \mathrm{K}\underbrace{\left[\mathrm{R} \,|\, \mathbf{t}\right]}_{\text{3x4 matrix}} \quad \text{and} \quad \mathbf{x} = \begin{bmatrix} x \\ y \\ w \end{bmatrix} \because \mathbf{X}' \text{ is not normalized.}$$

  - No problem! To get its projected point on the image plane, we can normalize it later, $\begin{bmatrix} x/w \\ y/w \\ 1 \end{bmatrix}$.

Camera Projection Model



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Camera Projection Model

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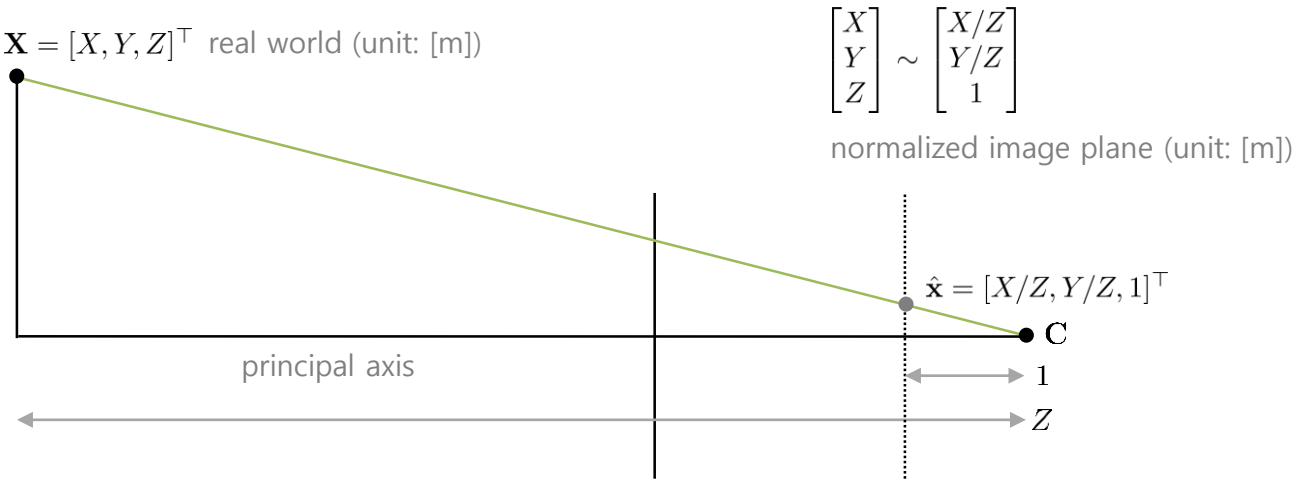
- **Why Homogeneous Coordinate?**
  - A linear transformation (rotation and translation) is applied by a single matrix multiplication.
  - A light ray can be represented as a point on the image plane.
  - An infinite point (a.k.a. ideal point) is numerically represented by $w = 0$.
  - A point and line are represented beautifully as like $\mathbf{l}^\top \mathbf{x} = 0$ or $\mathbf{x}^\top \mathbf{l} = 0$.
  - …



Camera Projection Model

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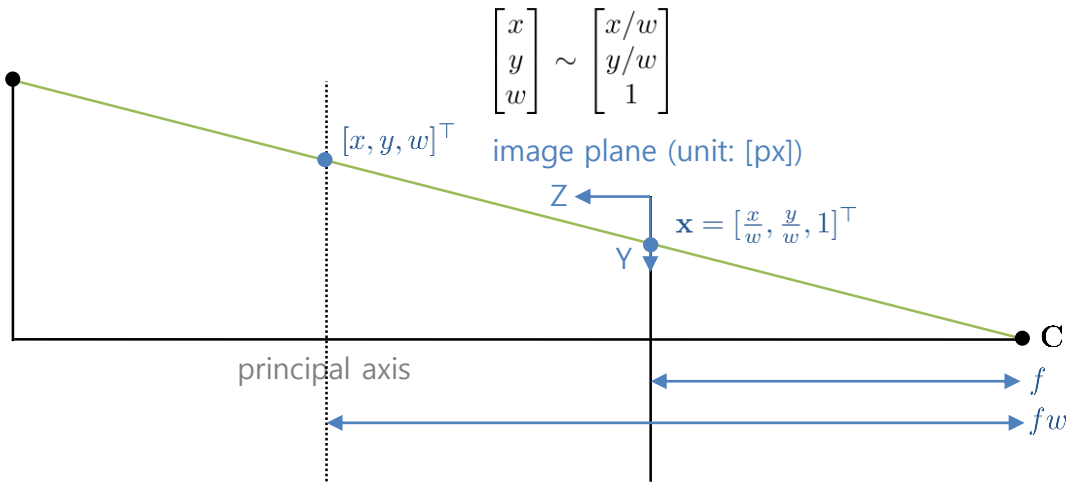
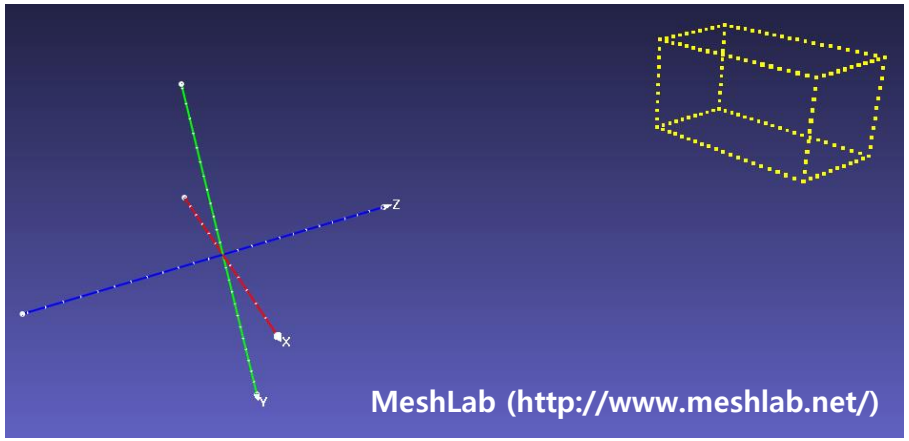


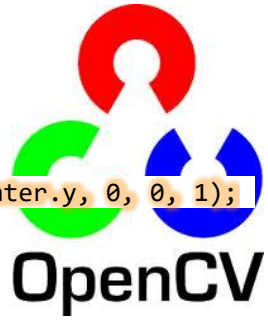
Image Formation using OpenCV


```
1.  #include "opencv_all.hpp"

2.  #define Rx(rx)  (cv::Mat_<double>(3, 3) << 1, 0, 0, 0, cos(rx), -sin(rx), 0, sin(rx), cos(rx))
3.  #define Ry(ry)  (cv::Mat_<double>(3, 3) << cos(ry), 0, sin(ry), 0, 1, 0, -sin(ry), 0, cos(ry))
4.  #define Rz(rz)  (cv::Mat_<double>(3, 3) << cos(rz), -sin(rz), 0, sin(rz), cos(rz), 0, 0, 0, 1)

5.  int main(void)
6.  {
7.      // The given camera configuration: focal length, principal point, image resolution, position, and orientation
8.      double camera_focal = 1000;
9.      cv::Point2d camera_center(320, 240);
10.     cv::Size camera_res(640, 480);
11.     cv::Point3d camera_pos[] = { cv::Point3d(0, 0, 0), cv::Point3d(-2, -2, 0), ... };
12.     cv::Point3d camera_ori[] = { cv::Point3d(0, 0, 0), cv::Point3d(-CV_PI / 12, CV_PI / 12, 0), ... };
13.     double camera_noise = 1;

14.     // Load a point cloud in the homogeneous coordinate
15.     FILE* fin = fopen("data/box.xyz", "rt");
16.     if (fin == NULL) return -1;
17.     cv::Mat X;
18.     while (!feof(fin))
19.     {
20.         double x, y, z;
21.         if (fscanf(fin, "%lf %lf %lf", &x, &y, &z) == 3)
22.         {
23.             X.push_back<double>(x);
24.             X.push_back<double>(y);
25.             X.push_back<double>(z);
26.             X.push_back<double>(1);
27.         }
28.     }
29.     fclose(fin);
30.     X = X.reshape(1, X.rows / 4).t(); // Convert to a 4 x N matrix
```



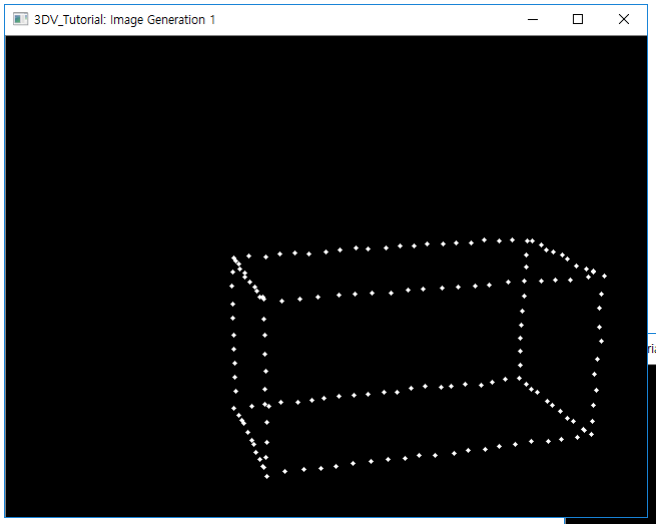
A point cloud: data/box.xyz

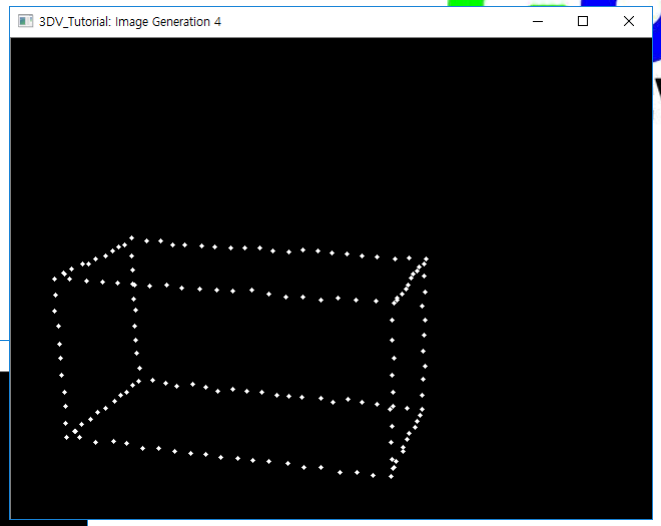
Image Formation using OpenCV

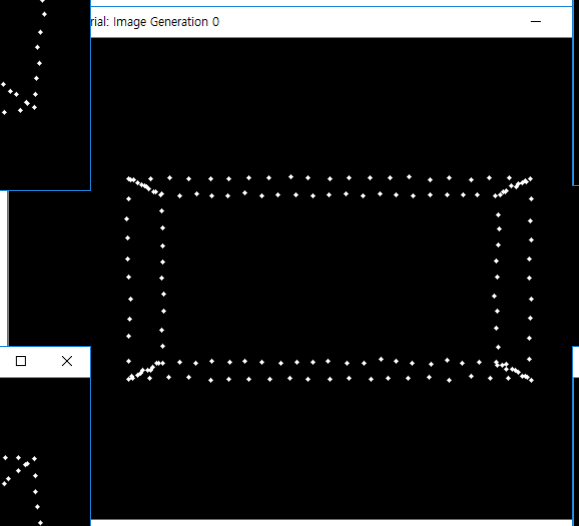


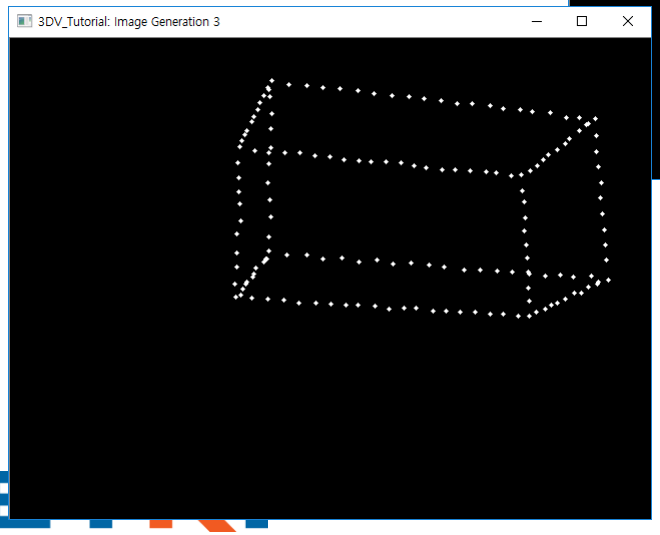

```
31.     // Generate images for each camera pose
32.     cv::Mat K = (cv::Mat_<double>(3, 3) << camera_focal, 0, camera_center.x, 0, camera_focal, camera_center.y, 0, 0, 1);
33.     for (int i = 0; i < sizeof(camera_pos) / sizeof(cv::Point3d); i++)
34.     {
35.         // Derive a projection matrix
36.         cv::Mat Rc = Rz(camera_ori[i].z) * Ry(camera_ori[i].y) * Rx(camera_ori[i].x);
37.         cv::Mat tc = (cv::Mat_<double>(3, 1) << camera_pos[i].x, camera_pos[i].y, camera_pos[i].z);
38.         cv::Mat Rt;
39.         cv::hconcat(Rc.t(), -Rc.t() * tc, Rt);
40.         cv::Mat P = K * Rt;

41.         // Project the points (c.f. OpenCV provide 'cv::projectPoints()' with consideration of distortion.)
42.         cv::Mat x = P * X;
43.         x.row(0) = x.row(0) / x.row(2);
44.         x.row(1) = x.row(1) / x.row(2);
45.         x.row(2) = 1;

46.         cv::Mat noise(2, x.cols, x.type());
47.         cv::randn(noise, cv::Scalar(0), cv::Scalar(camera_noise));
48.         x.rowRange(0, 2) = x.rowRange(0, 2) + noise; // Add noise

49.         // Show and store the points
50.         cv::Mat image = cv::Mat::zeros(camera_res, CV_8UC1);
51.         for (int c = 0; c < x.cols; c++)
52.         {
53.             cv::Point p(x.at<double>(0, c), x.at<double>(1, c));
54.             if (p.x >= 0 && p.x < camera_res.width && p.y >= 0 && p.y < camera_res.height)
55.                 cv::circle(image, p, 2, 255, -1);
56.         }
57.         cv::imshow(cv::format("3DV_Tutorial: Image Formation %d", i), image);

58.         FILE* fout = fopen(cv::format("image_formation%d.xyz", i).c_str(), "wt");
59.         if (fout == NULL) return -1;
60.         for (int c = 0; c < x.cols; c++)
61.             fprintf(fout, "%f %f 1\n", x.at<double>(0, c), x.at<double>(1, c));
62.         fclose(fout);
63.     }
64.     ...
65. }
```

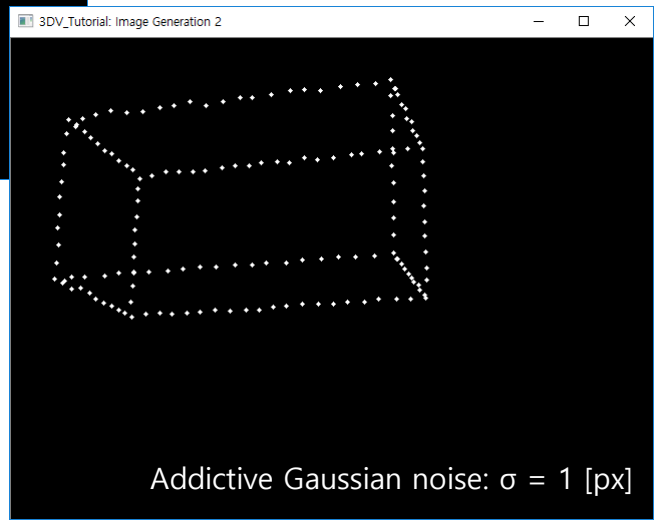
Image Formation using OpenCV



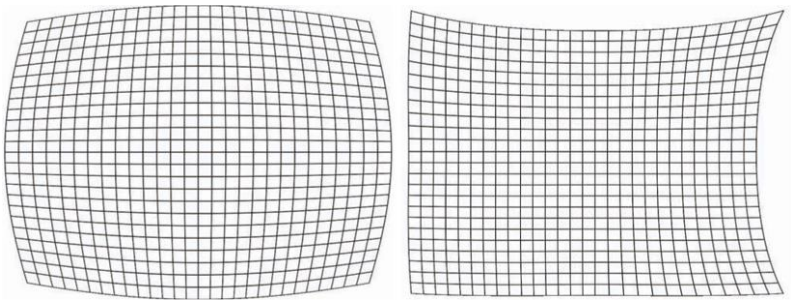








Camera Projection Model



Radial Distortion Tangential Distortion

- **Geometric Distortion**
  - Radial/Tangential Distortion Model

$$\begin{bmatrix}\hat{x}_d \\ \hat{y}_d\end{bmatrix} = (1 + k_1 r^2 + k_2 r^4 + \cdots)\begin{bmatrix}\hat{x} \\ \hat{y}\end{bmatrix} + \begin{bmatrix}2p_1\hat{x}\hat{y} + p_2(r^2 + 2\hat{x}^2) \\ 2p_2\hat{x}\hat{y} + p_1(r^2 + 2\hat{y}^2)\end{bmatrix}$$

where $r^2 = \hat{x}^2 + \hat{y}^2$

defined on the normalized image plane
- Radial distortion: k_1, k_2, ...
- Tangential distortion: p_1, p_2

  - Image Rectification
    - OpenCV `cv::undistort()` and `cv::undistortPoints()` (c.f. included in `imgproc` module)
    - Camera Distortion Correction: Theory and Practice, http://darkpgmr.tistory.com/31

correction

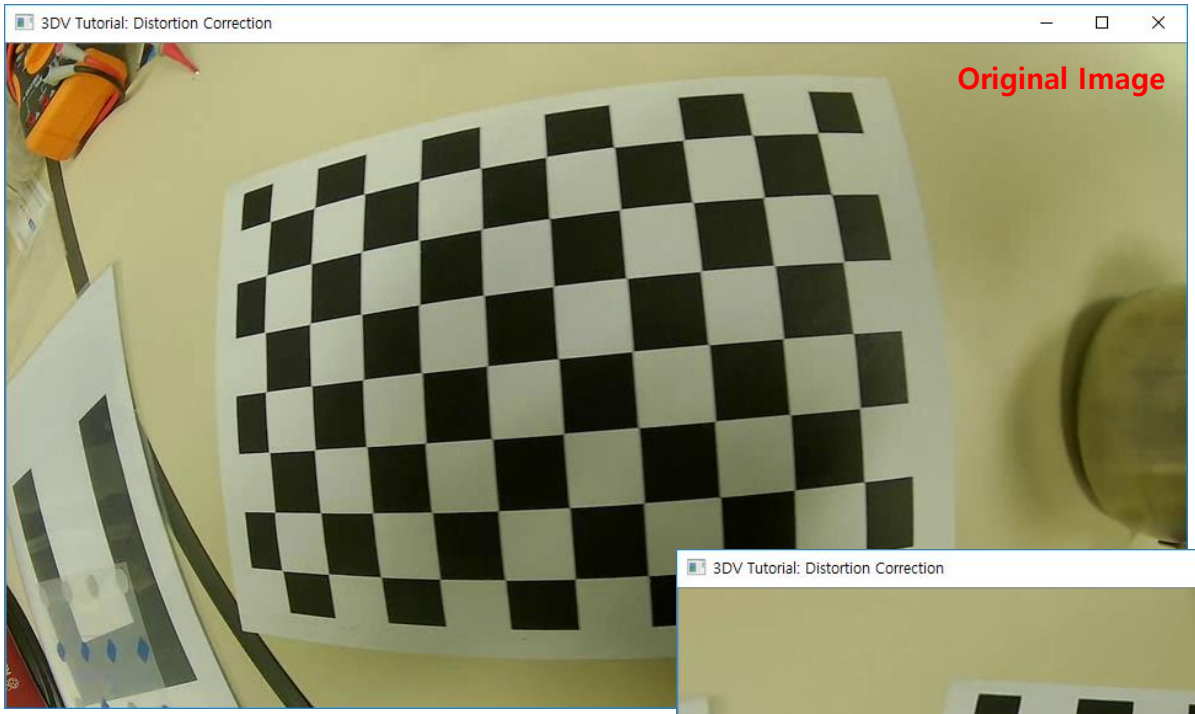
```
K1: 1.105763E-01
K2: 1.886214E-02
K3: 1.473832E-02
P1:-8.448460E-03
P2:-7.356744E-03
```

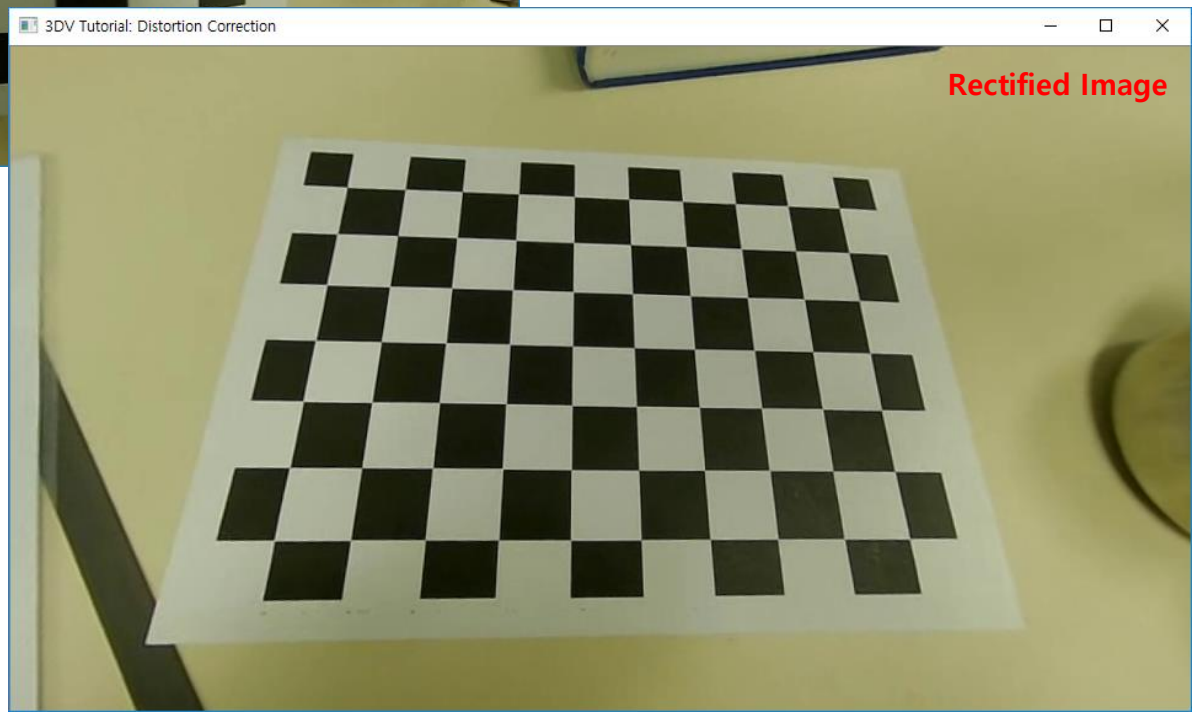
Images are from IEEE and http://alumni.media.mit.edu/~sbeck/.

