1 Basic

1.1 Gaussian Distributions

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right)$$

$$p(x) = (2\pi)^{-\frac{d}{2}} |\Sigma|^{-\frac{1}{2}} \exp\left(-\frac{1}{2}(x-\mu)^\intercal \Sigma^{-1}(x-\mu)\right)$$

$$p\left(\begin{bmatrix}\mathbf{a}_1\\ \mathbf{a}_2\end{bmatrix}\right) = \mathcal{N}\left(\begin{bmatrix}\mathbf{a}_1\\ \mathbf{a}_2\end{bmatrix} \mid \begin{bmatrix}\mathbf{u}_1\\ \mathbf{u}_2\end{bmatrix}, \begin{bmatrix}\Sigma_{11} & \Sigma_{12}\\ \Sigma_{21} & \Sigma_{22}\end{bmatrix}\right)$$

$$p(\mathbf{a}_2|\mathbf{a}_1 = \mathbf{z}) = \mathcal{N}\left(\mathbf{a}_2|\mathbf{u}_2 + \Sigma_{21}\Sigma_{11}^{-1}(\mathbf{z}-\mathbf{u}_1), \Sigma_{22} - \Sigma_{21}\Sigma_{11}^{-1}\Sigma_{12}\right)$$

1.2 Derivations

$$\frac{\mathrm{d}\mathbf{x}^\top\mathbf{B}}{\mathrm{d}x} = \mathbf{B}$$
$$\frac{\mathrm{d}\mathbf{x}^\top\mathbf{b}}{\mathrm{d}x} = \mathbf{b}$$
$$\frac{\mathrm{d}\mathbf{x}^\top\mathbf{x}}{\mathrm{d}x} = 2\mathbf{x}$$
$$\frac{\mathrm{d}\mathbf{x}^\top\mathbf{B}\mathbf{x}}{\mathrm{d}x} = 2\mathbf{B}\mathbf{x}$$
$$\frac{\partial\mathbf{x}^\top\mathbf{B}\mathbf{x}}{\partial x} = \mathbf{x}^\top\mathbf{x}$$
$$\frac{\mathrm{d}}{\mathrm{d}x}\mathbf{B}\mathbf{x} \rightarrow \frac{\mathrm{d}}{\mathrm{d}x}\mathbf{x}^\top\mathbf{B}^\top$$

1.3 Taylor approximation

$$f(x) \approx f(a) + (x-a)^\intercal \nabla f(a) + (x-a)^\intercal H_f(a)(x-a)$$

1.4 Sylvester's criterion

Checks whether a symmetric/hermitian matrix is positive semi definite: M is positive definite if and only if all the following matrices have a positive determinant:

- the upper left 1-by-1 corner of M
- the upper left 2-by-2 corner of M
- the upper left 3-by-3 corner of M

  $\vdots$
- M itself.

1.5 Determinant

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$|A| = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$