2.图像去畸变

见代码

3.双目视差的使用

见代码

4.矩阵运算微分

Q1

$\boldsymbol{A} \in R^{N\times N}, \boldsymbol{x} \in R^{N}$

$$\boldsymbol{A} = \begin{bmatrix} a11 & a12 & \cdots & a1n \\ a21 & a22 & \dots & a2n \\ \vdots & \vdots & & \vdots \\ an1 & an2 & \cdots & ann \end{bmatrix} \quad \boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$f(\mathrm{x}) = \boldsymbol{Ax} = \begin{bmatrix} a11x1 + a12x2 + \cdots + a1nxn \\ a21x1 + a22x2 + \cdots + a2nxn \\ \vdots \\ an1\mathrm{x}1 + \mathrm{an}2\mathrm{x}2 + \cdots + \mathrm{ann}xn \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}(1,:)\boldsymbol{x} \\ \boldsymbol{A}(2,:)\boldsymbol{x} \\ \vdots \\ \boldsymbol{A}(n,:)\boldsymbol{x} \end{bmatrix}$$

$$\frac{\boldsymbol{\partial}(\boldsymbol{Ax})}{\partial \boldsymbol{x^T}} = \begin{bmatrix} \frac{\boldsymbol{\partial} f_1(x)}{\partial \boldsymbol{x^T}} \\ \frac{\boldsymbol{\partial} f_2(x)}{\partial \boldsymbol{x^T}} \\ \vdots \\ \frac{\boldsymbol{\partial} f_n(x)}{\partial \boldsymbol{x^T}} \end{bmatrix} = \begin{bmatrix} \frac{\boldsymbol{\partial} f_1(x)}{\partial x_1} & \cdots & \frac{\boldsymbol{\partial} f_1(x)}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\boldsymbol{\partial} f_n(x)}{\partial x_1} & \cdots & \frac{\boldsymbol{\partial} f_n(x)}{\partial x_n} \end{bmatrix} = \begin{bmatrix} a11 & a12 & \cdots & a1n \\ a21 & a22 & \dots & a2n \\ \vdots & \vdots & & \vdots \\ an1 & an2 & \cdots & ann \end{bmatrix} = \boldsymbol{A}$$

Q2

Method 1

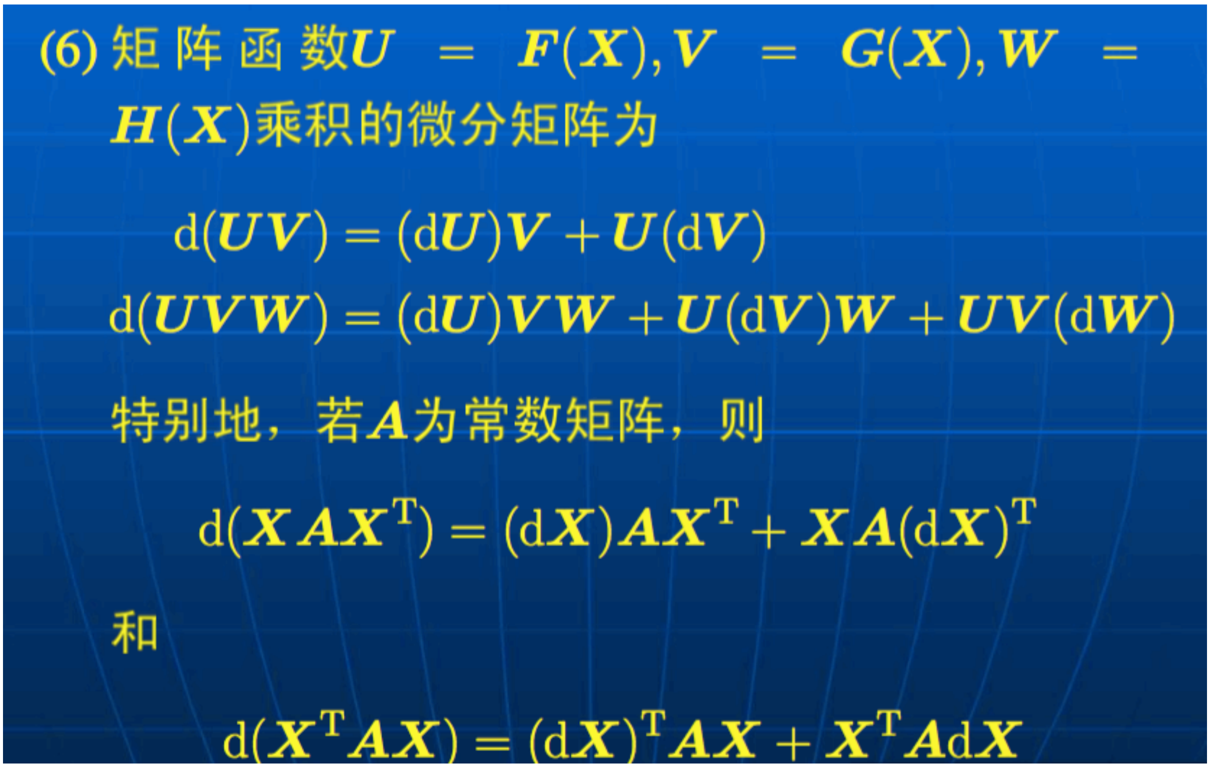
$f(\mathrm{x}) = \boldsymbol{x}^T\boldsymbol{Ax} =$

