# A PROJECT

# ON

# FINDING GENERAL SOLUTION OF HOMOGENOUS EQUATIONS

# USING PYTHON

## BITS Pilani-Dubai Campus, DIAC

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**A Report**

**on**

# FINDING GENERAL SOLUTION OF HOMOGENOUS EQUATIONS

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# Acknowledgement

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I further wish to thank Dr. A. Somasundaram, for clearing doubts and providing an in-depth view of the concepts used in this project.

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# 1. Aim

Solve 3 models of $2^{nd}$ order homogenous equations to find their respective general solutions.

a. Solve y'' - 8y' + 17y = 0 where y(0)= -4 & y'(0)= -1 and find the values of the constants

b. Solve y'' + 11y' + 24y = 0 where y(0) = 0 & y'(0) = -7 and find the values of the constants

c. Solve y'' - 4y' +4y = 0 where y(0)= 12 & y'(0)= 3 and find the values of the constants

# 2. Homogenous differential equations-

A homogenous differential equation is a differential equation which contains a homogenous function.

$$\boldsymbol{\frac{dy}{dx} = F(x, y)}$$

F (x, y) is the homogenous function and $\frac{dy}{dx}$ is the derivative of the function.

### a. Homogenous function-

A homogenous function is a function, F (x, y), where each variable of the equation has some constant, k not equal to 0.

$$\boldsymbol{F(kx, ky\,) = \; k^n F(x, y)}$$

Here *n* is the degree of homogeneity of the function.

### b. $2^{nd}$ order homogenous differential equation

It is a differential equation which includes nth order derivatives up until n=2. These equations upon solving for a particular root, equate to zero. For example, the equation given below is a 2nd order differential equation of y in terms of x,

$$\boldsymbol{a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0}$$

In this equation, *a, b* and *c* are constants of the respective derivative.

### c. General solutions

General solution is the solution of a differential equation which contains the required arbitrary constants. It gives the complete solution of the ordered differential equation.

## 3. Solving homogenous differential equations with constant coefficients

Consider a 2nd order equation of the form,

$$\boldsymbol{\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = 0} \qquad \textbf{(1)}$$

Where the derivatives are of the form y in terms of x and *P* & *Q* are constant coefficients.

Let $y = e^{mx}$ be substituted in (1),

$$\boldsymbol{m^2e^{mx} + \ Pme^{mx} + \ Qe^{mx} = 0}$$

Here m is the variable to be determined.

Dividing both sides by $e^{mx}$,

$$\Rightarrow \boldsymbol{m^2 + \ Pm + \ Q = 0} \qquad (2)$$

Solving (2), the roots obtained are *m1* & *m2.*
The solution of the 1st root is,

$$y_1 = e^{m_1 x}$$

Similarly, the solution of the 2nd root is,

$$y_2 = e^{m_2 x}$$

The general solution of the (1) varies depending on the type of roots obtained. On this basis, there are 3 ways to form the general solution ($y_g$) of the equation:

**a. When the roots are real and distinct**

$$y_g = c_1 e^{m_1 x} + c_2 e^{m_2 x}$$

**b. When the roots are real and equal**

$$y_g = e^{mx}(c_1 + c_2 x)$$

**c. Roots are imaginary and complex**

$$y_g = e^{ax}(c_1 \cos bx + c_2 \sin bx)$$

$$where\ a\ is\ the\ real\ part\ and\ b\ is\ the\ imaginary\ part\ of\ the\ root,$$

$$m = a \pm ib$$

# 4. Applications of solving differential equations

There are many real-life applications of solving differential equations. The most common use is to get accurate measurement results in the field of science. A few of the applications are:

a. Calculate the flow of electricity

b. Measure the velocity at which a pendulum oscillates
c. Newton’s law of cooling

# 5. Software used

The software used to implement the models is Python.
Python is a general-purpose programming language with a variety of libraries and packages that are useful in mathematical applications. For this project, the following packages have been imported:

## 5.1. Sympy

Sympy is a Python library for symbolic mathematics. It consists of numerous functions that help in representing and solving algebraic equations and functions.
The functions implemented in the code from this library are:

i. **Symbol()**

The function Symbol() creates the parameter String into a constant or variable.

ii. **diff()**

It is used for finding the derivative of a given function (the first parameter) in terms of the second parameter.

iii. **subs()**

subs() is a function which substitutes a variable with another value for a defined function.

