Q2

1.为何说bundle adjustment is slow 是不对的

The claimed slowness is almost always due to the unthinking use of a general-purpose optimization routine that completely ignores the problem structure and sparseness. Real bundle routines are much more efficient than this, and usually considerably more efficient and flexible than the newly suggested method.

2. BA中有哪些需要注意参数化的地方? Poses和Point各有那些参数化方式? 有何优缺点

Although they are in principle equivalent, different parametrizations often have profoundly different numerical behaviors which greatly affect the speed and reliability of the adjustment iteration. The most suitable parametrizations for optimization are as uniform, finite and well-behaved as possible *near the current state estimate*.

3D points:

If a 3D $(X\ Y\ Z)^\top$ parametrization (or equivalently a homogeneous affine $(X\ Y\ Z\ 1)^\top$ one) is used for very distant 3D points, large X, Y, Z displacements are needed to change the image significantly. *I.e.*, in (X Y Z) space the cost function becomes very flat and steps needed for cost adjustment become very large for distant points. In comparison, with a homogeneous projective parametrization $(X\ Y\ Z\ W)^\top$, the behavior near infinity is natural, finite and well-conditioned so long as the normalization keeps the homogeneous 4-vector finite at infinity (by sending $W \to 0$ there).

Affine parametrization $(X\ Y\ Z\ 1)^\top$ is acceptable for points near the origin with close-range convergent camera geometries, but it is disastrous for distant ones because it artificially cuts away half of the natural parameter space, and hides the fact by sending the resulting edge to infinite parameter values. Instead, you should use a homogeneous parametrization $(X\ Y\ Z\ W)^\top$ for distant points.

Rotations: Similarly, experience suggests that quasi-global 3 parameter rotation parametrizations such as Euler angles cause numerical problems unless one can be certain to avoid their singularities and regions of uneven coverage. Rotations should be parametrized using either quaternions subject to $\|\boldsymbol{q}\|^2 = 1$, or local perturbations $\boldsymbol{R}\,\delta\boldsymbol{R}$ or $\delta\boldsymbol{R}\,\boldsymbol{R}$ of an existing rotation R, where $\delta\boldsymbol{R}$ can be any well- behaved 3 parameter small rotation approximation, *e.g.* $\delta\boldsymbol{R} = (\boldsymbol{I} + [\,\delta\boldsymbol{r}\,]_\times)$, the Rodriguez formula, local Euler angles, *etc*.

3.你能看到那些方向在后续工作中有体现? 请举例说明

Bundle adjustment 的Network Structure 发展为 图优化

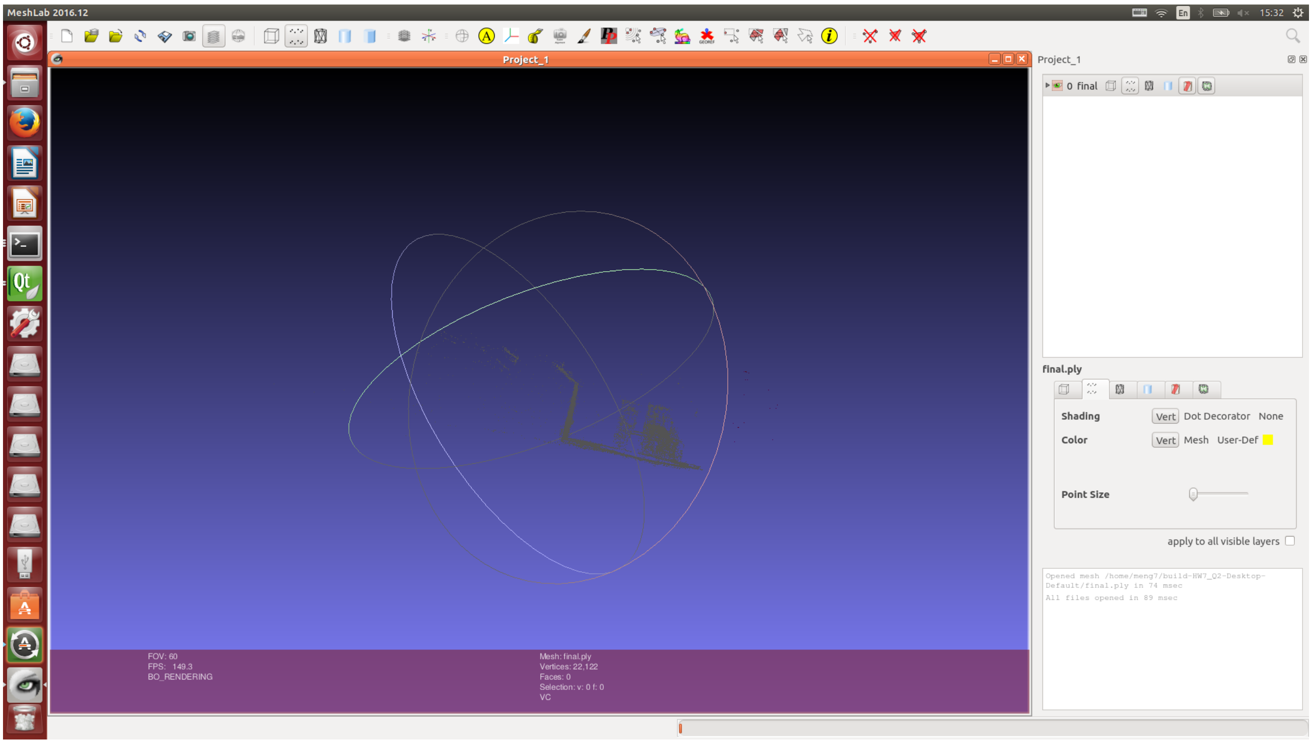
The abstract structure of the measurement network can be characterized graphically by the **network graph** (top left), which shows which features are seen in which images, and the **parameter connection graph** (top right) which details the sparse structure by showing which parameter blocks have direct interactions. Blocks are linked if and only if they jointly influence at least one observation.

