Supplemental Materials

1 Performance plateau in the slow-predator domain

The present study included the results of numerical solutions that showed a performance plateau in the slow-predator domain, where the prey is faster than the predator. Here we define the boundary to this plateau, where an equivalent escape performance is achieved for a large range of escape angles α (Fig. 2A). For each escape angle in this range, the prey performs equally well by not allowing the predator to approach any closer than the distance at which the prey initiates an escape.

The distance between predator and prey varies with time, as given by the following equation:

$$D^2 = ((X_0 - Ut) + Vt\cos\alpha)^2 + (Vt\sin\alpha)^2, \tag{1}$$

where U and V are respectively the predator and prey speeds that do not vary with time, and X_0 is the initial position of the prey (i.e. $D^2 = X_0^2$ at $t = 0$). As explained in our article, the distance function may be solved for the time at which the minimum distance between predator and prey is achieved. The minimum distance distance occurs at $t = 0$ if the distance function increases monotonically with time. All escape angles for which this is true will yield an ideal minimum distance ($\frac{D}{X_0} = 1$). We will consider whether this is true of the following range of angles, which is bounded by the two solutions to Eqn. 1 reported by Weihs and Webb (1984):

$$0 \leq \alpha \leq \arccos(K), \tag{2}$$

where $K = U/V$. For our purposes, it is helpful to formulate this inequality as follows:

$$K \leq \cos\alpha \leq 1. \tag{3}$$

Toward this aim, it suffices to show that the distance function is increasing for all positive time values. This may be achieved by proving that the derivative of Eqn. 1 with respect to time is greater than or equal to zero. This derivative is given by the following equation:

$$\frac{\partial D^2}{\partial t} = 2(t(U^2 + V^2) - UX_0 + V(X_0 - 2tU)\cos\alpha). \tag{4}$$

It is helpful to rewrite this expression as a linear function of time as follows:

$$\frac{\partial D^2}{\partial t} = 2(U^2 + V^2 - 2UV\cos\alpha)t + 2X_0(V\cos\alpha - U), \tag{5}$$

In order to show that Eqn. 5 is nonnegative for $t \geq 0$, it suffices to show that the slope is positive and the intercept (when $t = 0$) is nonnegative.

- Positive slope.

  Eqn. 3 implies that $V\cos\alpha \leq V$. Multiplying this inequality by $-2U$ and adding $U^2 + V^2$ on both sides yields:

  $$U^2 + V^2 - 2UV\cos\alpha \geq V^2 - 2UV + U^2 \tag{6}$$

  The right hand side of this inequality is equivalent to $(V - U)^2$. Because $K < 1$, it follows that $V > U$. Thus,

  $$U^2 + V^2 - 2UV\cos\alpha \geq (V - U)^2 > 0 \tag{7}$$

- Nonnegative intercept.

  Because we consider only the situation where $X_0 > 0$, we simply need to show that $V\cos\alpha - U \geq 0$.

  Eqn. 3 implies that $KV \leq V\cos\alpha$. Since $K = U/V$, we have

  $$U \leq V\cos\alpha. \tag{8}$$

  We can rewrite this inequality as

  $$V\cos\alpha - U \geq 0. \tag{9}$$

Eqns. 6 and 9 together show that the distance is always increasing for $0 \leq \alpha \leq \arccos(K)$ and $K < 1$. An analogous argument applies for $-\arccos(K) \leq \alpha < 0$. The preceding proof shows that the minimum distance occurs at $t = 0$ and is given by $D^2 = X_0^2$. This defines a performance plateau for the prey as a wide range of angles yield equally successful escapes.

2 Initial Lateral Displacement

The distance function given by Eqn. 1 is based on the assumption that the predator is headed directly at the initial position of the prey. However, previous experiments have shown that predators often fail to perfectly align their approach toward the prey. To model this situation, here we introduce a lateral initial position to the distance function.

2.1 Distance function with initial lateral displacement

This general form of the distance function is now given by the following:

$$D^2 = ((X_0 - Ut) + Vt\cos\alpha)^2 + (Y_0 + Vt\sin\alpha)^2. \tag{10}$$

Note that the introduction of an initial lateral position (Y_0) allows us to rewrite the equation in polar coordinates, which simplifies our analysis below. We can rewrite Eqn. 10 in polar coordinates (R, θ) by setting $R_0^2 = X_0^2 + Y_0^2$, and $\theta_0 = \arctan(Y_0/X_0)$. Here we assume that $Y_0 \geq 0$, but the final results are presented for the more general case. This yields the following:

$$D_0^2 = R_0^2 + (1 + K^2)t^2V^2 - 2Vt(KVt\cos\alpha - R_0\cos(\alpha - \theta_0) + KR_0\cos\theta_0). \tag{11}$$