## 5.2. cmath

cmath is a module in python associated with complex numbers.

i. **.real**

Returns the real part of a complex number.

ii. **.imag**

Returns the imaginary part of a complex number.

## 5.3. math

math module in python provides functions which can be applied on floating point numbers.

i. **cos()**

creates a cos(x) function where x is the parameter passed which is the value of cosine.

ii. **sin()**

creates a sin(x) function where x is the parameter passed which is the value of sine.

iii. **exp()**

returns E raised to the power passed as parameter. Here, E is the natural base of logarithms.

# 6. Solutions of chosen models

The following program displays a menu listing the chosen models and takes the model number as input from the user and displays the solution with steps as output via if-else statements. The entire program is placed within a loop such that the user can revisit the menu and view other models too.

The coefficients of the second order homogenous equation as taken as 3 variables. The derivatives are assigned to variables using the Symbols function from Sympy. The roots of the assumed second order equation are found through the discriminant formula for 2nd order equations where there exists an equation, $ax^2 + by + c = 0$, where a, b and c are constants.

The formula to find the discriminant is,

$$d = b^2 - 4ac$$

which is substituted in the following formula to find the roots of the corresponding equation.

$$x = \frac{-b \pm \sqrt{d}}{2a}$$

Based on the type of roots generated, the roots are substituted in the relevant general solution formula. The general formula contains an arbitrary constant for each root present.

To find the constants of the solved model's general solution, the values given for the equation and its first derivative are substituted and utilised. The final solution, after substituting the solved constants is then displayed.

## 6.1. Model 1

Problem: **Solve $\mathbf{y'' - 8y' + 17y = 0}$ where $\mathbf{y(0) = -4}$ & $\mathbf{y'(0) = -1}$ and find the values of the constants.**

Let $y = e^{mx}$ where *m* is the variable to be determined.
Then, $y' = me^{mx}$ and $y'' = m^2 e^{mx}$.

Substituting *y, y′ and y′′* in the given differential equation,

$$\mathbf{y'' - 8y' + 17y = 0}$$

$$\Rightarrow \quad m^2 e^{mx} - 8m \;\; e^{mx} + 17e^{mx} = 0$$

Dividing both sides of the equation with $e^{mx}$,the following quadratic equation is formed.

$$\Rightarrow \quad \boldsymbol{m^2 - 8m \;\; + 17 = 0} \qquad\qquad (\mathbf{1})$$

Let **a= 1, b=-8 and c=17** from (1). Calculating the roots of *m* using the discriminant formula,

$$d = 8^2 - 4(1)(17) = -4$$

The roots of equation (1) are,

$$m_1 = \frac{-(-8) + \sqrt{-4}}{2(1)} = 4 + i$$

and

$$m_2 = \frac{-(-8) - \sqrt{-4}}{2(1)} = 4 - i$$

It is to be noted that the found roots are imaginary and complex. Hence, the general solution of (1) is,

$$\boldsymbol{y(x) = e^{4x}(c_1 \cos x + c_2 \sin x)} \qquad (\mathbf{2})$$

Substituting $x = 0$ in (2),

$$\boldsymbol{y(0) = -4 = c1} \qquad (\mathbf{3})$$

Taking the first order derivative of (2),

$$\boldsymbol{y'(x) = 4c_1e^{4x}cosx - c_1e^{4x}sinx + 4c_2e^{4x}sinx + c_2e^{4x}cosx}$$

Substituting $x = 0$ in y'(x),

$$\boldsymbol{y'(0) = -1 = 4c_1 + c_2} \qquad (4)$$

Substituting (3) in (4),

$$\boldsymbol{c_2} = -1 - 4(-4)$$

$$\boldsymbol{c_2 = 15}$$

Inserting the values of $\boldsymbol{c_1}$ & $\boldsymbol{c_2}$ into (2), the general solution of $\mathrm{y}'' - 8\mathrm{y}' + 17\mathrm{y} = 0$ is,

$$\boldsymbol{y(x) = e^{4x}(-4\cos x + 15\sin x)}$$

### 6.2. Model 2

Problem: **Solve $\mathbf{y'' + 11y' + 24y = 0}$ , where $\mathbf{y(0) = 0}$ & $\mathbf{y'(0) = -7}$ and find the values of the constants.**

Let $y = e^{mx}$ where *m* is the variable to be determined.