$$\sum_{i=1}^{n} \sum_{j=1}^{n} aijx_ix_j = \sum_{j=1}^{n} akjx_kx_j + \sum_{i=1}^{n} aikx_ix_k$$

$$\frac{\boldsymbol{\partial}(\boldsymbol{x}^T\boldsymbol{Ax})}{\partial x_k} = \sum_{j=1}^{n} akjx_j + \sum_{i=1}^{n} aikx_i = \mathbf{A}(\mathrm{k},:)\mathbf{x} + \mathbf{A}(:,\mathrm{k})^T\mathbf{x}$$

$$\frac{\boldsymbol{\partial}(\boldsymbol{x}^T\boldsymbol{Ax})}{\partial \boldsymbol{x}} = \boldsymbol{Ax} + \mathbf{A}^T\boldsymbol{x}$$

Method 2



$$f = \boldsymbol{x}^T \boldsymbol{A}\boldsymbol{x}$$

$$df = (\mathrm{d}\mathbf{x})^{\mathrm{T}} \boldsymbol{A}\boldsymbol{x} + \mathrm{x}^{\mathrm{T}}\mathrm{A}\,(\mathrm{d}\mathbf{x})$$

$$= (\boldsymbol{A}\boldsymbol{x})^{\mathrm{T}}(\mathrm{d}\mathbf{x}) + \mathrm{x}^{\mathrm{T}}\boldsymbol{A}\,(\mathrm{d}\mathbf{x})$$

$$= ((\boldsymbol{A}\boldsymbol{x})^{\mathrm{T}} + \mathrm{x}^{\mathrm{T}}\mathrm{A}\,)(\mathrm{d}\mathbf{x})$$

$$= \boldsymbol{x}^{\mathrm{T}}(\boldsymbol{A}^{\mathrm{T}} + \boldsymbol{A})\ (\mathrm{d}\mathbf{x})$$

$$\frac{\partial(\boldsymbol{x}^T \boldsymbol{A}\boldsymbol{x})}{\partial \boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \mathbf{A}^T \boldsymbol{x}$$

Q3

Prove $\boldsymbol{x}^T\boldsymbol{A}\boldsymbol{x} = \mathrm{tr}(\boldsymbol{A}\boldsymbol{x}\boldsymbol{x}^T)$

The trace of an n-by-n square matrix A is defined to be the sum of the elements on the main diagonal (the diagonal from the upper left to the lower right) of A, i.e.,

$$\mathrm{tr}(A) = \sum_{i=1}^{n} a_{ii} = a_{11} + a_{22} + \cdots + a_{nn}$$

where aii denotes the entry on the ith row and ith column of A.

