

Lin Alg HW

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1 Basic Computations

These are boring, but its good to know how vectors and matrix vector products work by just the numbers.

- Compute the sum $\begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 3 \\ 2 \end{bmatrix}$.
- Compute the sum $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix}$.
- Compute the value of $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

2 Linear Transformations

- Compute the range of the linear transformation $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 4 & 2 \\ 0 & 6 & 5 \end{bmatrix}$. Express your answer as the span of some vectors.
- For some linear transformation T , $T\left(\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 1 \\ 5 \end{bmatrix}$ and $T\left(\begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$.
Compute $T\left(\begin{bmatrix} 5 \\ 0 \\ -1 \end{bmatrix}\right)$.
- For two linear transformations T_1 and T_2 , is $T_1(T_2(\mathbf{v})) = T_2(T_1(\mathbf{v}))$ always true for all \mathbf{v} ? Explain why, and assume there are no issues with domain/range stuff.
- If two linear transformations T_1 and T_2 satisfy $T_1(T_2(\mathbf{v})) = \mathbf{0}$ for all \mathbf{v} , does one of T_1 or T_2 have to be the linear transformation that maps all vectors to $\mathbf{0}$? Assume there are no issues with domain/range stuff.

3 Least Squares, Projection

- Compute \mathbf{x} such that $\|\mathbf{Ax} - \mathbf{b}\|$ is minimized, where $A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 4 & 6 \\ 1 & 2 & 0 \end{bmatrix}$,
 $b = \begin{bmatrix} 3 \\ -1 \\ 5 \end{bmatrix}$, and the norm is the L2 norm.
- Using the previous question, compute the projection of \mathbf{b} onto the the plane spanned by \mathbf{v}_1 and \mathbf{v}_2 , where $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 4 \\ 2 \end{bmatrix}$.
- Using the previous parts, what is the distance from b to $\text{span}\{\mathbf{v}_1, \mathbf{v}_2\}$?