Then, $y' = me^{mx}$ and $y'' = m^2e^{mx}$.

Substituting *y, y' and y''* in the given differential equation,

$$\mathbf{y'' + 11y' + 24y = 0}$$

$$\Rightarrow \quad m^2e^{mx} + 11m \ e^{mx} + 24e^{mx} = 0$$

Dividing both sides of the equation with $e^{mx}$,the following quadratic equation is formed.

$$\Rightarrow \quad \boldsymbol{m^2 + 11m + 24 = 0} \qquad \textbf{(1)}$$

Let **a= 1, b=11 and c=24** from (1). Calculating the roots of *m* using the discriminant formula,

$$d = 11^2 - 4(1)(24) = 25$$

The roots of equation (1) are,

$$m_1 = \frac{-(11) + \sqrt{25}}{2(1)} = -3$$

and

$$m_2 = \frac{-(11) - \sqrt{25}}{2(1)} = -8$$

It is to be noted that the found roots are real and distinct. Hence, the general solution of (1) is,

$$\boldsymbol{y(x) = c_1 e^{-3x} + c_2 e^{-8x}} \qquad \textbf{(2)}$$

Substituting $x = 0$ in (2),

$$\boldsymbol{y(0) = 0 = c1 + c2} \qquad \textbf{(3)}$$

Taking the first order derivative of (2),

$$\boldsymbol{y'(x) = -3c_1 e^{-3x} - 8\, c_2 e^{-8x}}$$

Substituting $x = 0$ in y’(x),

$$\boldsymbol{y'(0) = -7 = -3c_1 - 8c_2} \qquad \textbf{(4)}$$

Substituting (3) in (4),

$$\boldsymbol{-7 = -3c_1 - 8(-c_1) \quad (from\ (3))}$$

$$\boldsymbol{5c_1 = -7 \quad \Rightarrow c_1 = -1.4}$$

$$\Rightarrow \boldsymbol{c_2 = -c_1 = 1.4}$$

Inserting the values of $\boldsymbol{c_1}$ & $\boldsymbol{c_2}$ into (2), the general solution of $\mathrm{y}'' - 8\mathrm{y}' + 17\mathrm{y} = 0$ is,

$$\boldsymbol{y(x) = 1.4e^{-3x} - 1.4\,e^{-8x}}$$

### 6.3. Model 3

Problem: Solve $\mathrm{y}'' - 4\mathrm{y}' + 4\mathrm{y} = 0$ , where $\mathrm{y}(0) = 12$ & $\mathrm{y}'(0) = 3$ and find the values of the constants.

Let $y = e^{mx}$ where *m* is the variable to be determined.

Then, $y' = me^{mx}$ and $y'' = m^2e^{mx}$.

Substituting *y, y' and y''* in the given differential equation,

$$\mathbf{y'' - 4\,y' + 4y = 0}$$

$$\Rightarrow \quad m^2e^{mx} - 4m \quad e^{mx} + 4e^{mx} = 0$$

Dividing both sides of the equation with $e^{mx}$,the following quadratic equation is formed.

$$\Rightarrow \quad \boldsymbol{m^2 - 4m \quad + 4 = 0} \qquad\qquad (\mathbf{1})$$

Let **a= 1, b=-4 and c=4** from (1). Calculating the roots of *m* using the discriminant formula,

$$d = -4^2 - 4(1)(4) = 0$$

The roots of equation (1) are,

$$m_1 = \frac{-(-4) + \sqrt{0}}{2(1)} = 2$$

and

$$m_2 = \frac{-(-4) - \sqrt{0}}{2(1)} = 2$$

It is to be noted that the found roots are real but not distinct. Hence, the general solution of (1) is,

$$\boldsymbol{y(x) = c_1 e^{2x} + x c_2 e^{2x}} \qquad \textbf{(2)}$$

Substituting $x = 0$ in (2),

$$\boldsymbol{y(0) = 12 = c1} \qquad \textbf{(3)}$$

Taking the first order derivative of (2),

$$\boldsymbol{y'(x) = 2c_1 e^{2x} + 2x c_2 e^{2x} + c_2 e^{2x}}$$

Substituting $x = 0$ in y′(x),

$$\boldsymbol{3 = 2c_1 + c_2}$$

$$\boldsymbol{\Rightarrow c_2 = 3 - 2c_1}$$

$$\boldsymbol{\Rightarrow c_2 = 3 - 2(12) = -27}$$

Inserting the values of $\boldsymbol{c_1}$ & $\boldsymbol{c_2}$ into (2), the general solution of $y'' - 8y' + 17y = 0$ is,

