An Analytic Toy Problem for Computing Global Error

Here's a very simple DAE

$$F(\dot{x}, x, p, t) = \left\{ \begin{array}{c} y' + p \cdot a(t)y(t) \\ t/200 - a(t) \end{array} \right\} = 0$$

The analytic solution for y is

$$Y(t) = \frac{y_0}{\exp\left(\frac{pt^2}{200}\right)}$$

Now say our metric is $g(p) = y(t_f)$. Then

$$\frac{dg}{dp} = \frac{dy(t_f)}{dp} = \frac{-y_0 t_f^2}{200\exp\left(\frac{pt^2}{200}\right)}$$

The global error is

$$e(t) = Y(t) - y(t).$$

The adjoint equation is

$$\begin{aligned} \left(\begin{array}{c} \lambda^d \\ 0 \end{array} \right)' &= \left(\begin{array}{cc} (F^d_{x^d})^T & (F^a_{x^d})^T \\ (F^d_{x^a})^T & (F^a_{x^a})^T \end{array} \right) \left(\begin{array}{c} \lambda^T \\ \lambda^a \end{array} \right) \\ &= \left(\begin{array}{cc} p \cdot a(t) & 0 \\ p \cdot y & -1 \end{array} \right) \left(\begin{array}{c} \lambda^T \\ \lambda^a \end{array} \right) \end{aligned}$$

Running the matlab code

You will need the subdirectory *builtIns* as it contains the hacked ODE solvers that I created for extracting truncation eror. The driver code file is

[forSol, backSol]=testAnalyticDAE(doPlot)

where doPlot==1 will produce a couple of plots and doPlot==0 will suppress the plots. The code has some comments which should help you see what I am doing. As far as how I am extracting/estimating the truncation error, I think I'd rather explain and show in person.

Here is an example code output:

True Global Error: -1.4057e-05
Est. Global Error: 3.9762e-07 (Rel Error = -1.028e+00)
True dgdp: -1.0394e-01
Est. dpgp: -1.0394e-01 (Rel Error = 1.683e-05)

Feel free to send me questions.