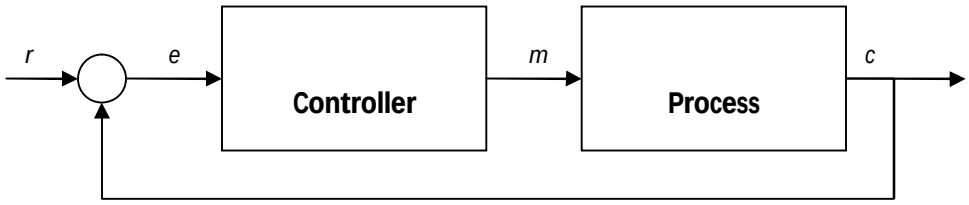
Appendix 3:
PID Implementation

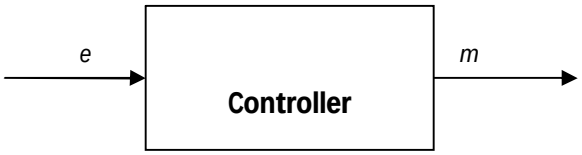


***r* = set point**

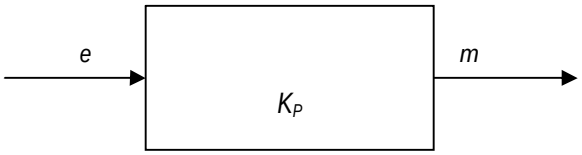
e* = error = *r* – *c

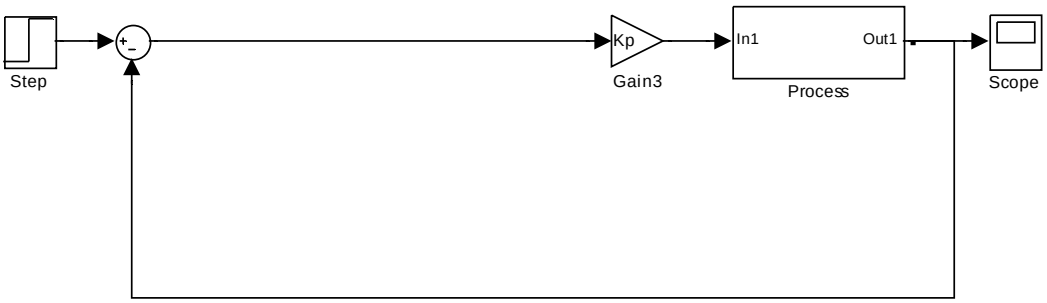
***m* = manipulated variable**

***c* = controlled variable**



P Controller



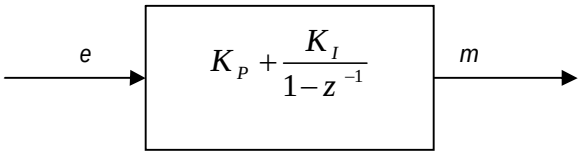


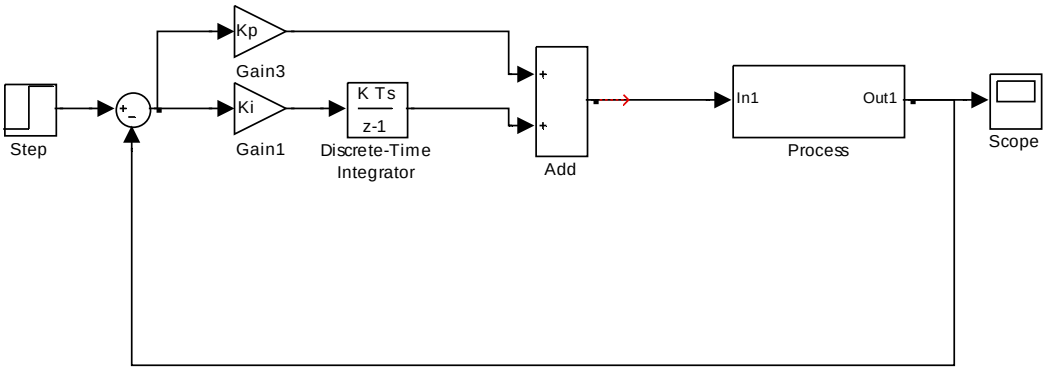
$$\frac{m}{e} = K_P$$

$$m = K_P e$$

```
c = get_c();
e = r-c;
m = Kp*e;
```

PI Controller 1





$$\frac{m}{e} = \left(K_P + \frac{K_I}{1-z^{-1}} \right)$$

$$m = \left(K_P + \frac{K_I}{1-z^{-1}} \right) e$$

$$m\left(1-z^{-1}\right) = \left(K_P\left(1-z^{-1}\right) + K_I\right) e$$

$$m - mz^{-1} = K_P e - K_P e z^{-1} + K_I e$$

$$m = K_P e - K_P e z^{-1} + K_I e + mz^{-1}$$

$$m = \left(K_P + K_I\right) e - K_P e z^{-1} + mz^{-1}$$