Distortion Correction using OpenCV


```
1.  #include "opencv_all.hpp"

2.  int main(void)
3.  {
4.      cv::Mat K = (cv::Mat_<double>(3, 3) << 432.7390364738057, 0, 476.0614994349778, ..., 0, 0, 1);
5.      cv::Mat dist_coeff = (cv::Mat_<double>(5, 1) << -0.2852754904152874, 0.1016466459919075, ...);

6.      // Open an video
7.      cv::VideoCapture video;
8.      if (!video.open("data/chessboard.avi")) return -1;

9.      // Run distortion correction
10.     bool show_rectify = true;
11.     cv::Mat map1, map2;
12.     while (true)
13.     {
14.         // Grab an image from the video
15.         cv::Mat image;
16.         video >> image;
17.         if (image.empty()) break;

18.         // Rectify geometric distortion (c.f. 'cv::undistort()' can be applied for one-time remapping.)
19.         if (show_rectify)
20.         {
21.             if (map1.empty() || map2.empty())
22.                 cv::initUndistortRectifyMap(K, dist_coeff, cv::Mat(), cv::Mat(), image.size(), CV_32FC1, map1, map2);
23.             cv::remap(image, image, map1, map2, cv::InterpolationFlags::INTER_LINEAR);
24.         }

25.         // Show the image
26.         cv::imshow("3DV Tutorial: Distortion Correction", image);
27.         int key = cv::waitKey(1);
28.         if (key == 27) break;                                   // 'ESC' key: Exit
29.         else if (key == 9) show_rectify = !show_rectify;        // 'Tab' key: Toggle rectification
30.         ...
31.     }

32.     video.release();
33.     return 0;
34. }
```

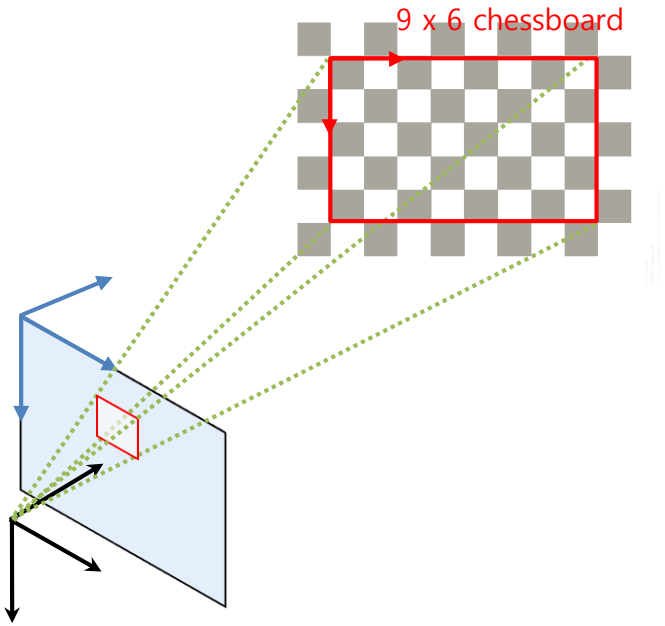
Distortion Correction using OpenCV





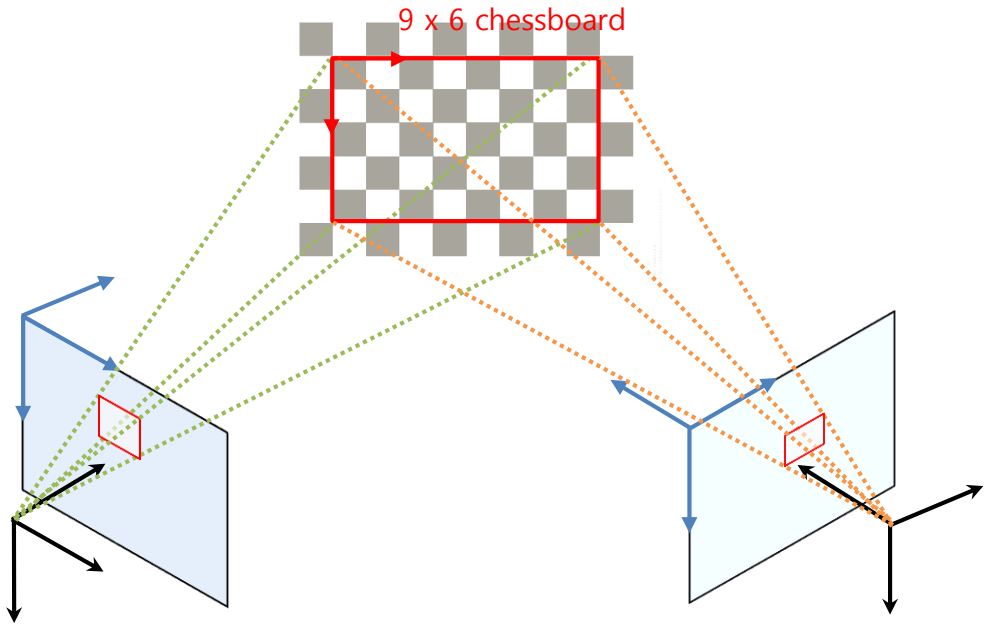
General 2D-3D Geometry

- **Camera Calibration**
  - Known: 3D points $\mathbf{X}_1, \mathbf{X}_2, \cdots, \mathbf{X}_n$ and their projected points $\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_n$
  - Unknown: 5 (intrinsic w/o distortion) + 6 (extrinsic) DoF
  - Constraints: *n* x projection $\mathbf{x}_i = \mathrm{P}\mathbf{X}_i$



General 2D-3D Geometry

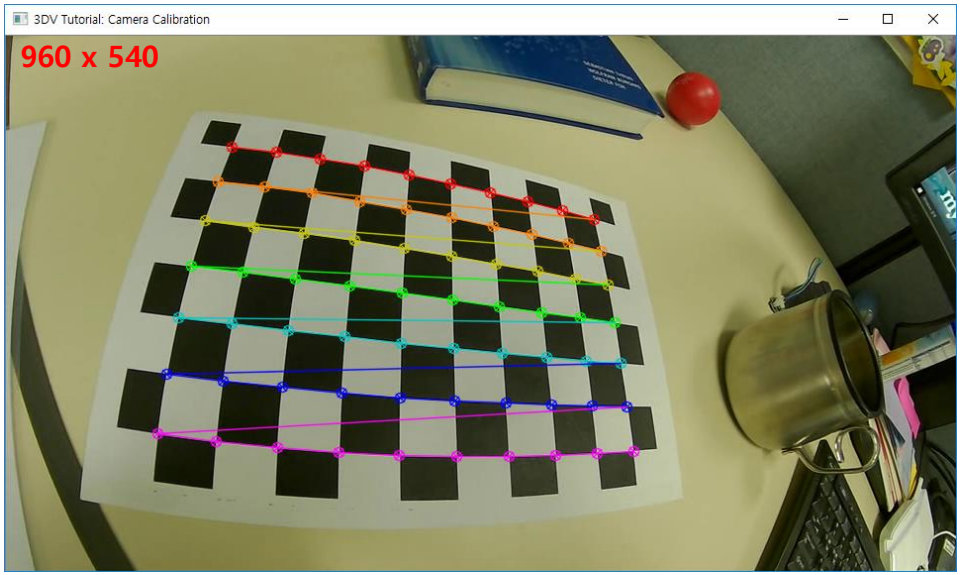
- **Camera Calibration**
  - Known: 3D points $\mathbf{X}_1, \mathbf{X}_2, \cdots, \mathbf{X}_n$ and their projected points from each camera $\mathbf{x}_i^j$
  - Unknown: 5 (intrinsic w/o distortion) + *m* x 6 (extrinsic) DoF
  - Constraints: *n* x *m* x projection $\mathbf{x}_i^j = \mathrm{K}\left[\mathrm{R}_j \,|\, \mathbf{t}_j\right]\mathbf{X}_i$
  - Solutions
    - OpenCV `cv::calibrateCamera()` and `cv::initCameraMatrix2D()`
    - Camera Calibration Toolbox for MATLAB, http://www.vision.caltech.edu/bouguetj/calib_doc/
    - GML C++ Camera Calibration Toolbox, http://graphics.cs.msu.ru/en/node/909
    - DarkCamCalibrator, http://darkpgmr.tistory.com/139



Camera Calibration using OpenCV


```cpp
#include "opencv_all.hpp"

int main(void)
{
    bool select_images = true;
    cv::Size board_pattern(10, 7);
    double board_cellsize = 0.025;

    // Select images
    std::vector<cv::Mat> images;
    ...

    // Find 2D corner points from the given images
    std::vector<std::vector<cv::Point2f> > image_points;
    for (size_t i = 0; i < images.size(); i++)
    {
        std::vector<cv::Point2f> pts;
        if (cv::findChessboardCorners(images[i], board_pattern, pts))
            image_points.push_back(pts);
    }
    if (image_points.empty()) return -1;

    // Prepare 3D points of the chess board
    std::vector<std::vector<cv::Point3f> > object_points(1);
    for (int r = 0; r < board_pattern.height; r++)
        for (int c = 0; c < board_pattern.width; c++)
            object_points[0].push_back(cv::Point3f(board_cellsize * c, board_cellsize * r, 0));
    object_points.resize(image_points.size(), object_points[0]); // Copy

    // Calibrate the camera
    cv::Mat K = cv::Mat::eye(3, 3, CV_64F);
    cv::Mat dist_coeff = cv::Mat::zeros(4, 1, CV_64F);
    std::vector<cv::Mat> rvecs, tvecs;
    double rms = cv::calibrateCamera(object_points, image_points, images[0].size(), K, dist_coeff, rvecs, tvecs);

    // Report calibration results
    ...
    return 0;
}
```

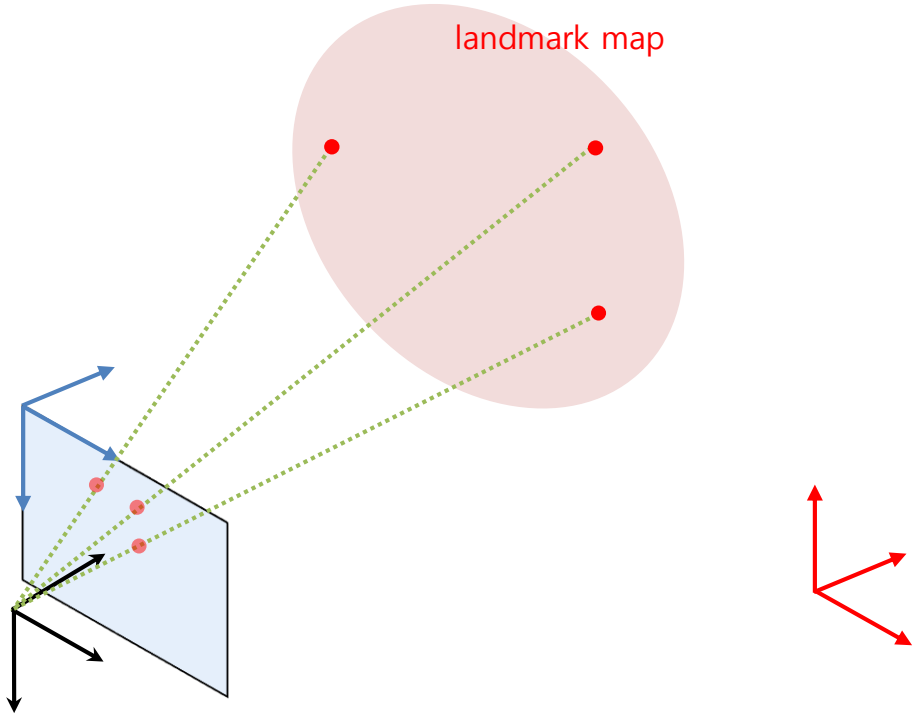
Camera Calibration using OpenCV




```
## Camera Calbration Results
* The number of applied images = 22
* RMS error = 0.473353
* Camera matrix (K) =
  [432.7390364738057, 0, 476.0614994349778]
  [0, 431.2395555913084, 288.7602152621297]
  [0, 0, 1]
* Distortion coefficient (k1, k2, p1, p2, k3, ...) =
  [-0.2852754904152874, 0.1016466459919075,
   -0.0004420196146339175, 0.0001149909868437517,
   -0.01803978785585194]
```

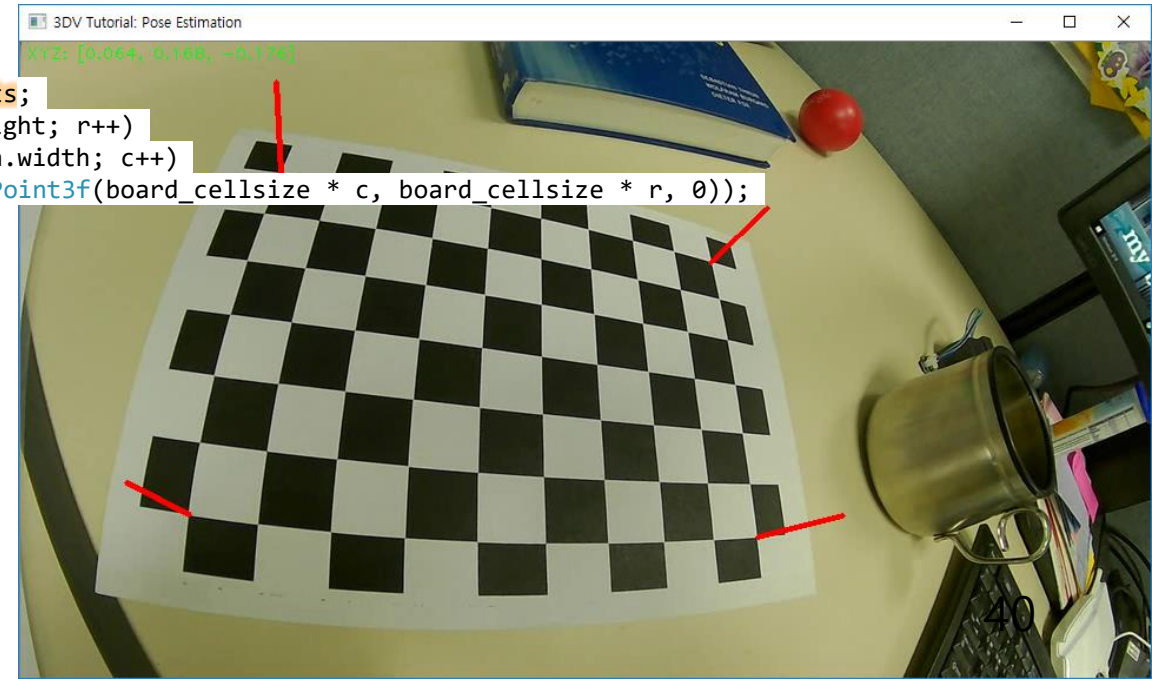
General 2D-3D Geometry

- **Absolute Camera Pose Estimation** (Perspective-n-Point; PnP)
  - Known: 3D points $\mathbf{X}_1, \mathbf{X}_2, \cdots, \mathbf{X}_n$, their projected points $\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_n$, and camera matrix K
  - Unknown: Camera pose (6 DoF)
  - Constraints: n x projection $\mathbf{x}_i = \mathrm{K}\left[\mathrm{R} \,|\, \mathbf{t}\right]\mathbf{X}_i$
  - Solutions ($n \geq 3$)
    - OpenCV `cv::solvePnP()` and `cv::solvePnPRansac()`
    - Efficient PnP (EPnP), http://cvlab.epfl.ch/EPnP/



Pose Estimation using OpenCV


```
1. #include "opencv_all.hpp"

2. int main(void)
3. {
4.     cv::Mat K = (cv::Mat_<double>(3, 3) << 432.7390364738057, 0, 476.0614994349778, ...);
5.     cv::Mat dist_coeff = (cv::Mat_<double>(5, 1) << -0.2852754904152874, 0.1016466459919075, ...);
6.     cv::Size board_pattern(10, 7);
7.     double board_cellsize = 0.025;

8.     // Open an video
9.     cv::VideoCapture video;
10.    if (!video.open("data/chessboard.avi")) return -1;

11.    // Prepare four sticks for simple AR
12.    std::vector<cv::Point3f> stick_points;
13.    stick_points.push_back(cv::Point3f(0, 0, 0));
14.    stick_points.push_back(cv::Point3f(0, 0, -board_cellsize * 2));
15.    ...

16.    // Run pose estimation
17.    std::vector<cv::Point3f> object_points;
18.    for (int r = 0; r < board_pattern.height; r++)
19.        for (int c = 0; c < board_pattern.width; c++)
20.            object_points.push_back(cv::Point3f(board_cellsize * c, board_cellsize * r, 0));
```



