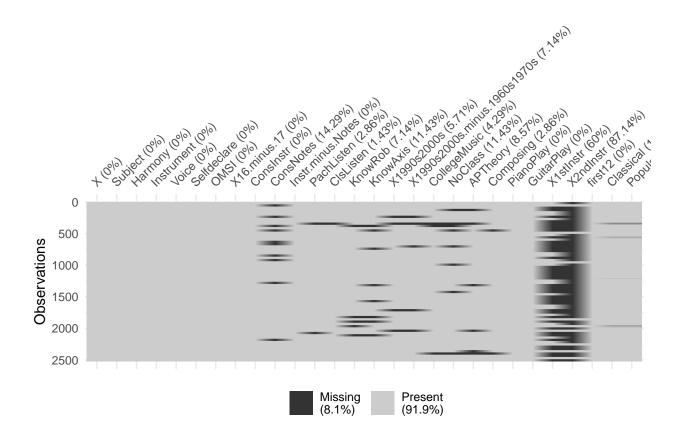
# Appendix

Mitchell Pudil 11/30/2019

## **EDA** and Variable Transformations

Let's begin by looking at the missing data



We notice there are several columns that have missing data.

A couple of the variables we will be using throughout the study are KnowRob and PachListen. However, both of these variables have many missing data points. However, nobody has both missing. We can exploit these observations and fill in the missing data by the average of the levels in the other.

First, determine the relationship between the two variables

```
summary(lm(ratings$KnowRob ~ ratings$PachListen))
```

```
##
## Call:
## lm(formula = ratings$KnowRob ~ ratings$PachListen)
##
```

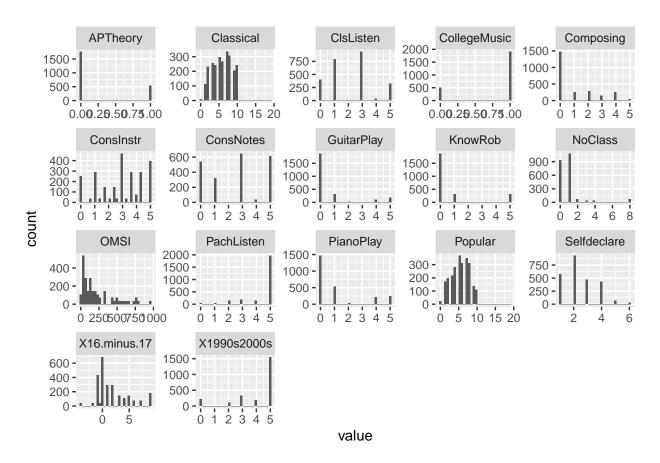
```
## Residuals:
##
      Min
               1Q Median
                              30
                                     Max
## -0.9308 -0.9308 -0.9308 0.0692 4.0692
## Coefficients:
##
                    Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                    ## ratings$PachListen 0.28805
                                0.03275 8.795 < 2e-16 ***
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 1.71 on 2266 degrees of freedom
    (252 observations deleted due to missingness)
## Multiple R-squared: 0.03301,
                                 Adjusted R-squared: 0.03258
## F-statistic: 77.35 on 1 and 2266 DF, p-value: < 2.2e-16
summary(lm(ratings$PachListen ~ ratings$KnowRob))
##
## Call:
## lm(formula = ratings$PachListen ~ ratings$KnowRob)
##
## Residuals:
      Min
               10 Median
                              3Q
                                     Max
## -4.4329 -0.0058 0.5671 0.5671 0.5671
## Coefficients:
##
                  Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                  4.43286 0.02490 178.057 <2e-16 ***
## ratings$KnowRob 0.11460
                             0.01303 8.795 <2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 1.079 on 2266 degrees of freedom
    (252 observations deleted due to missingness)
## Multiple R-squared: 0.03301,
                                 Adjusted R-squared: 0.03258
## F-statistic: 77.35 on 1 and 2266 DF, p-value: < 2.2e-16
Then fill in missing data
for(i in 1:nrow(ratings)){
 if(is.na(ratings$PachListen[i])) {
   ratings$PachListen[i] <- round(4.43 + 0.114*ratings$KnowRob[i])</pre>
 }
 if(is.na(ratings$KnowRob[i])) {
   ratings$KnowRob[i] <- max(round(-0.5 + 0.28*ratings$PachListen[i]), 0)
 }
}
```

Plot Numeric Variables

```
ratings[,-c(1:5, 11, 15, 17, 24, 25, 26)] %>%
  keep(is.numeric) %>%
  gather() %>%
  ggplot(aes(value)) +
   facet_wrap(~ key, scales = "free") +
   geom_histogram()
```

## `stat\_bin()` using `bins = 30`. Pick better value with `binwidth`.

## Warning: Removed 1278 rows containing non-finite values (stat\_bin).

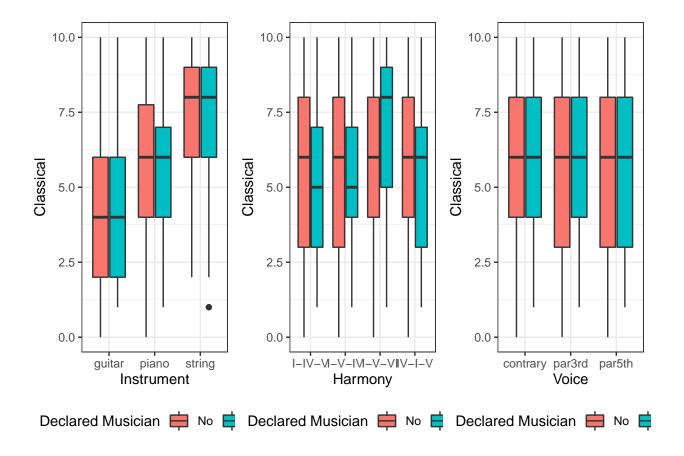


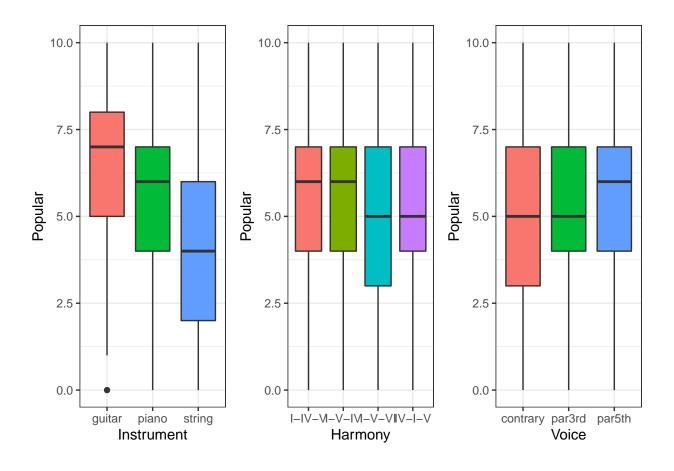
OMSI Score and classical/popular outliers