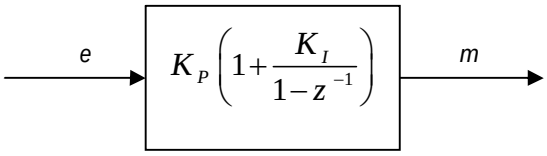
$$m(k) = \left(K_P + K_I\right) e(k) - K_P e(k-1) + m(k-1)$$

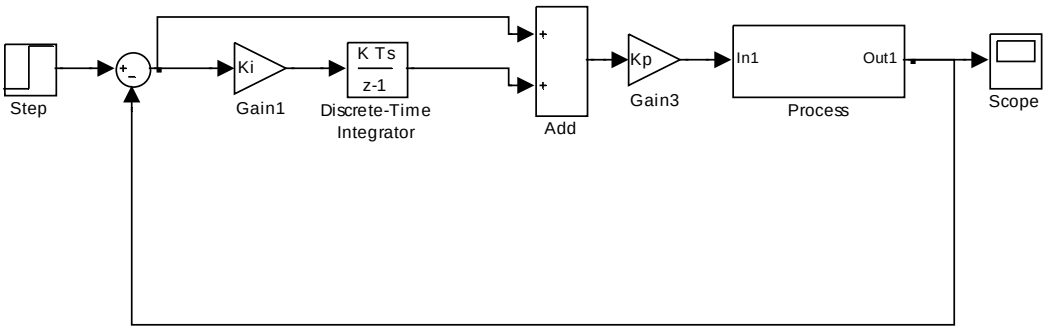
```
c = get_c();
e = r-c;
e_sum = e_sum + e;
m = Kp*e + Ki*e_sum;
```

OR

```
e_prev = e;
m_prev = m;
c = get_c();
e = r-c;
m = (Kp + Ki)*e – Kp*e_prev + m_prev;
```

PI Controller 2





$$\frac{m}{e}=K_P\left(1+\frac{K_I}{1-z^{-1}}\right)$$

$$m=K_P\left(1+\frac{K_I}{1-z^{-1}}\right)e$$

$$m\left(1-z^{-1}\right)=K_P\left(\left(1-z^{-1}\right)+K_I\right)e$$

$$m-mz^{-1}=K_Pe-K_Pez^{-1}+K_PK_Ie$$

$$m=K_Pe-K_Pez^{-1}+K_PK_Ie+mz^{-1}$$

$$m=\left(K_P+K_PK_I\right)e-K_Pez^{-1}+mz^{-1}$$

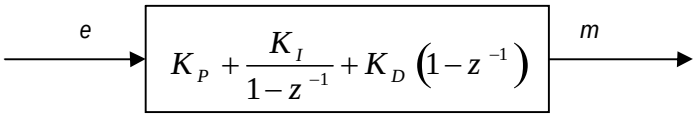
$$m(k)=\left(K_P+K_PK_I\right)e(k)-K_Pe(k-1)+m(k-1)$$

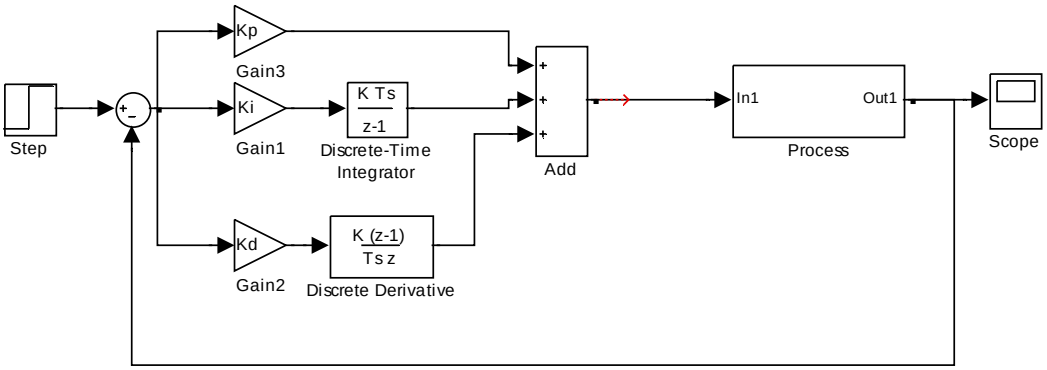
```
c = get_c();
e = r-c;
e_sum = e_sum + e;
m = Kp*(e + Ki*e_sum);
```