$$\boldsymbol{y(x) = 12e^{2x} - x(27)e^{2x}}$$

# 7. Source code

```
import math
from sympy import *

yn='y'

while(yn=='y'):
    print("\nTHE 3 MODELS CHOSEN FOR DEMONSTRATING HOW GENERAL SOLUTIONS OF HOMOGENOUS EQUATIONS WITH CONSTANT COEFFCIENTS ARE FOUND ARE THE FOLLOWING-")
    print("1-Solve y'' - 8y' + 17y = 0 where y(0)= -4 & y'(0)= -1 and find the values of the constants")
    print("2-Solve y'' + 11y' + 24y = 0 where y(0) = 0 & y'(0) = -7 and find the values of the constants")
    print("3-Solve y'' - 4y' +4y = 0 where y(0)= 12 & y'(0)= 3 and find the values of the constants")

    n= int(input("Enter choice to see solution: "))

    if (n==1):
        '''
        Solve y'' - 8y' + 17y = 0 where y(0)= -4 & y'(0)= -1 and find the values of the constants
        '''

        #coefficients of each derivative
        a=1
        b= -8
        c= 17

        #calculating roots of the equation
        d= (b**2)- (4*a*c)
        r1= ((-b)+ (d)**(0.5))/2
        r2= ((-b)-(d)**(0.5))/2
        print("Assuming y'' - 8y' + 17y = 0  as a quadratic equation where\nm^2 = y'' \nm = y'\ny= 1\nHence the equation to be solved is:\n m^2 -8m +17 = 0")
        print("\nThe roots of equation are: ",r1," & ",r2,"\n")
```

```
        x=Symbol('x')
        c1=Symbol('c1')
        c2=Symbol('c2')

        if (d<0):
            print("The roots are complex.")
            real= r1.real
            img= r1.imag
            print("Real part of root= ",real)
            print("Imaginary part of root= ", img)

            y=  (c1*cos(img*x)+ c2*sin(img*x))*exp(real*x)

            y_dx= diff(y, x)
            print("\ny(x)= ", y)
            print("y'(x)= ", y_dx)

            #substituting 0 in both y(x) & y'(x)
            yg0= y.subs(x,0)
            yg0_dx= y_dx.subs(x,0)
            print("\nSubstituting x=0 in y(x) & yg(x), the resulting
equations are,")
            print("y(0) = -4 = ",yg0,"          (1)")
            print("y'(0) = -1 = ",yg0_dx,"          (2)")

            print("\nFrom solving equations (1) & (2),")
            c1= -4

            c2= c2.subs(c2, -1-4*c1)
            yg=  (c1*cos(img*x)+ c2*sin(img*x))*exp(real*x)
            print("Values of constants are\nc1 = ",c1,"\nc2 = ",c2)
            print("\nSubstituting values of c1 & c2 in y(x), the
general solution of y'' - 8y' + 17y = 0  is,\ny(x)= ", yg)
```

```
    elif (n==2):
            '''
            Solve y'' + 11y' + 24y = 0 where y(0) = 0 & y'(0) = -7
and find the values of the constants
            '''
            #coefficients of each derivative
            a=1
            b= 11
            c= 24


            #calculating roots of the equation
            d= (b**2)- (4*a*c)
            print(d)
            r1= ((-b)+ (d)**(0.5))/2