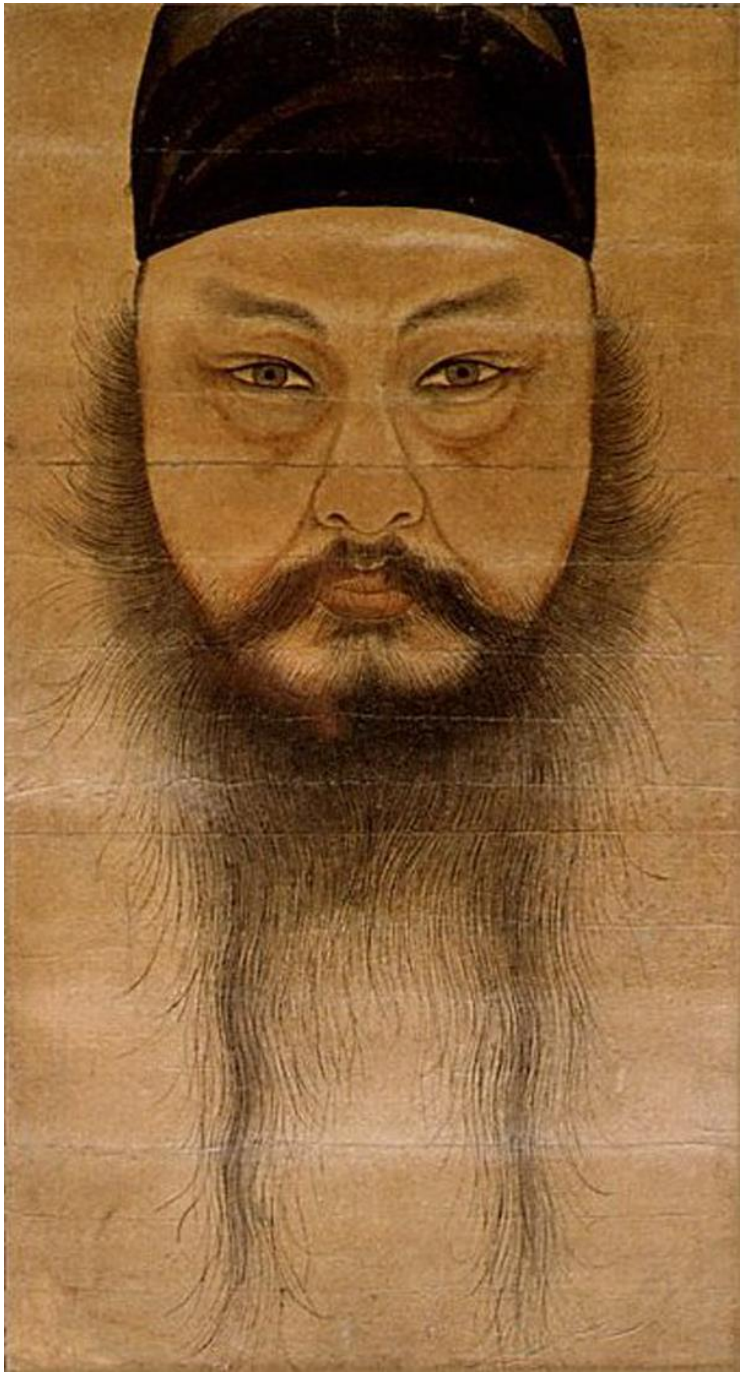
ETRI

Pose Estimation using OpenCV


```
21.     while (true)
22.     {
23.         // Grab an image from the video
24.         cv::Mat image;
25.         video >> image;
26.         if (image.empty()) break;

27.         // Estimate camera pose
28.         std::vector<cv::Point2f> image_points;
29.         bool complete = cv::findChessboardCorners(image, board_pattern, image_points, ...);
30.         if (complete)
31.         {
32.             cv::Mat rvec, tvec;
33.             cv::solvePnP(object_points, image_points, K, dist_coeff, rvec, tvec);

34.             // Draw four sticks
35.             std::vector<cv::Point2f> line_points;
36.             cv::projectPoints(stick_points, rvec, tvec, K, dist_coeff, line_points);
37.             for (int i = 0; i < 8; i += 2)
38.                 cv::line(image, line_points[i], line_points[i + 1], cv::Scalar(0, 0, 255), 4);

39.             // Print camera position
40.             cv::Mat R;
41.             cv::Rodrigues(rvec, R);
42.             cv::Mat p = -R.t() * tvec;
43.             cv::String info = cv::format("XYZ: [%.3f, %.3f, %.3f]", p.at<double>(0), p.at<double>(1), p.at<double>(2));
44.             cv::putText(image, info, cv::Point(5, 15), cv::FONT_HERSHEY_PLAIN, 1, cv::Scalar(0, 255, 0));
45.         }

46.         // Show the image
47.         ...
48.     }

49.     video.release();
50.     return 0;
51. }
```

ETRI

Two-view Geometry

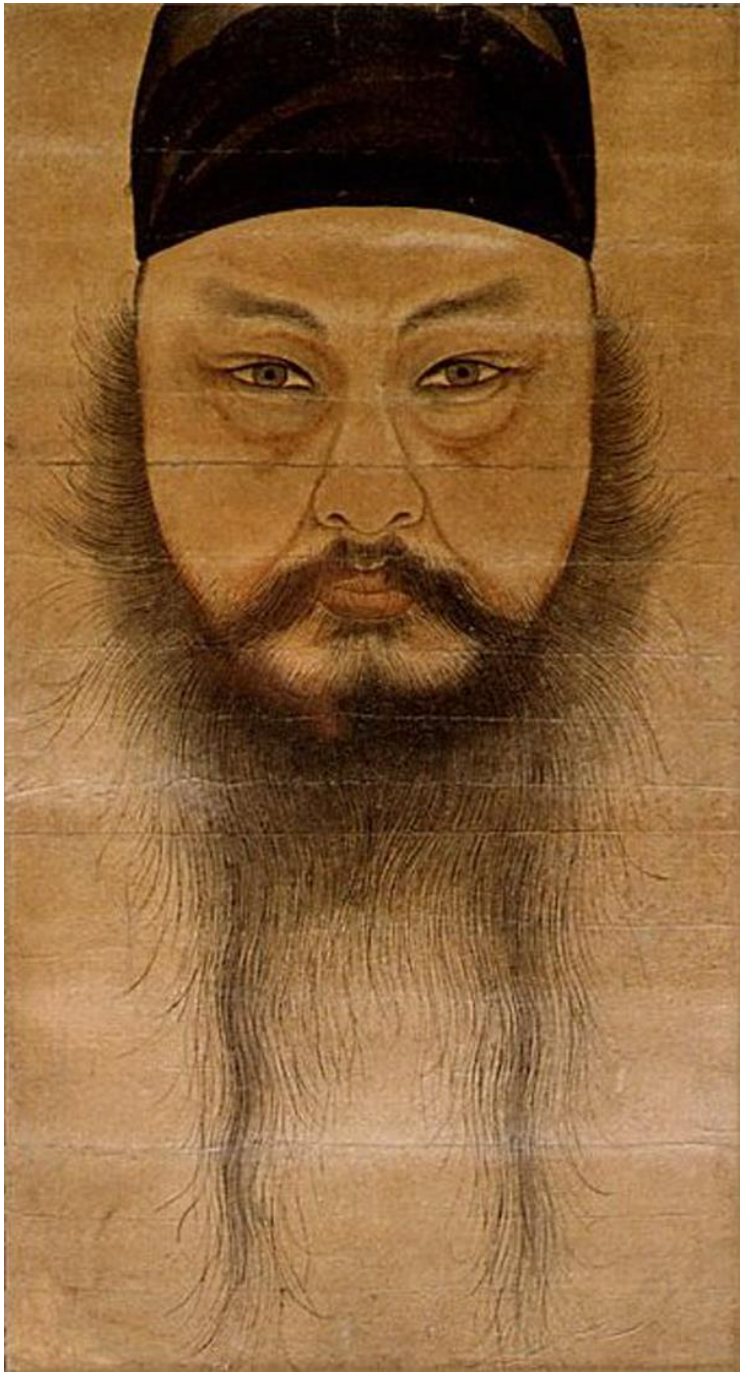


윤두서(1668-1715) 자화상, 국보 제240호
Korean National Treasure No. 240

Two-view Geometry

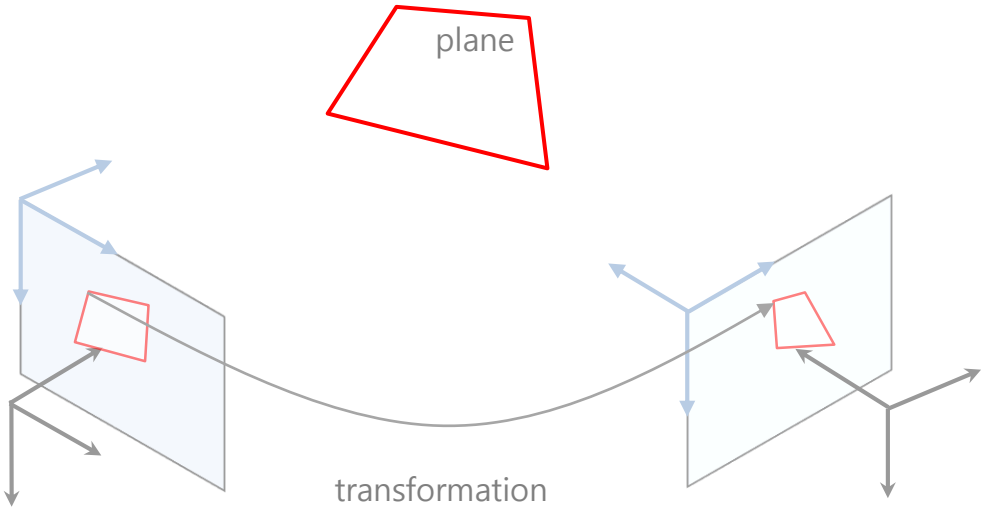
Planar 2D-2D Geometry (Projective Geometry)

General 2D-2D Geometry (Epipolar Geometry)

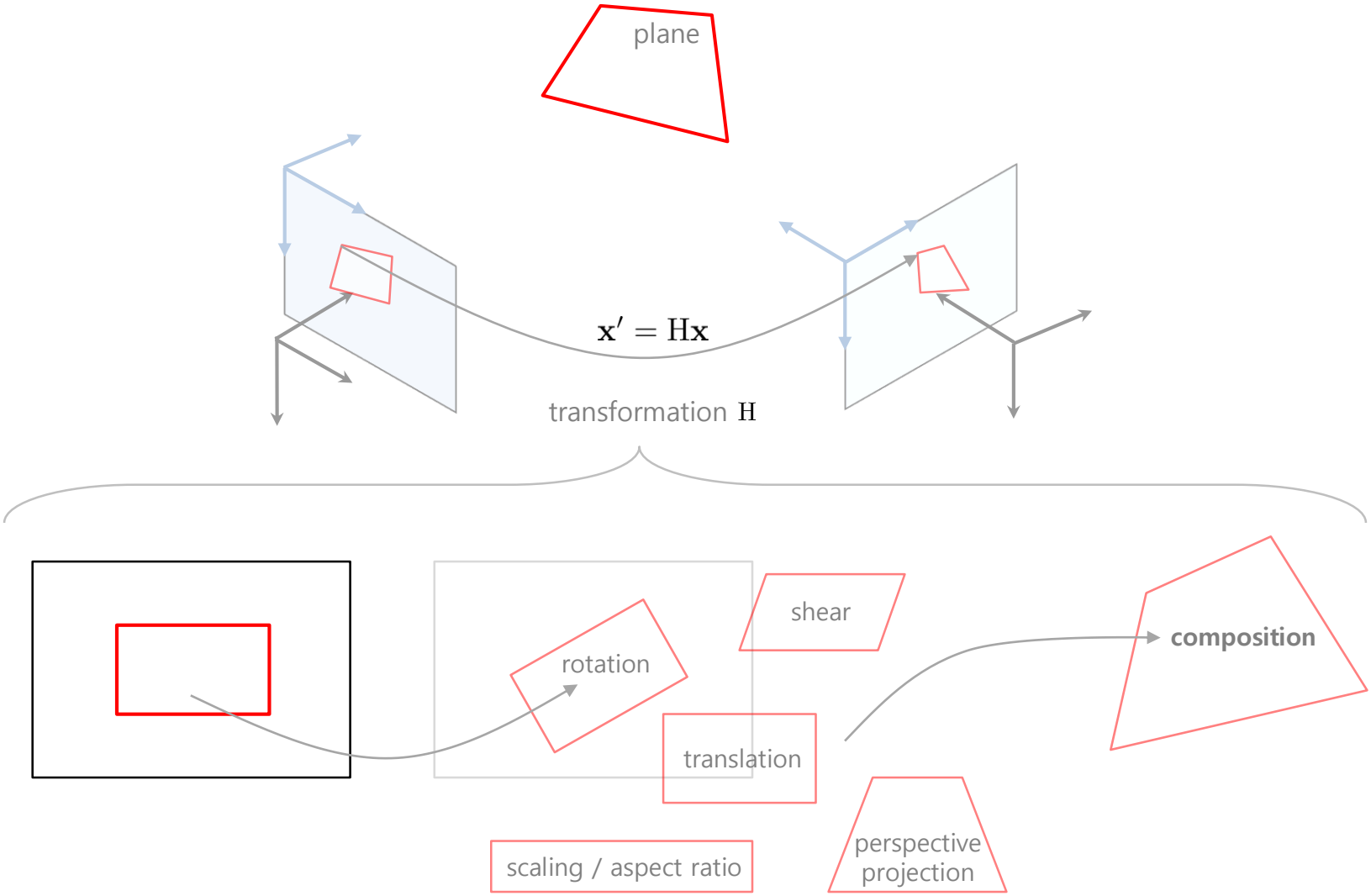


윤두서(1668-1715) 자화상, 국보 제240호
Korean National Treasure No. 240

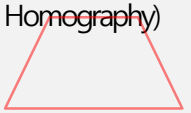
Planar 2D-2D Geometry (Projective Geometry)



Planar 2D-2D Geometry (Projective Geometry)



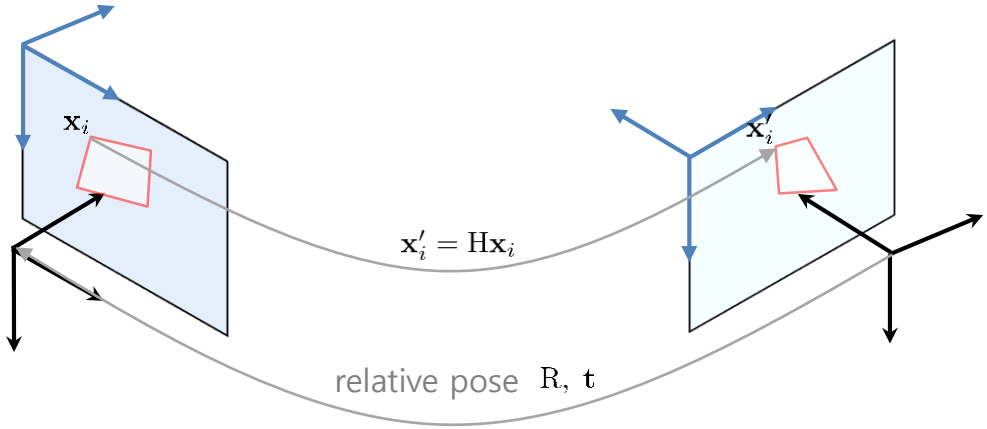
Planar 2D-2D Geometry (Projective Geometry)

	**Euclidean Transform** (a.k.a. Rigid Transform) 	**Similarity Transform** 	**Affine Transform** 	**Projective Transform** (a.k.a. Planar Homography) 
Matrix Forms H	$\begin{bmatrix} \cos\theta & -\sin\theta & t_x \\ \sin\theta & \cos\theta & t_y \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} s\cos\theta & -s\sin\theta & t_x \\ s\sin\theta & s\cos\theta & t_y \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} a_{11} & a_{12} & t_x \\ a_{21} & a_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} a_{11} & a_{12} & t_x \\ a_{21} & a_{22} & t_y \\ v_1 & v_2 & 1 \end{bmatrix}$
DoF	3	4	6	8
Transformations - rotation - translation - scaling - aspect ratio - shear - perspective projection	 O O	 O O O	 O O O O O	 O O O O O O
Invariants - length - angle - ratio of lengths - parallelism - incidence - cross ratio	 O O O O O	 O O O O O	 O O O	 O O

OpenCV Functions			`cv::getAffineTransform()`
			`cv::estimateRigidTransform()`
			-
			`cv::warpAffine()`
				`cv::getPerspectiveTransform()`
				-
				`cv::findHomography()`
				`cv::warpPerspective()`

Planar 2D-2D Geometry (Projective Geometry)

- **Planar Homography Estimation**
  - Known: Point correspondence $(\mathbf{x}_1, \mathbf{x}'_1), \cdots, (\mathbf{x}_n, \mathbf{x}'_n)$
  - Unknown: Planar Homography (8 DoF)
  - Constraints: n x projective transformation $\mathbf{x}'_i = \mathrm{H}\mathbf{x}_i$
  - Solutions ($n \geq 4$)
    - OpenCV `cv::getPerspectiveTransform()` and `cv::findHomography()`
    - c.f. More simplified transformations need less number of minimal correspondence.
      - Affine ($n \geq 3$), similarity ($n \geq 2$), Euclidean ($n \geq 2$)
  - [Note] Planar homography can be decomposed as relative camera pose R, $\mathbf{t}$.
    - OpenCV `cv::decomposeHomographyMat()`
    - c.f. However, the decomposition needs to know camera matrices.



Perspective Correction using OpenCV


```
1. #include "opencv_all.hpp"

2. void MouseEventHandler(int event, int x, int y, int flags, void* param)
3. {
4.     ...
5.         std::vector<cv::Point> *points_src = (std::vector<cv::Point> *)param;
6.         points_src->push_back(cv::Point(x, y));
7.     ...
8. }

9. int main(void)
10.{
11.    cv::Size card_size(450, 250);

12.    // Prepare the rectified points
13.    std::vector<cv::Point> points_dst;
14.    points_dst.push_back(cv::Point(0, 0));
15.    points_dst.push_back(cv::Point(card_size.width, 0));
16.    points_dst.push_back(cv::Point(0, card_size.height));
17.    points_dst.push_back(cv::Point(card_size.width, card_size.height));

18.    // Load an image
19.    cv::Mat original = cv::imread("data/sunglok.jpg");
20.    if (original.empty()) return -1;

21.    // Get the matched points from a user's mouse
22.    std::vector<cv::Point> points_src;
23.    cv::namedWindow("3DV Tutorial: Perspective Correction");
24.    cv::setMouseCallback("3DV Tutorial: Perspective Correction", MouseEventHandler, &points_src);
25.    while (points_src.size() < 4)
26.    {
27.        ...
28.    }
29.    if (points_src.size() < 4) return -1;

30.    // Calculate planar homography and rectify perspective distortion
31.    cv::Mat H = cv::findHomography(points_src, points_dst);
32.    cv::Mat rectify;
33.    cv::warpPerspective(original, rectify, H, card_size);
```