OR

```
e_prev = e;
m_prev = m;
c = get_c();
e = r-c;
m = (Kp + Kp*Ki)*e – Kp*e_prev + m_prev;
```

PID Controller 1





$$\frac{m}{e} = \left(K_P + \frac{K_I}{1-z^{-1}} + K_D\left(1-z^{-1}\right)\right)$$

$$m = \left(K_P + \frac{K_I}{1-z^{-1}} + K_D\left(1-z^{-1}\right)\right)e$$

$$m\left(1-z^{-1}\right) = \left(K_P\left(1-z^{-1}\right) + K_I + K_D\left(1-z^{-1}\right)^2\right)e$$

$$m - mz^{-1} = K_P e - K_P e z^{-1} + K_I e + K_D\left(1-2z^{-1}+z^{-2}\right)e$$

$$m = K_P e - K_P e z^{-1} + K_I e + mz^{-1} + K_D e - 2K_D e z^{-1} + K_D e z^{-2}$$

$$m = \left(K_P + K_I + K_D\right)e - \left(K_P + 2K_D\right)e z^{-1} + mz^{-1} + K_D e z^{-2}$$

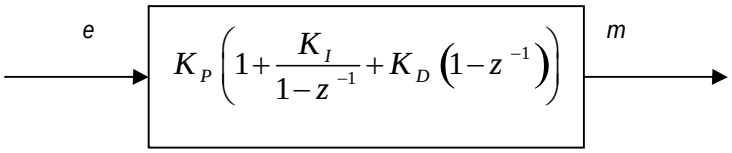
$$m(k) = \left(K_P + K_I + K_D\right)e(k) - \left(K_P + 2K_D\right)e(k-1) + m(k-1) + K_D e(k-2)$$

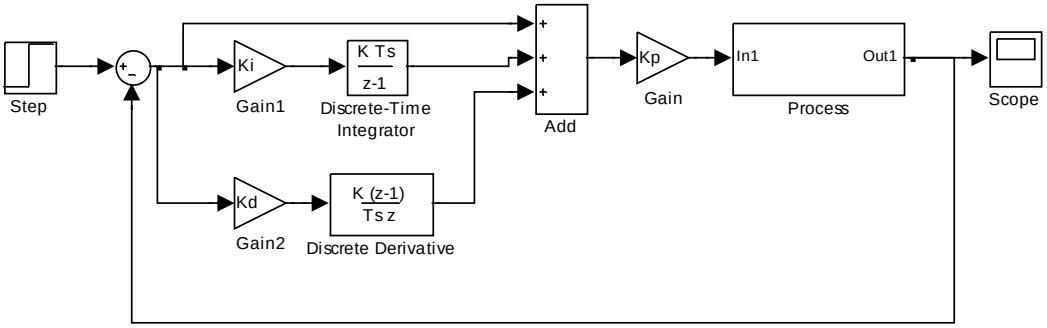
```
e_prev = e;
c = get_c();
e = r-c;
e_sum = e_sum + e;
m = Kp*e + Ki*e_sum + Kd*(e-e_prev);
```

OR

```
e_prev_prev = e_prev;
e_prev = e;
m_prev = m;
c = get_c();
e = r-c;
m = (Kp+Ki+Kd)*e–(Kp+2Kd)*e_prev+m_prev+Kd*e_prev_prev;
```

PID Controller 2





$$\frac{m}{e}=K_P\left(1+\frac{K_I}{1-z^{-1}}+K_D\left(1-z^{-1}\right)\right)$$

$$m=K_P\left(1+\frac{K_I}{1-z^{-1}}+K_D\left(1-z^{-1}\right)\right)e$$

$$m\left(1-z^{-1}\right)=K_P\left(\left(1-z^{-1}\right)+K_I+K_D\left(1-z^{-1}\right)^2\right)e$$

$$m-mz^{-1}=K_Pe-K_Pez^{-1}+K_PK_Ie+K_PK_D\left(1-2z^{-1}+z^{-2}\right)e$$

$$m=K_Pe-K_Pez^{-1}+K_PK_Ie+mz^{-1}+K_PK_De-2K_PK_Dez^{-1}+K_PK_Dez^{-2}$$

$$m=\left(K_P+K_PK_I+K_PK_D\right)e-\left(K_P+2K_PK_D\right)ez^{-1}+mz^{-1}+K_PK_Dez^{-2}$$