According to given condition

$$\boldsymbol{Ax} \in R^{N\times 1}, \boldsymbol{x}^T \in R^{1\times N} ,$$

$\mathrm{tr}(\boldsymbol{A}\boldsymbol{x}\boldsymbol{x}^T)$ can be calculated as

$$\mathrm{tr}(\boldsymbol{A}\boldsymbol{x}\boldsymbol{x}^T)$$

$$= x_1 \sum_{\mathrm{j}=1}^{n} a1jx_{\mathrm{j}} + x_2 \sum_{\mathrm{j}=1}^{n} a2jx_{\mathrm{j}} + \cdots + x_{\mathrm{n}} \sum_{\mathrm{j}=1}^{n} anjx_{\mathrm{j}}$$

$$\boldsymbol{x}^T\boldsymbol{A}\boldsymbol{x} = \mathbf{x}^{\mathrm{T}}(\mathbf{Ax}) = [x1 \quad x2 \quad \cdots \quad xn] \begin{bmatrix} a11x1 + a12x2 + \cdots + a1nxn \\ a21x1 + a22x2 + \cdots + a2nxn \\ \vdots \\ an1\mathrm{x}1 + \mathrm{an}2\mathrm{x}2 + \cdots + \mathrm{ann}xn \end{bmatrix}$$

$$= x_1 \sum_{\mathrm{j}=1}^{n} a1jx_{\mathrm{j}} + x_2 \sum_{\mathrm{j}=1}^{n} a2jx_{\mathrm{j}} + \cdots + x_{\mathrm{n}} \sum_{\mathrm{j}=1}^{n} anjx_{\mathrm{j}}$$

Therefore, $\boldsymbol{x}^T\boldsymbol{A}\boldsymbol{x} = \mathrm{tr}(\boldsymbol{A}\boldsymbol{x}\boldsymbol{x}^T)$

5.高斯牛顿法的曲线拟合实验

见代码

6.批量最大似然估计

Q1

According to the given information, we know that

$$\mathbf{x} = (\mathbf{x0}, \dots, \mathbf{xk}),$$
$$\mathbf{v} = (\mathbf{v1}, \dots, \mathbf{vk}) = (\breve{\mathbf{x0}}, \mathbf{v1}, \dots, \mathbf{vk}),$$
$$\mathbf{y} = (\mathbf{y0}, \dots, \mathbf{yk}),$$

where k = 1, 2, ..., n.

Now we have a lifted column **z** consisting of all the known data and **x** consisting of all the states:

$$\mathbf{z} = [\mathbf{v1}, \mathbf{v2}, \mathbf{v3}, \mathbf{y1}, \mathbf{y2}, \mathbf{y3}]^{\mathbf{T}},$$
$$\mathbf{x} = [\mathbf{x0}, \mathbf{x1}, \mathbf{x2}, \mathbf{x3}]^{\mathbf{T}}$$

The error is defined as

$$\mathbf{e} = \mathbf{z} - \mathbf{Hx}$$
$$= \begin{bmatrix} \mathbf{v1} \\ \mathbf{v2} \\ \mathbf{v3} \\ \mathbf{y1} \\ \mathbf{y2} \\ \mathbf{y3} \end{bmatrix} - \mathbf{H} \begin{bmatrix} \mathbf{x0} \\ \mathbf{x1} \\ \mathbf{x2} \\ \mathbf{x3} \end{bmatrix}$$