Perspective Correction using OpenCV

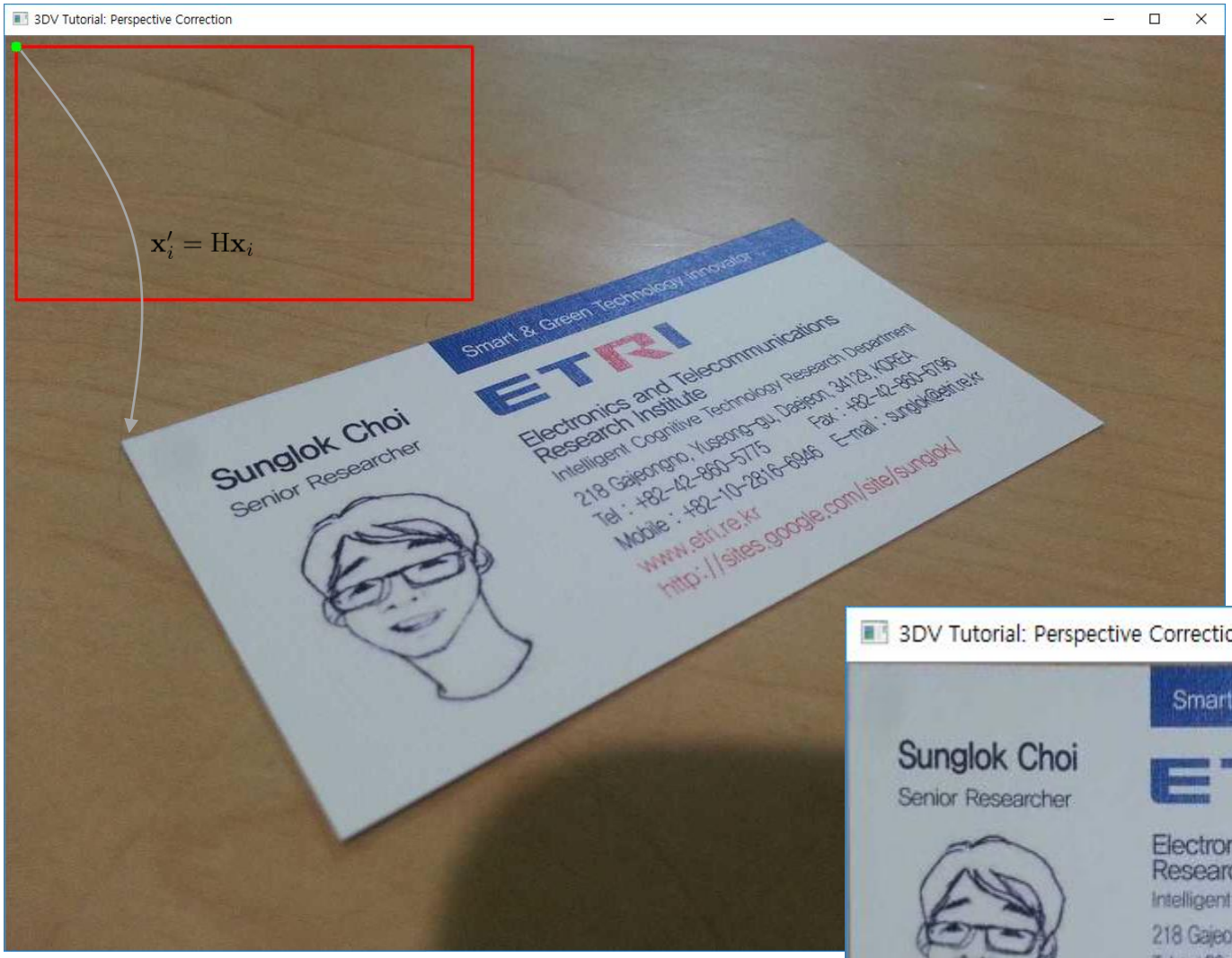


Image Stitching using OpenCV

```
1.  #include "opencv_all.hpp"

2.  int main(void)
3.  {
4.      // Load two images (c.f. We assume that two images have the same size and type)
5.      cv::Mat image1 = cv::imread("data/hill01.jpg");
6.      cv::Mat image2 = cv::imread("data/hill02.jpg");
7.      if (image1.empty() || image2.empty()) return -1;

8.      // Retrieve matching points
9.      cv::Mat gray1, gray2;
10.     ...
11.     cv::Ptr<cv::FeatureDetector> detector = cv::xfeatures2d::SURF::create();  // SURF features
12.     std::vector<cv::KeyPoint> keypoint1, keypoint2;
13.     detector->detect(gray1, keypoint1);
14.     detector->detect(gray2, keypoint2);
15.     cv::Ptr<cv::FeatureDetector> extractor = cv::xfeatures2d::SURF::create(); // SURF descriptors
16.     cv::Mat descriptor1, descriptor2;
17.     extractor->compute(gray1, keypoint1, descriptor1);
18.     extractor->compute(gray2, keypoint2, descriptor2);
19.     cv::FlannBasedMatcher matcher; // Approximate Nearest Neighbors (ANN) matcher
20.     std::vector<cv::DMatch> match;
21.     matcher.match(descriptor1, descriptor2, match);

22.     // Calculate planar homography and merge them
23.     std::vector<cv::Point2f> points1, points2;
24.     for (size_t i = 0; i < match.size(); i++)
25.     {
26.         points1.push_back(keypoint1.at(match.at(i).queryIdx).pt);
27.         points2.push_back(keypoint2.at(match.at(i).trainIdx).pt);
28.     }
29.     cv::Mat H = cv::findHomography(points2, points1, cv::RANSAC);
30.     cv::Mat merge;
31.     cv::warpPerspective(image2, merge, H, cv::Size(image1.cols * 2, image1.rows));
32.     merge.colRange(0, image1.cols) = image1 * 1; // Copy

33.     // Show the merged image
34.     ...
35. }
```

+

=

Video Stabilization using OpenCV


```
1. #include "opencv_all.hpp"

2. int main(void)
3. {
4.     // Open an video and get the reference image and feature points
5.     cv::VideoCapture video;
6.     if (!video.open("data/traffic.avi")) return -1;

7.     cv::Mat gray_ref;
8.     video >> gray_ref;
9.     if (gray_ref.empty())
10.    {
11.        video.release();
12.        return -1;
13.    }
14.    if (gray_ref.channels() > 1) cv::cvtColor(gray_ref, gray_ref, CV_RGB2GRAY);

15.    std::vector<cv::Point2f> point_ref;
16.    cv::goodFeaturesToTrack(gray_ref, point_ref, 2000, 0.01, 10);
17.    if (point_ref.size() < 4)
18.    {
19.        video.release();
20.        return -1;
21.    }
```

A shaking CCTV video: data/traffic.avi

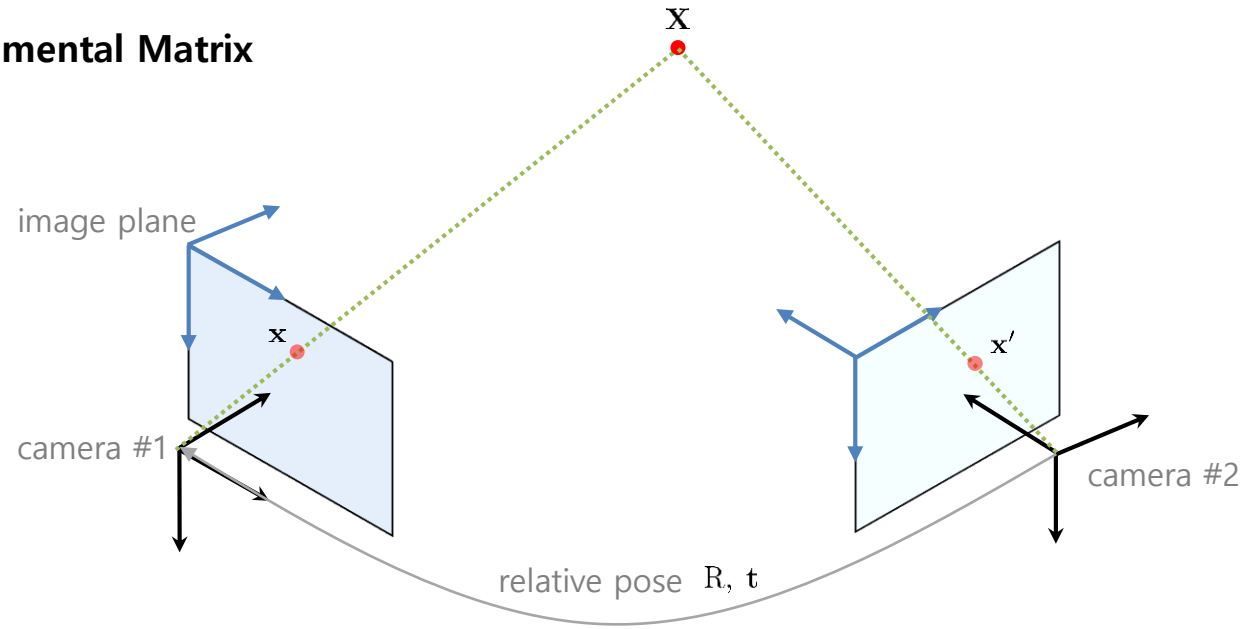
Video Stabilization using OpenCV


```
22.      // Run and show video stabilization
23.      while (true)
24.      {
25.          // Grab an image from the video
26.          cv::Mat image, gray;
27.          video >> image;
28.          if (image.empty()) break;
29.          if (image.channels() > 1) cv::cvtColor(image, gray, CV_RGB2GRAY);
30.          else                      gray = image.clone();

31.          // Extract optical flow and calculate planar homography
32.          std::vector<cv::Point2f> point;
33.          std::vector<uchar> m_status;
34.          cv::Mat err, inlier_mask;
35.          cv::calcOpticalFlowPyrLK(gray_ref, gray, point_ref, point, m_status, err);
36.          cv::Mat H = cv::findHomography(point, point_ref, inlier_mask, cv::RANSAC);

37.          // Synthesize a stabilized image
38.          cv::Mat warp;
39.          cv::warpPerspective(image, warp, H, cv::Size(image.cols, image.rows));

40.          // Show the original and rectified images together
41.          for (size_t i = 0; i < point_ref.size(); i++)
42.          {
43.              if (inlier_mask.at<uchar>(i) > 0) cv::line(image, point_ref[i], point[i], cv::Scalar(0, 0, 255));
44.              else cv::line(image, point_ref[i], point[i], cv::Scalar(0, 255, 0));
45.          }
46.          cv::hconcat(image, warp, image);
47.          cv::imshow("3DVT Tutorial: Video Stabilization", image);
48.          if (cv::waitKey(1) == 27) break; // 'ESC' key
49.      }

50.      video.release();
51.      return 0;
52.  }
```

ETRI

General 2D-2D Geometry (Epipolar Geometry)

- **Fundamental Matrix**



Epipolar Constraint $\mathbf{x}'^{\top}\mathrm{F}\mathbf{x} = 0$

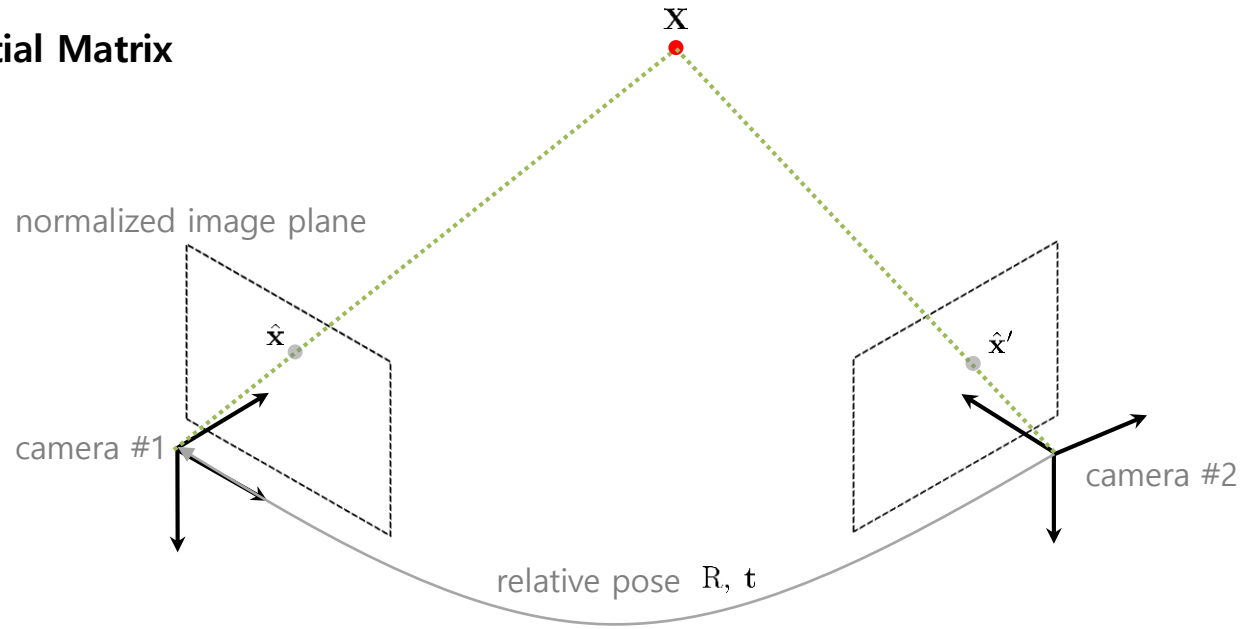
$\mathbf{x} = \mathrm{K}\hat{\mathbf{x}}$

$\hat{\mathbf{x}}'^{\top}\mathrm{K}'^{\top}\mathrm{FK}\hat{\mathbf{x}} = 0$

$\hat{\mathbf{x}}'^{\top}\mathrm{E}\hat{\mathbf{x}} = 0 \quad (\mathrm{E} = \mathrm{K}'^{\top}\mathrm{FK})$ on the normalized image plane

General 2D-2D Geometry (Epipolar Geometry)

- **Essential Matrix**



Epipolar Constraint $\hat{\mathbf{x}}'^{\top}\mathrm{E}\hat{\mathbf{x}} = 0 \quad (\mathrm{E} = [\mathbf{t}]_{\times}\mathrm{R})$

c.f. $[\mathbf{t}]_{\times} = \begin{bmatrix} 0 & -t_z & t_y \\ t_z & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix}$

$\hat{\mathbf{x}}' \cdot (\mathbf{t} \times \hat{\mathbf{x}}') = 0$

$\hat{\mathbf{x}}' \cdot (\mathbf{t} \times \hat{\mathbf{x}}') = \hat{\mathbf{x}}' \cdot \mathbf{t} \times \mathrm{R}\hat{\mathbf{x}}$

$\hat{\mathbf{x}}'\cdot$

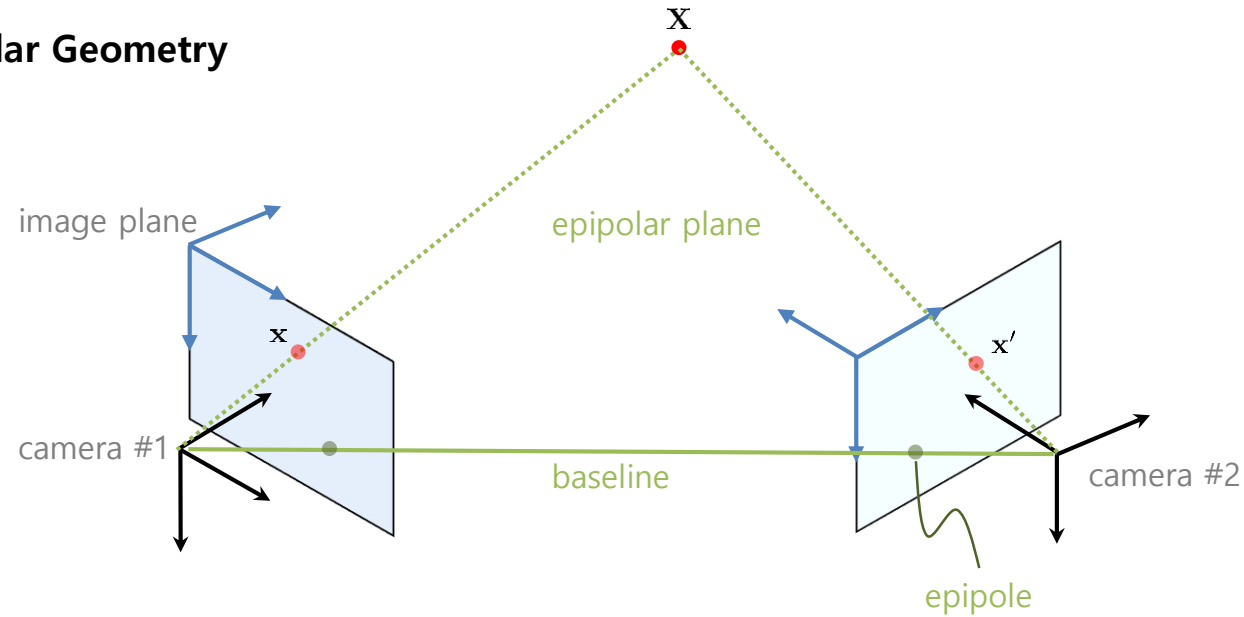
$\mathbf{t} \times \hat{\mathbf{x}}' = \mathbf{t} \times \mathrm{R}\hat{\mathbf{x}}$

$\mathbf{t}\times$

$\hat{\mathbf{x}}' = \mathrm{R}\hat{\mathbf{x}} + \mathbf{t}$

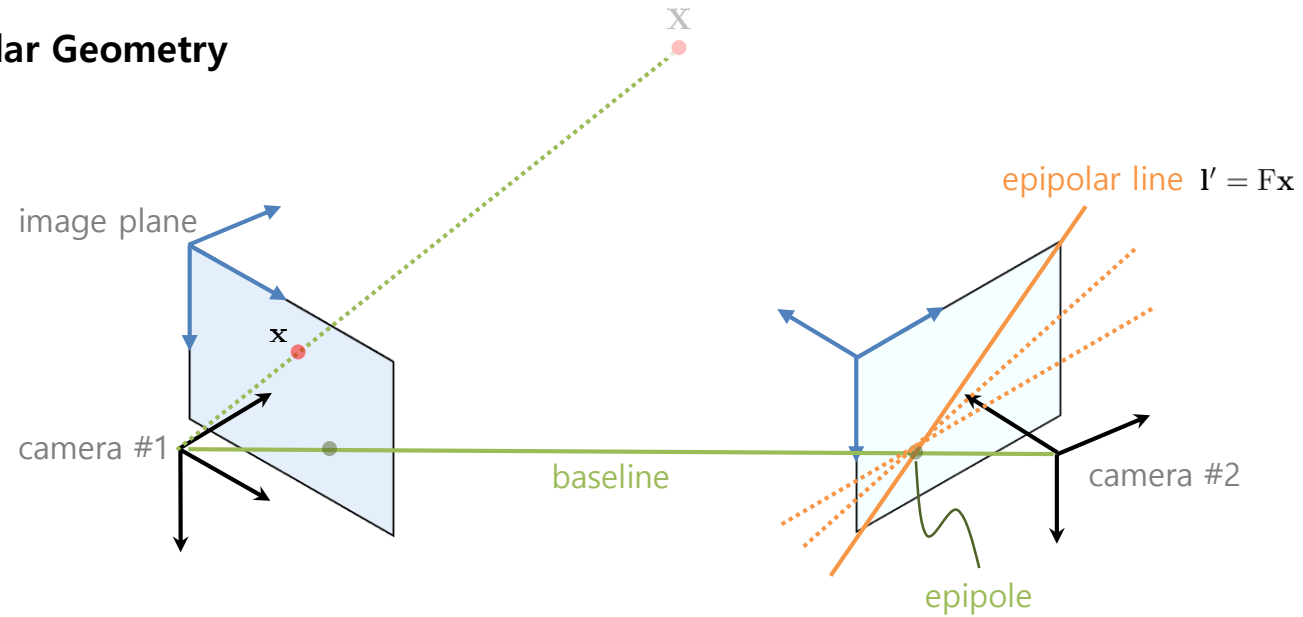
General 2D-2D Geometry (Epipolar Geometry)

- **Epipolar Geometry**



General 2D-2D Geometry (Epipolar Geometry)

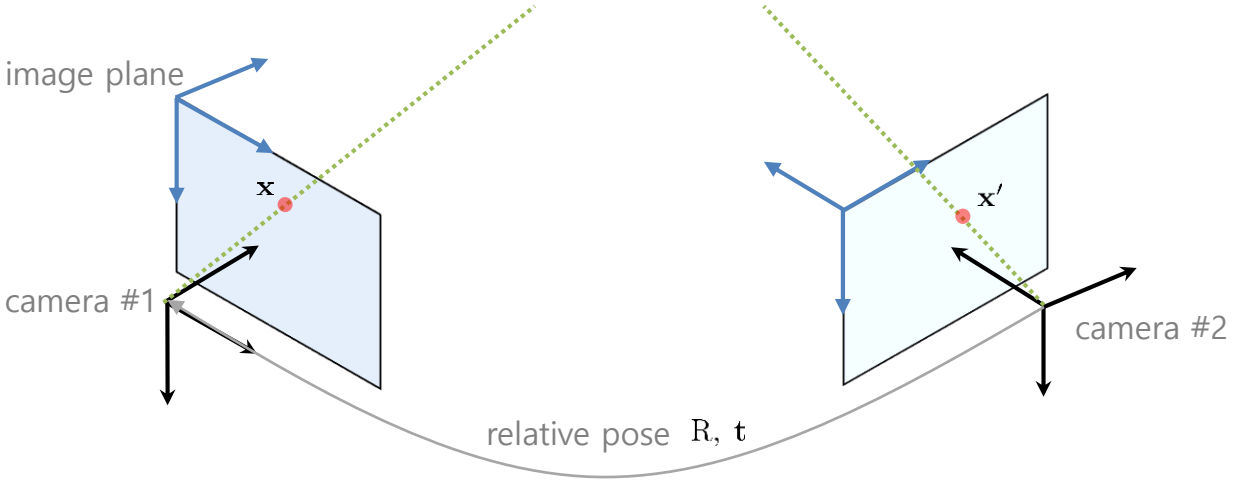
- **Epipolar Geometry**



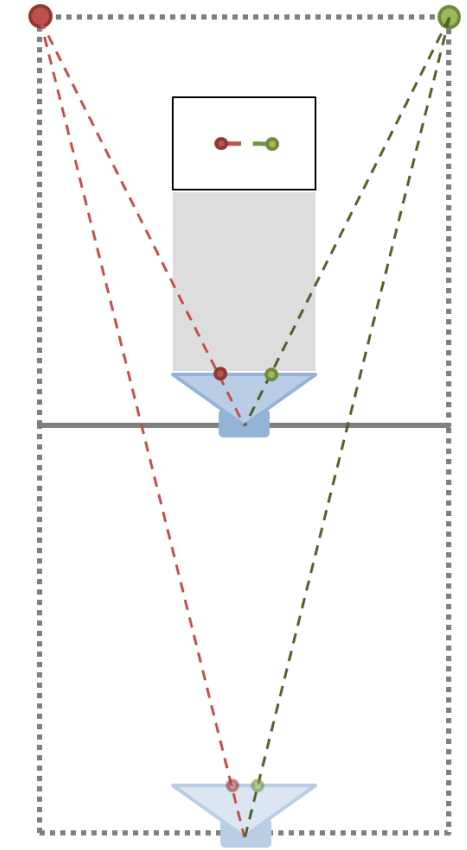
General 2D-2D Geometry (Epipolar Geometry)