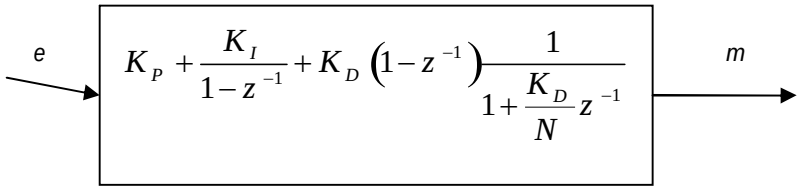
$$m(k)=\left(K_P+K_PK_I+K_PK_D\right)e(k)-\left(K_P+2K_PK_D\right)e(k-1)+m(k-1)+K_PK_De(k-2)$$

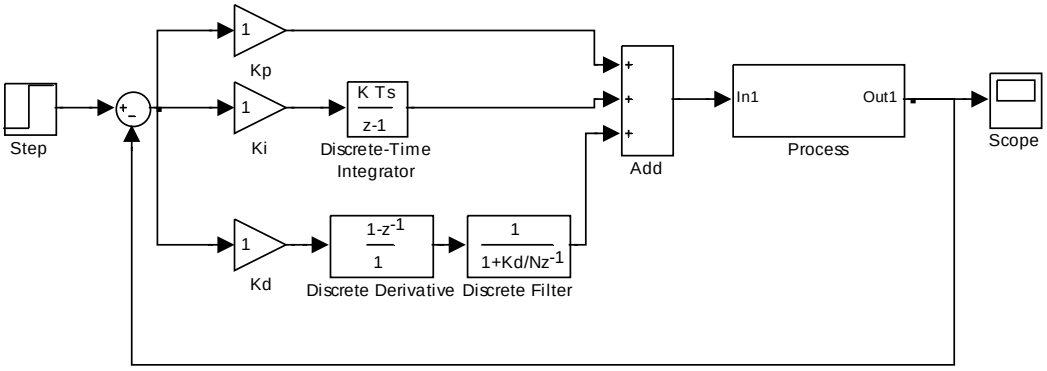
```
e_prev = e;
c = get_c();
e = r-c;
e_sum = e_sum + e;
m = Kp*(e + Ki*e_sum + Kd*(e-e_prev));
```

OR

```
e_prev_prev = e_prev;
e_prev = e;
m_prev = m;
c = get_c();
e = r-c;
m = (Kp + Kp*Ki + Kp*Kd)*e – (Kp + 2Kp*Kd)*e_prev +
      m_prev + Kp*Kd*e_prev_prev;
```

PID Controller 3





$$\frac{m}{e} = \left(K_P + \frac{K_I}{1-z^{-1}} + K_D\left(1-z^{-1}\right)\frac{1}{1+\frac{K_D}{N}z^{-1}} \right)$$

$$m = \left(K_P + \frac{K_I}{1-z^{-1}} + K_D\left(1-z^{-1}\right)\frac{1}{1+\frac{K_D}{N}z^{-1}} \right)e$$

$$m\left(1-z^{-1}\right)\left(1+\frac{K_D}{N}z^{-1}\right) = \left(K_P\left(1-z^{-1}\right)\left(1+\frac{K_D}{N}z^{-1}\right) + K_I\left(1+\frac{K_D}{N}z^{-1}\right) + K_D\left(1-z^{-1}\right)^2 \right)e$$

$$m\left(1+\frac{K_D}{N}z^{-1}-z^{-1}-\frac{K_D}{N}z^{-2}\right) = \left(K_P\left(1+\frac{K_D}{N}z^{-1}-z^{-1}-\frac{K_D}{N}z^{-2}\right) + K_I + \frac{K_I K_D}{N}z^{-1} + K_D\left(1-2z^{-1}+z^{-2}\right) \right)e$$

$$m\left(1+\left(\frac{K_D}{N}-1\right)z^{-1}-\frac{K_D}{N}z^{-2}\right) = \left(K_P + \frac{K_P K_D}{N}z^{-1} - K_P z^{-1} - \frac{K_P K_D}{N}z^{-2} + K_I + \frac{K_I K_D}{N}z^{-1} + K_D - 2K_D z^{-1} + K_D z^{-2} \right)e$$

$$m\left(1+\left(\frac{K_D}{N}-1\right)z^{-1}-\frac{K_D}{N}z^{-2}\right) = \left(\left(K_P+K_I+K_D\right) + \left(\frac{K_D}{N}\left(K_P+K_I\right) - K_P - 2K_D\right)z^{-1} + \left(K_D - \frac{K_P K_D}{N}\right)z^{-2} \right)e$$

$$m(k) + \left(\frac{K_D}{N}-1\right)m(k-1) - \frac{K_D}{N}m(k-2) = \left(K_P+K_I+K_D\right)e(k) + \left(\frac{K_D}{N}\left(K_P+K_I\right) - K_P - 2K_D\right)e(k-1) + \left(K_D - \frac{K_P K_D}{N}\right)e(k-2)$$