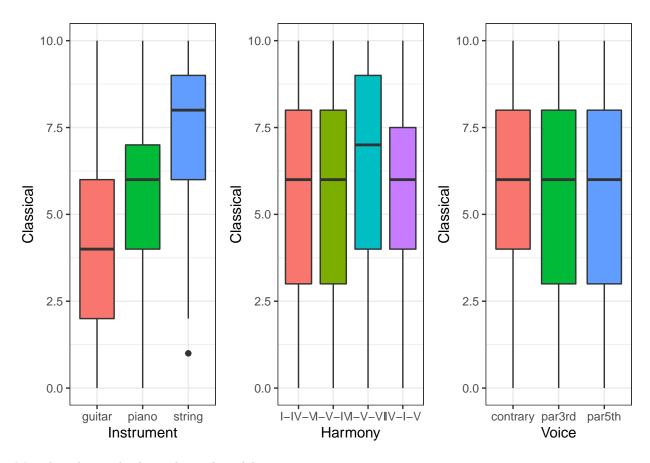
```
powerTransform(OMSI ~ 1, data=ratings) # Should use log in model

## Estimated transformation parameter
## Y1
## 0.08565721

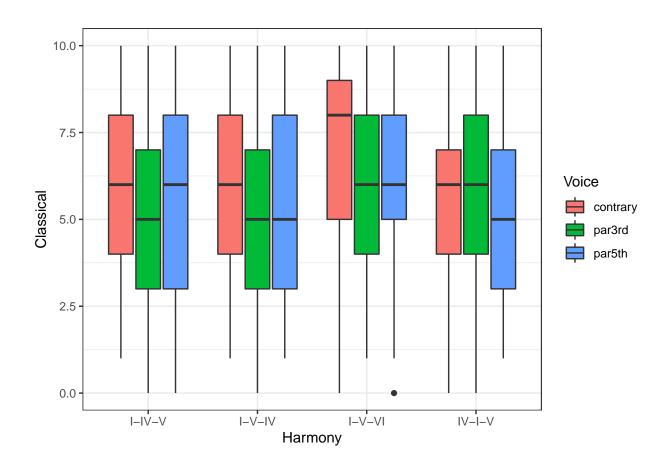
ratings$binaryrob <- ifelse(ratings$KnowRob==5, 1, 0)
ratings <- subset(ratings, Popular <= 10 & Classical <= 10)</pre>
```

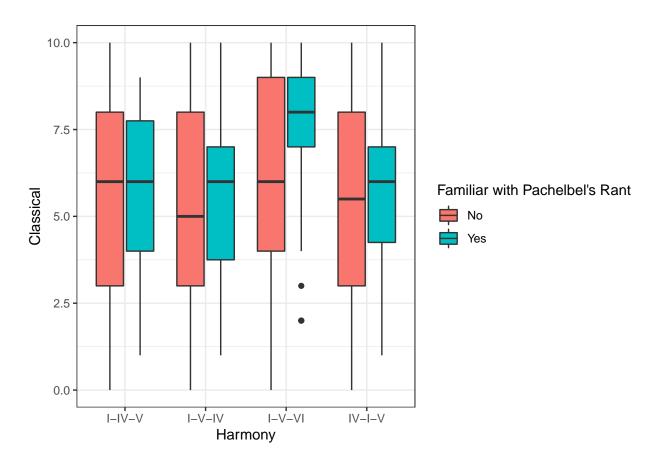




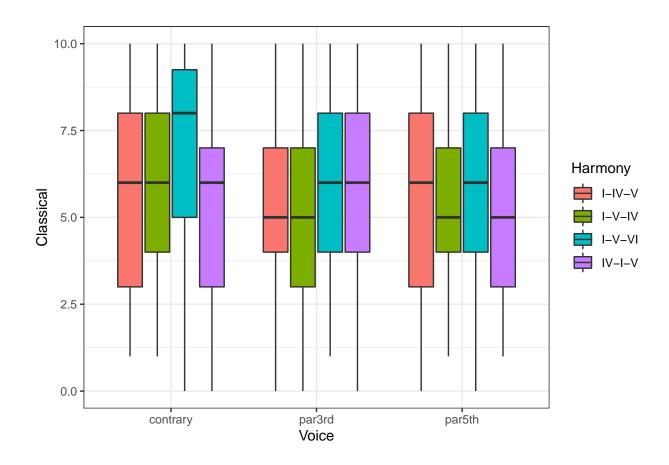


More boxplots to back up classical model





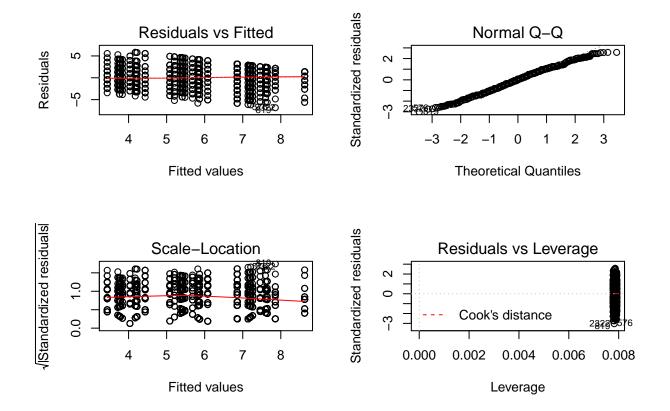
```
ggplot(ratings2, aes(x=Voice, y=Classical, fill=Harmony)) + geom_boxplot() +
theme_bw() + labs(fill= "Harmony")
```



### Classical Model

par(mfrow=c(2,2))
plot(step.model)

Note: The following is a simplified version of the steps that were taken to produce the final model. First, determine best linear model.



Determine if random intercept is important

```
## Data: ratings2
## Models:
## step.model: Classical ~ Instrument + Voice + Harmony + Voice: Harmony
## lmer.intercept.only: Classical ~ Harmony * Voice + Instrument + (1 | Subject)
                                   BIC logLik deviance Chisq Chi Df
##
                       Df
                             AIC
## step.model
                       15 7996.0 8078.3 -3983.0
                                                  7966.0
##
  lmer.intercept.only 16 7535.4 7623.2 -3751.7
                                                 7503.4 462.64
                                                                     1
##
                      Pr(>Chisq)
## step.model
## lmer.intercept.only
                       < 2.2e-16 ***
##
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

Pr(>Chisq) << 0.05, and AIC much smaller with intercept model, so we will update model.

Now compare with more random effects (note: no other covariates yet).