- **Relative Camera Pose Estimation**
  - Known: Point correspondence $(\mathbf{x}_1, \mathbf{x}'_1), \cdots, (\mathbf{x}_n, \mathbf{x}'_n)$ and camera matrices K, K'
  - Unknown: Rotation and translation (5 DoF)
  - Constraints: n x epipolar constraint $\mathbf{x}'^{\top}\mathrm{F}\mathbf{x} = 0$
  - Solutions (OpenCV) — up-to scale → scale ambiguity
    - **Fundamental matrix**: 7/8-point algorithm
      - **Estimation**: `cv::findFundamentalMat()` → 1 solution
      - **Conversion to E**: $\mathrm{E} = \mathrm{K}'^{\top}\mathrm{F}\mathrm{K}$
    - **Essential matrix**: 5-point algorithm
      - **Estimation**: `cv::findEssentialMat()` → k solutions
      - **Decomposition**: `cv::decomposeEssentialMat()` → 4 solutions
      - **Decomposition with positive-depth check**: `cv::recoverPose()` → 1 solution

F
$\mathrm{E} = \mathrm{K}'^{\top}\mathrm{F}\mathrm{K}$
E
$\mathrm{E} = [\mathbf{t}]_{\times}\mathrm{R}$
$\mathrm{R},\ \mathbf{t}$

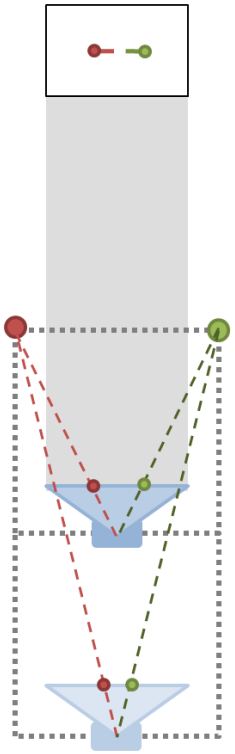


Epipolar Geometry: Scale Ambiguity



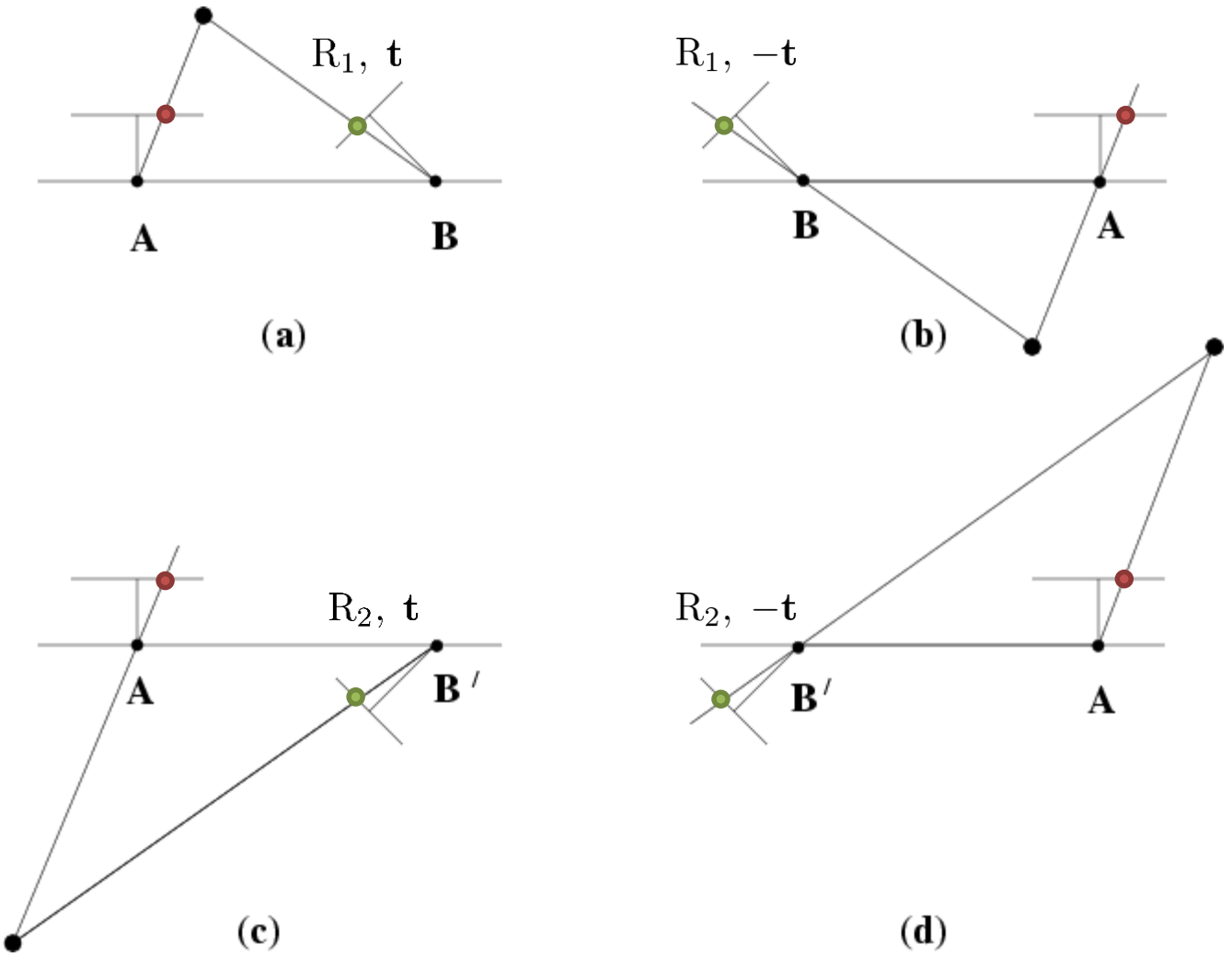
(a) 2-meter-wide tunnel

optical flow on an image



(b) 1-meter-wide tunnel

Epipolar Geometry: Relative Pose Ambiguity



Images are from *Multiple View Geometry in Computer Vision* (by Hartley and Zisserman).

General 2D-2D Geometry (Epipolar Geometry)

- **Relative Camera Pose Estimation**
  - Known: Point correspondence $(\mathbf{x}_1, \mathbf{x}'_1), \cdots, (\mathbf{x}_n, \mathbf{x}'_n)$ and camera matrices K, K'
  - Unknown: Rotation and translation (5 DoF)
  - Constraints: n x epipolar constraint $\mathbf{x}'^{\top}\mathrm{F}\mathbf{x} = 0$
  - Solutions (OpenCV)

intrinsic & extrinsic camera parameters

- **Fundamental matrix**: 7/8-point algorithm **(7 DoF)**

$$\mathrm{F} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix} \quad \& \quad \begin{matrix} \mathrm{rank}(\mathrm{F}) = 2 \\ \det(\mathrm{F}) = 0 \end{matrix}$$

  - **Estimation**: `cv::findFundamentalMat()` → 1 solution
  - **Conversion to E**: $\mathrm{E} = \mathrm{K}'^{\top}\mathrm{F}\mathrm{K}$
  - **Degenerate cases**: No translation, correspondence from a single plane

extrinsic camera parameters

- **Essential matrix**: 5-point algorithm **(5 DoF)**

$$2\mathrm{E}\mathrm{E}^{\top}\mathrm{E} - \mathrm{tr}(\mathrm{E}\mathrm{E}^{\top})\mathrm{E} = 0$$

  - **Estimation**: `cv::findEssentialMat()` → k solutions
  - **Decomposition**: `cv::decomposeEssentialMat()` → 4 solutions
  - **Decomposition with positive-depth check**: `cv::recoverPose()` → 1 solution
  - **Degenerate cases**: No translation $(\because \mathrm{E} = [\mathbf{t}]_{\times}\mathrm{R})$

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General 2D-2D Geometry (Epipolar Geometry)

- **Relative Camera Pose Estimation**
  - Known: Point correspondence $(\mathbf{x}_1, \mathbf{x}'_1), \cdots, (\mathbf{x}_n, \mathbf{x}'_n)$ and camera matrices K, K'
  - Unknown: Rotation and translation (5 DoF)
  - Constraints: n x epipolar constraint $\mathbf{x}'^{\top}\mathrm{F}\mathbf{x} = 0$
  - Solutions (OpenCV)

intrinsic & extrinsic camera parameters

- **Fundamental matrix**: 7/8-point algorithm **(7 DoF)**

$$\mathrm{F} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix} \quad \& \quad \begin{matrix} \mathrm{rank}(\mathrm{F}) = 2 \\ \det(\mathrm{F}) = 0 \end{matrix}$$

  - **Estimation**: `cv::findFundamentalMat()` → 1 solution
  - **Conversion to E**: $\mathrm{E} = \mathrm{K}'^{\top}\mathrm{F}\mathrm{K}$
  - **Degenerate cases**: No translation, correspondence from a single plane

extrinsic camera parameters

- **Essential matrix**: 5-point algorithm **(5 DoF)**

$$2\mathrm{E}\mathrm{E}^{\top}\mathrm{E} - \mathrm{tr}(\mathrm{E}\mathrm{E}^{\top})\mathrm{E} = 0$$

  - **Estimation**: `cv::findEssentialMat()` → k solutions
  - **Decomposition**: `cv::decomposeEssentialMat()` → 4 solutions
  - **Decomposition with positive-depth check**: `cv::recoverPose()` → 1 solution
  - **Degenerate cases**: No translation $(\because \mathrm{E} = [\mathbf{t}]_{\times}\mathrm{R})$

intrinsic & extrinsic camera parameters + plane normal

- **Planar homography**: 4-point algorithm **(8 DoF)**
  - **Estimation**: `cv::findHomography()` → 1 solutions
  - **Conversion to the normalized coordinate**: $\hat{\mathrm{H}} = \mathrm{K}'^{-1}\mathrm{H}\mathrm{K}$
  - **Decomposition**: `cv::decomposeHomographyMat()` → 4 solutions
  - **Degenerate cases**: Correspondence **not** from a single plane

$$\mathrm{H} = \mathrm{K}'\hat{\mathrm{H}}\mathrm{K}^{-1}$$

$$\hat{\mathrm{H}} = \lambda\left(\mathrm{R} + \frac{1}{d}\mathbf{t}\mathbf{n}^{\top}\right) \longleftarrow \hat{\mathbf{x}}' = \mathrm{R}\hat{\mathbf{x}} + \mathbf{t} \ \text{ and } \ \frac{1}{d}\mathbf{t}\mathbf{n}^{\top} = 1 \ \ (\because n_x\hat{x} + n_y\hat{y} + n_z\hat{z} + d = 0)$$

ETRI

Visual Odometry (Epipolar) using OpenCV

```
1.  #include "opencv_all.hpp"

2.  int main(void)
3.  {
4.      bool use_5pt = false;
5.      double camera_focal = 718.8560;
6.      cv::Point2d camera_center(607.1928, 185.2157);

7.      // Open a file to write camera trajectory
8.      FILE* camera_traj = fopen("visual_odometry_epipolar.xyz", "wt");
9.      if (camera_traj == NULL) return -1;

10.     // Open an video and get the initial image
11.     cv::VideoCapture video;
12.     if (!video.open("data/KITTI_00_L/%06d.png")) return -1;

13.     cv::Mat gray_prev;
14.     video >> gray_prev;
15.     ...

16.     // Run and store monocular visual odometry
17.     cv::Mat camera_pose = cv::Mat::eye(4, 4, CV_64F);
18.     while (true)
19.     {
20.         // Grab an image from the video
21.         cv::Mat image, gray;
22.         video >> image;
23.         . . .

24.         // Extract optical flow
25.         std::vector<cv::Point2f> point_prev, point;
26.         cv::goodFeaturesToTrack(gray_prev, point_prev, 2000, 0.01, 10);
27.         std::vector<uchar> m_status;
28.         cv::Mat err;
29.         cv::calcOpticalFlowPyrLK(gray_prev, gray, point_prev, point, m_status, err);
30.         gray_prev = gray;
```

KITTI Odometry Dataset (#: 00, Left): data/KITTI_00_L/%06d.png

Visual Odometry (Epipolar) using OpenCV

```
31.         // Calculate relative pose
32.         cv::Mat E, inlier_mask;
33.         if (use_5pt)
34.         {
35.             E = cv::findEssentialMat(point_prev, point, camera_focal, camera_center, cv::RANSAC, 0.99, 1,
                                         inlier_mask);
36.         }
37.         else
38.         {
39.             cv::Mat F = cv::findFundamentalMat(point_prev, point, cv::FM_RANSAC, 1, 0.99, inlier_mask);
40.             cv::Mat K = (cv::Mat_<double>(3, 3) << camera_focal, 0, camera_center.x, 0, camera_focal,
                                                       camera_center.y, 0, 0, 1);
41.             E = K.t() * F * K;
42.         }
43.         cv::Mat R, t;
44.         int inlier_num = cv::recoverPose(E, point_prev, point, R, t, camera_focal, camera_center, inlier_mask);

45.         // Accumulate pose
46.         cv::Mat T = cv::Mat::eye(4, 4, R.type());
47.         T(cv::Range(0, 3), cv::Range(0, 3)) = R * 1.0;
48.         T(cv::Range(0, 3), cv::Range(3, 4)) = t * 1.0;
49.         camera_pose = camera_pose * T.inv();

50.         // Show the image and write camera pose
51.         if (image.channels() < 3) cv::cvtColor(image, image, CV_GRAY2RGB);
52.         for (size_t i = 0; i < point_prev.size(); i++)
53.         {
54.             if (inlier_mask.at<uchar>(i) > 0) cv::line(image, point_prev[i], point[i], cv::Scalar(0, 0, 255));
55.             else cv::line(image, point_prev[i], point[i], cv::Scalar(0, 255, 0));
56.         }
57.         cv::imshow("3DVT Tutorial: Monocular Visual Odometry", image);
58.         fprintf(camera_traj, "%.6f %.6f %.6f\n", camera_pose.at<double>(0, 3), camera_pose.at<double>(1, 3),
                                                     camera_pose.at<double>(2, 3));
59.         if (cv::waitKey(1) == 27) break; // 'ESC' key
60.     }

61.     video.release();
62.     fclose(camera_traj);
63.     return 0;
64. }
```

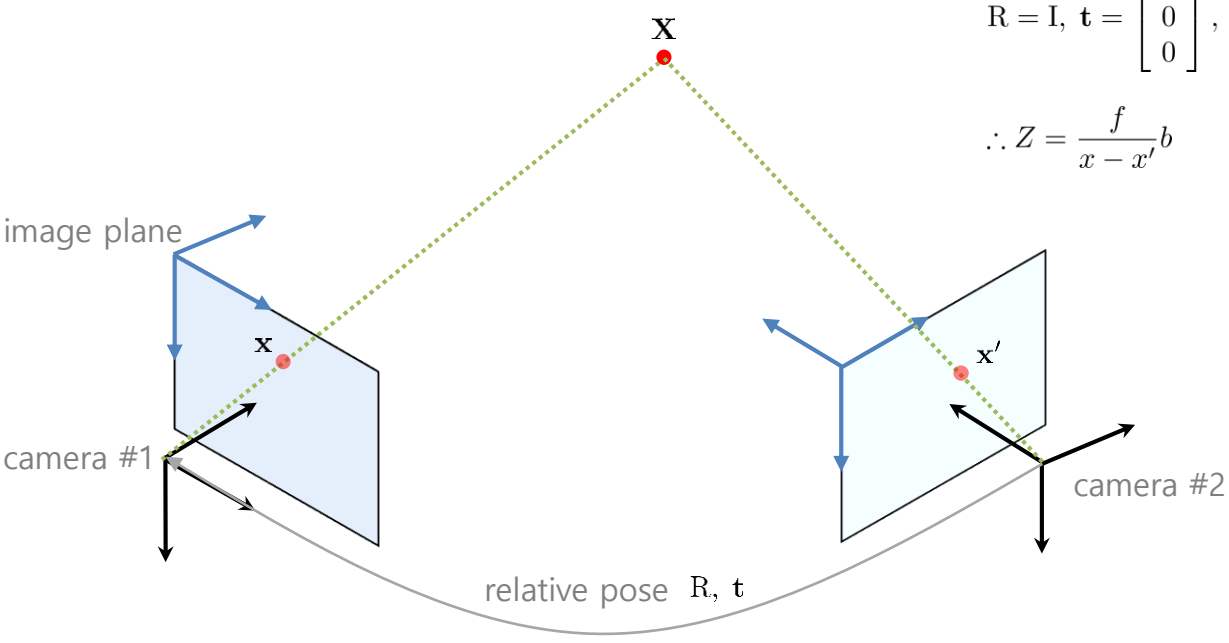
General 2D-2D Geometry (Epipolar Geometry)

- **Relative Point Localization (Triangulation)**
  - Known: Point correspondence, camera matrices, and relative pose
  - Unknown: Position of a 3D point (3 DoF)
  - Constraints: 2 x projection $\mathbf{x} = \mathrm{K}\left[\mathrm{I}\,|\,\mathbf{0}\right]\mathbf{X},\ \mathbf{x}' = \mathrm{K}'\left[\mathrm{R}\,|\,\mathbf{t}\right]\mathbf{X}$
  - Solution (OpenCV): `cv::triangulatePoints()`

Special case) Stereo cameras

$$\mathrm{R} = \mathrm{I},\ \mathbf{t} = \begin{bmatrix} -b \\ 0 \\ 0 \end{bmatrix},\ \mathrm{K} = \mathrm{K}' = \begin{bmatrix} f & 0 & c_x \\ 0 & f & c_y \\ 0 & 0 & 1 \end{bmatrix}$$