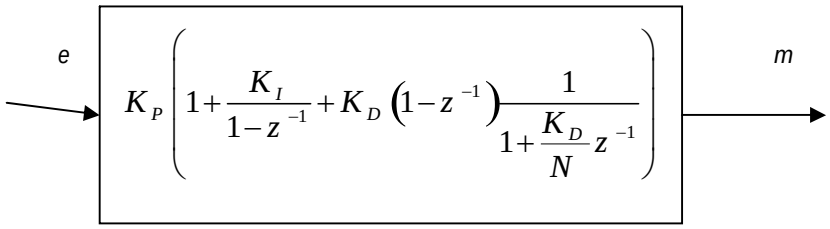
$$m(k) = \left(K_P+K_I+K_D\right)e(k) + \left(\frac{K_D}{N}\left(K_P+K_I\right) - K_P - 2K_D\right)e(k-1) + \left(K_D - \frac{K_P K_D}{N}\right)e(k-2) - \left(\frac{K_D}{N}-1\right)m(k-1) + \frac{K_D}{N}m(k-2)$$

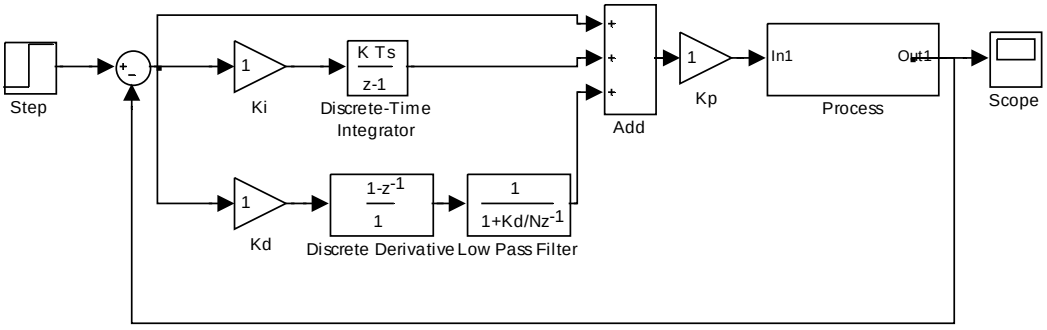
```
e_prev = e;
c = get_c();
e = r-c;
e_sum = e_sum + e;
m1_prev = m1;
m1 = Kd*(e-e_prev) – Kd/N*m1_prev;
m = Kp*e + Ki*e_sum + m1;
```

OR

```
e_prev_prev = e_prev;
e_prev = e;
m_prev_prev = m_prev;
m_prev = m;
c = get_c();
e = r-c;
m = (Kp+Ki+Kd)*e–(Kd/N*(Kp+Ki)-Kp-2Kd)*e_prev +
     (Kd-Kp*Kd/N)*e_prev_prev – (Kd/N-1)*m_prev +
Kd/N*m_prev_prev;
```

PID Controller 4





$$\frac{m}{e} = K_P\left(1+\frac{K_I}{1-z^{-1}}+K_D\left(1-z^{-1}\right)\frac{1}{1+\frac{K_D}{N}z^{-1}}\right)$$

$$m = K_P\left(1+\frac{K_I}{1-z^{-1}}+K_D\left(1-z^{-1}\right)\frac{1}{1+\frac{K_D}{N}z^{-1}}\right)e$$

$$m\left(1-z^{-1}\right)\left(1+\frac{K_D}{N}z^{-1}\right) = K_P\left(\left(1-z^{-1}\right)\left(1+\frac{K_D}{N}z^{-1}\right)+K_I\left(1+\frac{K_D}{N}z^{-1}\right)+K_D\left(1-z^{-1}\right)^2\right)e$$

$$m\left(1+\frac{K_D}{N}z^{-1}-z^{-1}-\frac{K_D}{N}z^{-2}\right) = K_P\left(\left(1+\frac{K_D}{N}z^{-1}-z^{-1}-\frac{K_D}{N}z^{-2}\right)+K_I+\frac{K_IK_D}{N}z^{-1}+K_D\left(1-2z^{-1}+z^{-2}\right)\right)e$$

$$m\left(1+\left(\frac{K_D}{N}-1\right)z^{-1}-\frac{K_D}{N}z^{-2}\right) = \left(K_P+\frac{K_PK_D}{N}z^{-1}-K_Pz^{-1}-\frac{K_PK_D}{N}z^{-2}+K_PK_I+\frac{K_PK_IK_D}{N}z^{-1}+K_PK_D-2K_PK_Dz^{-1}+K_PK_Dz^{-2}\right)e$$