```
lmer.voice <- lmer(Classical ~Instrument + Harmony*Voice + (1 | Subject) +</pre>
                     (0 + Voice|Subject), data=ratings2, REML=FALSE,
                   control = lmerControl(optimizer= "bobyqa"))
## Warning: Model failed to converge with 1 negative eigenvalue: -1.2e+00
lmer.instrument <- lmer(Classical ~Instrument + Harmony*Voice + (1 | Subject) +</pre>
                          (0 + Instrument | Subject),
                        data=ratings2, REML=FALSE, control = lmerControl(optimizer= "bobyqa"))
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : unable to evaluate scaled gradient
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge: degenerate Hessian with 1
## negative eigenvalues
lmer.voice.instrument <- lmer(Classical ~ Instrument + Harmony*Voice + (1 | Subject) +</pre>
                                 (0 + Voice | Subject) +
                                 (0 + Instrument | Subject), data=ratings2, REML=FALSE,
                               control = lmerControl(optimizer = "bobyqa"))
## Warning in optwrap(optimizer, devfun, getStart(start, rho$lower, rho$pp), :
## convergence code 1 from bobyqa: bobyqa -- maximum number of function
## evaluations exceeded
lmer.voice.harmony <- lmer(Classical ~ Instrument + Harmony*Voice + (1 | Subject) +</pre>
                              (0 + Harmony | Subject) +
                              (0 + Voice | Subject), data=ratings2, REML=FALSE, control = lmerControl(opt
lmer.voice.instrument.harmony <- lmer(Classical ~ Instrument + Harmony*Voice + (1 | Subject) +</pre>
                                         (0 + Voice | Subject) +
                                       (0 + Instrument | Subject) + (0 + Harmony | Subject), data=rating
                                       control = lmerControl(optimizer = "bobyqa"))
## Warning in optwrap(optimizer, devfun, getStart(start, rho$lower, rho$pp), :
## convergence code 1 from bobyqa: bobyqa -- maximum number of function
## evaluations exceeded
## Warning: Model failed to converge with 4 negative eigenvalues: -3.3e-02
## -1.6e-01 -7.7e+00 -6.6e+01
anova (lmer.voice, lmer.instrument, lmer.voice.instrument, lmer.voice.harmony, lmer.voice.instrument.harmony
## Data: ratings2
## Models:
## lmer.voice: Classical ~ Instrument + Harmony * Voice + (1 | Subject) + (0 +
```

```
## lmer.voice:
                  Voice | Subject)
## lmer.instrument: Classical ~ Instrument + Harmony * Voice + (1 | Subject) + (0 +
                        Instrument | Subject)
## lmer.instrument:
## lmer.voice.instrument: Classical ~ Instrument + Harmony * Voice + (1 | Subject) + (0 +
                             Voice | Subject) + (0 + Instrument | Subject)
## lmer.voice.instrument:
## lmer.voice.harmony: Classical ~ Instrument + Harmony * Voice + (1 | Subject) + (0 +
## lmer.voice.harmony:
                          Harmony | Subject) + (0 + Voice | Subject)
## lmer.voice.instrument.harmony: Classical ~ Instrument + Harmony * Voice + (1 | Subject) + (0 +
## lmer.voice.instrument.harmony:
                                      Voice | Subject) + (0 + Instrument | Subject) + (0 + Harmony |
## lmer.voice.instrument.harmony:
                                      Subject)
                                       AIC
                                              BIC logLik deviance
                                                                      Chisq
                                 22 7547.2 7667.9 -3751.6
## lmer.voice
                                                           7503.2
                                                           7195.2 308.0413
                                 22 7239.2 7359.9 -3597.6
## lmer.instrument
## lmer.voice.instrument
                                28 7250.1 7403.7 -3597.1
                                                           7194.1
                                                                     1.0547
## lmer.voice.harmony
                                32 7512.9 7688.5 -3724.5
                                                           7448.9
                                                                     0.0000
## lmer.voice.instrument.harmony 38 7168.7 7377.1 -3546.3
                                                           7092.7 356.2545
##
                                Chi Df Pr(>Chisq)
## lmer.voice
## lmer.instrument
                                      0
                                            <2e-16 ***
## lmer.voice.instrument
                                      6
                                            0.9835
## lmer.voice.harmony
                                      4
                                            1.0000
## lmer.voice.instrument.harmony
                                            <2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

The lmer with the instrument random effect works best (note that some models were not shown because of non-convergence).

Add covariates

Stepwise regression to determine best model with covariates

```
# summary(final_fm)
```

Determine if we should change random effects

```
lmer.fe.voice <- lmer(Classical ~ Harmony*KnowRob + Voice + Instrument + musician +</pre>
                            PianoPlay + X16.minus.17 + ConsNotes +
                            Harmony:Voice + Harmony:musician + Instrument:musician +
                         (1 | Subject) + (0 + Voice | Subject), data=ratings2, REML=FALSE,
                         control = lmerControl(optimizer = "bobyqa"))
lmer.fe.harm <- lmer(Classical ~ Harmony*KnowRob + Voice + Instrument + musician +</pre>
                            PianoPlay + X16.minus.17 + ConsNotes +
                            Harmony:Voice + Harmony:musician + Instrument:musician +
                         (1 | Subject) + (0 + Harmony | Subject), data=ratings2, REML=FALSE,
                         control = lmerControl(optimizer = "bobyqa"))
## Warning: Model failed to converge with 1 negative eigenvalue: -1.6e+02
lmer.fe.voice.harm.inst <- lmer(Classical ~ Harmony*KnowRob + Voice + Instrument + musician +</pre>
                            PianoPlay + X16.minus.17 + ConsNotes +
                            Harmony:Voice + Harmony:musician + Instrument:musician +
                         (1 | Subject) + (0 + Harmony | Subject) + (0 + Voice | Subject) +
                           (0 + Instrument | Subject), data=ratings2, REML=FALSE,
                         control = lmerControl(optimizer = "bobyqa"))
anova(lmer.fe.int, lmer.fe.voice, lmer.fe.harm, lmer.fe.voice.harm.inst)
## Data: ratings2
## Models:
## lmer.fe.voice: Classical ~ Harmony * KnowRob + Voice + Instrument + musician +
## lmer.fe.voice:
                      PianoPlay + X16.minus.17 + ConsNotes + Harmony: Voice + Harmony: musician +
## lmer.fe.voice:
                      Instrument:musician + (1 | Subject) + (0 + Voice | Subject)
## lmer.fe.int: Classical ~ Harmony * KnowRob + Voice + Instrument + musician +
## lmer.fe.int:
                    PianoPlay + X16.minus.17 + ConsNotes + (0 + Harmony | Subject) +
## lmer.fe.int:
                    Harmony:Voice + Harmony:musician + Instrument:musician +
## lmer.fe.int:
                    (1 | Subject)
## lmer.fe.harm: Classical ~ Harmony * KnowRob + Voice + Instrument + musician +
## lmer.fe.harm:
                     PianoPlay + X16.minus.17 + ConsNotes + Harmony: Voice + Harmony: musician +
## lmer.fe.harm:
                     Instrument:musician + (1 | Subject) + (0 + Harmony | Subject)
## lmer.fe.voice.harm.inst: Classical ~ Harmony * KnowRob + Voice + Instrument + musician +
                                PianoPlay + X16.minus.17 + ConsNotes + Harmony:Voice + Harmony:musician
## lmer.fe.voice.harm.inst:
## lmer.fe.voice.harm.inst:
                                Instrument:musician + (1 | Subject) + (0 + Harmony | Subject) +
                                (0 + Voice | Subject) + (0 + Instrument | Subject)
## lmer.fe.voice.harm.inst:
##
                                 AIC
                                        BIC logLik deviance
                                                                Chisq Chi Df
                           35 7494.1 7686.1 -3712.1
## lmer.fe.voice
                                                      7424.1
## lmer.fe.int
                           39 7480.5 7694.5 -3701.3
                                                      7402.5 21.594
                                                                           4
## lmer.fe.harm
                           39 7480.5 7694.5 -3701.3
                                                      7402.5
                                                              0.000
                                                                           0
## lmer.fe.voice.harm.inst 51 7158.5 7438.3 -3528.3
                                                      7056.5 345.985
                                                                          12
##
                           Pr(>Chisq)
## lmer.fe.voice
## lmer.fe.int
                            0.0002414 ***
## lmer.fe.harm
                            1.0000000
## lmer.fe.voice.harm.inst < 2.2e-16 ***
## ---
```

```
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

It now appears that we should use voice, harmony, and instrument random effects.

#### lmer.fe.voice.harm.inst