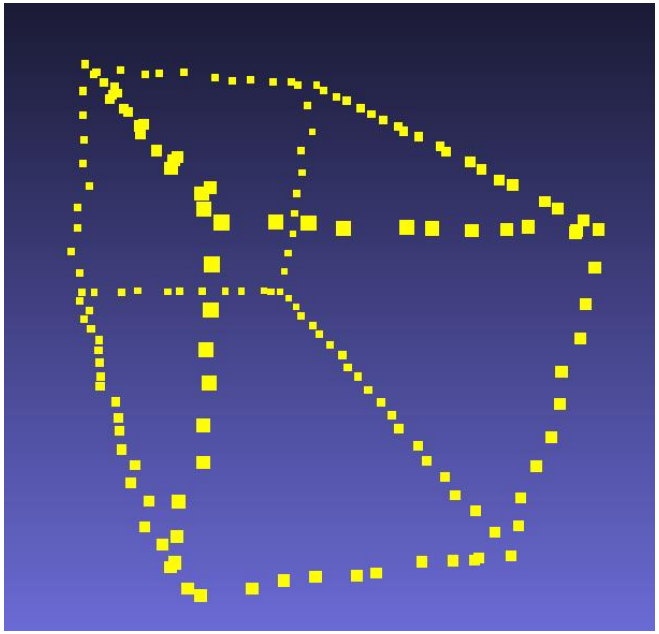
$$\therefore Z = \frac{f}{x - x'} b$$



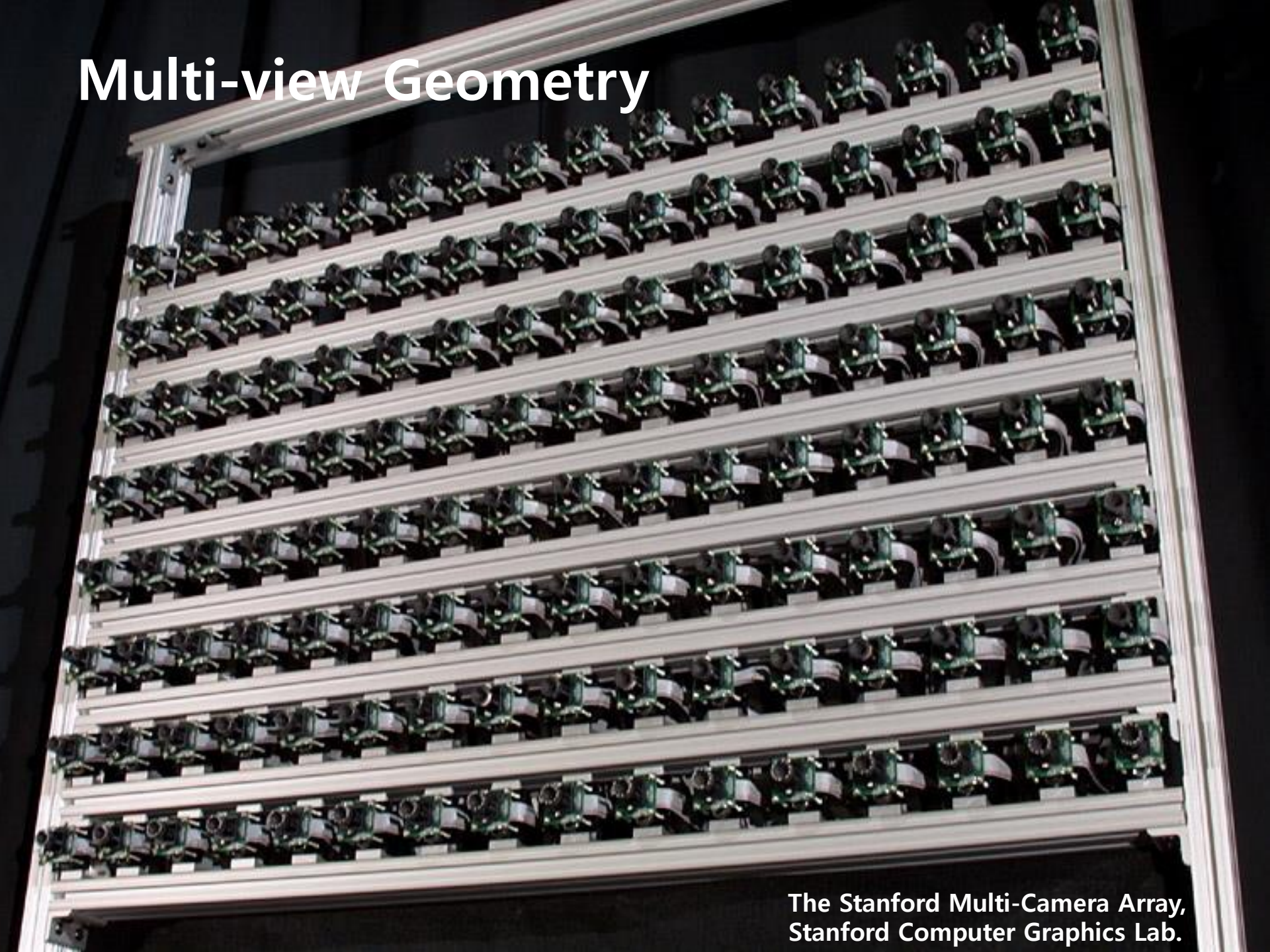
Triangulation using OpenCV


```
1.  #include "opencv_all.hpp"

2.  int main(void)
3.  {
4.      double camera_focal = 1000;
5.      cv::Point2d camera_center(320, 240);

6.      // Load two views of 'box.xyz'
7.      std::vector<cv::Point2d> points0, points1;
8.      FILE* fin0 = fopen("image_formation0.xyz", "rt");
9.      FILE* fin1 = fopen("image_formation1.xyz", "rt");
10.     ...

11.     // Esitmate relative pose of two views
12.     cv::Mat F = cv::findFundamentalMat(points0, points1, cv::FM_8POINT);
13.     cv::Mat K = (cv::Mat_<double>(3, 3) << camera_focal, ...);
14.     cv::Mat E = K.t() * F * K;
15.     cv::Mat R, t;
16.     cv::recoverPose(E, points0, points1, K, R, t);

17.     // Reconstruct 3D points of 'box.xyz' (triangulation)
18.     cv::Mat P0 = K * cv::Mat::eye(3, 4, CV_64F);
19.     cv::Mat Rt, X;
20.     cv::hconcat(R, t, Rt);
21.     cv::Mat P1 = K * Rt;
22.     cv::triangulatePoints(P0, P1, points0, points1, X);
23.     X.row(0) = X.row(0) / X.row(3);
24.     X.row(1) = X.row(1) / X.row(3);
25.     X.row(2) = X.row(2) / X.row(3);
26.     X.row(3) = 1;

27.     // Store the 3D points
28.     FILE* fout = fopen("triangulation.xyz", "wt");
29.     if (fout == NULL) return -1;
30.     for (int c = 0; c < X.cols; c++)
31.         fprintf(fout, "%f %f %f\n", X.at<double>(0, c), X.at<double>(1, c), X.at<double>(2, c));
32.     fclose(fout);
33.     return 0;
34. }
```

Result: triangulation.xyz

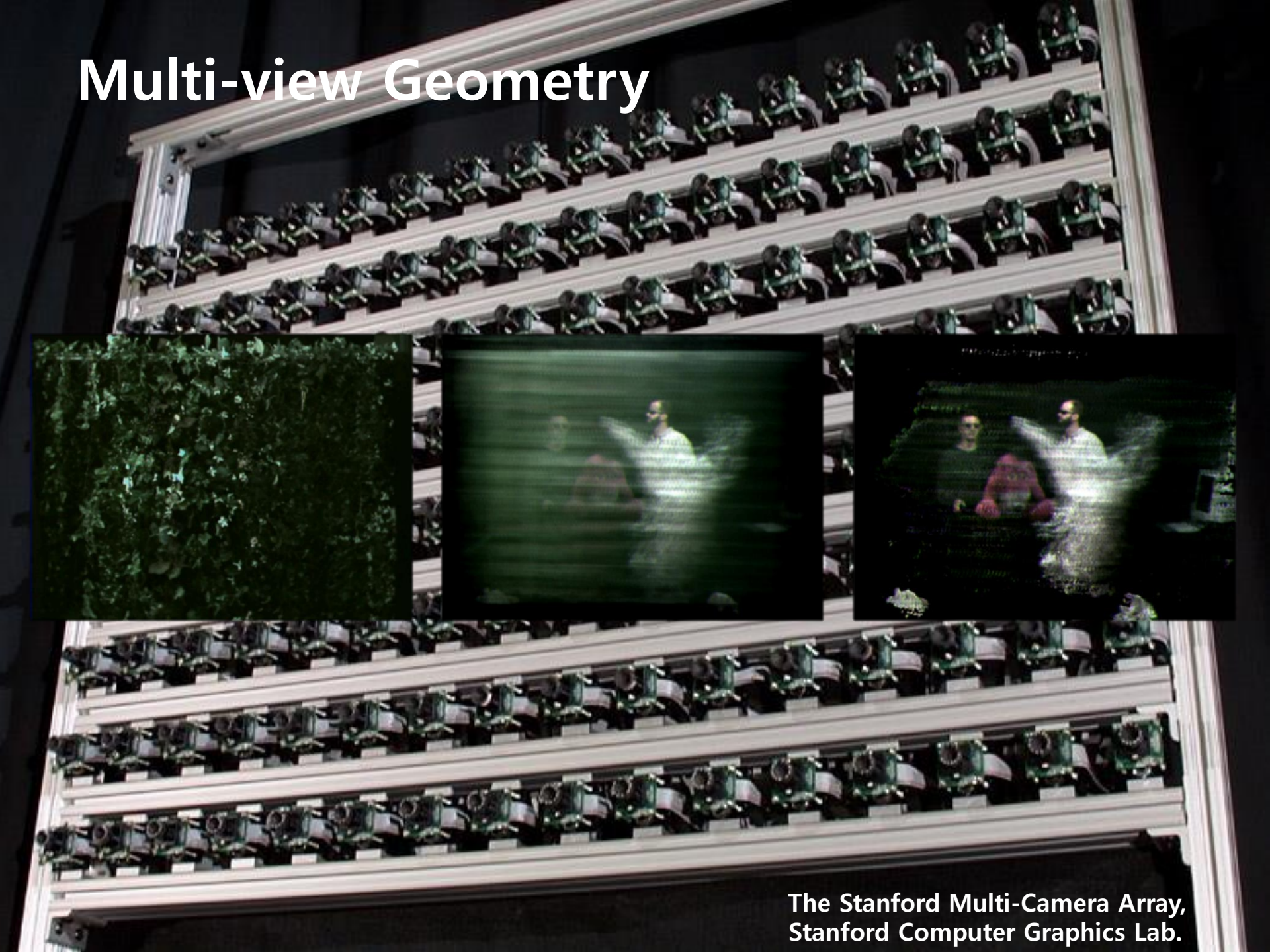


Multi-view Geometry



The Stanford Multi-Camera Array,
Stanford Computer Graphics Lab.

Multi-view Geometry

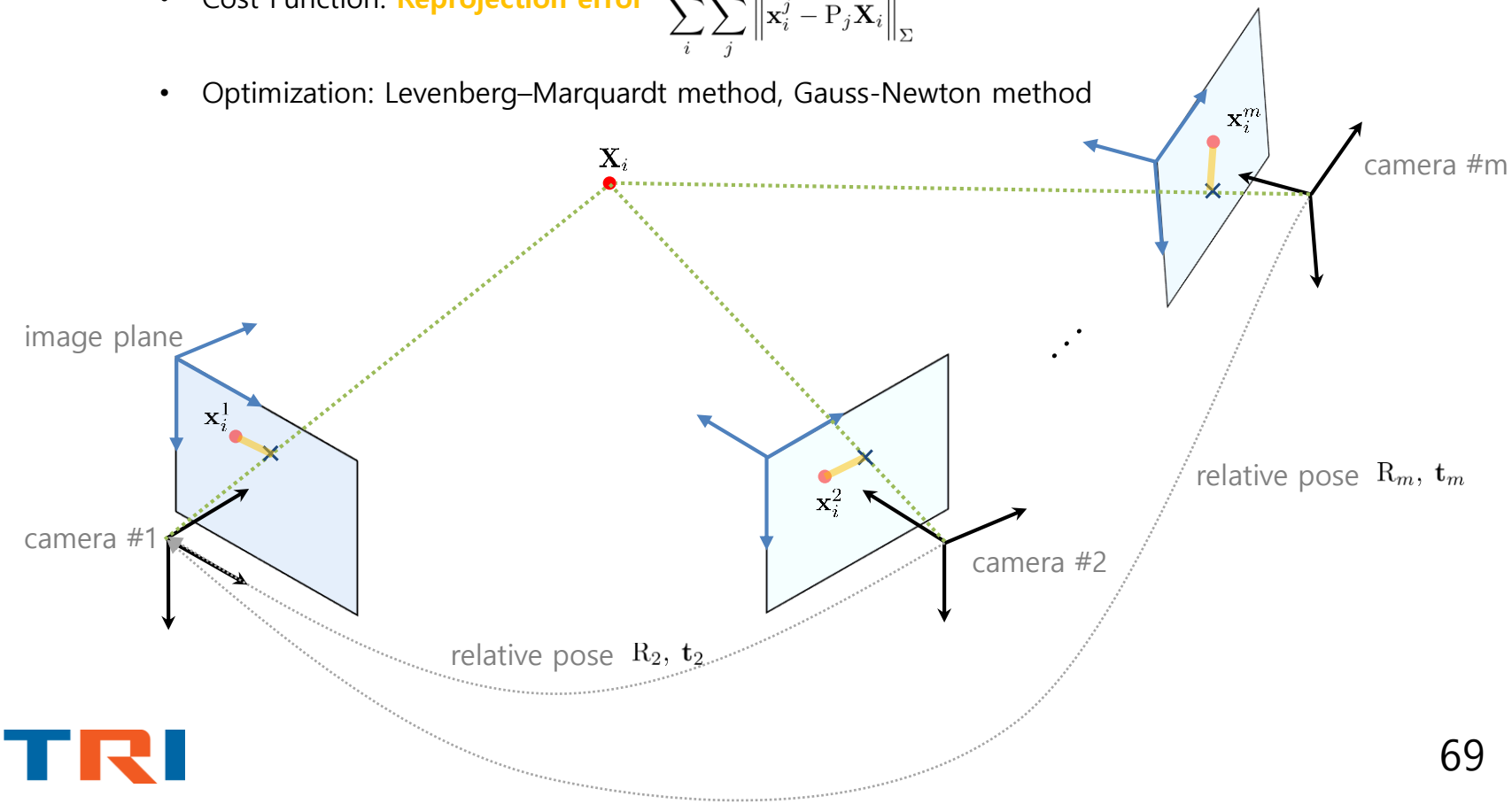


The Stanford Multi-Camera Array,
Stanford Computer Graphics Lab.

Bundle Adjustment

- **Bundle Adjustment**
  - Known: Point correspondence, camera matrices, position of 3D points, and each camera's relative pose
  - Unknown: Position of 3D points and each camera's relative pose ($6n + 3m$ DoF)
  - Constraints: n x m x projection $\mathbf{x}_i^j = \mathrm{P}_j \mathbf{X}_i = \mathrm{K}_j \left[\mathrm{R}_j \,|\, \mathbf{t}_j \right] \mathbf{X}_i$
  - Solution: **Non-linear least-square optimization**
    - Cost Function: **Reprojection error** $\sum_i^n \sum_j^m \left\| \mathbf{x}_i^j - \mathrm{P}_j \mathbf{X}_i \right\|_\Sigma^2$
    - Optimization: Levenberg–Marquardt method, Gauss-Newton method

c.f. initial values



Bundle Adjustment

- **Bundle Adjustment**

  c.f. initial values

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    - Optimization: Levenberg–Marquardt method, Gauss-Newton method

- **Bundle Adjustment and Optimization Tools**
  - OpenCV: No function (but cvsba is available as a sba wrapper.)
  - Bundle adjustment: sba (Sparse Bundle Adjustment), SSBA (Simple Sparse Bundle Adjustment), pba (Multicore Bundle Adjustment)
    - Incremental structure-from-motion: Bundler, CMPMVS, VisualSFM
  - Optimization: g2o (General Graph Optimization), iSAM (Incremental Smoothing and Mapping), GTSAM, Ceres Solver (Google)

Bundle Adjustment using cvsba


```
1.  #include "opencv_all.hpp"
2.  #include "cvsba.h"

3.  int main(void)
4.  {
5.      double camera_focal = 1000;
6.      cv::Point2d camera_center(320, 240);
7.      int n_views = 5;

8.      // Load multiple views of 'box.xyz'
9.      std::vector<std::vector<cv::Point2d> > xs;
10.     for (int i = 0; i < n_views; i++)
11.     {
12.         FILE* fin = fopen(cv::format("image_formation%d.xyz", i).c_str(), "rt");
13.         ...
14.     }

15.     // Assume that all feature points are visible
16.     std::vector<int> visible_all(xs.front().size(), 1);
17.     std::vector<std::vector<int> > visibility(n_views, visible_all);

18.     // Prepare each camera projection matrix
19.     std::vector<cv::Mat> Ks, dist_coeffs, Rs, ts;
20.     cv::Mat K = (cv::Mat_<double>(3, 3) << camera_focal, 0, camera_center.x, ...);
21.     Ks.resize(n_views, K);                                  // K for all cameras
22.     dist_coeffs.resize(n_views, cv::Mat::zeros(5, 1, CV_64F));  // dist_coeff for all cameras
23.     Rs.push_back(cv::Mat::eye(3, 3, CV_64F));               // R for the first camera (index: 0)
24.     ts.push_back(cv::Mat::zeros(3, 1, CV_64F));             // t for the first camera (index: 0)

25.     // Estimate relative pose of the inital two views (epipolar geometry)
26.     cv::Mat F = cv::findFundamentalMat(xs[0], xs[1], cv::FM_8POINT);
27.     cv::Mat E = K.t() * F * K;
28.     cv::Mat R, t;
29.     cv::recoverPose(E, xs[0], xs[1], K, R, t);
30.     Rs.push_back(R);                                        // R for the second camera
31.     ts.push_back(t);                                        // t for the second camera
```

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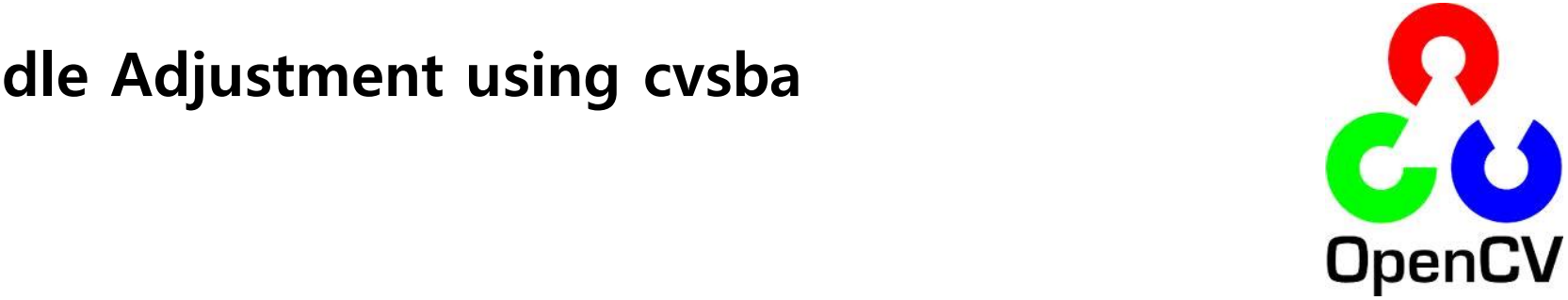
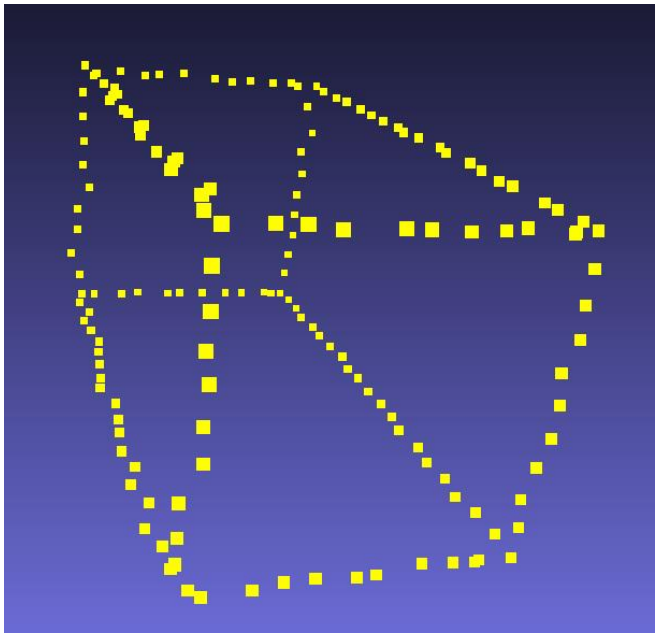
Bundle Adjustment using cvsba


```
32.     // Reconstruct the initial 3D points of 'box.xyz' (triangulation)
33.     cv::Mat P0 = K * cv::Mat::eye(3, 4, CV_64F);
34.     cv::Mat Rt, X;
35.     cv::hconcat(R, t, Rt);
36.     cv::Mat P1 = K * Rt;
37.     cv::triangulatePoints(P0, P1, xs[0], xs[1], X);
38.     std::vector<cv::Point3d> Xs;
39.     X.row(0) = X.row(0) / X.row(3);
40.     X.row(1) = X.row(1) / X.row(3);
41.     X.row(2) = X.row(2) / X.row(3);
42.     X.row(3) = 1;
43.     for (int c = 0; c < X.cols; c++)
44.         Xs.push_back(cv::Point3d(X.at<double>(0, c), X.at<double>(1, c), X.at<double>(2, c)));

45.     // Estimate the initial relative pose of other views (PnP)
46.     for (int i = 2; i < n_views; i++)
47.     {
48.         cv::Mat rvec;
49.         cv::solvePnP(Xs, xs[i], Ks[i], dist_coeffs[i], rvec, t);
50.         cv::Rodrigues(rvec, R);
51.         Rs.push_back(R);                                    // R for the third and other cameras
52.         ts.push_back(t);                                    // t for the third and other cameras
53.     }

54.     // Optimize camera pose and 3D points (bundle adjustment)
55.     try
56.     {
57.         cvsba::Sba sba;
58.         cvsba::Sba::Params param;
59.         param.type = cvsba::Sba::MOTIONSTRUCTURE;
60.         param.fixedIntrinsics = 5;
61.         param.fixedDistortion = 5;
62.         param.verbose = true;
63.         sba.setParams(param);
64.         double error = sba.run(Xs, xs, visibility, Ks, Rs, ts, dist_coeffs);
65.     }
66.     catch (cv::Exception) { }
67.     // Store the 3D points
```