            r2= ((-b)-(d)**(0.5))/2
            print("Assuming y’’ + 11y’ + 24y = 0 as a quadratic
equation where\nm^2 = y’’ \nm = y' \ny= 1\nHence the equation to be
solved is: \n m^2 +11m +24 = 0")
            print("\nThe roots of equation are: ",r1," & ",r2)
            x=Symbol('x')
            c1=Symbol('c1')
            c2=Symbol('c2')
            y= c1*exp(r1*x)+ c2*exp(r2*x)
            y_dx= diff(y,x)
            print("\ny(x)= ", y)
            print("y’ (x) = ", y_dx)


            #substituting 0 in both y(x) & y’(x)
            yg0= y.subs(x,0)
            yg0_dx= y_dx.subs(x,0)
            print("\nSubstituting x=0 in y(x) & y’(x), the resulting
equations are, ")
            print("y(0) = 0 = ",yg0,"       (1)")
            print("y’(0) = -7 = ",yg0_dx,"       (2)")
```

```
            print("\nFrom solving equations (1) & (2),")
            c1= -7/5
            c2= -c1

            yg= c1*exp(r1*x)+ c2*exp(r2*x)
            print("Values of constants are\nc1 = ",c1," \nc2 = ",c2)
            print("\nSubstituting values of c1 &c2 in y(x), the
general solution of y’’ +11y’ + 24y =0 is, \ny(x) = ", yg)


    if (n==3):
            '''
            Solve y'' - 4y' +4y = 0 where y(0)= 12 & y'(0)= 3 and
find the values of the constants
            '''
            #coefficients of each derivative
            a=1
            b= -4
            c= 4

            #calculating roots of the equation
            d= (b**2)- (4*a*c)
            r1= ((-b)+ (d)**(0.5))/2
            r2= ((-b)-(d)**(0.5))/2
            print("Assuming y'' - 4y' +4y = 0 as a quadratic
equation where\nm^2 = y’’ \nm = y' \ny= 1\n Hence the equation to be
solved is: \n m^2 +3m -10 = 0")
            print("\n\nThe roots of equation are: ",r1," & ",r2)

            x=Symbol('x')
            c1=Symbol('c1')
            c2=Symbol('c2')
            y= c1*exp(r1*x)+ x*c2*exp(r2*x)
            y_dx= diff(y,x)
```

```
            print("y(x)= ", y)
            print("y’ (x) = ", y_dx)


            #substituting 0 in both y(x) & y’(x)
            yg0= y.subs(x,0)
            yg0_dx=y_dx.subs(x,0)
            print("\nSubstituting x=0 in y(x) & y’(x), the resulting
equations are, ")
            print("y(0) = 12 = ",yg0,"           (1)")
            print("y’(0) = 3 = ",yg0_dx,"        (2)")


            print("\nFrom solving equations (1) & (2),")
            c1= 12
            c2= c2.subs(c2, -3-2*c1)
            yg= c1*exp(r1*x)+ x*c2*exp(r2*x)
            print("Values of constants are\nc1 = ",c1," \nc2 = ",c2)
            print("\nSubstituting values of c1 &c2 in y(x), the
general solution of y’’ +3y’ - 10y =0 is, \ny(x) = ", yg)



    yn= input("\n\nWould you like to view the other models (y / n)?
")
```