```
## Linear mixed model fit by maximum likelihood ['lmerModLmerTest']
  Formula: Classical ~ Harmony * KnowRob + Voice + Instrument + musician +
##
       PianoPlay + X16.minus.17 + ConsNotes + Harmony: Voice + Harmony: musician +
##
       Instrument:musician + (1 | Subject) + (0 + Harmony | Subject) +
       (0 + Voice | Subject) + (0 + Instrument | Subject)
##
##
      Data: ratings2
                           logLik deviance
         AIC
                                             df.resid
##
                    BIC
##
    7158.531
              7438.320 -3528.265
                                   7056.531
                                                  1732
   Random effects:
    Groups
                                Std.Dev.
                                          Corr
##
              Name
                                1.991e-05
##
    Subject
              (Intercept)
##
    Subject.1 HarmonyI-IV-V
                                8.449e-01
##
              HarmonyI-V-IV
                                1.055e+00 0.95
              HarmonyI-V-VI
                                1.109e+00 0.54
##
                                                 0.72
##
              HarmonyIV-I-V
                                9.126e-01 0.96 0.89 0.63
##
    Subject.2 Voicecontrary
                                5.597e-01
##
              Voicepar3rd
                                5.946e-01 0.78
##
              Voicepar5th
                                4.535e-01 0.87
                                                 0.99
##
    Subject.3 Instrumentguitar 9.266e-01
##
              Instrumentpiano 1.118e+00 0.17
              Instrumentstring 9.902e-01 -0.99 -0.02
##
##
    Residual
                                1.555e+00
##
   Number of obs: 1783, groups: Subject, 50
   Fixed Effects:
##
                  (Intercept)
                                            HarmonyI-V-IV
                                                 0.163046
##
                    4.251337
##
               HarmonyI-V-VI
                                            HarmonyIV-I-V
                    0.456219
                                                -0.304244
##
##
                      KnowRob
                                              Voicepar3rd
##
                    0.062385
                                                -0.276201
                 Voicepar5th
                                          Instrumentpiano
##
##
                    -0.208396
                                                 1.886213
##
            Instrumentstring
                                                 musician
##
                    3.765677
                                                -0.281390
##
                   PianoPlay
                                             X16.minus.17
##
                    0.281173
                                                -0.067127
##
                    ConsNotes
                                   HarmonyI-V-IV:KnowRob
##
                    -0.125301
                                                 0.002246
##
       HarmonyI-V-VI:KnowRob
                                   HarmonyIV-I-V:KnowRob
##
                     0.296622
                                                 0.012698
   HarmonyI-V-IV:Voicepar3rd
                               HarmonyI-V-VI:Voicepar3rd
##
                    -0.489231
##
                                                -0.702249
   HarmonyIV-I-V:Voicepar3rd
                               HarmonyI-V-IV:Voicepar5th
##
                    0.670296
                                                -0.229224
   HarmonyI-V-VI:Voicepar5th
                               HarmonyIV-I-V:Voicepar5th
##
##
                    -0.558935
                                                 0.153321
##
      HarmonyI-V-IV:musician
                                  HarmonyI-V-VI:musician
##
                    0.032438
                                                 1.048662
```

```
##
      HarmonyIV-I-V:musician
                                Instrumentpiano: musician
##
                    0.073178
                                               -0.552774
## Instrumentstring:musician
##
                   -0.776579
## convergence code 0; 1 optimizer warnings; 0 lme4 warnings
Relevel Voice so we can compare contrary
#ratings3 <- within(ratings2, Voice <- relevel(Voice, ref = 2))</pre>
final.classical.releveled <- lmer(Classical ~ Voice + Harmony*binaryrob+ Instrument + musician +
                            PianoPlay + X16.minus.17 + ConsNotes +
                             Harmony: Voice + Harmony: musician + Instrument: musician -1 +
                          (1 | Subject) + (0 + Harmony | Subject) + (0 + Voice | Subject) +
                            (0 + Instrument | Subject), data=ratings2, REML=FALSE,
                         control = lmerControl(optimizer = "bobyqa"))
summary(final.classical.releveled)
## Linear mixed model fit by maximum likelihood . t-tests use
     Satterthwaite's method [lmerModLmerTest]
## Formula:
  Classical ~ Voice + Harmony * binaryrob + Instrument + musician +
       PianoPlay + X16.minus.17 + ConsNotes + Harmony: Voice + Harmony: musician +
       Instrument:musician - 1 + (1 | Subject) + (0 + Harmony |
##
##
       Subject) + (0 + Voice | Subject) + (0 + Instrument | Subject)
      Data: ratings2
##
## Control: lmerControl(optimizer = "bobyqa")
##
##
        AIC
                       logLik deviance df.resid
                 BTC
     7158.9
              7438.7 -3528.5
##
                                7056.9
                                            1732
##
## Scaled residuals:
       Min
                10 Median
                                 3Q
                                        Max
## -4.6199 -0.5791 0.0274 0.5502 3.4722
##
## Random effects:
   Groups
                                Variance Std.Dev. Corr
##
              Name
    Subject
              (Intercept)
                                0.0000
                                         0.0000
                               0.7012
    Subject.1 HarmonyI-IV-V
                                         0.8374
##
##
              HarmonyI-V-IV
                                1.0994
                                         1.0485
                                                  0.95
##
              HarmonyI-V-VI
                                1.2252
                                         1.1069
                                                  0.55 0.72
              HarmonyIV-I-V
                                0.8187
                                         0.9048
                                                  0.96 0.89 0.63
##
    Subject.2 Voicecontrary
                                         0.5718
##
                                0.3270
##
              Voicepar3rd
                                0.3710
                                         0.6091
                                                  0.79
              Voicepar5th
                                0.2198
                                         0.4688
                                                  0.88 0.99
##
    Subject.3 Instrumentguitar 0.8665
##
                                         0.9309
##
              Instrumentpiano 1.2835
                                         1.1329
                                                   0.19
##
              Instrumentstring 0.9769
                                         0.9884
                                                  -0.98 -0.01
  Residual
                                2.4174
                                         1.5548
## Number of obs: 1783, groups: Subject, 50
##
```

## Fixed effects:

```
##
                                Estimate Std. Error
                                                             df t value
                                                                 11.412
## Voicecontrary
                                 4.25630
                                             0.37298
                                                       75.78880
                                                       73.28222
                                                                  10.638
## Voicepar3rd
                                 3.97996
                                             0.37411
## Voicepar5th
                                             0.37009
                                                       73.74761
                                                                  10.937
                                 4.04774
## HarmonyI-V-IV
                                 0.15959
                                             0.21461
                                                      274.60966
                                                                   0.744
## HarmonyI-V-VI
                                 0.48746
                                             0.27394
                                                       99.71232
                                                                   1.779
## HarmonyIV-I-V
                                -0.30602
                                             0.20957
                                                      328.02620
                                                                  -1.460
## binaryrob
                                 0.21105
                                             0.48389
                                                       50.95506
                                                                   0.436
## Instrumentpiano
                                 1.88616
                                             0.27702
                                                       49.57512
                                                                   6.809
## Instrumentstring
                                 3.76562
                                             0.38061
                                                       49.99122
                                                                   9.894
## musician
                                -0.26233
                                             0.46140
                                                       54.25792
                                                                  -0.569
## PianoPlay
                                 0.28063
                                             0.09539
                                                       50.39704
                                                                   2.942
## X16.minus.17
                                -0.06652
                                             0.05152
                                                       50.37173
                                                                 -1.291
                                                       50.36919
## ConsNotes
                                -0.12151
                                             0.08407
                                                                  -1.445
## HarmonyI-V-IV:binaryrob
                                             0.34466
                                 0.06262
                                                       88.01617
                                                                   0.182
## HarmonyI-V-VI:binaryrob
                                 1.49544
                                             0.50601
                                                       51.24660
                                                                   2.955
## HarmonyIV-I-V:binaryrob
                                             0.32757
                                                       97.13036
                                                                   0.326
                                 0.10677
## Voicepar3rd:HarmonyI-V-IV
                                -0.48903
                                             0.25507 1474.27460
                                                                  -1.917
## Voicepar5th:HarmonyI-V-IV
                                -0.22936
                                             0.25526 1473.61199
                                                                  -0.899
## Voicepar3rd:HarmonyI-V-VI
                                -0.70261
                                             0.25516 1475.16277
                                                                  -2.754
## Voicepar5th:HarmonyI-V-VI
                                -0.55938
                                             0.25539 1474.78989
                                                                  -2.190
## Voicepar3rd:HarmonyIV-I-V
                                 0.67045
                                             0.25507 1474.64461
                                                                   2.628
                                             0.25508 1474.55014
## Voicepar5th:HarmonyIV-I-V
                                 0.15331
                                                                   0.601
## HarmonyI-V-IV:musician
                                 0.02522
                                             0.23910
                                                       85.63812
                                                                   0.105
## HarmonyI-V-VI:musician
                                 1.08171
                                             0.35287
                                                       50.79003
                                                                   3.065
## HarmonyIV-I-V:musician
                                 0.06815
                                             0.22853
                                                       96.26374
                                                                   0.298
## Instrumentpiano:musician
                                -0.55238
                                                       50.10386
                                             0.41888
                                                                  -1.319
##
  Instrumentstring:musician
                                -0.77650
                                             0.57379
                                                       49.99350
                                                                 -1.353
##
                              Pr(>|t|)
## Voicecontrary
                               < 2e-16 ***
## Voicepar3rd
                               < 2e-16 ***
## Voicepar5th
                               < 2e-16 ***
## HarmonyI-V-IV
                               0.45773
## HarmonyI-V-VI
                               0.07821
## HarmonyIV-I-V
                               0.14518
## binaryrob
                               0.66456
## Instrumentpiano
                              1.24e-08 ***
## Instrumentstring
                              2.30e-13 ***
## musician
                               0.57201
## PianoPlay
                               0.00492 **
## X16.minus.17
                               0.20254
## ConsNotes
                               0.15455
## HarmonyI-V-IV:binaryrob
                               0.85625
## HarmonyI-V-VI:binaryrob
                               0.00471 **
## HarmonyIV-I-V:binaryrob
                               0.74517
## Voicepar3rd:HarmonyI-V-IV
                               0.05540
## Voicepar5th:HarmonyI-V-IV
                               0.36904
## Voicepar3rd:HarmonyI-V-VI
                               0.00597 **
## Voicepar5th:HarmonyI-V-VI
                               0.02866 *
## Voicepar3rd:HarmonyIV-I-V
                               0.00867 **
## Voicepar5th:HarmonyIV-I-V
                               0.54793
## HarmonyI-V-IV:musician
                               0.91626
## HarmonyI-V-VI:musician
                               0.00348 **
## HarmonyIV-I-V:musician
                               0.76619
```

```
## Instrumentpiano:musician 0.19326
## Instrumentstring:musician 0.18205
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## convergence code: 0
## boundary (singular) fit: see ?isSingular
```

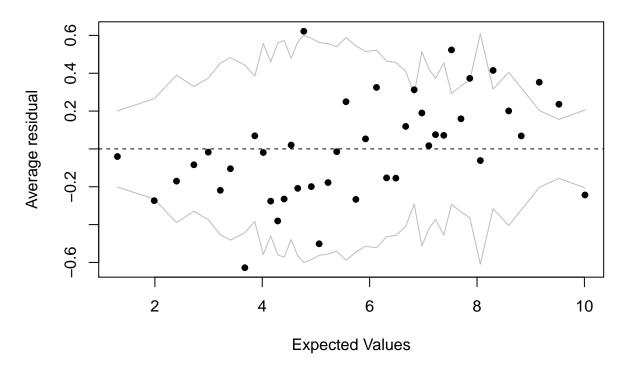
See paper for regression output.

#### Check errors for Classical Model

We'll start by looking at the binned residuals