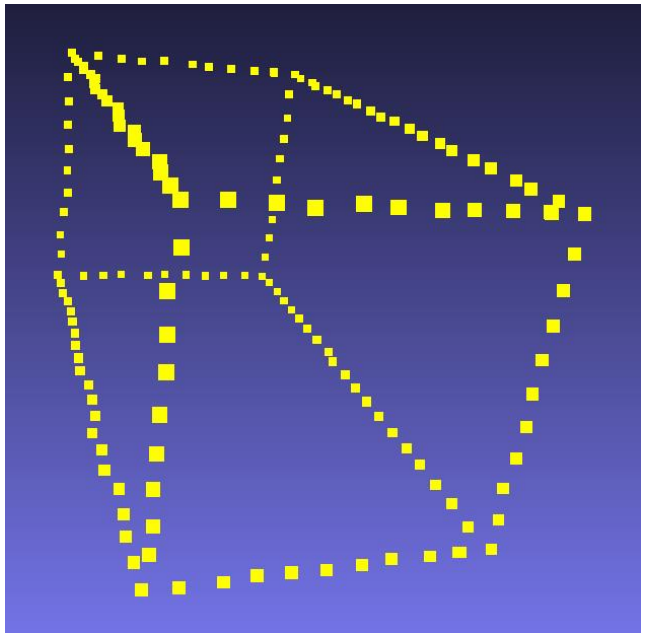
Bundle Adjustment using cvsba



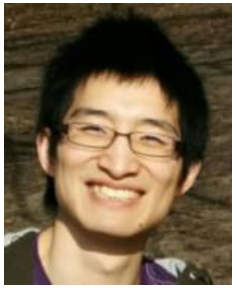
Result: triangulation.xyz

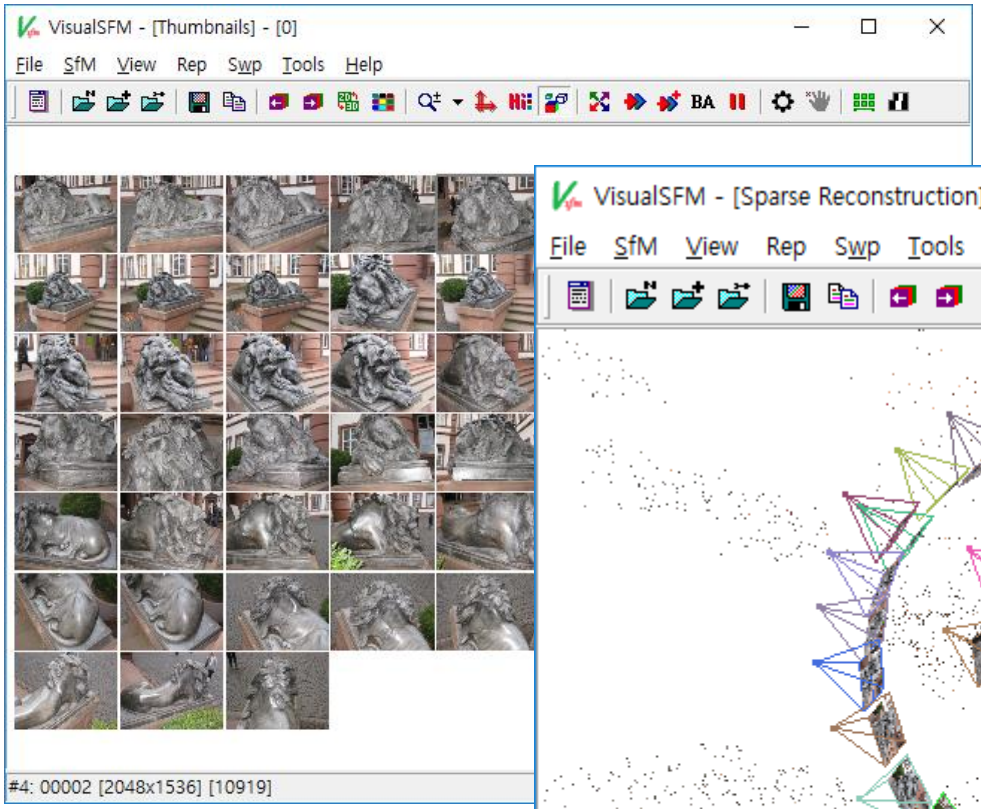


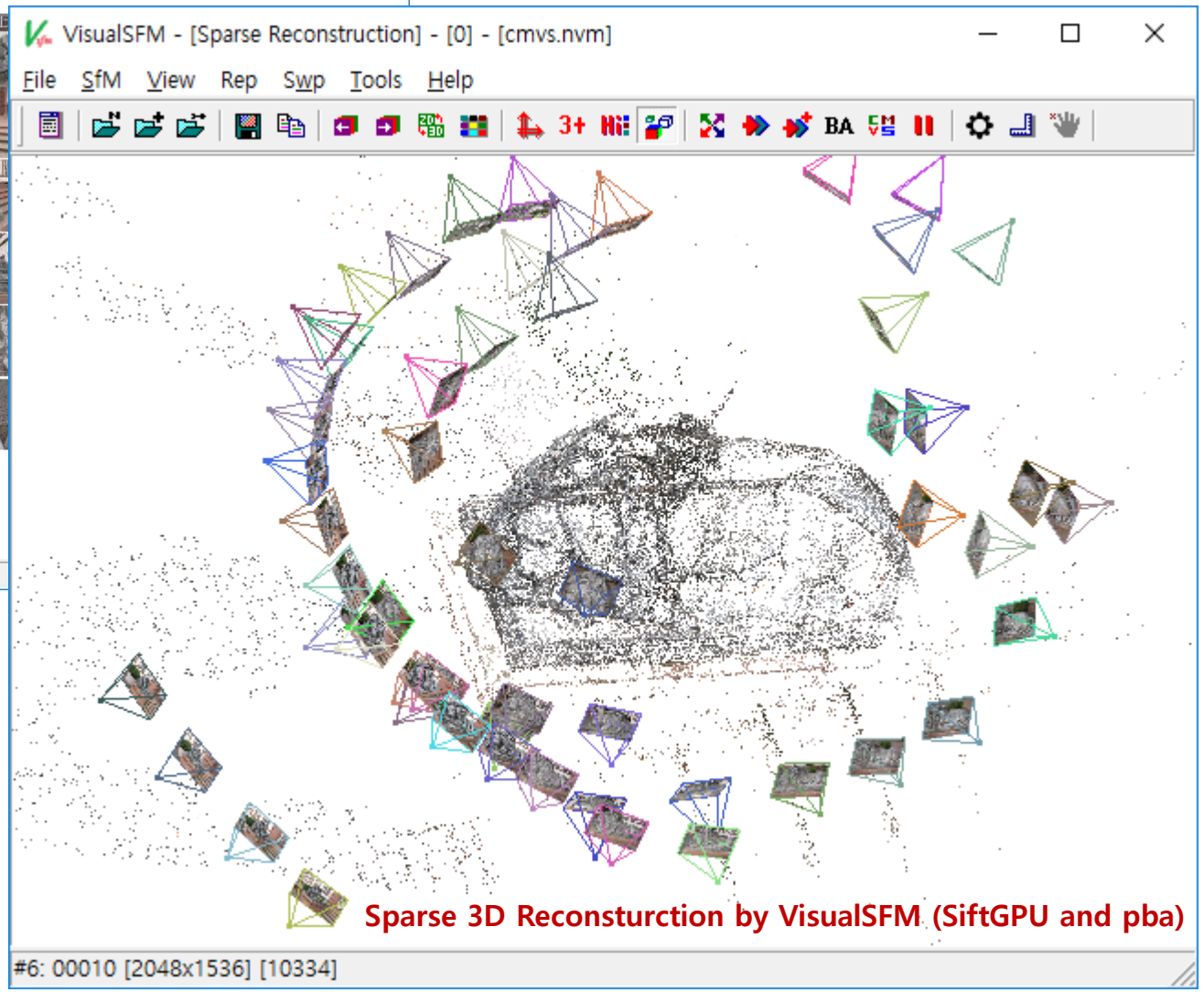
Result: bundle_adjustment.xyz



3D Reconstruction using VisualSFM







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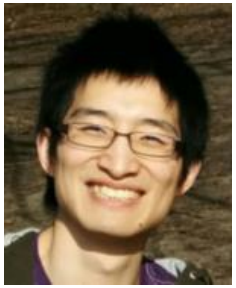
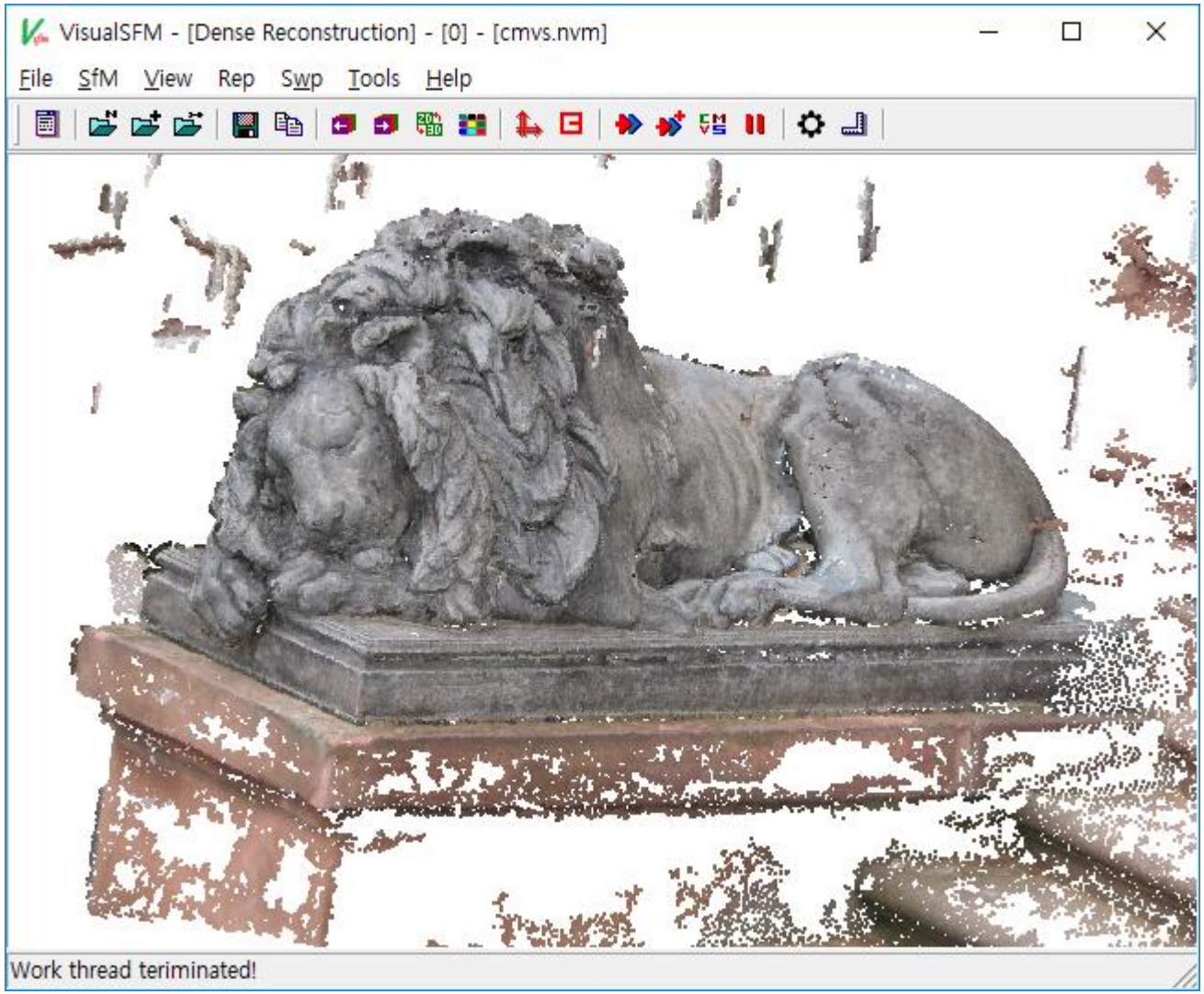
3D Reconstruction using VisualSFM





Dense 3D Reconsturction by VisualSFM (CMVS and PMVS2)

Multiple-view Geometry: Applications

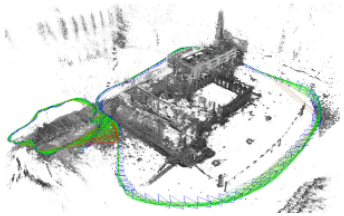
- **Structure-from-Motion (SfM)** → 3D Reconstruction, Photo Browsing
  - Photo Toursim, PhotoSynth, PhotoCity
  - Building Rome in a Day, Building Rome on a Cloudless Day
- **Visual SLAM** → Augmented Reality, Navigation (Mapping and Localization)
  - PTAM (Parallel Tracking and Mapping), ORB-SLAM, LSD-SLAM, DTAM (Dense Tracking and Mapping)
    - ORB-SLAM for Windows
- **Visual Odometry** → Navigation (Localization)
  - LIBVISO2 (C++ Library for Visual Odometry 2), SVO (Semi-direct Monocular Visual Odometry)

Structure-from-Motion

image set

Visual SLAM

image sequences



Visual Odometry

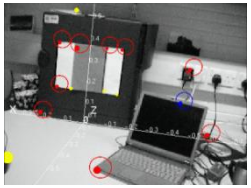
without global mapping
without loop closing

Multiple-view Geometry: Visual SLAM

- **15 Years of Visual SLAM**

SLAM in Robotics

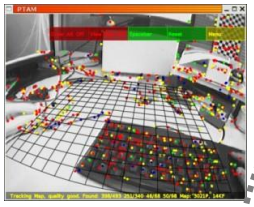
Monocular Camera



MonoSLAM (2003): sparse features, filtering

v.s.

SfM in Computer Vision

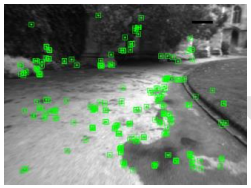


PTAM (2007): less sparse features, optimization

RGB-D Camera

KinectFusion (2011): dense volumetric

DTAM (2011): dense volumetric



ORB-SLAM (2014) v.s. **LSD-SLAM (2014)**

less sparse features

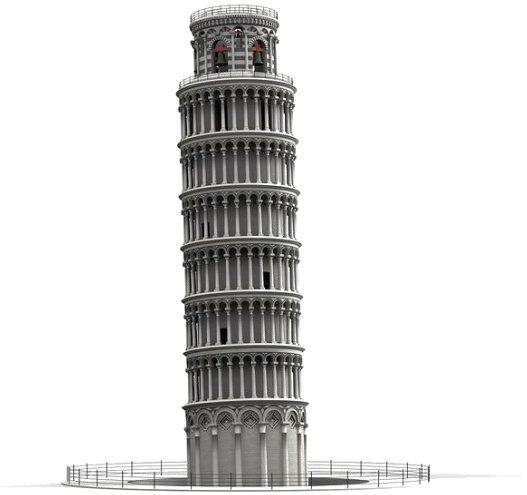
direct, semi-dense

ElasticFusion (2015) / **DynamicFusion (2015)**

dense surfels

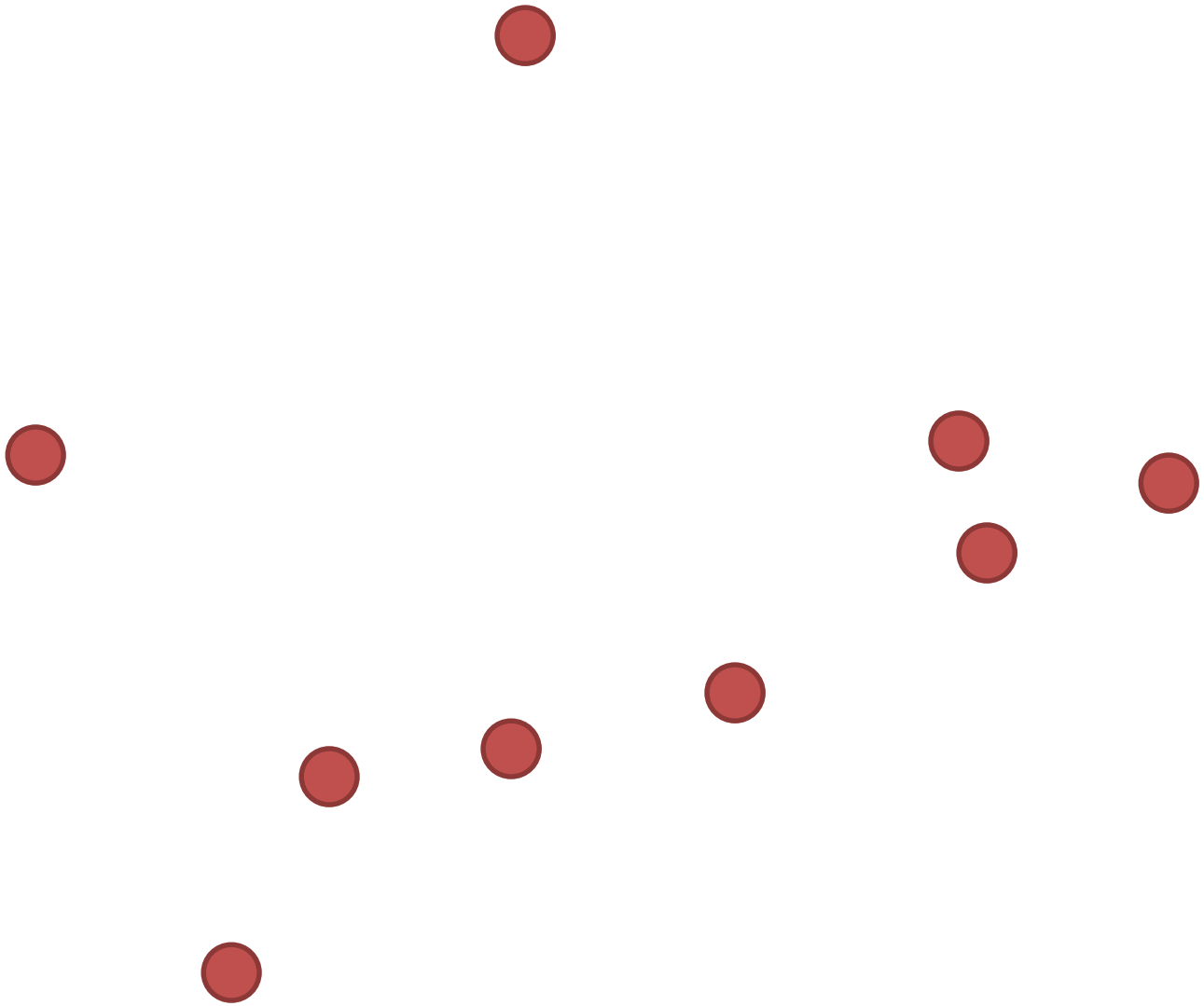
dynamic dense

Correspondence Problem

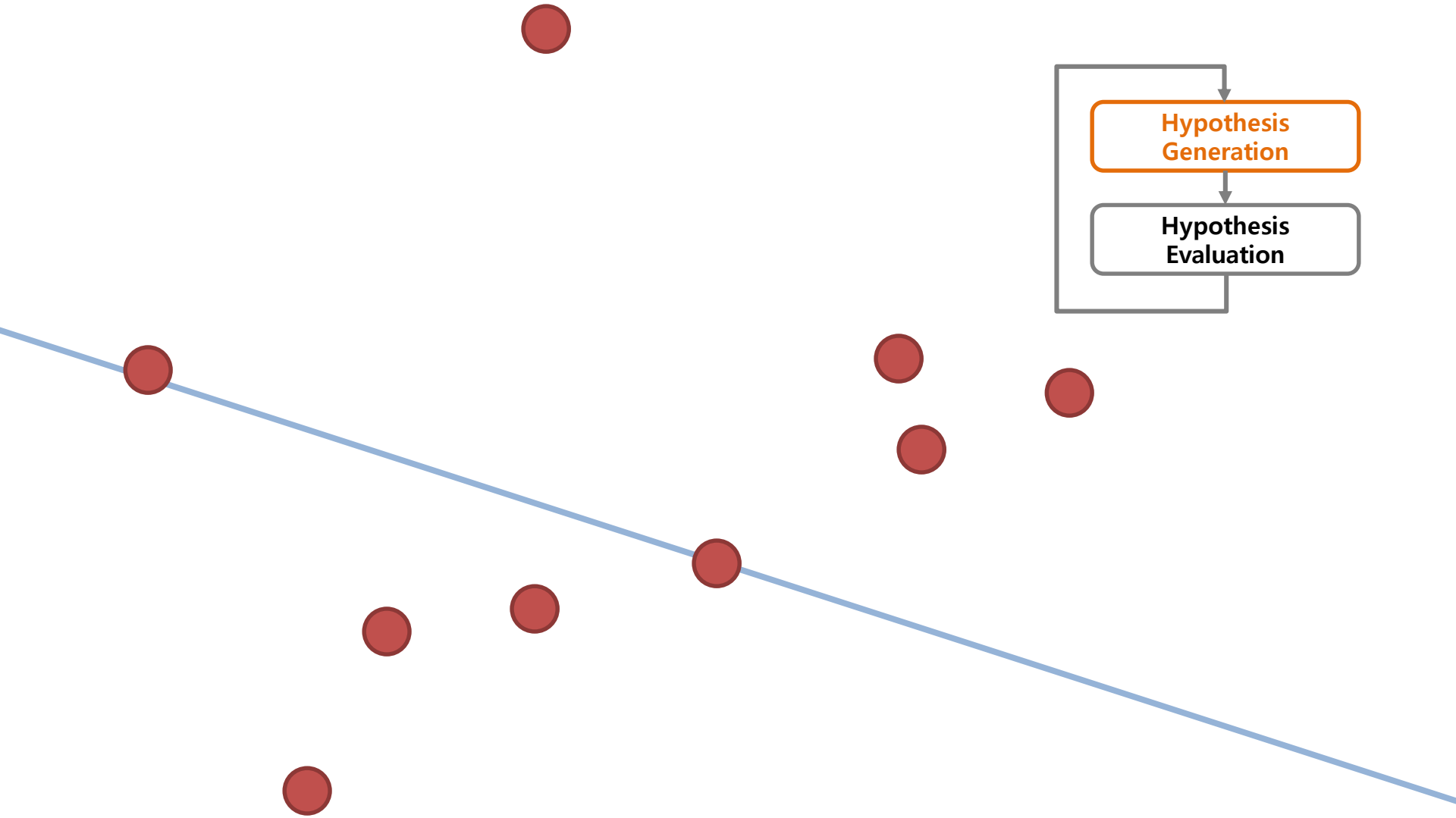


Images are from pixabay.