$$m\left(1+\left(\frac{K_D}{N}-1\right)z^{-1}-\frac{K_D}{N}z^{-2}\right) = \left(\left(K_P+K_PK_I+K_PK_D\right)+\left(\frac{K_PK_D}{N}\left(1+K_I\right)-K_P-2K_PK_D\right)z^{-1}+\left(K_PK_D\left(1-\frac{1}{N}\right)\right)z^{-2}\right)e$$

$$m(k)+\left(\frac{K_D}{N}-1\right)m(k-1)-\frac{K_D}{N}m(k-2) = \left(K_P+K_PK_I+K_PK_D\right)e(k)+\left(\frac{K_PK_D}{N}\left(1+K_I\right)-K_P-2K_PK_D\right)e(k-1)+\left(K_PK_D\left(1-\frac{1}{N}\right)\right)e(k-2)$$

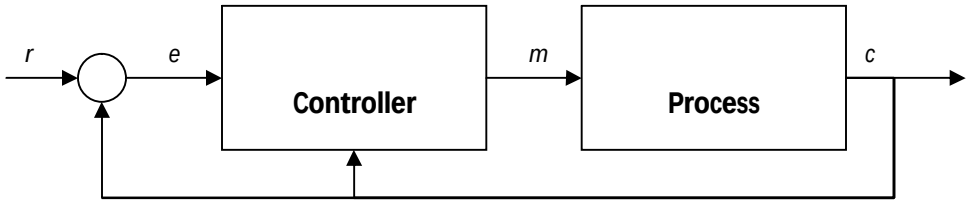
$$m(k) = \left(K_P+K_PK_I+K_PK_D\right)e(k)+\left(\frac{K_PK_D}{N}\left(1+K_I\right)-K_P-2K_PK_D\right)e(k-1)+\left(K_PK_D\left(1-\frac{1}{N}\right)\right)e(k-2)-\left(\frac{K_D}{N}-1\right)m(k-1)+\frac{K_D}{N}m(k-2)$$

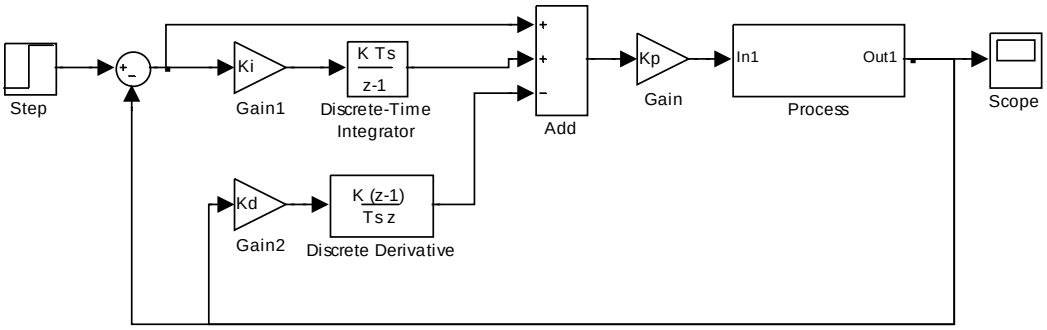
```
e_prev = e;
c = get_c();
e = r-c;
e_sum = e_sum + e;
m1_prev = m1;
m1 = Kd*(e-e_prev) – Kd/N*m1_prev;
m = Kp*(e + Ki*e_sum + m1);
```

OR

```
e_prev_prev = e_prev;
e_prev = e;
m_prev_prev = m_prev;
m_prev = m;
c = get_c();
e = r-c;
m = Kp(1+Ki+Kd)*e–Kp*(Kd/N*(1+Ki)-1-2Kd)*e_prev +
     Kp*Kd(1-1/N)*e_prev_prev – (Kd/N-1)*m_prev +
Kd/N*m_prev_prev;
```

Enhancement 1.0 : Derivative Action Suppressing for Error Step Changes





$$m = K_P\left(\left(1+\frac{K_I}{1-z^{-1}}\right)e-\left(K_D\left(1-z^{-1}\right)\right)\right)c$$

$$m\left(1-z^{-1}\right)=K_P\left(\left(1-z^{-1}+K_I\right)e-\left(K_D\left(1-z^{-1}\right)^2\right)c\right)$$

$$m-mz^{-1}=K_P\left(e-ez^{-1}+K_Ie-K_Dc\left(1-2z^{-1}+z^{-2}\right)\right)$$

$$m-mz^{-1}=K_P\left(e-ez^{-1}+K_Ie-K_D\left(c-2cz^{-1}+cz^{-2}\right)\right)$$

$$m=K_P\left(\left(1+K_I\right)e-ez^{-1}-K_D\left(c-2cz^{-1}+cz^{-2}\right)\right)+mz^{-1}$$