```
modelc <- final.classical.releveled
binnedplot(fitted(modelc),resid(modelc))</pre>
```

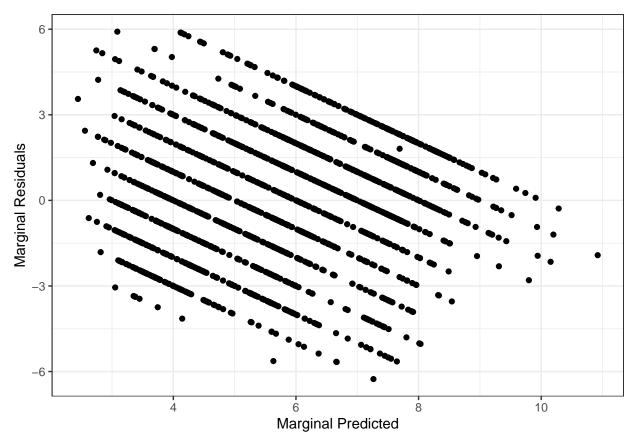
### **Binned residual plot**



It appears that the majority of the residuals are within the bin.

Next, we look at marginal fitted values vs. residuals

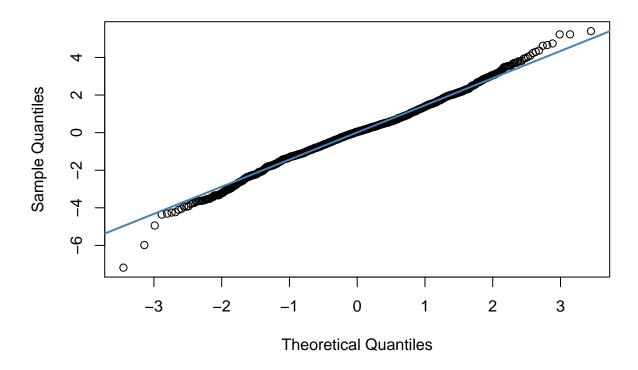
```
ggplot(mapping=aes(yhat.marg(modelc), r.marg(modelc))) + geom_point() +
labs(x="Marginal Predicted", y="Marginal Residuals") + theme_bw()
```



In the marginal models residuals plot above, we don't care about trends, but are more focused on the spread of the points. Therefore, the marginal residuals plot above looks good.

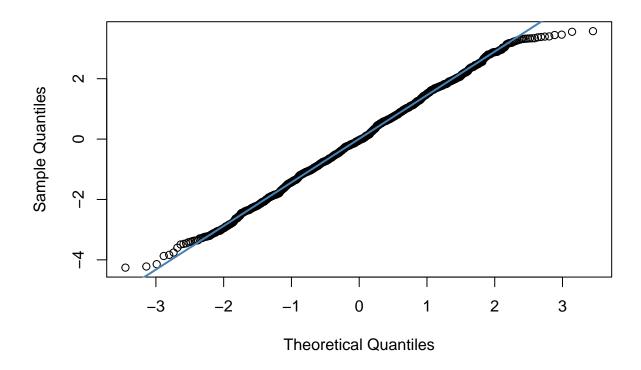
Now we will look at the QQ plot for conditional residuals.

```
qqnorm(r.cond(modelc))
qqline(r.reff(modelc), col = "steelblue", lwd = 2)
```



And the QQ plot for the random effects

```
qqnorm(r.reff(modelc))
qqline(r.reff(modelc), col = "steelblue", lwd = 2)
```



Both QQ plots look linear, suggesting normality of residuals.

We now move to determining the popular ratings model.

### Popular Ratings Model

We'll start by running a stepwise regression to determine the optimal fixed effects.

```
ConsInstr + ConsNotes + X1990s2000s + CollegeMusic + NoClass +
   APTheory + Composing + Harmony:musician + Instrument:musician +
    KnowRob:PachListen + (1 | Subject) + (0 + Harmony | Subject), data=ratings2,
                     control= lmerControl(optimizer = "bobyga"), REML = FALSE)
## boundary (singular) fit: see ?isSingular
## Warning: Model failed to converge with 1 negative eigenvalue: -2.9e+00
lmer.pop.voice <- lmer(Popular ~ Harmony + Voice + Instrument + musician + KnowRob +</pre>
   PachListen + log(OMSI) + PianoPlay + GuitarPlay + X16.minus.17 +
   ConsInstr + ConsNotes + X1990s2000s + CollegeMusic + NoClass +
    APTheory + Composing + Harmony:musician + Instrument:musician +
    KnowRob:PachListen + (1 | Subject) + (0 + Voice | Subject), data=ratings2,
                     control= lmerControl(optimizer = "bobyqa"), REML = FALSE)
## boundary (singular) fit: see ?isSingular
lmer.pop.harm.inst <- lmer(Popular ~ Harmony + Voice + Instrument + musician + KnowRob +</pre>
   PachListen + log(OMSI) + PianoPlay + GuitarPlay + X16.minus.17 +
   ConsInstr + ConsNotes + X1990s2000s + CollegeMusic + NoClass +
    APTheory + Composing + Harmony:musician + Instrument:musician +
   KnowRob:PachListen + (1 | Subject) + (0 + Harmony | Subject) +
                       (0 + Instrument | Subject), data=ratings2,
                     control= lmerControl(optimizer = "bobyqa"), REML = FALSE)
## boundary (singular) fit: see ?isSingular
## Warning: Model failed to converge with 2 negative eigenvalues: -1.8e-05
## -8.4e+00
lmer.pop.harm.voice <- lmer(Popular ~ Harmony + Voice + Instrument + musician + KnowRob +</pre>
   PachListen + log(OMSI) + PianoPlay + GuitarPlay + X16.minus.17 +
    ConsInstr + ConsNotes + X1990s2000s + CollegeMusic + NoClass +
   APTheory + Composing + Harmony:musician + Instrument:musician +
   KnowRob:PachListen +
                       (0 + Voice | Subject), data=ratings2,
                     control= lmerControl(optimizer = "bobyqa"), REML = FALSE)
## boundary (singular) fit: see ?isSingular
lmer.pop.voice.inst <- lmer(Popular ~ Harmony + Voice + Instrument + musician + KnowRob +</pre>
   PachListen + log(OMSI) + PianoPlay + GuitarPlay + X16.minus.17 +
    ConsInstr + ConsNotes + X1990s2000s + CollegeMusic + NoClass +
   APTheory + Composing + Harmony:musician + Instrument:musician +
    KnowRob:PachListen +
                       (0 + Instrument | Subject), data=ratings2,
                     control= lmerControl(optimizer = "bobyqa"), REML = FALSE)
```

There were a few models that failed to converge, but we can still compare the models that did converge.