Random Sample Consensus (RANSAC)



Random Sample Consensus (RANSAC)



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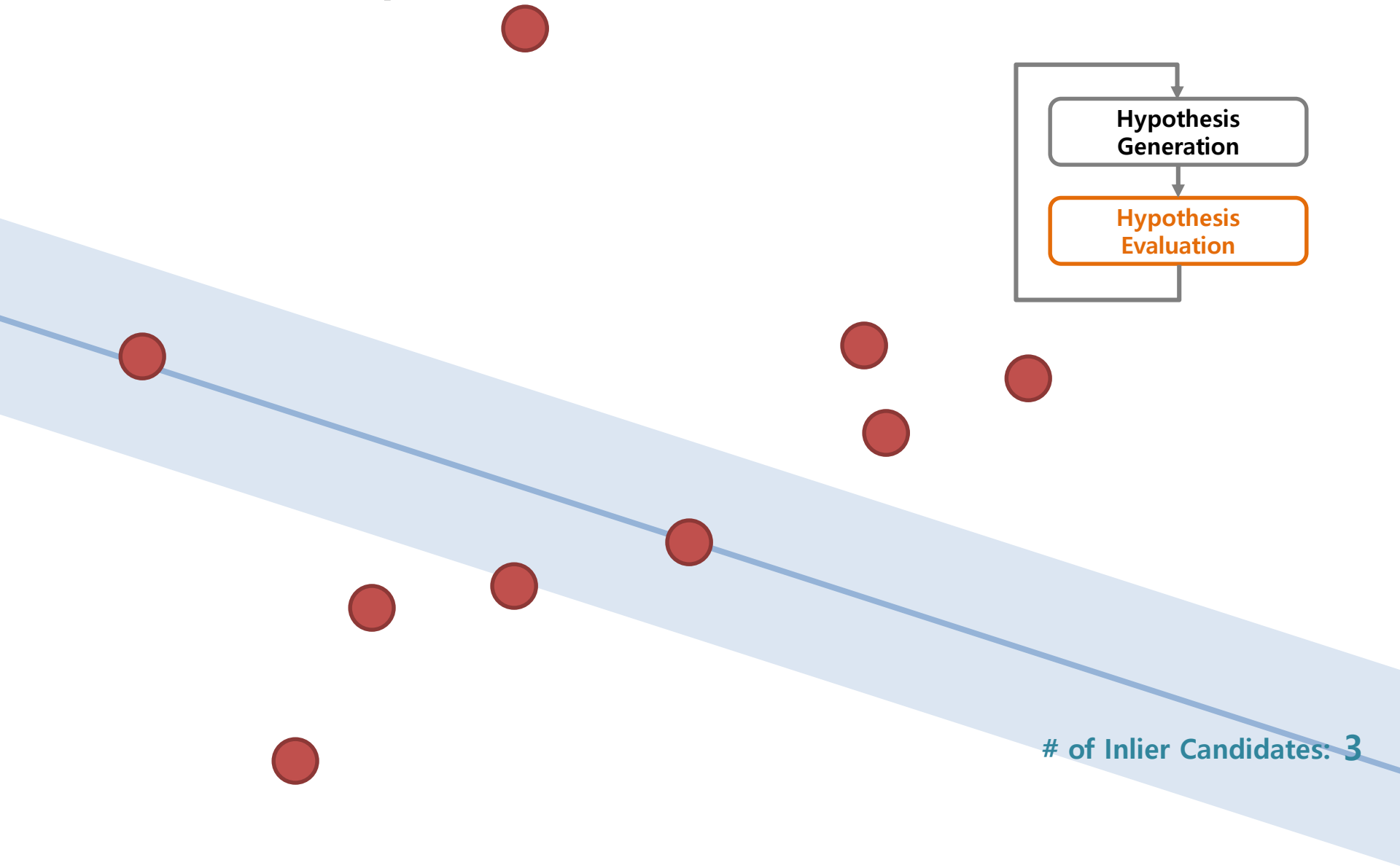

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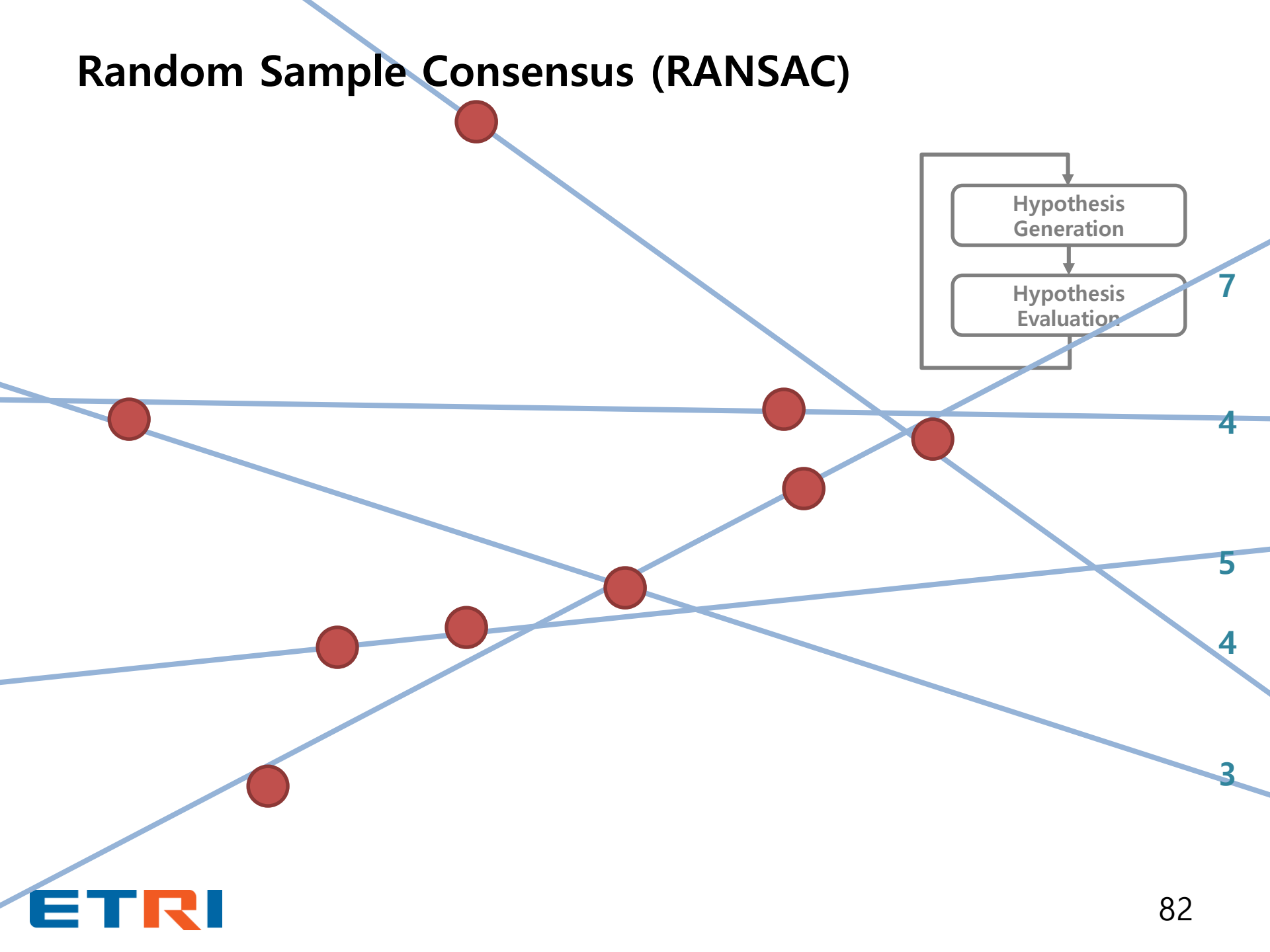


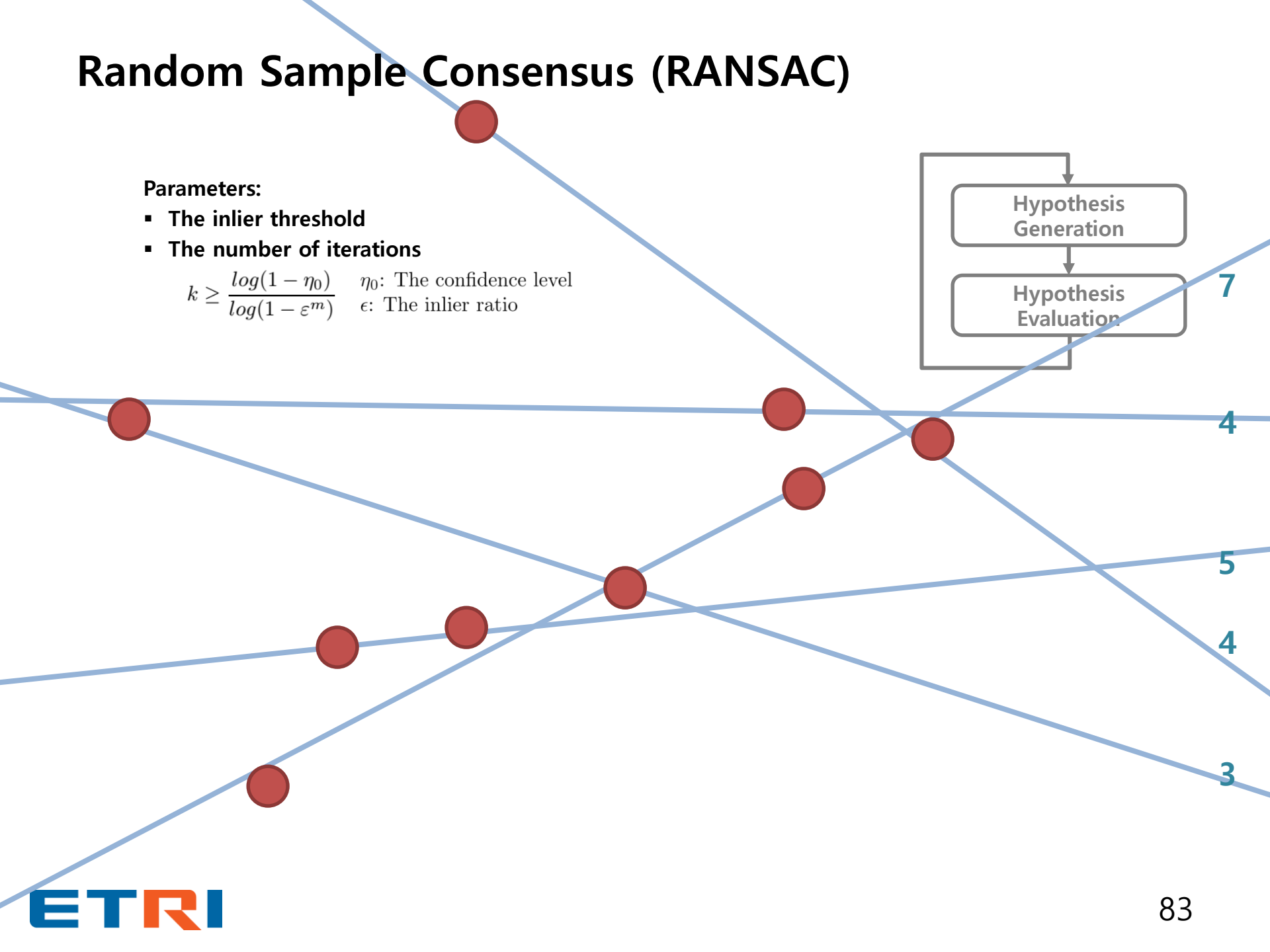
Random Sample Consensus (RANSAC)

Parameters:

- **The inlier threshold**
- **The number of iterations**

$$k \geq \frac{log(1-\eta_0)}{log(1-\varepsilon^m)}$$

η_0: The confidence level
ϵ: The inlier ratio



Line Fitting with RANSAC


```
1.  #include "opencv_all.hpp"

2.  #define CONVERT_LINE(line) (cv::Vec3d(line(0), -line(1), -line(0) * line(2) + line(1) * line(3)))

3.  int main(void)
4.  {
5.      int ransac_trial = 50;
6.      double ransac_thresh = 3.0;
7.      int ransac_n_sample = 2;
8.      int sim_n_data = 1000;
9.      double sim_inlier_ratio = 0.5, sim_inlier_noise = 1.0;
10.     cv::Vec3d truth(1.0 / sqrt(2.0), 1.0 / sqrt(2.0), -240.0); // The line model: a*x + b*y + c = 0

11.     // Generate data
12.     std::vector<cv::Point2d> data;
13.     cv::RNG rng;
14.     for (int i = 0; i < sim_n_data; i++)
15.     {
16.         if (rng.uniform(0.0, 1.0) < sim_inlier_ratio)
17.         {
18.             double x = rng.uniform(0.0, 480.0);
19.             double y = (truth(0) * x + truth(2)) / -truth(1);
20.             x += rng.gaussian(sim_inlier_noise);
21.             y += rng.gaussian(sim_inlier_noise);
22.             data.push_back(cv::Point2d(x, y)); // Inlier
23.         }
24.         else data.push_back(cv::Point2d(rng.uniform(0.0, 640.0), rng.uniform(0.0, 480.0))); // Outlier
25.     }

26.     // Estimate a line using RANSAC ...
27.     // Estimate a line using least-squares method (for reference) ...

28.     // Display estimates
29.     printf("* The Truth: %.3f, %.3f, %.3f\n", truth(0), truth(1), truth(2));
30.     printf("* Estimate (RANSAC): %.3f, %.3f, %.3f (Score: %d)\n", best_line(0), best_line(1), ..., best_score);
31.     printf("* Estimate (LSM): %.3f, %.3f, %.3f\n", lsm_line(0), lsm_line(1), lsm_line(2));

32.     return 0;
33. }
```

Line Fitting with RANSAC


```
1.      // Estimate a line using RANSAC
2.      int best_score = -1;
3.      cv::Vec3d best_line;
4.      for (int i = 0; i < ransac_trial; i++)
5.      {
6.          // Step 1: Hypothesis generation
7.          std::vector<cv::Point2d> sample;
8.          for (int j = 1; j < ransac_n_sample; j++)
9.          {
10.             int index = rng.uniform(0, data.size());
11.             sample.push_back(data[index]);
12.         }
13.         cv::Vec4d vvxy;
14.         cv::fitLine(sample, vvxy, CV_DIST_L2, 0, 0.01, 0.01);
15.         cv::Vec3d line = CONVERT_LINE(vvxy);

16.         // Step 2: Hypothesis evaluation
17.         int score = 0;
18.         for (size_t j = 0; j < data.size(); j++)
19.         {
20.             double error = fabs(line(0) * data[j].x + line(1) * data[j].y + line(2));
21.             if (error < ransac_thresh) score++;
22.         }

23.         if (score > best_score)
24.         {
25.             best_score = score;
26.             best_line = line;
27.         }
28.     }

29.     // Estimate a line using least-squares method (for reference)
30.     cv::Vec4d vvxy;
31.     cv::fitLine(data, vvxy, CV_DIST_L2, 0, 0.01, 0.01);
32.     cv::Vec3d lsm_line = CONVERT_LINE(vvxy);
```

Line Fitting Result

```
* The Truth: 0.707, 0.707, -240.000
* Estimate (RANSAC): 0.712, 0.702, -242.170 (Score: 434)
* Estimate (LSM): 0.748, 0.664, -314.997
```

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Summary: Overview

Slides and example codes are available:
https://github.com/sunglok/3dv_tutorial

- **What is 3D Vision?**
- **Single-view Geometry**
  - **Camera Projection Model**
    - A Pinhole Camera Model (**Simple 2D-3D Geometry**)
    - Homogenous Coordinates
    - Geometric Distortion and Rectification
  - **General 2D-3D Geometry**
    - Camera Calibration
    - Absolute Camera Pose Estimation (PnP Problem)
- **Two-view Geometry**
  - **Planar 2D-2D Geometry (Projective Geometry)**
    - Planar Homography
  - **General 2D-2D Geometry (Epipolar Geometry)**
    - Fundamental/Essential Matrix
    - Relative Camera Pose Estimation
    - Relative Point Localization (Triangulation)
- **Multi-view Geometry**
  - Bundle Adjustment (Non-linear Optimization)
  - Applications: Structure-from-motion, Visual SLAM, and Visual Odometry
- **Correspondence Problem**
  - Random Sample Consensus (RANSAC)

$$\therefore \mathbf{x} = \mathrm{P}\mathbf{X} \quad \left(\mathrm{P} = \mathrm{K}\left[\mathrm{R} \,|\, \mathbf{t}\right]\right)$$
$$\mathbf{x} = \mathrm{K}\hat{\mathbf{x}}$$

$$\therefore \mathbf{x}' = \mathrm{H}\mathbf{x}$$
$$\hat{\mathbf{x}}' = \hat{\mathrm{H}}\hat{\mathbf{x}} \quad \left(\hat{\mathrm{H}} = \mathrm{R} + \frac{1}{d}\mathbf{t}\mathbf{n}^\top\right)$$

$$\therefore \mathbf{x}'^\top \mathrm{F}\mathbf{x} = 0$$
$$\hat{\mathbf{x}}'^\top \mathrm{E}\hat{\mathbf{x}} = 0 \quad \left(\mathrm{E} = [\mathbf{t}]_\times \mathrm{R}\right)$$

$$\sum_i^n \sum_j^m \left\| \mathbf{x}_i^j - \mathrm{P}_j \mathbf{X}_i \right\|_\Sigma^2$$

Summary: Examples

Slides and example codes are available:
https://github.com/sunglok/3dv_tutorial

- Single-view Geometry
  - Camera Projection Model
    - Simple Camera Calibration and Object Localization
    - Image Formation: `image_formation.cpp`
    - Geometric Distortion Correction: `distortion_correction.cpp`
  - General 2D-3D Geometry
    - Camera Calibration: `camera_calibration.cpp`
    - Pose Estimation and Its Application to Augmented Reality: `pose_estimation.cpp`
- Two-view Geometry
  - Planar 2D-2D Geometry (Projective Geometry)
    - Perspective Distortion Correction: `perspective_correction.cpp`
    - Planar Image Stitching: `image_stitching.cpp`
    - 2D Video Stabilization: `video_stabilization.cpp`
  - General 2D-2D Geometry (Epipolar Geometry)
    - Monocular Visual Odometry (Epipolar Version): `visual_odometry_epipolar.cpp`
    - Triangulation (Two-view Reconstruction): `triangulation.cpp`
- Multi-view Geometry
  - Bundle Adjustment: `bundle_adjustment.cpp`
  - Sparse and Dense 3D Reconstruction using VisualSFM
- Correspondence Problem
  - Line Fitting with RANSAC: `ransac_line.cpp`