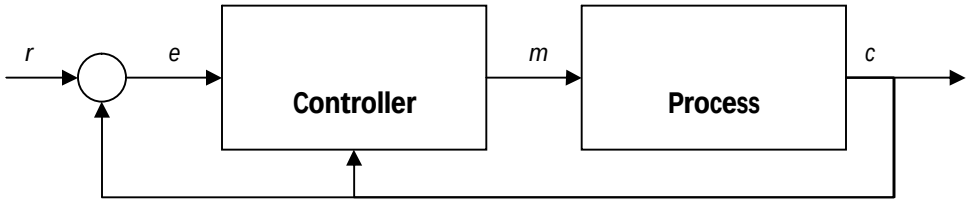
$$m(k)=K_P\left(\left(1+K_I\right)e(k)-e(k-1)-K_D\left(c(k)-2c(k-1)+c(k-2)\right)\right)+m(k-1)$$

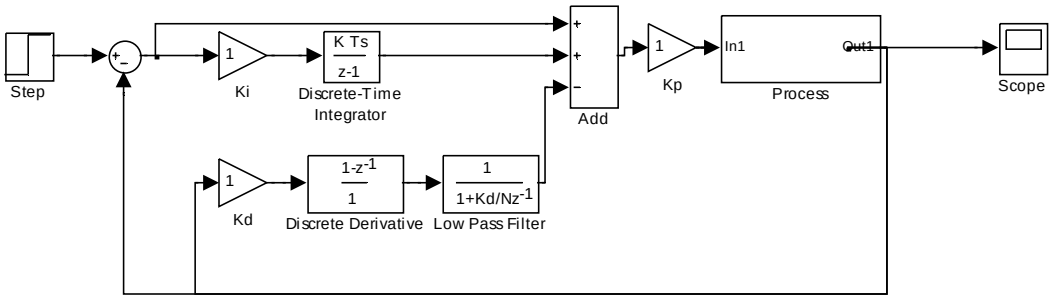
```
c_prev = c;
c = get_c();
e = r-c;
e_sum = e_sum + e;
m = Kp*(e + Ki*e_sum – Kd*(c – c_prev));
```

OR

```
c_prev_prev = c_prev;
c_prev = c;
m_prev = m;
c = get_c();
e = r-c;
m = Kp*((1+Ki)*e – e_prev – Kd*(c – 2*c_prev +
     c_prev_prev)) + m_prev;
```

Enhancement 1.1 : Derivative Action Suppressing for Error Step Changes with Low Pass Filter





$$m = K_P\left(\left(1+\frac{K_I}{1-z^{-1}}\right)e - \left(K_D\left(1-z^{-1}\right)\frac{1}{1+\frac{K_D}{N}z^{-1}}\right)\right)c$$

$$m\left(1-z^{-1}\right) = K_P\left(\left(1-z^{-1}+K_I\right)e - \left(K_D\left(1-z^{-1}\right)^2\frac{1}{1+\frac{K_D}{N}z^{-1}}\right)c\right)$$

$$m\left(1-z^{-1}\right)\left(1+\frac{K_D}{N}z^{-1}\right) = K_P\left(\left(1-z^{-1}+K_I\right)\left(1+\frac{K_D}{N}z^{-1}\right)e - \left(K_D\left(1-z^{-1}\right)^2\right)c\right)$$

$$m\left(1+\frac{K_D}{N}z^{-1}-z^{-1}-\frac{K_D}{N}z^{-2}\right) = K_P\left(\left(1+K_I+\left(K_I+\frac{K_IK_D}{N}-1\right)z^{-1}-\frac{K_D}{N}z^{-2}\right)e - K_D\left(1-2z^{-1}+z^{-2}\right)c\right)$$

$$m\left(1+\left(\frac{K_D}{N}-1\right)z^{-1}-\frac{K_D}{N}z^{-2}\right) = K_P\left(\left(\left(1+K_I\right)+\left(K_I+\frac{K_IK_D}{N}-1\right)z^{-1}-\frac{K_D}{N}z^{-2}\right)e - K_D\left(1-2z^{-1}+z^{-2}\right)c\right)$$

$$m(k)+\left(\frac{K_D}{N}-1\right)m(k-1)-\frac{K_D}{N}m(k-2) = K_P\left(1+K_I\right)e(k)+K_P\left(K_I+\frac{K_IK_D}{N}-1\right)e(k-1)-\frac{K_PK_D}{N}e(k-2)$$
$$-K_PK_Dc(k)+2K_PK_Dc(k-1)-K_PK_Dc(k-2)$$