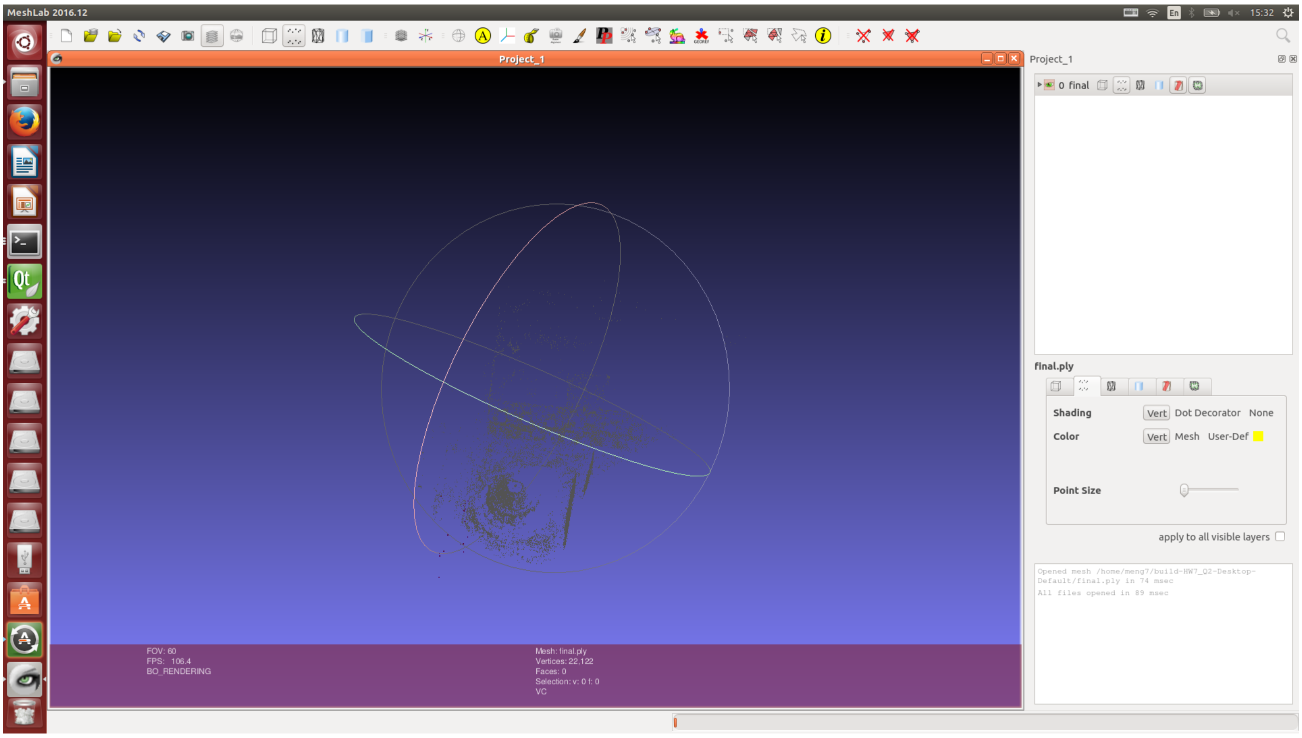
而图优化中 图由顶点和边组成

顶点表示优化变量 边表示constraint 比如观测方程

2.2

Result





Q3

3.1

1.任意一点由周围(包括自己)一共16个小块组成, error为16维, 每一维表示某个点周围的某一个小块对应当前位姿下的当前图像上相应点的相应小块的灰度差

2.每个error都关联优化变量相机位姿 和 优化变量空间三维点坐标

3.**误差对于相机的雅克比**

首先求投影方程关于位姿的雅克比 (2x6)