The matrix **H** can be calculated as

$$\mathbf{H} = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Q2

$$\boldsymbol{J}_{\boldsymbol{v},\boldsymbol{k}}(\boldsymbol{x}) = \frac{1}{2}(\boldsymbol{x}_k - \boldsymbol{x}_{k-1} - \boldsymbol{v}_k)^T \mathbf{Q}_{\boldsymbol{k}}^{-\mathbf{1}}(\boldsymbol{x}_k - \boldsymbol{x}_{k-1} - \boldsymbol{v}_k), \quad k = 1, \dots, n,$$

$$\boldsymbol{J}_{\boldsymbol{y},\boldsymbol{k}}(\boldsymbol{x}) = \frac{1}{2}(\boldsymbol{y}_k - \boldsymbol{x}_k)^T \mathbf{R}_{\boldsymbol{k}}^{-\mathbf{1}}(\boldsymbol{y}_k - \boldsymbol{x}_k), \quad k = 1, \dots, n,$$

$$\boldsymbol{J}(\boldsymbol{x}) = \sum_{k=1}^{n} (\boldsymbol{J}_{\boldsymbol{v},\boldsymbol{k}}(\boldsymbol{x}) + \boldsymbol{J}_{\boldsymbol{y},\boldsymbol{k}}(\boldsymbol{x})), \quad k = 1, \dots, n,$$

To simplify the problem, the error can be determined as

$$\mathbf{e} = \mathbf{z} - \mathbf{H}\mathbf{x}$$

Therefore, we can find that,

$$\boldsymbol{J}(\boldsymbol{x}) = \frac{1}{2}(\boldsymbol{z} - \boldsymbol{H}\mathbf{x})^T \boldsymbol{W}^{-\mathbf{1}}(\boldsymbol{z} - \boldsymbol{H}\mathbf{x}), \quad k = 1, \dots, n,$$

where $\mathbf{W} = \begin{bmatrix} Q1 & 0 & 0 & 0 & 0 & 0 \\ 0 & Q2 & 0 & 0 & 0 & 0 \\ 0 & 0 & Q3 & 0 & 0 & 0 \\ 0 & 0 & 0 & R1 & 0 & 0 \\ 0 & 0 & 0 & 0 & R2 & 0 \\ 0 & 0 & 0 & 0 & 0 & R3 \end{bmatrix} = \begin{bmatrix} Q & 0 & 0 & 0 & 0 & 0 \\ 0 & Q & 0 & 0 & 0 & 0 \\ 0 & 0 & Q & 0 & 0 & 0 \\ 0 & 0 & 0 & R & 0 & 0 \\ 0 & 0 & 0 & 0 & R & 0 \\ 0 & 0 & 0 & 0 & 0 & R \end{bmatrix}$

Q3

The problem can be written as

$$\mathbf{x}^* = argmin\,\boldsymbol{J}(\boldsymbol{x})$$

Since $\boldsymbol{J}(\boldsymbol{x})$ is a paraboloid, we can find the minimum of $\boldsymbol{J}(\boldsymbol{x})$ by setting the partial derivative with respect to the variable x, to zero:

$$\mathbf{H}^T\mathbf{W}^{-1}(\boldsymbol{z} - \boldsymbol{H}\boldsymbol{x}) = 0$$

Therefore,

$$(\mathbf{H}^T\mathbf{W}^{-1}\mathbf{H})\mathbf{x} = \mathbf{H}^T\mathbf{W}^{-1}\boldsymbol{z}$$
$$\mathbf{x} = (\mathbf{H}^T\mathbf{W}^{-1}\mathbf{H})^{-1}\,\mathbf{H}^T\mathbf{W}^{-1}\boldsymbol{z}$$

The necessary and sufficient condition for having a solution for this equation is

$$rank(\mathbf{H}^T\mathbf{W}^{-1}\mathbf{H}) = \mathrm{N}(\mathrm{K}+1)$$

Since **W**$^{-1}$ is real symmetric positive-definite, we only need

$$rank(\mathbf{H}^T\mathbf{H}) = \mathrm{N}(\mathrm{K}+1)$$

According to the information provided, the prior knowledge of the initial state is not provided.

$$\mathbf{rank}(\mathbf{H}^T\mathbf{H}) = \mathbf{rank}(\mathbf{H}^T) = \mathbf{rank}\begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} = 4 = 3 + 1$$

Therefore, when Q>0 , R>0 (ensure that the previous item is invertible), there will always be a unique solution for **x**.