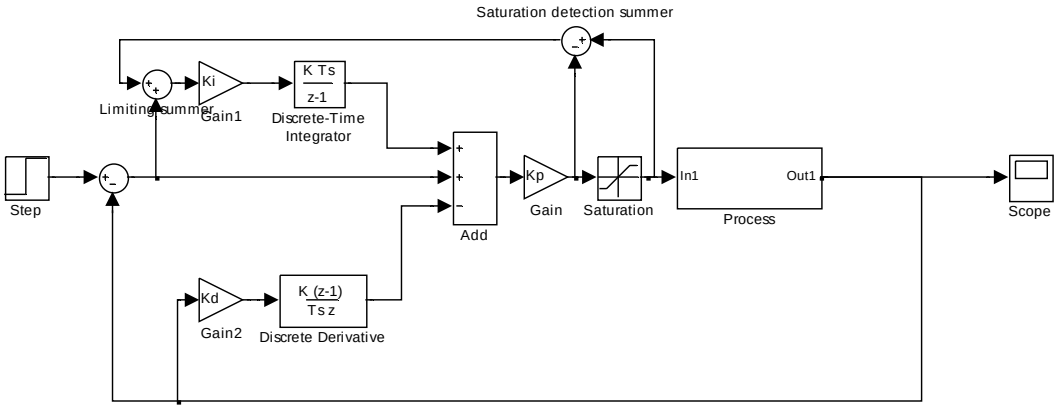
$$m(k) = K_P\left(1+K_I\right)e(k)+K_P\left(K_I+\frac{K_IK_D}{N}-1\right)e(k-1)-\frac{K_PK_D}{N}e(k-2)-K_PK_Dc(k)+2K_PK_Dc(k-1)-K_PK_Dc(k-2)$$
$$-\left(\frac{K_D}{N}-1\right)m(k-1)+\frac{K_D}{N}m(k-2)$$

```
c_prev = c;
c1_prev = c1;
c = get_c();
e = r-c;
e_sum = e_sum + e;
c1 = Kd*(e-e_prev) – Kd/N*c1_prev;
m = Kp*(e + Ki*e_sum –c1);
```

OR

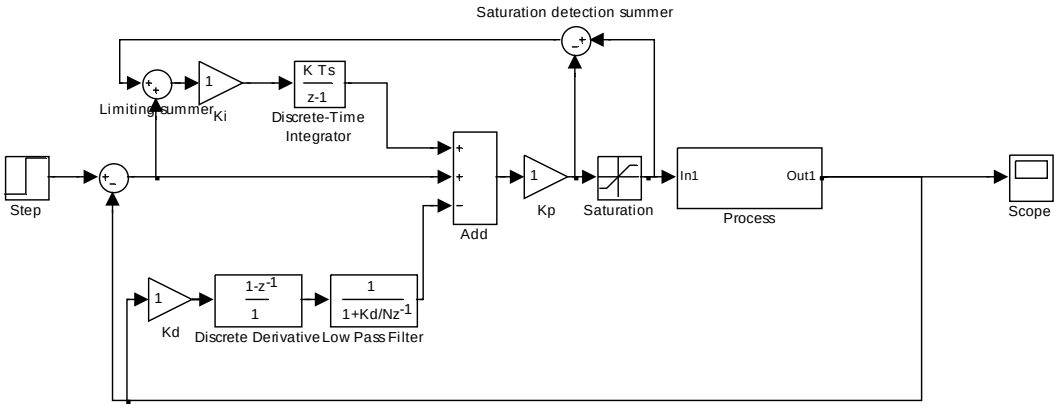
```
c_prev_prev = c_prev;
c_prev = c;
m_prev_prev = m_prev;
m_prev = m;
c = get_c();
e = r-c;

m = Kp*((1+Ki)*e + (Ki+Ki*Kd/N-1)*e_prev –
     Kp*Kd/N*e_prev_prev) – Kd*(c -2*c_prev+c_prev_prev))   -
(Kd/N-1)*m_prev + Kd/N*m_prev_prev;
```

Enhancement 2.0 : Anti-Wind Up Mechanism




```
c_prev = c;
c = get_c();
e = r-c;
e_mod = e + m_diff;
e_sum = e_sum + e_mod;
m = Kp*(e + Ki*e_sum – Kd*(c – c_prev));
m_sat = sat(m);
m_diff = m_sat – m;
```

Enhancement 2.1 : Anti-Wind Up Mechanism with Low Pass Filter



```
c_prev = c;
c1_prev = c1;
c = get_c();
e = r-c;
e_mod = e + m_diff;
e_sum = e_sum + e_mod;
c1 = Kd*(e-e_prev) – Kd/N*c1_prev;
m = Kp*(e + Ki*e_sum – c1);
m_sat = sat(m);
m_diff = m_sat – m;
```