投影方程关于当前相机坐标系下三维点的导数

$$\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{q}} = \begin{bmatrix} \frac{\partial u}{\partial X} & \frac{\partial u}{\partial Y} & \frac{\partial u}{\partial Z} \\ \frac{\partial v}{\partial X} & \frac{\partial v}{\partial Y} & \frac{\partial v}{\partial Z} \end{bmatrix} = \begin{bmatrix} \frac{f_x}{\mathrm{Z}} & 0 & -\frac{f_x X}{Z^2} \\ 0 & \frac{f_y}{Z} & -\frac{f_y Y}{Z^2} \end{bmatrix}.$$

变换后的三维点对变换的导数组成

$$\frac{\partial \boldsymbol{q}}{\partial \delta \boldsymbol{\xi}} = \left[\boldsymbol{I}, -\boldsymbol{q}^{\wedge}\right].$$

再求出每一个点的小块处相应的像素梯度, 16个小块一共组成了16x2的Matrix

三项相乘即可

$\partial \boldsymbol{u} \quad \partial \boldsymbol{q} \quad \partial \delta \boldsymbol{\xi}$

- **第一部分：图像梯度**
- **第二部分：像素对投影点导数（见上一章）**

$$\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{q}} = \begin{bmatrix} \frac{\partial u}{\partial X} & \frac{\partial u}{\partial Y} & \frac{\partial u}{\partial Z} \\ \frac{\partial v}{\partial X} & \frac{\partial v}{\partial Y} & \frac{\partial v}{\partial Z} \end{bmatrix} = \begin{bmatrix} \frac{f_x}{Z} & 0 & -\frac{f_x X}{Z^2} \\ 0 & \frac{f_y}{Z} & -\frac{f_y Y}{Z^2} \end{bmatrix}.$$

- **第三部分：投影点对位姿导数：** $\frac{\partial \boldsymbol{q}}{\partial \delta \boldsymbol{\xi}} = [\boldsymbol{I}, -\boldsymbol{q}^\wedge].$
- **综上：**

$$\boldsymbol{J} = -\frac{\partial \boldsymbol{I}_2}{\partial \boldsymbol{u}} \frac{\partial \boldsymbol{u}}{\partial \delta \boldsymbol{\xi}}. \qquad \frac{\partial \boldsymbol{u}}{\partial \delta \boldsymbol{\xi}} = \begin{bmatrix} \frac{f_x}{Z} & 0 & -\frac{f_x X}{Z^2} & -\frac{f_x XY}{Z^2} & f_x + \frac{f_x X^2}{Z^2} & -\frac{f_x Y}{Z} \\ 0 & \frac{f_y}{Z} & -\frac{f_y Y}{Z^2} & -f_y - \frac{f_y Y^2}{Z^2} & \frac{f_y XY}{Z^2} & \frac{f_y X}{Z} \end{bmatrix}$$

误差对于空间点的雅克比

投影方程关于当前相机坐标系下三维点的导数

$$\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{q}} = \begin{bmatrix} \frac{\partial u}{\partial X} & \frac{\partial u}{\partial Y} & \frac{\partial u}{\partial Z} \\ \frac{\partial v}{\partial X} & \frac{\partial v}{\partial Y} & \frac{\partial v}{\partial Z} \end{bmatrix} = \begin{bmatrix} \frac{f_x}{Z} & 0 & -\frac{f_x X}{Z^2} \\ 0 & \frac{f_y}{Z} & -\frac{f_y Y}{Z^2} \end{bmatrix}.$$

当前相机下三维点关于空间点的导数 -> **R**

每一个点的小块处相应的像素梯度, 16个小块一共组成了16x2的Matrix

三项相乘即可

3.2

1. 可以采用如下形式

a homogeneous projective parametrization (X Y Z W)

2.选择4x4的效果还可以, 如果取更大的patch, 会有更多的误差项，因为每一个小

块对应一个灰度差作为error，可能会导致最终的误差很大，取小的patch可以是误差变小精度变高，但是可能鲁棒性会减小

3.特征点法的BA考虑的是误差对于位姿以及误差对于空间点的雅克比 通过缩小重投影后像素坐标的误差来优化空间点和位姿

直接法的BA还需要考虑所选点处的像素梯度 通过缩小重投影后相应坐标处的灰度值的误差来优化空间点和位姿

4.阀值通过卡方分布表获得，根据不同的自由度和误差的置信度来获得阀值

Result