To find the time at which Eqn. 11 is minimal, we find the roots of the derivative of Eqn. 11 with respect to t, which yields the following solution:

$$t_{\min} = \frac{R_0}{V}\frac{[K\cos\theta_0 - \cos(\alpha - \theta_0)]}{1 - 2K\cos\alpha + K^2} \tag{12}$$

The above is negative when $K\cos\theta_0 < \cos(\alpha - \theta_0)$. Rewriting this inequality gives the range of α for which the distance is solely increasing. Explicitly, this is given by the following:

$$\theta_0 - \arccos(K\cos\theta_0) < \alpha < \theta_0 + \arccos(K\cos\theta_0). \tag{13}$$

For these values of α, the minimum distance occurs at $t = 0$ and is thus equal to the initial distance R_0^2. This defines a performance plateau when $K < 1$. If we now substitute $t_{\min}$ for t in Eqn. 11, we get the minimum distance as a function of α with respect to parameters K and θ_0. This is given by the following:

$$\overline{D}^2_{\min} = \frac{D^2_{\min}}{R_0^2} = \frac{(\sin(\alpha - \theta_0) + K\sin\theta_0)^2}{K^2 - 2K\cos\alpha + 1}. \tag{14}$$

2.2 Finding values of α that optimize the minimum distance

To find the escape angle which yields the largest minimum distance, we solved the following equation:

$$0 = \frac{\partial \overline{D}_{\min}^{2}}{\partial \alpha} = \frac{2(K\cos\alpha - 1)(K\cos\theta_0 - \cos(\alpha - \theta_0))(K\sin\theta_0 + \sin(\alpha - \theta_0))}{(K^2 - 2K\cos\alpha + 1)^2} \tag{15}$$

The solutions to this equation are found by finding where the numerator is equal to zero which is done by considering the following three cases:

Case 1: $K\cos\alpha - 1 = 0$.

Solving this equation yields the following relationship:

$$\alpha_1 = \pm\arccos K^{-1} \tag{16}$$

This solution is valid for $K \geq 1$, which indicates that prey are equally effective if escaping at an optimal angle toward the left ($\alpha > 0$), or right ($\alpha < 0$) of the predator's heading.

Case 2: $K\cos\theta_0 - \cos(\alpha - \theta_0) = 0$.

A careful analysis is required for this case because the solution can include complex numbers for some combinations of K and θ_0. This equation may be formulated as follows:

$$\cos(\alpha - \theta_0) = K\cos\theta_0. \tag{17}$$

This equation imposes conditions on the values of K and θ_0 to yield solutions which are real numbers. Explicitly, this condition is given by the following

$$|K\cos\theta_0| \leq 1 \tag{18}$$

Eqn. 18 is always satisfied when $0 \leq K \leq 1$. If $K > 1$, then we must have that $|\cos\theta_0| \leq 1/K$. This leads to the following bound for θ_0:

$$\arccos(K^{-1}) \leq \theta_0 \leq \arccos(-K^{-1}). \tag{19}$$

Note that as the value of K increases, the allowable range for θ_0 decreases. With these restrictions in mind, we proceed to solve Eqn. 17. The solution is given by:

$$\alpha_2 = \theta_0 + \arccos(K\cos\theta_0),$$
$$\alpha_3 = \theta_0 - \arccos(K\cos\theta_0).$$

Therefore, the following equation defines the boundaries of the performance plateau for $K < 1$:

$$|\alpha - \theta_0| \leq \arccos(K\cos\theta_0). \tag{20}$$

For $K > 1$, the solutions $\alpha_{2,3}$ are not optimal unless θ_0 simultaneously satisfies Eqn. 19.

Case 3: $K\sin\theta_0 + \sin(\alpha - \theta_0) = 0.$

Solving this equation yields the following:

$$\alpha_4 = \theta_0 - \arcsin(K\sin\theta_0),$$
$$\alpha_5 = \pi + \theta_0 + \arcsin(K\sin\theta_0).$$

For $K < 1$, α_4 is contained within the bounds defined by Eqn. 13. This means that the solution is contained within the performance plateau of the slow–predator domain. When $K > 1$, α_4 is a local minimum. Because we seek to find the angle which yields the greatest minimum distance, α_4 is not optimal when $K > 1$. Similarly, the solution given by α_5 is a local minimum for all values of K, so this solution does not yield an optimal escape.