```
## Data: ratings2
## Models:
## lmer.pop.int: Popular ~ Harmony + Voice + Instrument + musician + KnowRob +
## lmer.pop.int:
                     PachListen + log(OMSI) + PianoPlay + GuitarPlay + X16.minus.17 +
## lmer.pop.int:
                     ConsInstr + ConsNotes + X1990s2000s + CollegeMusic + NoClass +
                     APTheory + Composing + Harmony:musician + Instrument:musician +
## lmer.pop.int:
## lmer.pop.int:
                     KnowRob:PachListen + (1 | Subject)
## lmer.pop.harm.voice: Popular ~ Harmony + Voice + Instrument + musician + KnowRob +
## lmer.pop.harm.voice:
                            PachListen + log(OMSI) + PianoPlay + GuitarPlay + X16.minus.17 +
## lmer.pop.harm.voice:
                            ConsInstr + ConsNotes + X1990s2000s + CollegeMusic + NoClass +
## lmer.pop.harm.voice:
                            APTheory + Composing + Harmony:musician + Instrument:musician +
                            KnowRob:PachListen + (0 + Voice | Subject)
## lmer.pop.harm.voice:
## lmer.pop.voice.inst: Popular ~ Harmony + Voice + Instrument + musician + KnowRob +
## lmer.pop.voice.inst:
                            PachListen + log(OMSI) + PianoPlay + GuitarPlay + X16.minus.17 +
                            ConsInstr + ConsNotes + X1990s2000s + CollegeMusic + NoClass +
## lmer.pop.voice.inst:
## lmer.pop.voice.inst:
                            APTheory + Composing + Harmony: musician + Instrument: musician +
                            KnowRob:PachListen + (0 + Instrument | Subject)
## lmer.pop.voice.inst:
## lmer.pop.voice: Popular ~ Harmony + Voice + Instrument + musician + KnowRob +
## lmer.pop.voice:
                       PachListen + log(OMSI) + PianoPlay + GuitarPlay + X16.minus.17 +
## lmer.pop.voice:
                       ConsInstr + ConsNotes + X1990s2000s + CollegeMusic + NoClass +
## lmer.pop.voice:
                       APTheory + Composing + Harmony:musician + Instrument:musician +
## lmer.pop.voice:
                       KnowRob:PachListen + (1 | Subject) + (0 + Voice | Subject)
## lmer.pop.harm: Popular ~ Harmony + Voice + Instrument + musician + KnowRob +
## lmer.pop.harm:
                      PachListen + log(OMSI) + PianoPlay + GuitarPlay + X16.minus.17 +
                      ConsInstr + ConsNotes + X1990s2000s + CollegeMusic + NoClass +
## lmer.pop.harm:
## lmer.pop.harm:
                      APTheory + Composing + Harmony: musician + Instrument: musician +
## lmer.pop.harm:
                      KnowRob:PachListen + (1 | Subject) + (0 + Harmony | Subject)
## lmer.pop.harm.inst: Popular ~ Harmony + Voice + Instrument + musician + KnowRob +
                           PachListen + log(OMSI) + PianoPlay + GuitarPlay + X16.minus.17 +
## lmer.pop.harm.inst:
                           ConsInstr + ConsNotes + X1990s2000s + CollegeMusic + NoClass +
## lmer.pop.harm.inst:
## lmer.pop.harm.inst:
                           APTheory + Composing + Harmony:musician + Instrument:musician +
                           KnowRob:PachListen + (1 | Subject) + (0 + Harmony | Subject) +
## lmer.pop.harm.inst:
## lmer.pop.harm.inst:
                           (0 + Instrument | Subject)
##
                       Df
                             AIC
                                    BIC logLik deviance
                                                           Chisq Chi Df
## lmer.pop.int
                       30 7525.8 7690.3 -3732.9
                                                  7465.8
## lmer.pop.harm.voice 35 7532.6 7724.6 -3731.3
                                                  7462.6
                                                           3.166
                                                                       5
## lmer.pop.voice.inst 35 7306.7 7498.7 -3618.4
                                                  7236.7 225.883
                                                                       0
## lmer.pop.voice
                       36 7534.6 7732.1 -3731.3
                                                  7462.6
                                                           0.000
                                                                       1
## lmer.pop.harm
                       40 7510.9 7730.3 -3715.4
                                                  7430.9 31.736
                                                                       4
## lmer.pop.harm.inst 46 7262.3 7514.6 -3585.1
                                                  7170.3 260.592
                                                                       6
                       Pr(>Chisq)
##
## lmer.pop.int
## lmer.pop.harm.voice
                           0.6744
## lmer.pop.voice.inst
                        < 2.2e-16 ***
## lmer.pop.voice
                           1.0000
## lmer.pop.harm
                        2.166e-06 ***
## lmer.pop.harm.inst
                        < 2.2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

The smallest BIC overall appears to be the one where we used voice and instrument random effects. Let's

do one more check with fixed effects to see if there are any more variables we should drop.

```
step fm <- step(lmer.pop.voice.inst)</pre>
final_fm <- get_model(step_fm)</pre>
# summary(final_fm)
final fm
## Linear mixed model fit by maximum likelihood ['lmerModLmerTest']
## Formula: Popular ~ Harmony + Instrument + musician + (0 + Instrument |
##
       Subject) + Harmony:musician
##
      Data: ratings2
##
         AIC
                   BIC
                          logLik deviance
  7285.833 7379.096 -3625.917 7251.833
                                                 1766
##
## Random effects:
  Groups
            Name
                              Std.Dev. Corr
##
## Subject Instrumentguitar 1.102
             Instrumentpiano 1.506
##
                                        0.54
             Instrumentstring 1.710
                                        0.22 0.73
##
##
  Residual
                              1.705
## Number of obs: 1783, groups: Subject, 50
```

HarmonyI-V-IV

Instrumentpiano

-0.13095

-1.09293

musician HarmonyI-V-IV:musician HarmonyI-V-VI:musician

0.18816

## Fixed Effects:

## HarmonyIV-I-V:musician

(Intercept)

HarmonyIV-I-V

6.84581

-0.25397

0.13023

0.04709

##

##

##

##

##

##

The stepwise regression model suggests to only use the variables Harmony, Voice, musician, Instrument, and the interaction Harmony:musician.

Since adding back in Voice and Harmony random effects would cause negative eigenvalues, the final model then is:

HarmonyI-V-VI

Instrumentstring

0.08099

-2.76959

-0.75599

```
## Linear mixed model fit by maximum likelihood . t-tests use
     Satterthwaite's method [lmerModLmerTest]
## Formula: Popular ~ Voice + Harmony * musician + Instrument - 1 + (1 |
       Subject) + (0 + Voice | Subject) + (0 + Instrument | Subject)
##
##
      Data: ratings2
## Control: lmerControl(optimizer = "bobyqa")
##
##
        AIC
                       logLik deviance df.resid
##
     7296.3
              7438.9 -3622.2
                               7244.3
                                            1757
##
## Scaled residuals:
```

```
10 Median
                               3Q
## -3.5708 -0.6051 0.0168 0.6162 3.2564
##
## Random effects:
##
   Groups
                              Variance Std.Dev.
##
   Subject
                              1.863e-11 4.317e-06
              (Intercept)
##
    Subject.1 Voicecontrary
                              1.105e+00 1.051e+00
##
              Voicepar3rd
                              8.106e-01 9.003e-01 1.00
##
              Voicepar5th
                              1.099e+00 1.049e+00 0.99 1.00
##
   Subject.2 Instrumentguitar 1.956e-01 4.423e-01
##
              Instrumentpiano 1.359e+00 1.166e+00 -0.13
              Instrumentstring 2.008e+00 1.417e+00 -0.87 0.59
##
##
   Residual
                               2.889e+00 1.700e+00
## Number of obs: 1783, groups: Subject, 50
##
## Fixed effects:
##
                           Estimate Std. Error
                                                       df t value Pr(>|t|)
## Voicecontrary
                          6.776e+00 2.456e-01 8.454e+01
                                                           27.591 < 2e-16
                                                           29.773
## Voicepar3rd
                          6.941e+00 2.331e-01 7.804e+01
                                                                   < 2e-16
## Voicepar5th
                          6.990e+00 2.453e-01
                                                8.194e+01
                                                           28.494
## HarmonyI-V-IV
                         -1.310e-01 1.514e-01
                                                1.584e+03
                                                           -0.865
                                                                   0.38727
## HarmonyI-V-VI
                                                            0.530
                          8.033e-02 1.516e-01
                                                1.584e+03
                                                                   0.59623
## HarmonyIV-I-V
                         -2.540e-01 1.514e-01
                                                1.584e+03 -1.677
                                                                   0.09370
## musician
                          1.601e-03 3.326e-01
                                                7.419e+01
                                                            0.005
                                                                   0.99617
## Instrumentpiano
                         -1.093e+00 2.088e-01 4.983e+01
                                                          -5.236 3.29e-06
## Instrumentstring
                          -2.770e+00 2.750e-01 4.996e+01 -10.070 1.28e-13
## HarmonyI-V-IV:musician 1.869e-01 2.296e-01 1.585e+03
                                                            0.814
                                                                   0.41579
## HarmonyI-V-VI:musician -7.550e-01 2.296e-01 1.585e+03 -3.289
                                                                   0.00103
## HarmonyIV-I-V:musician 4.636e-02 2.294e-01 1.585e+03
                                                            0.202 0.83989
##
## Voicecontrary
## Voicepar3rd
                          ***
## Voicepar5th
## HarmonyI-V-IV
## HarmonyI-V-VI
## HarmonyIV-I-V
## musician
## Instrumentpiano
                          ***
## Instrumentstring
## HarmonyI-V-IV:musician
## HarmonyI-V-VI:musician **
## HarmonyIV-I-V:musician
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Correlation of Fixed Effects:
##
               Vccntr Vcpr3r Vcpr5t HrI-V-IV HrI-V-VI HrIV-I-V musicn
## Voicepar3rd 0.912
## Voicepar5th 0.916 0.912
## HrmnyI-V-IV -0.308 -0.325 -0.309
## HrmnyI-V-VI -0.307 -0.324 -0.308 0.499
## HrmnyIV-I-V -0.308 -0.325 -0.309 0.500
                                             0.499
## musician
              -0.595 -0.627 -0.596 0.228
                                             0.227
                                                      0.228
## Instrumntpn -0.196 -0.206 -0.197 0.000
                                            -0.001
                                                      0.000
                                                               0.000
```