# 8. Program Output

```
=============== RESTART: C:\Users\manoj\OneDrive\Desktop\MATH3.py ==============

THE 3 MODELS CHOSEN FOR DEMONSTRATING HOW GENERAL SOLUTIONS OF HOMOGENOUS EQUATIONS WITH CONSTANT COEFFCIENTS ARE FOUND ARE THE FOLLOWING-
1-Solve y'' - 8y' + 17y = 0 where y(0)= -4 & y'(0)= -1 and find the values of the constants
2-Solve y'' + 11y' + 24y = 0 where y(0) = 0 & y'(0) = -7 and find the values of the constants
3-Solve y'' - 4y' +4y = 0 where y(0)= 12 & y'(0)= 3 and find the values of the constants
Enter choice to see solution: 1
Assuming y'' - 8y' + 17y = 0  as a quadratic equation where
m^2 = y''
m = y'
y= 1
Hence the equation to be solved is:
 m^2 -8m +17 = 0

The roots of equation are:  (4+1j)  &  (4-1j)

The roots are complex.
Real part of root=  4.0
Imaginary part of root=  1.0

y(x)=  (c1*cos(1.0*x) + c2*sin(1.0*x))*exp(4.0*x)
y'(x)=  (-1.0*c1*sin(1.0*x) + 1.0*c2*cos(1.0*x))*exp(4.0*x) + 4.0*(c1*cos(1.0*x) + c2*sin(1.0*x))*exp(4.0*x)

Substituting x=0 in y(x) & yg(x), the resulting equations are,
y(0) = -4 =  c1           (1)
y'(0) = -1 =  4.0*c1 + 1.0*c2           (2)

From solving equations (1) & (2),
Values of constants are
c1 =  -4
c2 =  15

Substituting values of c1 & c2 in y(x), the general solution of y'' - 8y' + 17y = 0  is,
y(x)=  (15*sin(1.0*x) - 4*cos(1.0*x))*exp(4.0*x)


Would you like to view the other models (y / n)?  y
```

```
THE 3 MODELS CHOSEN FOR DEMONSTRATING HOW GENERAL SOLUTIONS OF HOMOGENOUS EQUATIONS WITH CONSTANT COEFFCIENTS ARE FOUND ARE THE FOLLOWING-
1-Solve y'' - 8y' + 17y = 0 where y(0)= -4 & y'(0)= -1 and find the values of the constants
2-Solve y'' + 11y' + 24y = 0 where y(0) = 0 & y'(0) = -7 and find the values of the constants
3-Solve y'' - 4y' +4y = 0 where y(0)= 12 & y'(0)= 3 and find the values of the constants
Enter choice to see solution: 2
25
Assuming y’’ + 11y’ + 24y = 0 as a quadratic equation where
m^2 = y’’
m = y'
y= 1
Hence the equation to be solved is:
 m^2 +11m +24 = 0

The roots of equation are:  -3.0  &  -8.0

y(x)=  c1*exp(-3.0*x) + c2*exp(-8.0*x)
y’ (x) =  -3.0*c1*exp(-3.0*x) - 8.0*c2*exp(-8.0*x)

Substituting x=0 in y(x) & y’(x), the resulting equations are,
y(0) = 0 =  c1 + c2              (1)
y’(0) = -7 =  -3.0*c1 - 8.0*c2          (2)

From solving equations (1) & (2),
Values of constants are
c1 =  -1.4
c2 =  1.4

Substituting values of c1 &c2 in y(x), the general solution of y’’ +11y’ + 24y =0 is,
y(x) =  1.4*exp(-8.0*x) - 1.4*exp(-3.0*x)


Would you like to view the other models (y / n)?  y

THE 3 MODELS CHOSEN FOR DEMONSTRATING HOW GENERAL SOLUTIONS OF HOMOGENOUS EQUATIONS WITH CONSTANT COEFFCIENTS ARE FOUND ARE THE FOLLOWING-
1-Solve y'' - 8y' + 17y = 0 where y(0)= -4 & y'(0)= -1 and find the values of the constants
2-Solve y'' + 11y' + 24y = 0 where y(0) = 0 & y'(0) = -7 and find the values of the constants
3-Solve y'' - 4y' +4y = 0 where y(0)= 12 & y'(0)= 3 and find the values of the constants
Enter choice to see solution: 3
Assuming y'' - 4y' +4y = 0 as a quadratic equation where
m^2 = y’’
m = y'
y= 1
 Hence the equation to be solved is:
```

```
THE 3 MODELS CHOSEN FOR DEMONSTRATING HOW GENERAL SOLUTIONS OF HOMOGENOUS EQUATIONS WITH CONSTANT COEFFCIENTS ARE FOUND ARE THE FOLLOWING-
1-Solve y'' - 8y' + 17y = 0 where y(0)= -4 & y'(0)= -1 and find the values of the constants
2-Solve y'' + 11y' + 24y = 0 where y(0) = 0 & y'(0) = -7 and find the values of the constants
3-Solve y'' - 4y' +4y = 0 where y(0)= 12 & y'(0)= 3 and find the values of the constants
Enter choice to see solution: 3
Assuming y'' - 4y' +4y = 0 as a quadratic equation where
m^2 = y''
m = y'
y= 1
 Hence the equation to be solved is:
 m^2 +3m -10 = 0


The roots of equation are:  2.0  &  2.0
y(x)=  c1*exp(2.0*x) + c2*x*exp(2.0*x)
y' (x) =  2.0*c1*exp(2.0*x) + 2.0*c2*x*exp(2.0*x) + c2*exp(2.0*x)

Substituting x=0 in y(x) & y'(x), the resulting equations are,
y(0) = 12 =  c1                         (1)
y'(0) = 3 =  2.0*c1 + c2                (2)

From solving equations (1) & (2),
Values of constants are
c1 =  12
c2 =  -27

Substituting values of c1 &c2 in y(x), the general solution of y'' +3y' - 10y =0 is,
y(x) =  -27*x*exp(2.0*x) + 12*exp(2.0*x)


Would you like to view the other models (y / n)?  n
>>>
```

# 9. References


- _(Python | Getting started with SymPy module , 2022)
- _(cmath — Mathematical functions for complex numbers, 2022)
- _(Svirin, n.d.)