Lin Alg HW

Alex

August 30, 2019

1 Basic Computations

These are boring, but its good to know how vectors and matrix vector products work by just the numbers.

- Compute the sum $\begin{bmatrix}1\\2\end{bmatrix} + \begin{bmatrix}3\\2\end{bmatrix}$.
- Compute the sum $\begin{bmatrix}1\\2\\3\end{bmatrix} + \begin{bmatrix}3\\2\\1\end{bmatrix} + \begin{bmatrix}4\\2\\0\end{bmatrix}$.
- Compute the value of $\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix}$.

2 Linear Transformations

- Compute the range of the linear transformation $\begin{bmatrix}1 & 2 & 3\\-1 & 4 & 2\\0 & 6 & 5\end{bmatrix}$. Express your answer as the span of some vectors.
- For some linear transformation T, $T(\begin{bmatrix}1\\2\\3\end{bmatrix}) = \begin{bmatrix}0\\1\\5\end{bmatrix}$ and $T(\begin{bmatrix}2\\-1\\-2\end{bmatrix}) = \begin{bmatrix}2\\0\\-1\end{bmatrix}$. Compute $T(\begin{bmatrix}5\\2\\1\end{bmatrix})$.
- For two linear transformations T_1 and T_2, is $T_1(T_2(\mathbf{v})) = T_2(T_1(\mathbf{v}))$ always true for all $\mathbf{v}$? Explain why, preferably intuitively rather than with an proof or example. Assume there are no issues with domain/range stuff.
- If two linear transformations T_1 and T_2 satisfy $T_1(T_2(\mathbf{v})) = \mathbf{0}$ for all $\mathbf{v}$, does one of T_1 or T_2 have to be the linear transformation that maps all vectors to $\mathbf{0}$? Assume there are no issues with domain/range stuff.

3 Least Squares, Projection

- Compute $\mathbf{x}$ such that $||\mathbf{Ax} - \mathbf{b}||$ is minimized, where $A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 4 & 6 \\ 1 & 2 & 0 \end{bmatrix}$, $b = \begin{bmatrix} 3 \\ -1 \\ 5 \end{bmatrix}$, and the norm is the L2 norm.
- Using the previous question, compute the projection of $\mathbf{b}$ onto the the plane spanned by $\mathbf{v_1}$ and $\mathbf{v_2}$, where $\mathbf{v_2} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ and $\mathbf{v_2} = \begin{bmatrix} 2 \\ 4 \\ 2 \end{bmatrix}$.
- Using the previous parts, what is the distance from b to span$\{\mathbf{v_1}, \mathbf{v_2}\}$?