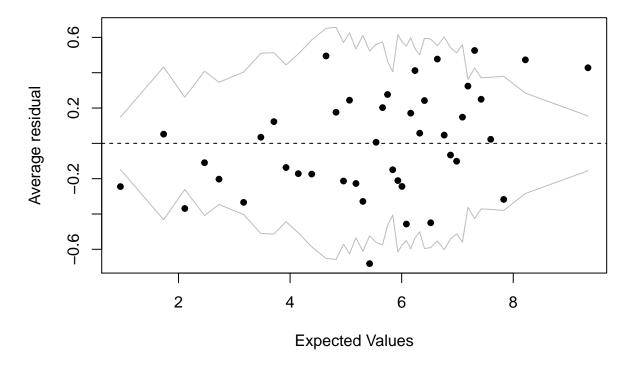
```
## Instrmntstr -0.292 -0.307 -0.292 0.000
                                              0.000
                                                        0.000
                                                                 0.000
## HrmnI-V-IV: 0.203 0.214 0.202 -0.659
                                             -0.329
                                                       -0.330
                                                                -0.344
## HrmnI-V-VI:
               0.202 0.214 0.202 -0.330
                                                       -0.330
                                                                -0.344
                                             -0.660
## HrmnIV-I-V:
               0.203 0.215 0.202 -0.330
                                             -0.330
                                                       -0.660
                                                                -0.344
##
               Instrmntp Instrmnts HI-V-IV: HI-V-VI:
## Voicepar3rd
## Voicepar5th
## HrmnyI-V-IV
## HrmnyI-V-VI
## HrmnyIV-I-V
## musician
## Instrumntpn
## Instrmntstr
               0.707
                0.001
## HrmnI-V-IV:
                          0.000
## HrmnI-V-VI:
                0.001
                          0.000
                                    0.499
## HrmnIV-I-V: 0.000
                          0.000
                                    0.499
                                             0.499
## convergence code: 0
## boundary (singular) fit: see ?isSingular
```

### Check errors for Popular Model

We'll start by looking at the binned residuals

```
modelp <- lmer.pop.final
binnedplot(fitted(modelp),resid(modelp))</pre>
```

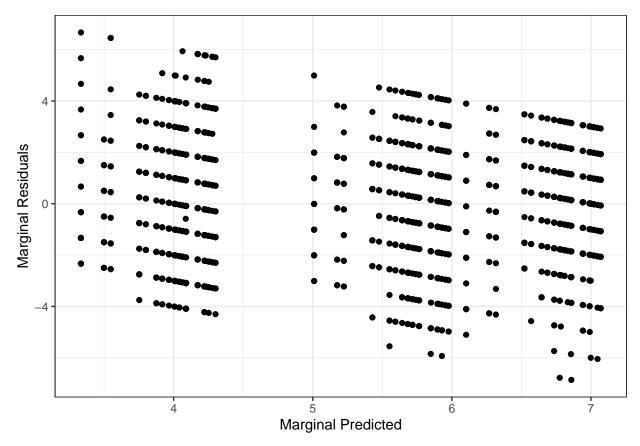
## **Binned residual plot**



It appears that the majority of the residuals are within the bin.

Next, we look at marginal fitted values vs. residuals

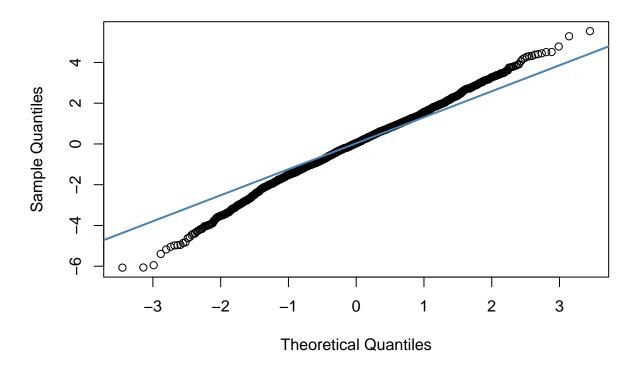
```
ggplot(mapping=aes(yhat.marg(modelp), r.marg(modelp))) + geom_point() +
labs(x="Marginal Predicted", y="Marginal Residuals") + theme_bw()
```



In the marginal models residuals plot above, we don't care about trends, but are more focused on the spread of the points. Therefore, the marginal residuals plot above looks good.

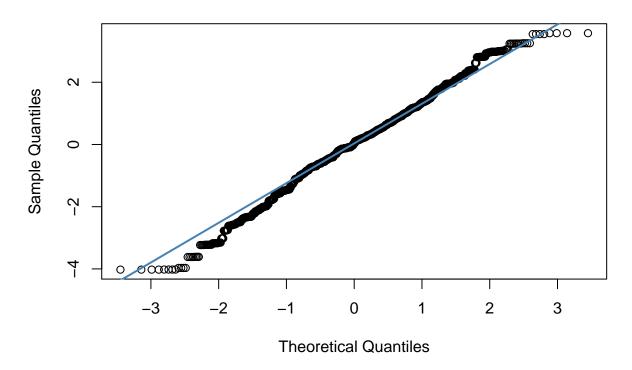
Now we will look at the QQ plot for conditional residuals.

```
qqnorm(r.cond(modelp))
qqline(r.reff(modelp), col = "steelblue", lwd = 2)
```



And the QQ plot for the random effects

```
qqnorm(r.reff(modelp))
qqline(r.reff(modelp), col = "steelblue", lwd = 2)
```



Both QQ plots look linear, suggesting normality of residuals.